Chiral two- and three-nucleon forces with explicit Delta degree of freedom

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in collaboration with

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Outline

- Introduction&Motivation
- 2-N forces with explicit Δ
- 3-N forces with explicit Δ
- πN scattering with explicit Δ
- Summary and Outlook

EFT with explicit $\Delta(1232)$

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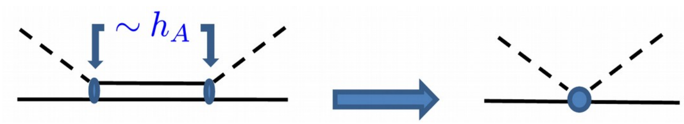
➔ Δ gives a large contribution to LEC (c_3,c_4) via resonance saturation
Bernard, Kaiser, Meißner '97



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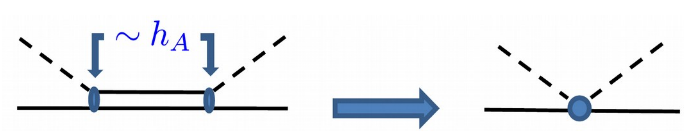
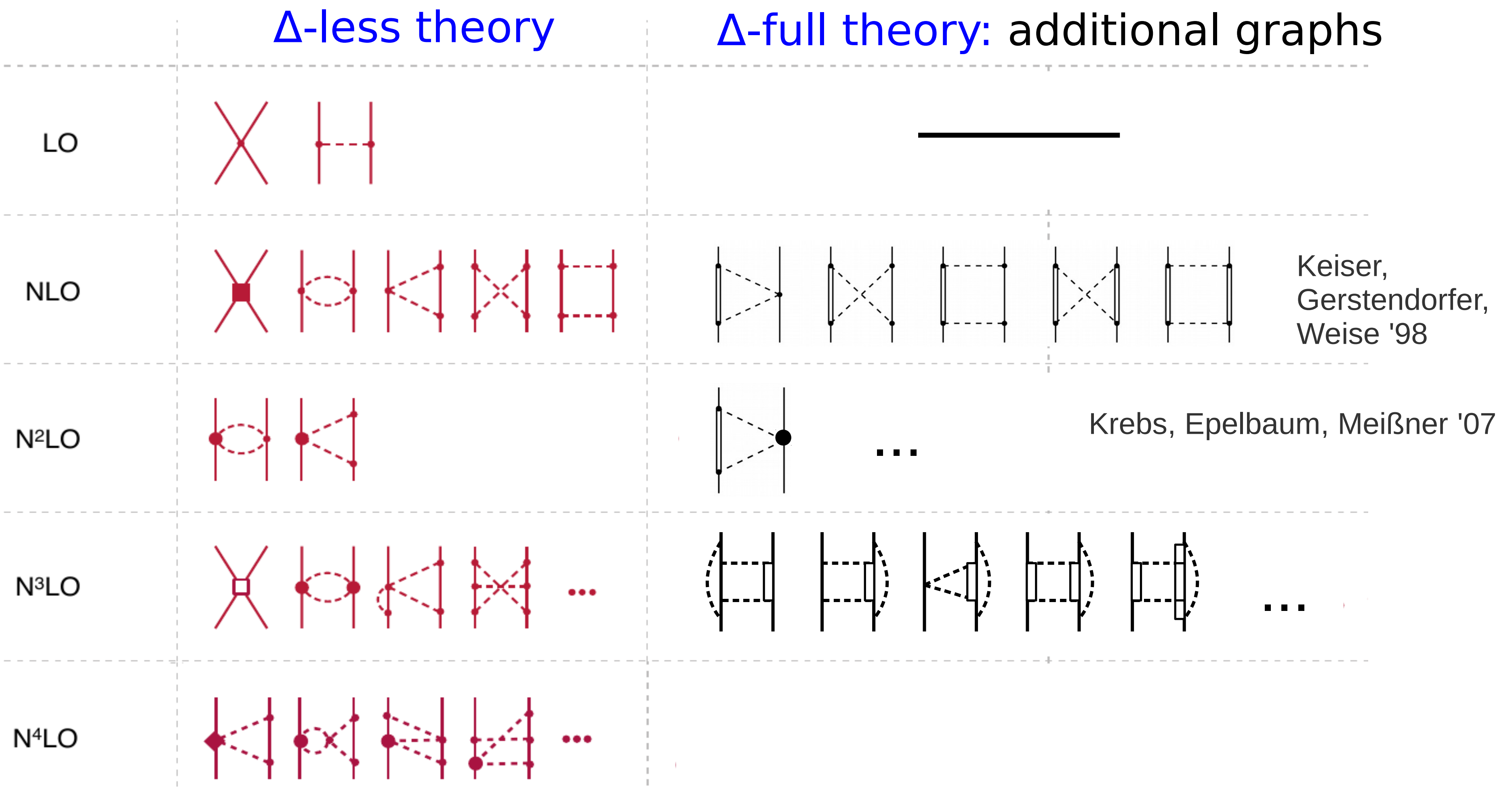


- Explicit decoupling of Δ makes comparison with Δ–less theory more transparent
  Bernard, Fearing, Hemmert, Meißner '98

finite parts of LECs can be always chosen such that $\lim_{\Delta \to \infty} = \Delta\text{–less}$
Appelquist, Carrazone '74 (Decoupling theorem)

Small scale expansion of 2NF



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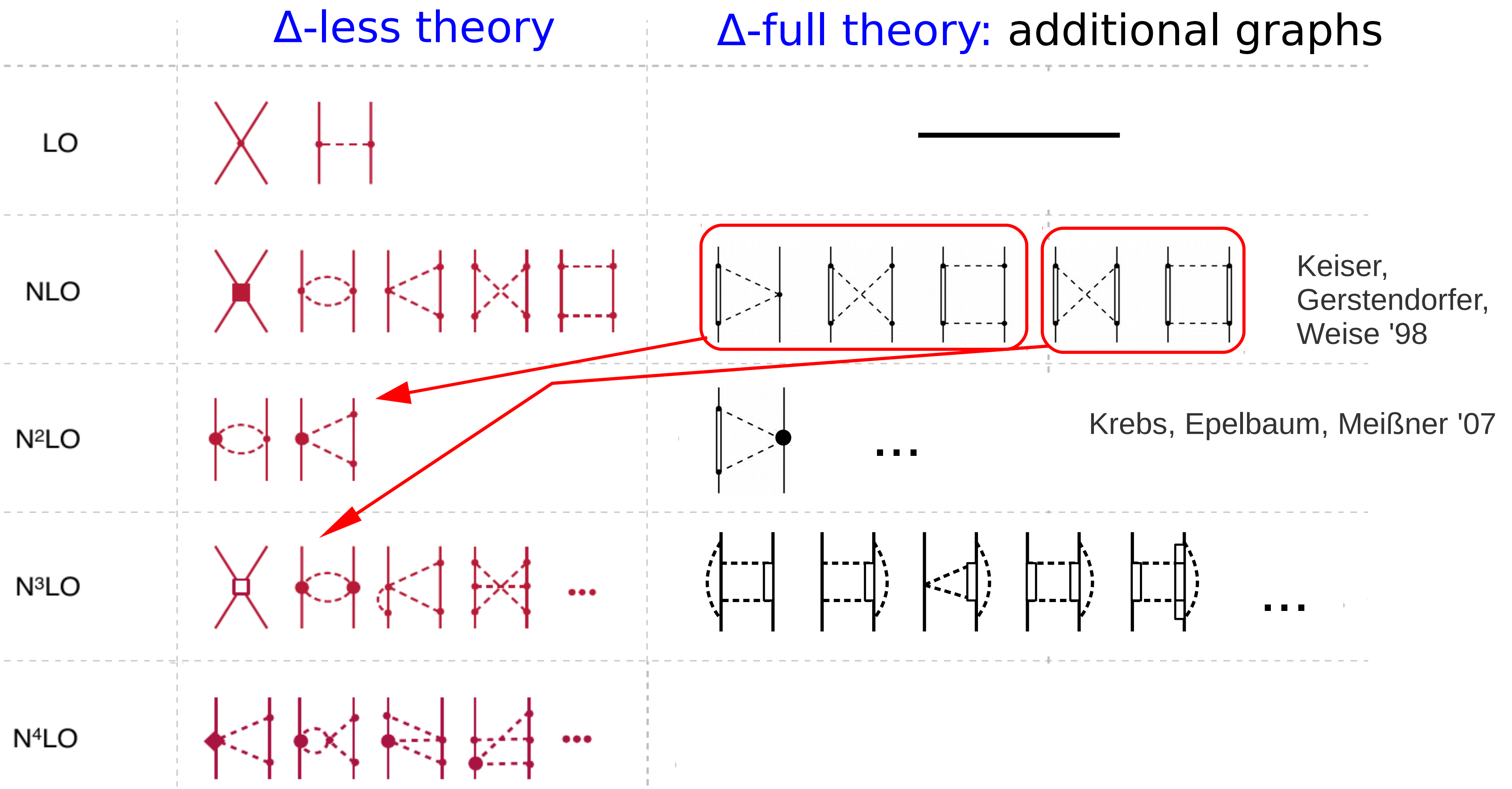

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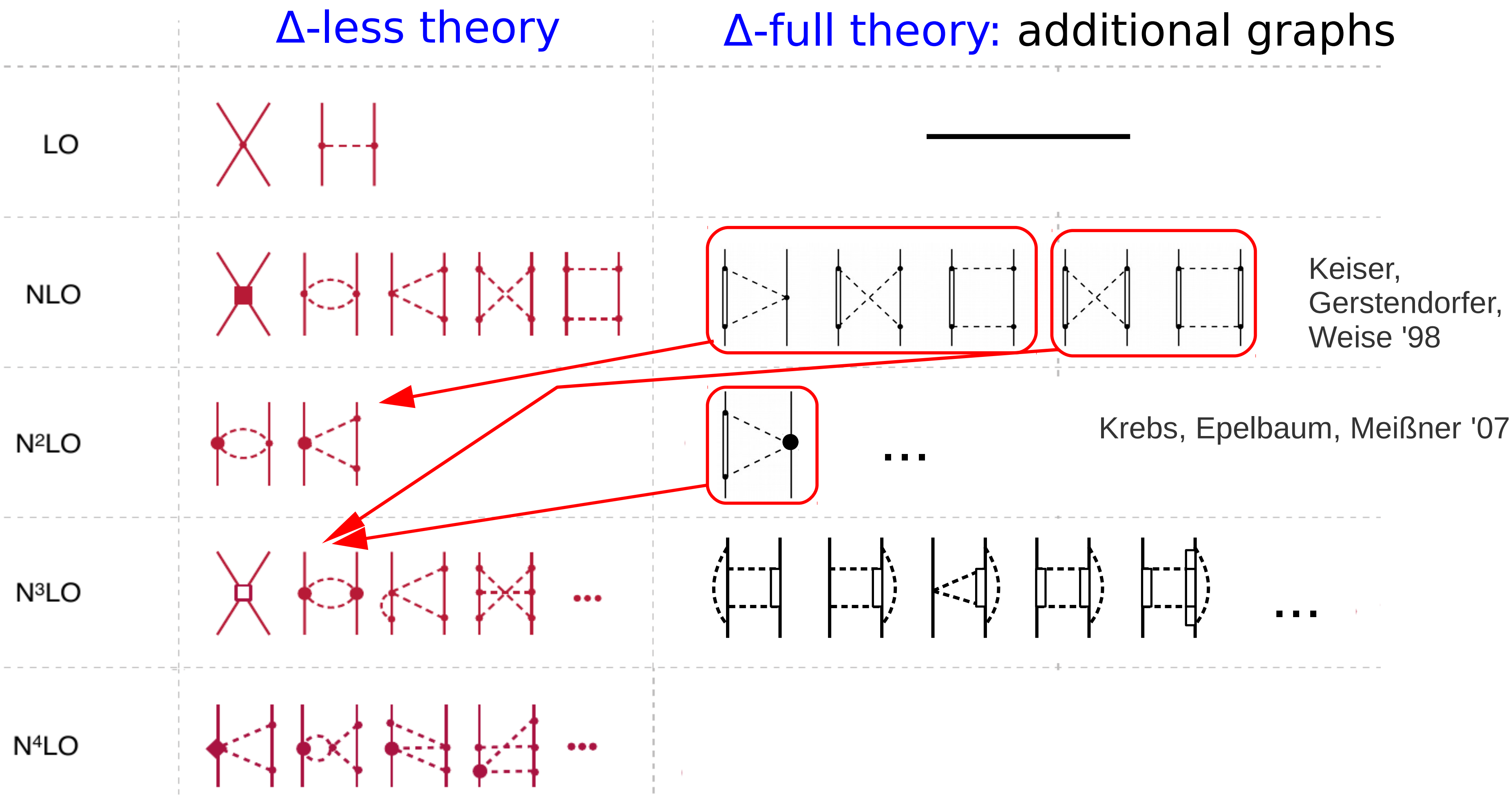

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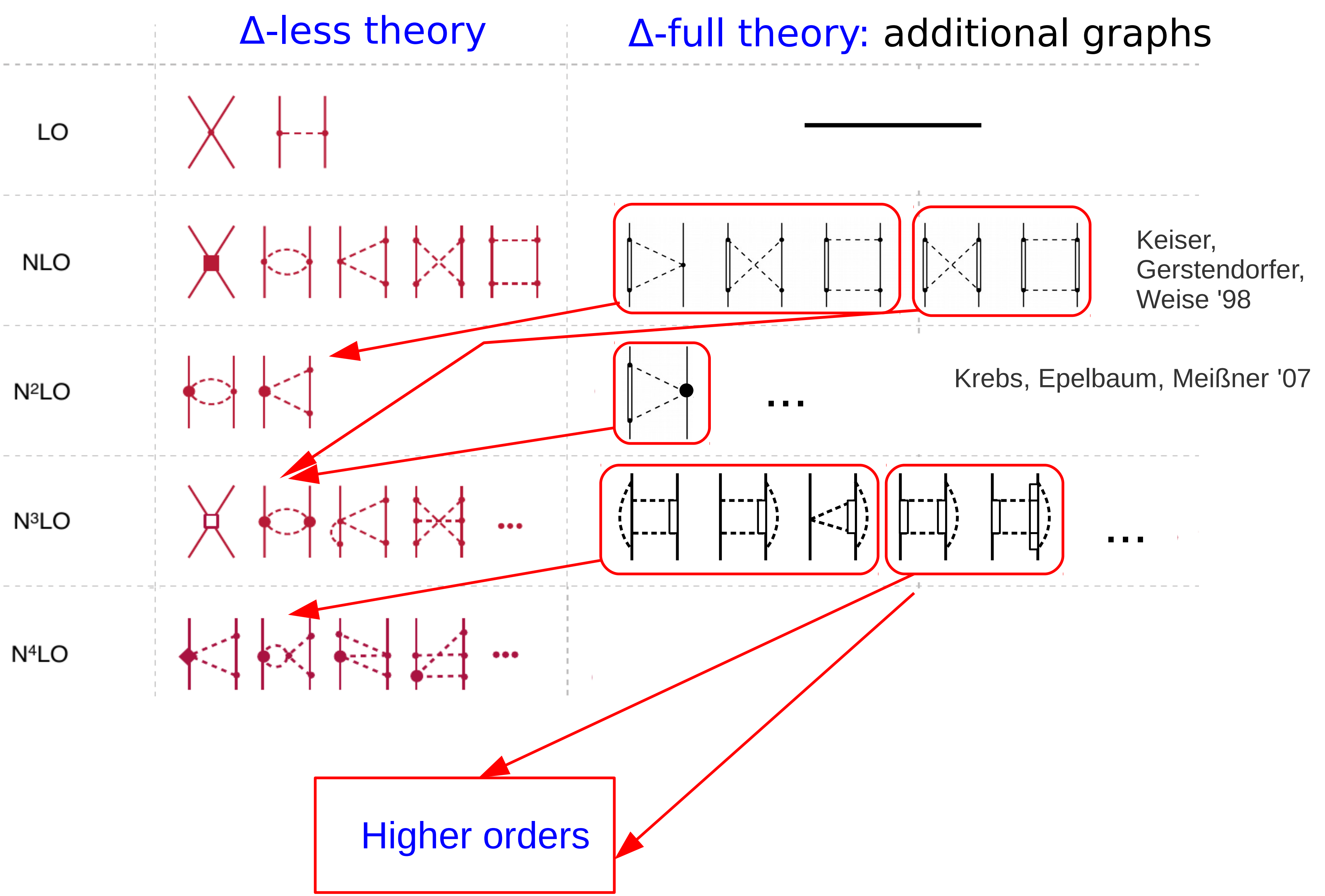
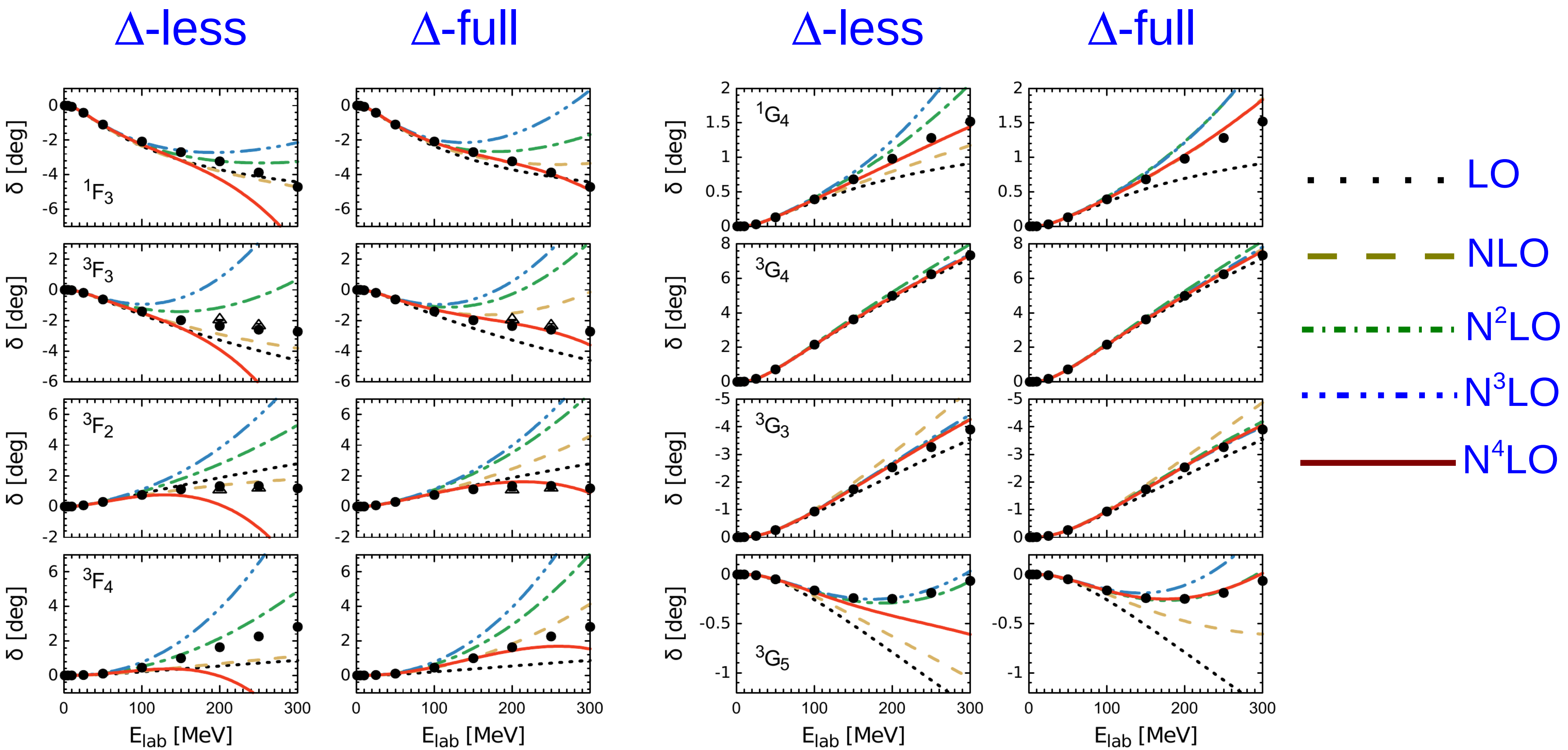


Preliminary results for N^3LO 2N forces with explicit Δ

- Only 2-pion-exchange contribution are considered (the long range part)
- $1/m_N$ corrections are not yet included
- Results for peripheral phases, no refitting of LEC's, no cut offs
- No additional parameters,
  h_A and g_1 ($\pi N\Delta$ and $\pi\Delta\Delta$)
  are extracted from the fit to πN scattering

F and G waves

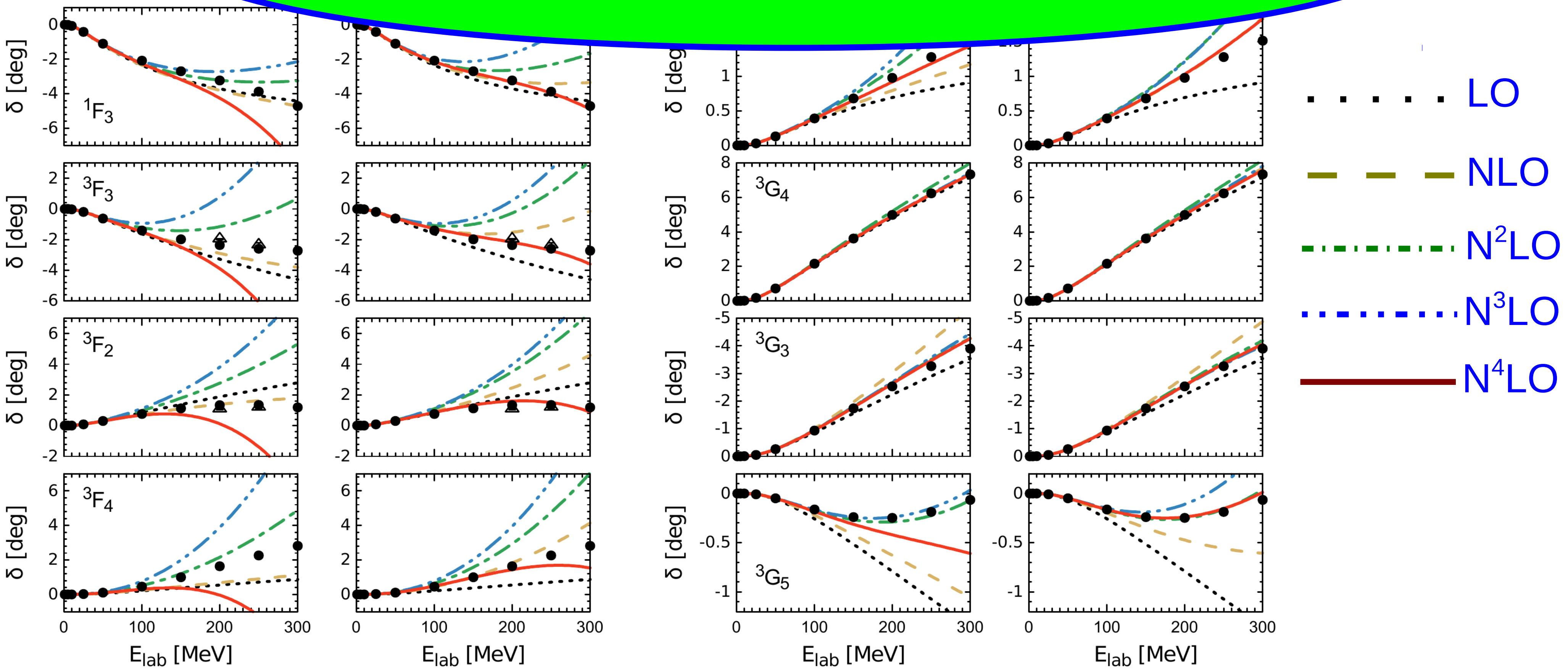


Data:Nijmegen PWA

F and G waves

Δ-

F-waves might be sensitive to the short-range physics



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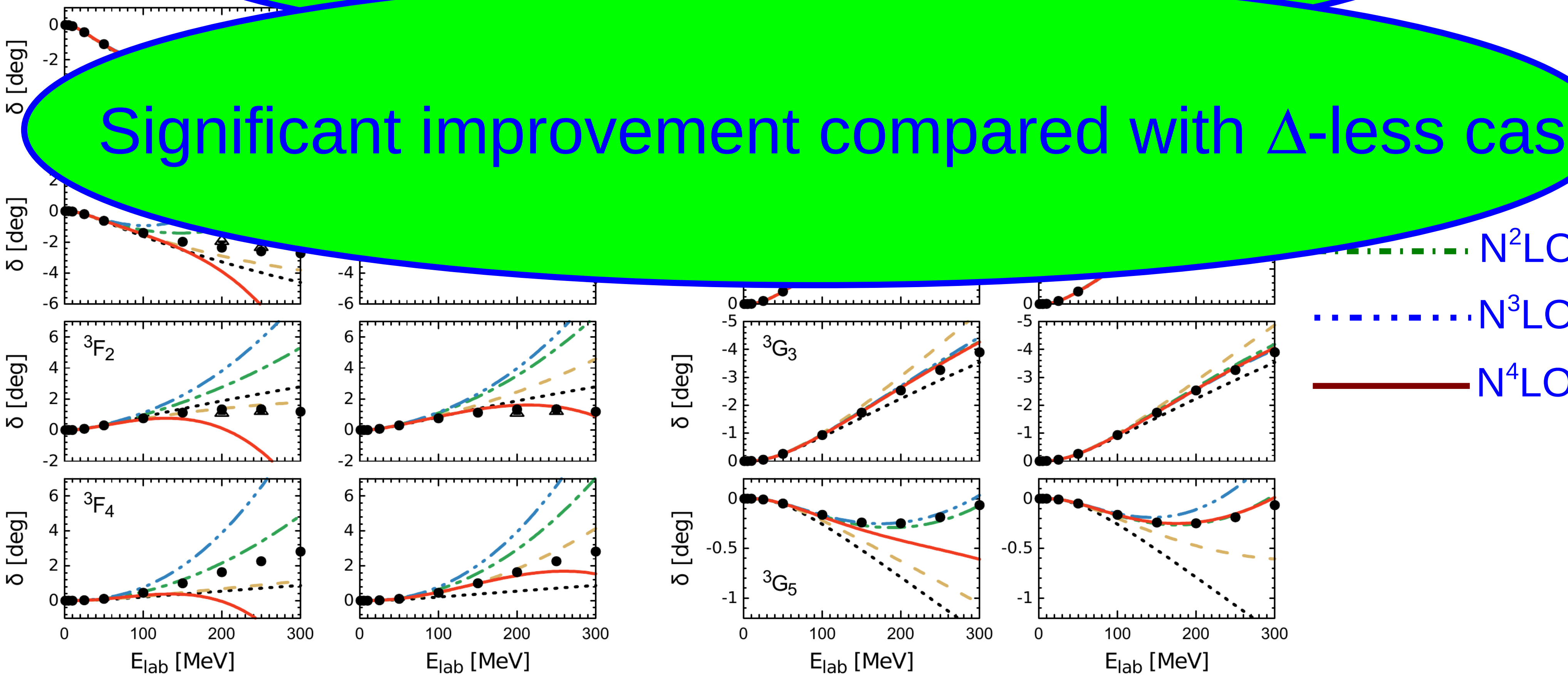
Significant improvement compared with Δ-less case



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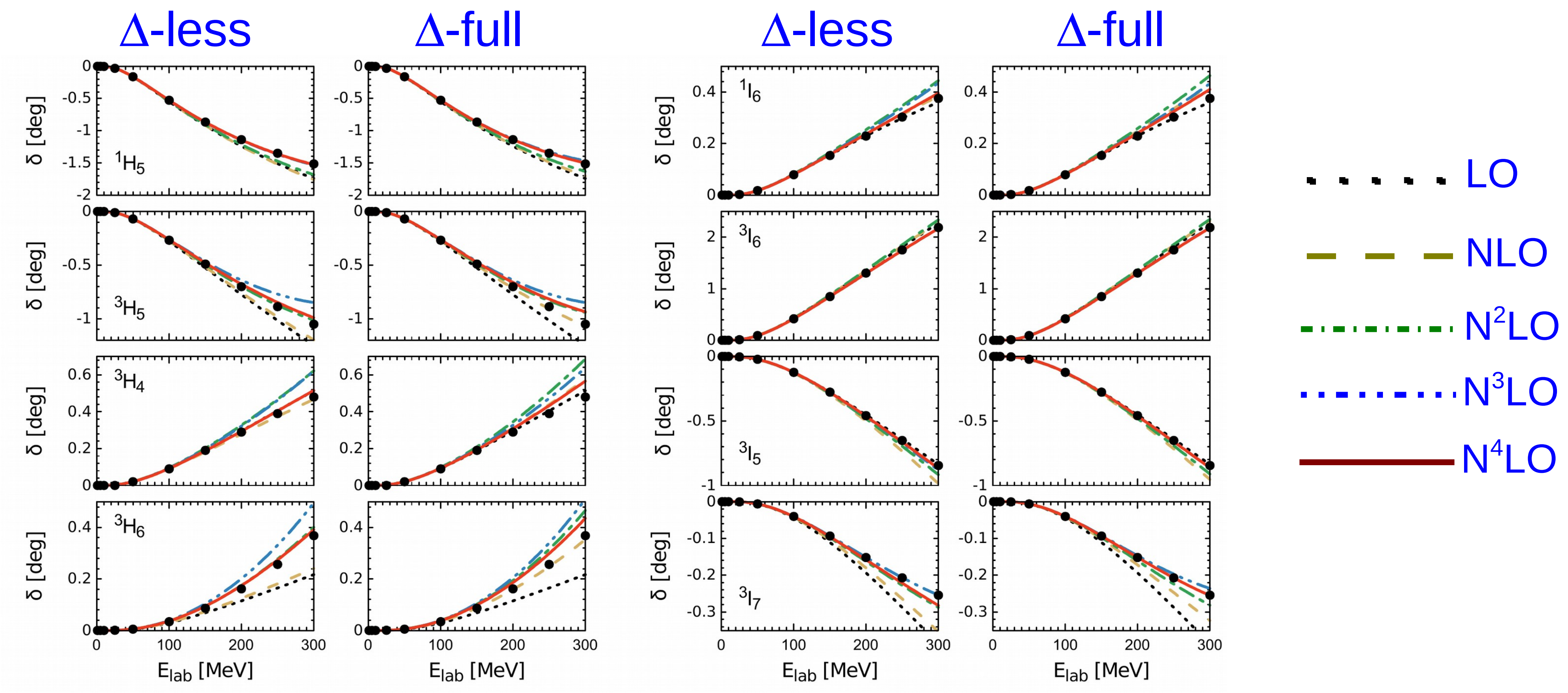
H and I waves



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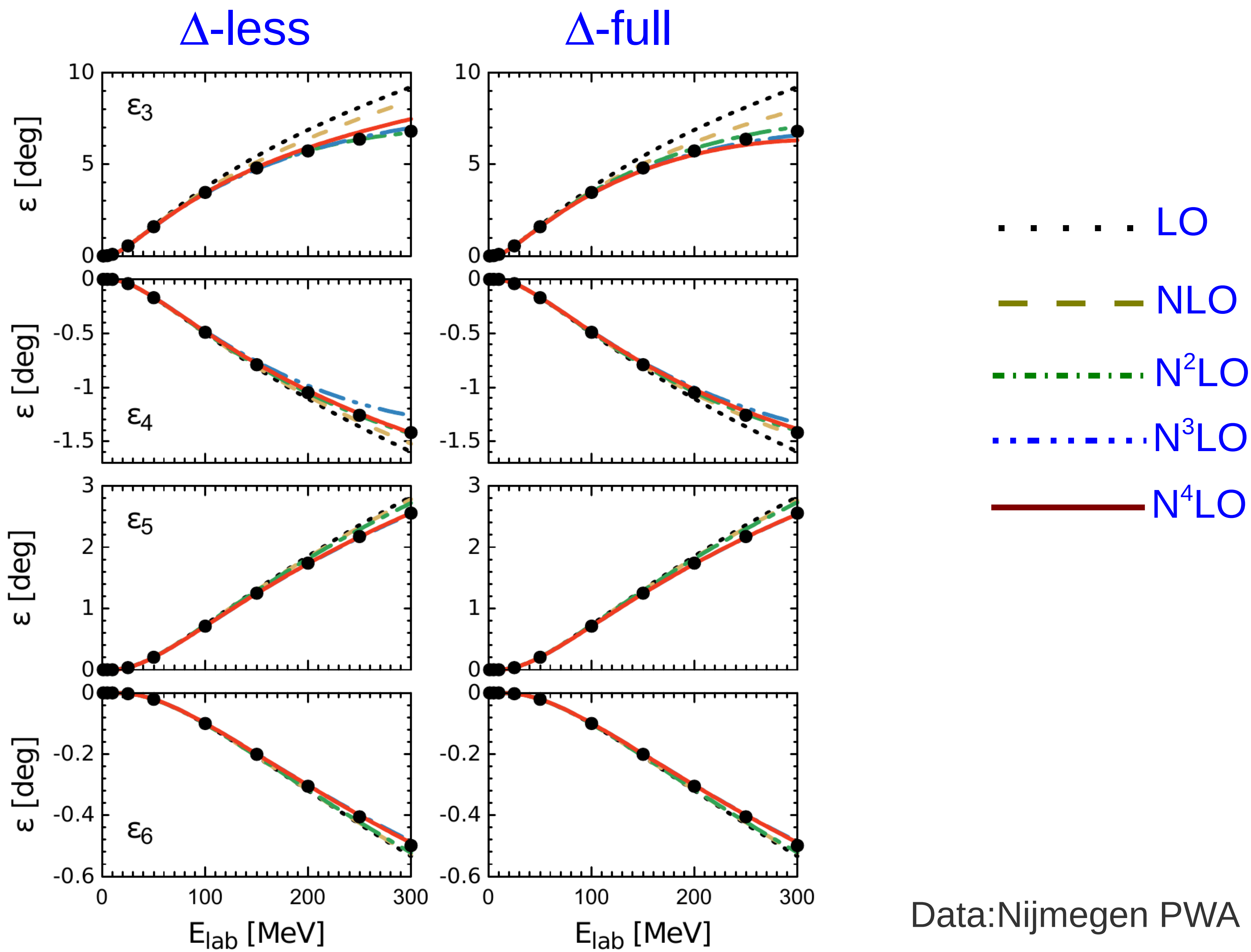
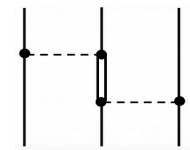
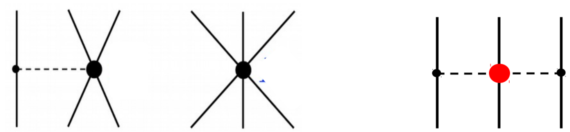
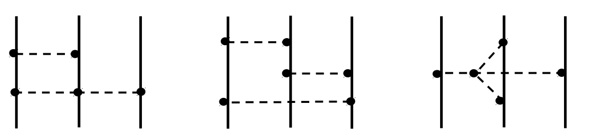
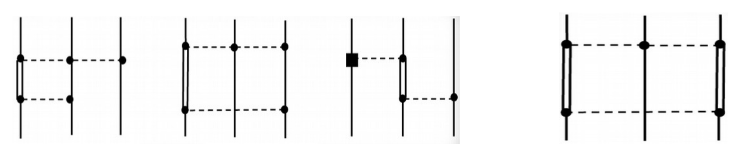
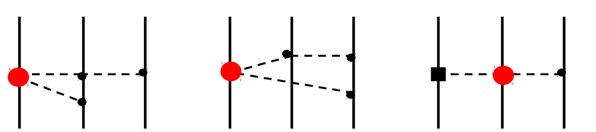
Mixing angles ε_3, ε_4, ε_5, ε_6



Data:Nijmegen PWA

Small scale expansion of 3NF

	Δ-less theory	Δ-full theory: additional graphs
NLO	—	
N^2LO		—
N^3LO	 ...	 ...
N^4LO	 ...	...

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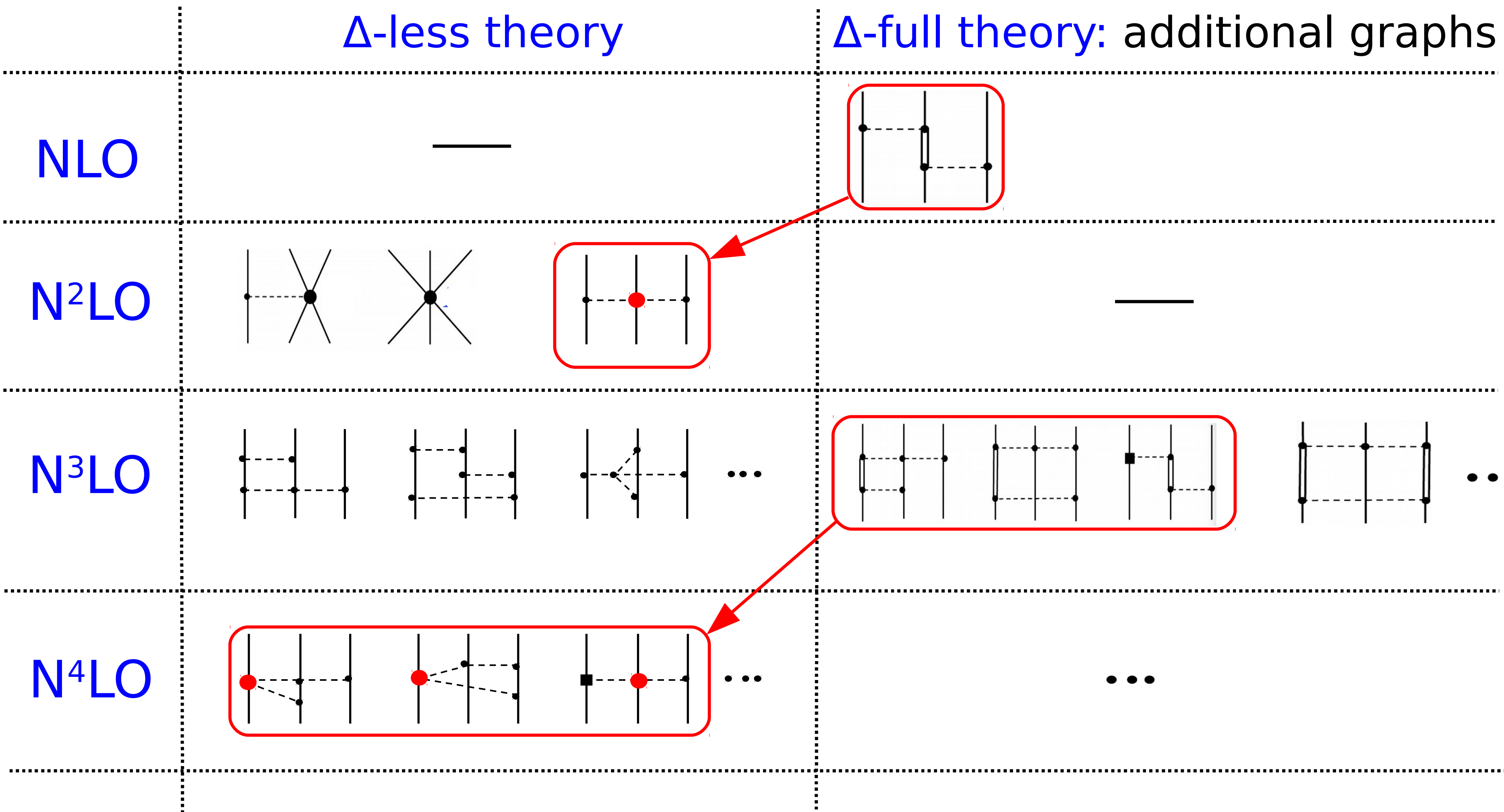

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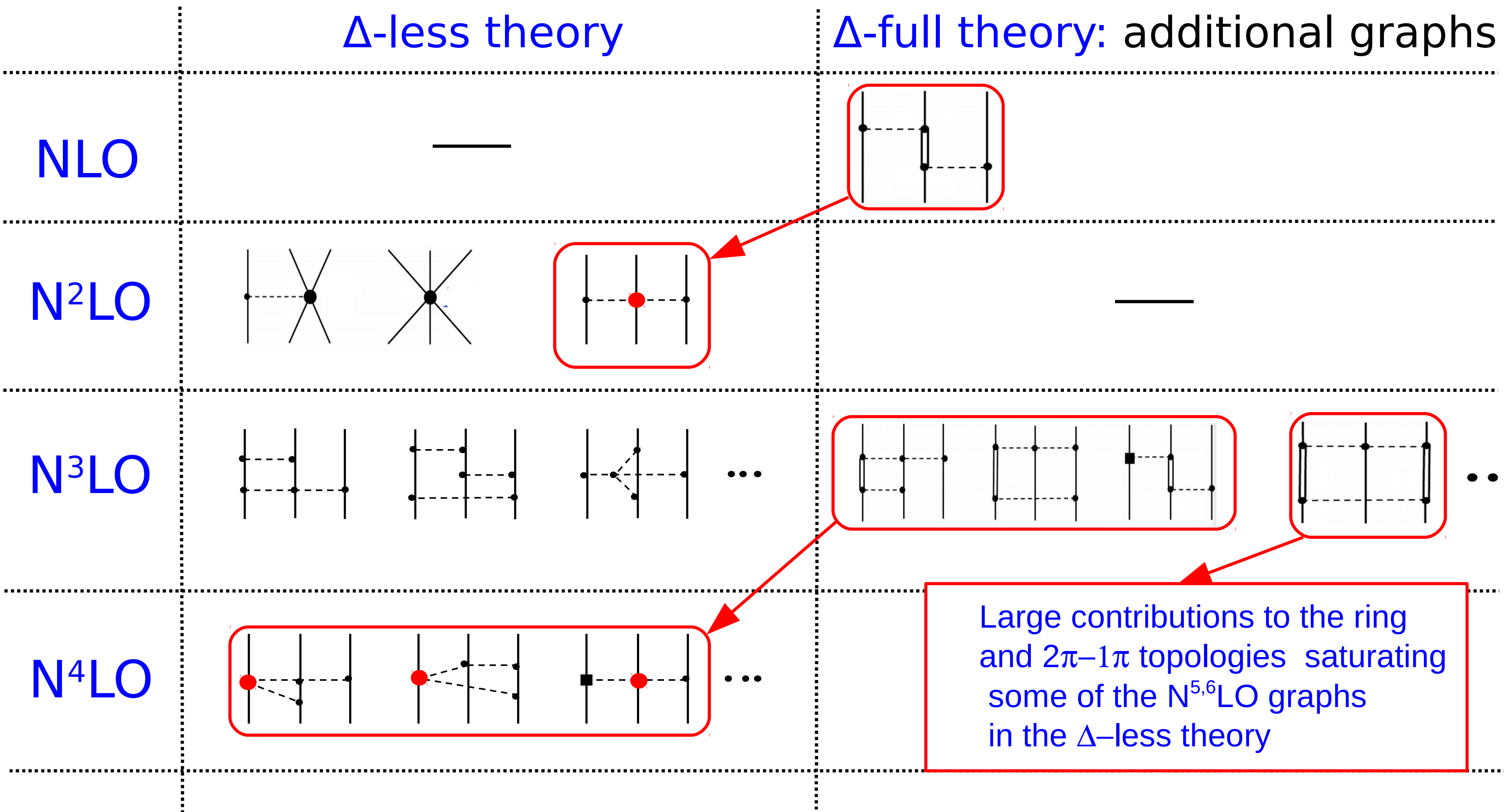
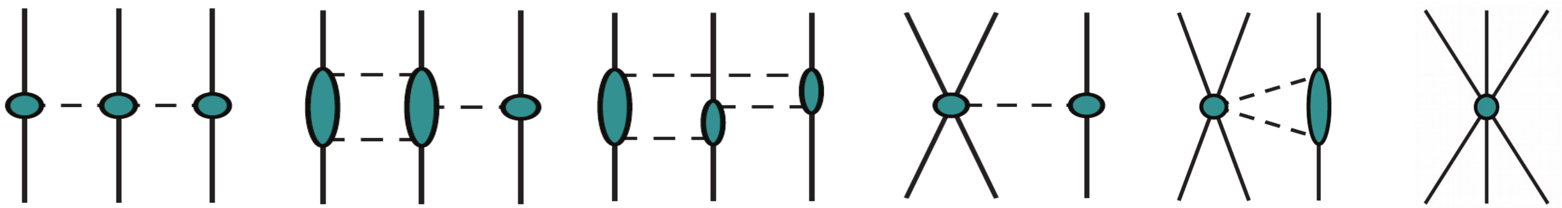


Long-range 3NF



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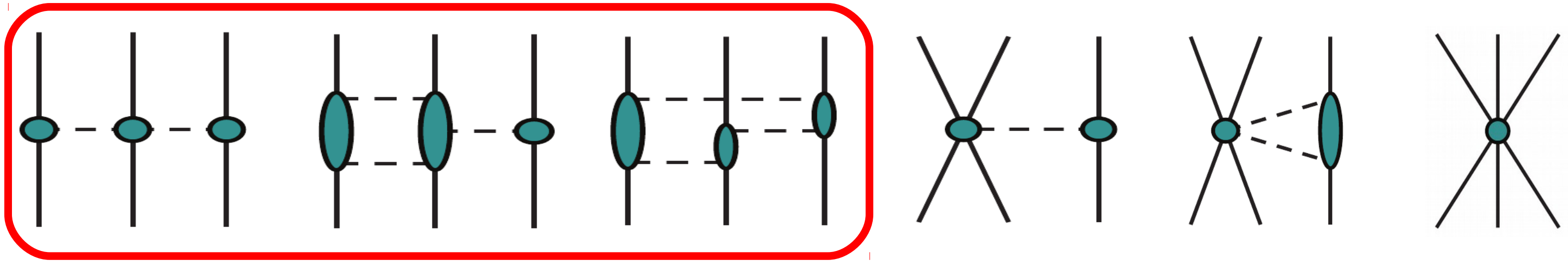
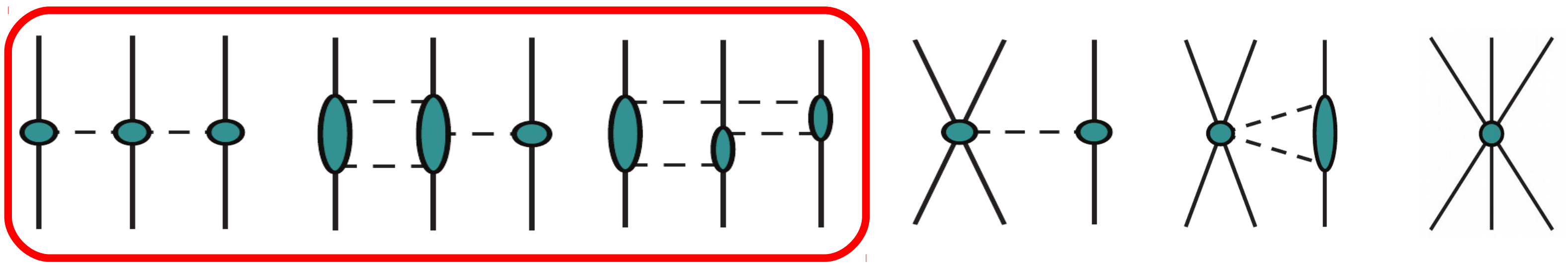


2π– exchange

2π–1π– exchange

ring

Long-range 3NF



2π– exchange

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ring

- ➔ Only the long range part considered (coordinate space)
- ➔ Scheme independent
- ➔ No unknown parameters

Most general structure of a local 3NF

Krebs, Gasparyan, Epelbaum '13

Up to N^4LO all considered contribution are local

Constraints:
- Locality
- Isospin symmetry
- Parity and time-reversal invariance

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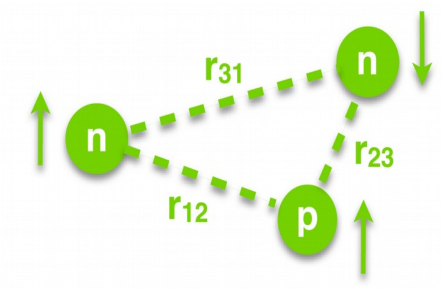

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$$
\begin{aligned}
\tilde{\mathcal{G}}_1 &= 1\,,\\
\tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,,\\
\tilde{\mathcal{G}}_3 &= \vec{\sigma}_1\cdot\vec{\sigma}_3\,,\\
\tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\vec{\sigma}_1\cdot\vec{\sigma}_3\,,\\
\tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,,\\
\tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot(\vec{\sigma}_2\times\vec{\sigma}_3)\,,\\
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_2\cdot(\hat{r}_{12}\times\hat{r}_{23})\,,\\
\tilde{\mathcal{G}}_8 &= \hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{23}\cdot\vec{\sigma}_3\,,\\
\tilde{\mathcal{G}}_9 &= \hat{r}_{23}\cdot\vec{\sigma}_3\,\hat{r}_{12}\cdot\vec{\sigma}_1\,,\\
\tilde{\mathcal{G}}_{10} &= \hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{12}\cdot\vec{\sigma}_3\,,\\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{23}\cdot\vec{\sigma}_2\,,\\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{12}\cdot\vec{\sigma}_2\,,\\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{12}\cdot\vec{\sigma}_1\,\hat{r}_{23}\cdot\vec{\sigma}_2\,,\\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{12}\cdot\vec{\sigma}_1\,\hat{r}_{12}\cdot\vec{\sigma}_2\,,\\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\hat{r}_{13}\cdot\vec{\sigma}_1\,\hat{r}_{13}\cdot\vec{\sigma}_3\,,\\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat{r}_{12}\cdot\vec{\sigma}_2\,\hat{r}_{12}\cdot\vec{\sigma}_3\,,\\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\hat{r}_{23}\cdot\vec{\sigma}_1\,\hat{r}_{12}\cdot\vec{\sigma}_3\,,\\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot\vec{\sigma}_3\,\vec{\sigma}_2\cdot(\hat{r}_{12}\times\hat{r}_{23})\,,\\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_3\cdot\hat{r}_{23}\,\hat{r}_{23}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\,,\\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot\hat{r}_{23}\,\vec{\sigma}_2\cdot\hat{r}_{23}\,\vec{\sigma}_3\cdot(\hat{r}_{12}\times\hat{r}_{23})\,,\\
\tilde{\mathcal{G}}_{21} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot\hat{r}_{13}\,\vec{\sigma}_3\cdot\hat{r}_{13}\,\vec{\sigma}_2\cdot(\hat{r}_{12}\times\hat{r}_{23})\,,\\
\tilde{\mathcal{G}}_{22} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec{\sigma}_1\cdot\hat{r}_{23}\,\vec{\sigma}_3\cdot\hat{r}_{12}\,\vec{\sigma}_2\cdot(\hat{r}_{12}\times\hat{r}_{23})\,,
\end{aligned}
$$

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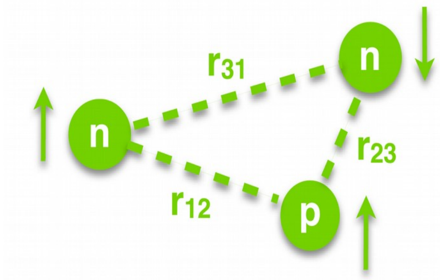
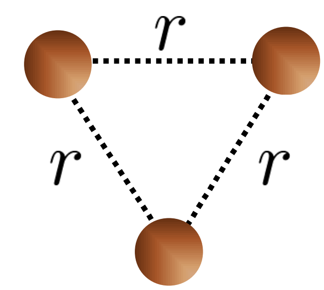


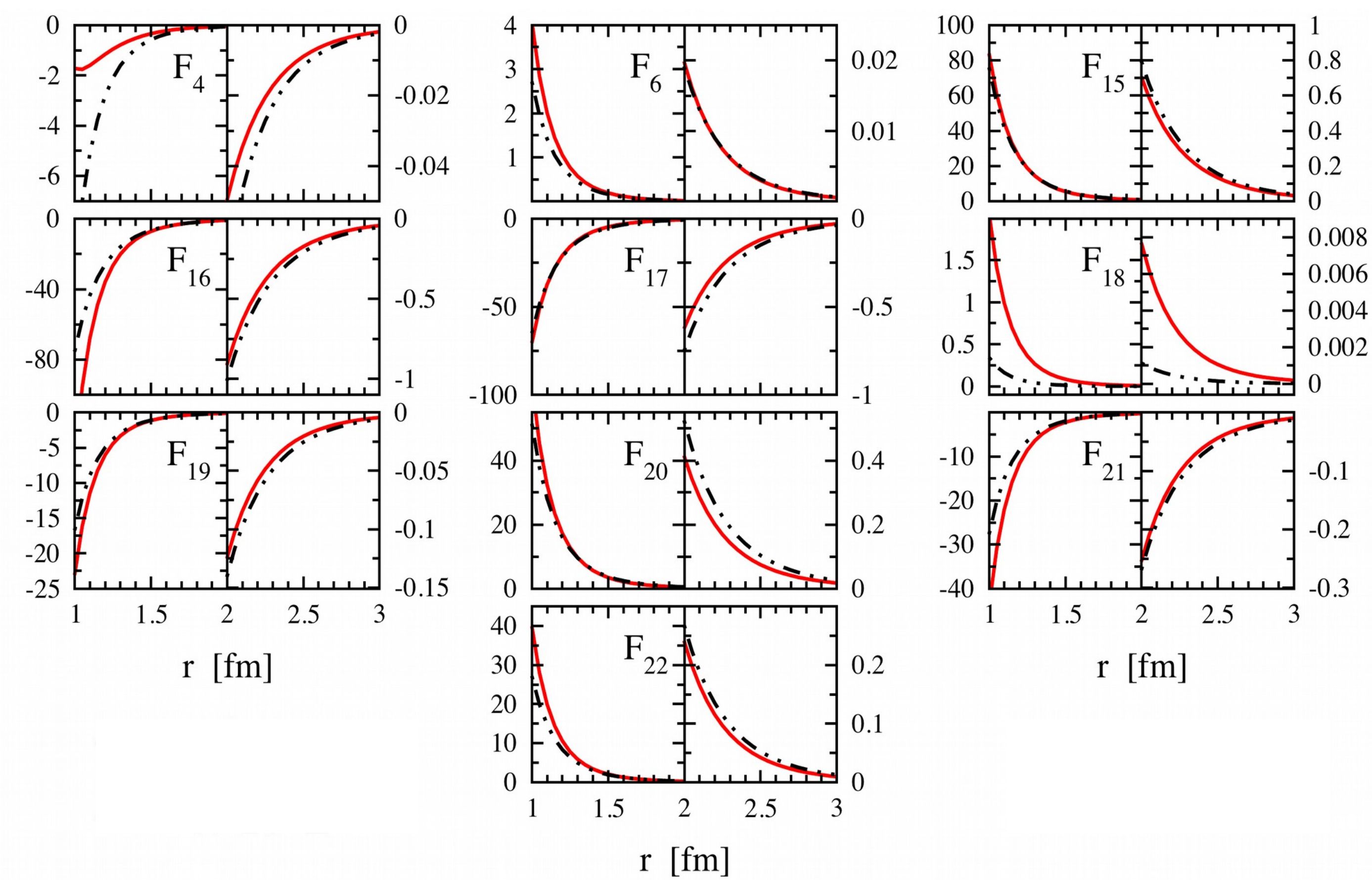
$$V_{3N} = \sum_{i=1}^{22} \tilde{\mathcal{G}}_i F_i(r_{12}, r_{23}, r_{31}) + 5\text{perm.}$$

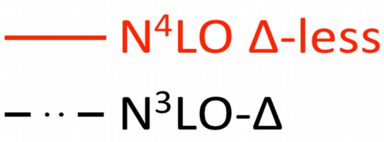
Two-pion-exhcange 3NF in Δ–full and Δ–less approach (preliminary)

Krebs, Gasparyan, Epelbaum, in preparation

TPE "structure functions" F_i in MeV"
in equilateral-triangle configuration







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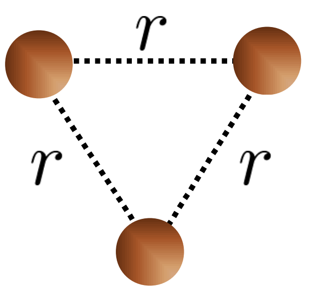

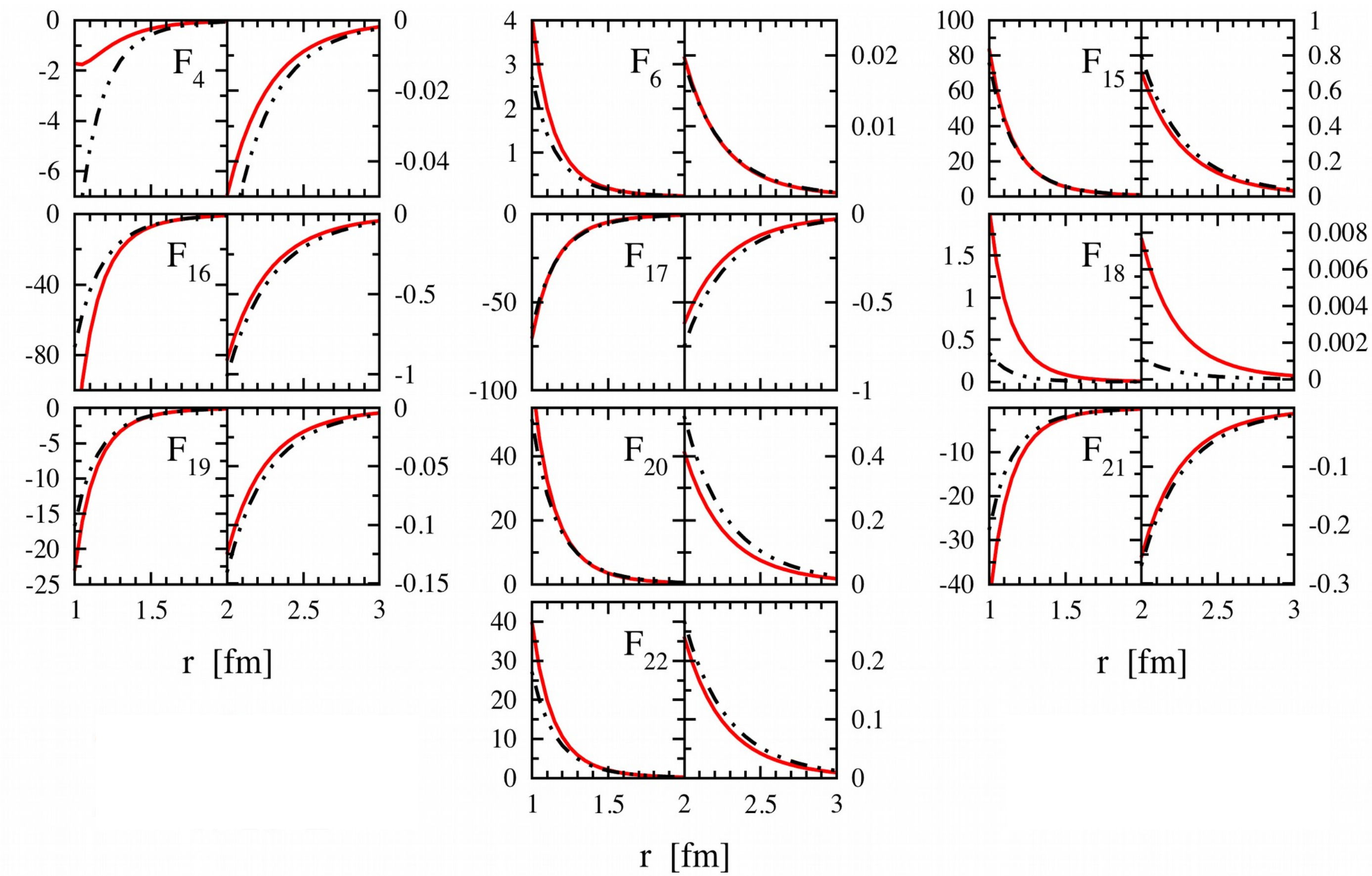

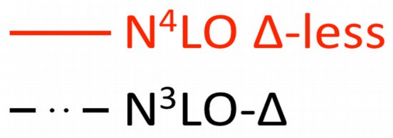

➔ similar results for large contributions

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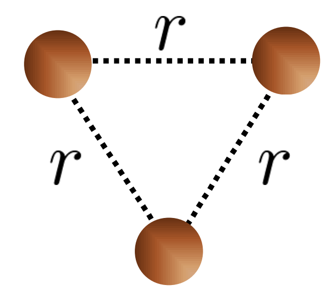
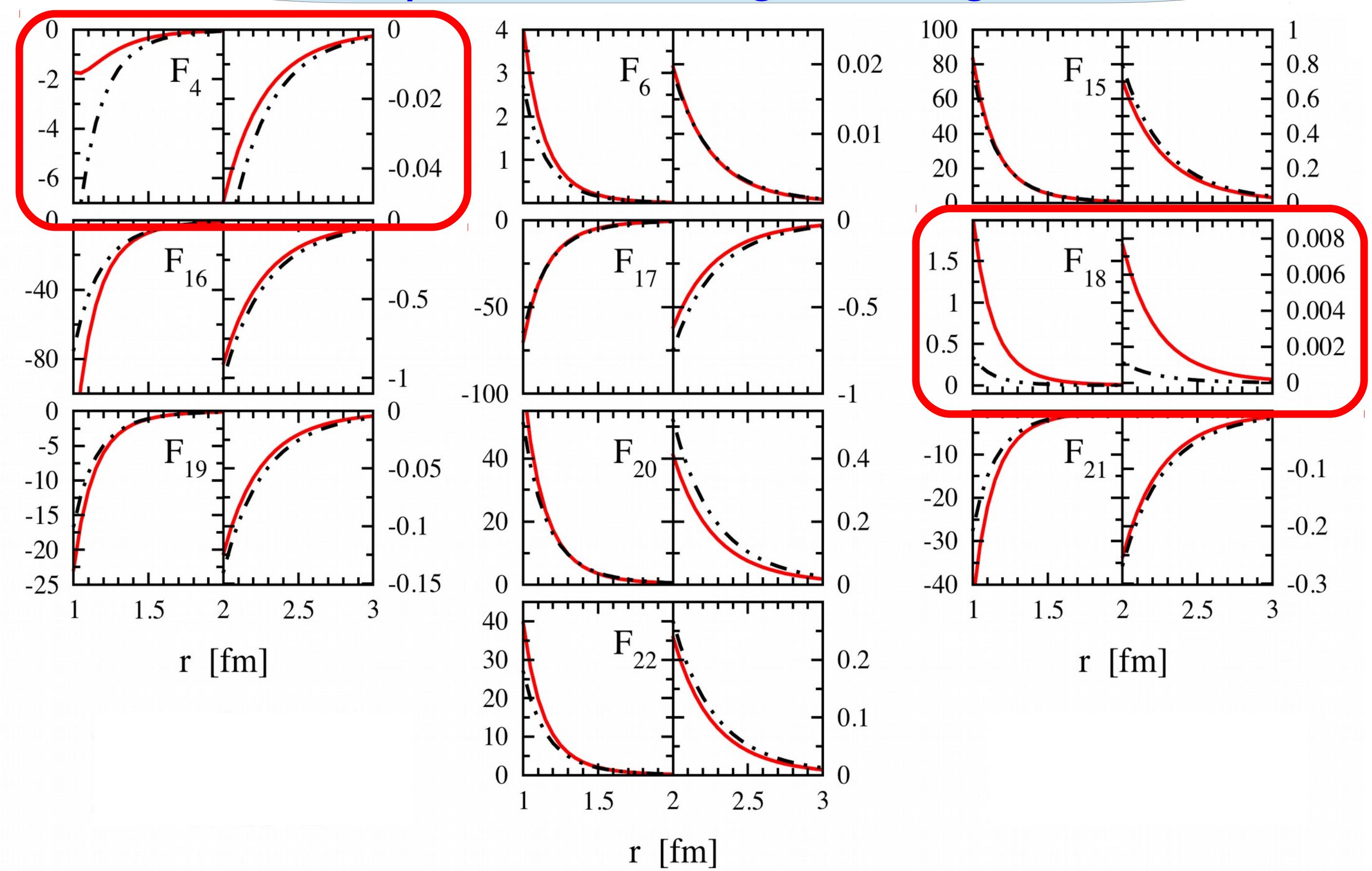
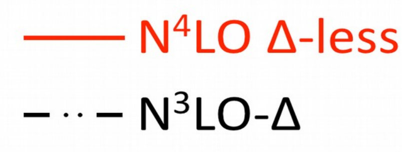
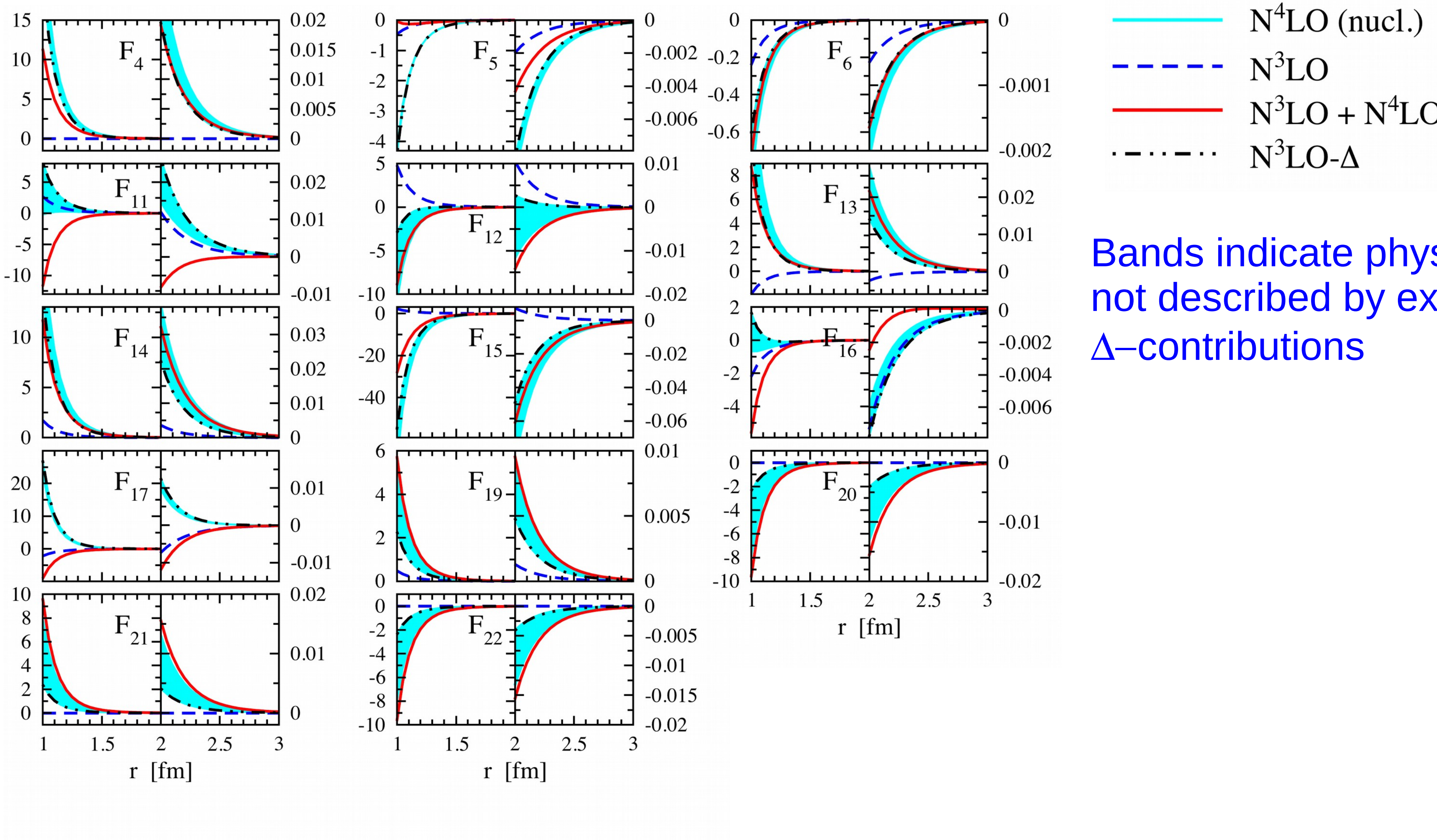






- ➔ similar results for large contributions
- ➔ slightly different for small contributions

Two-pion-one-pion-exhcange 3NF in Δ–full and Δ–less approach (preliminary)



Bands indicate physics not described by explicit Δ–contributions

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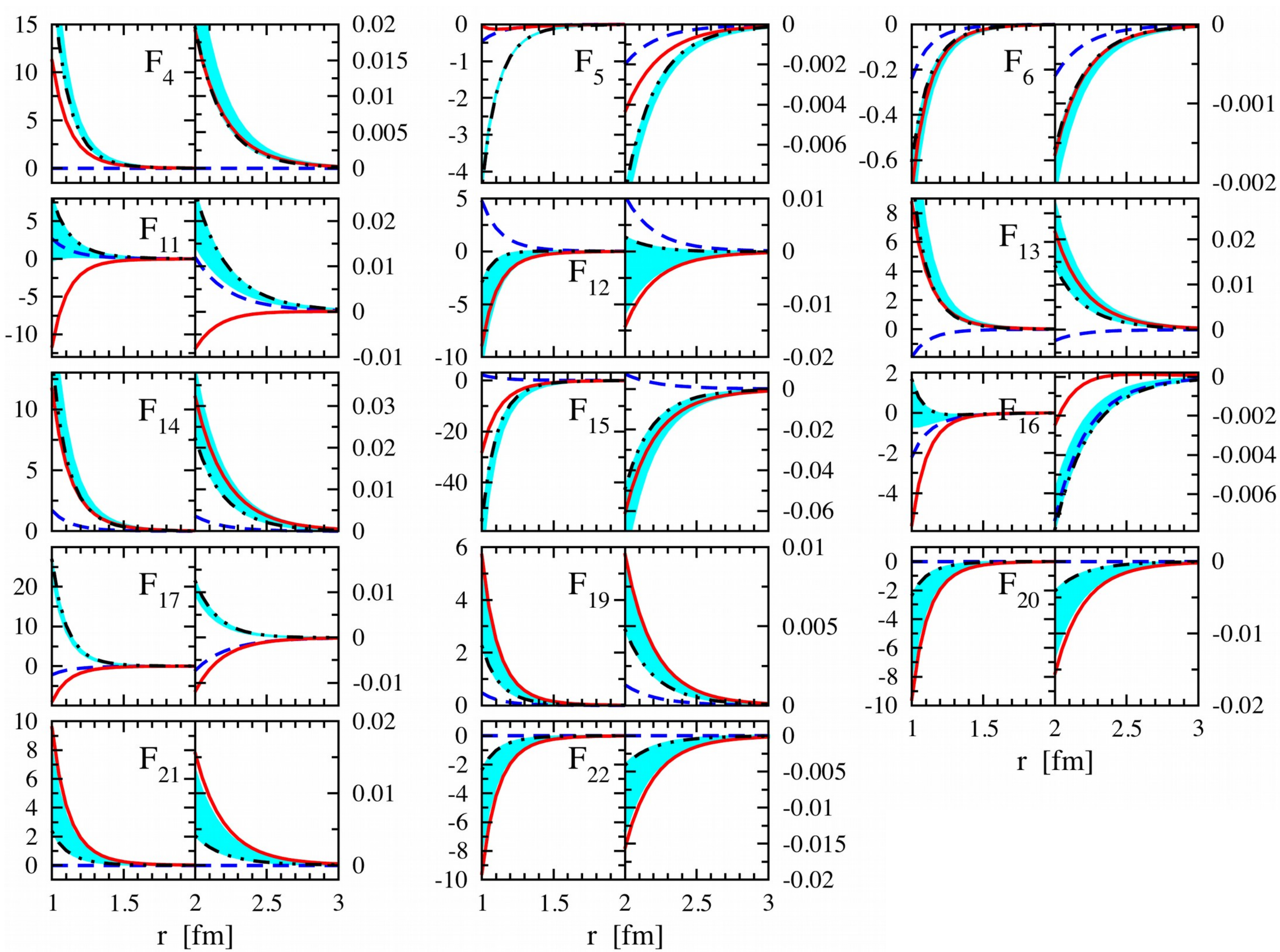

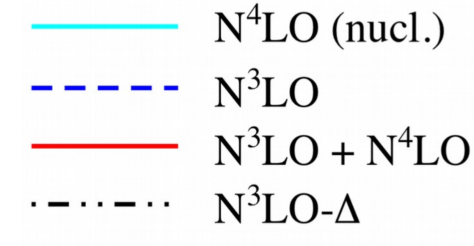

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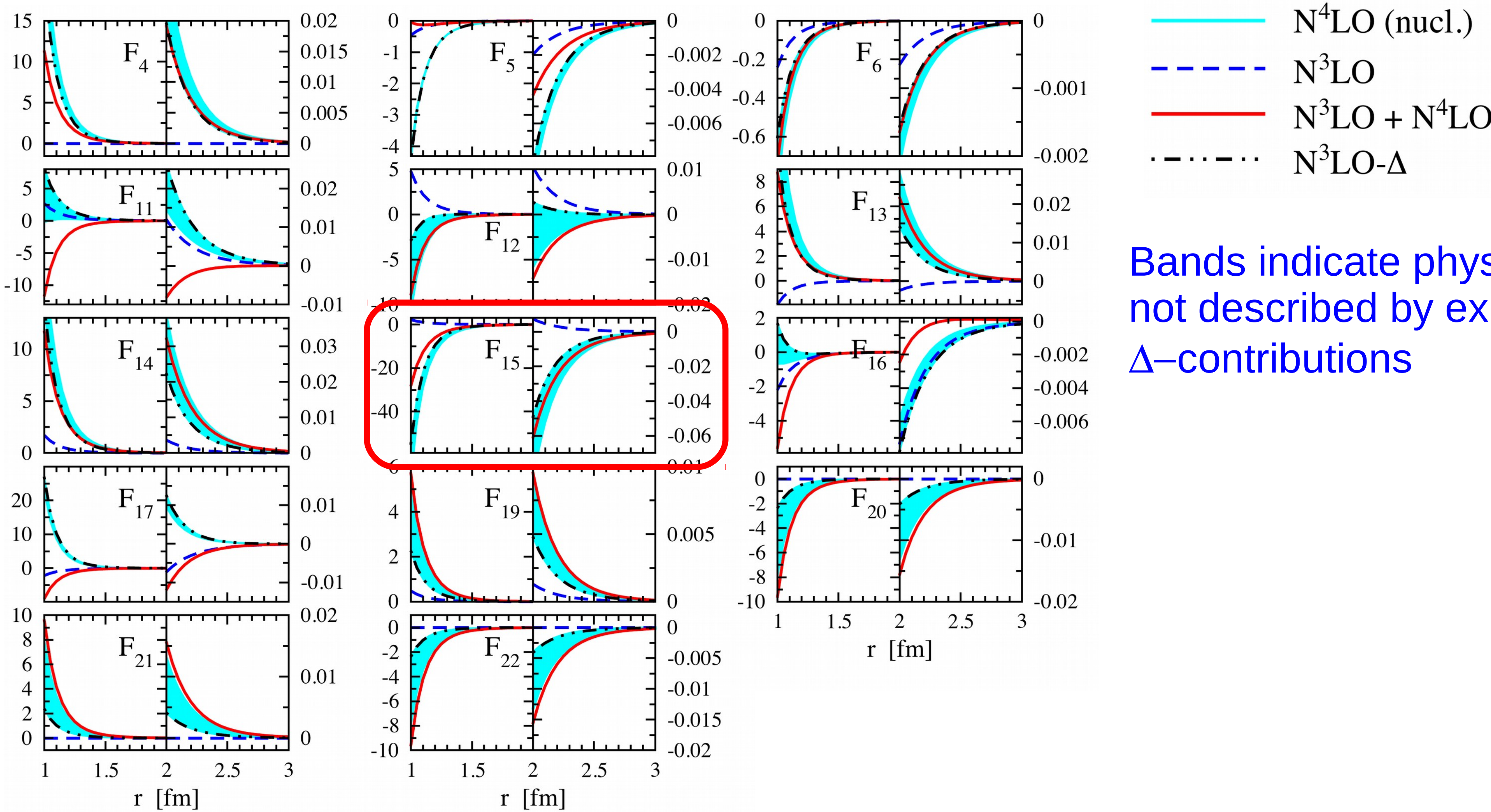

Bands indicate physics not described by explicit Δ–contributions

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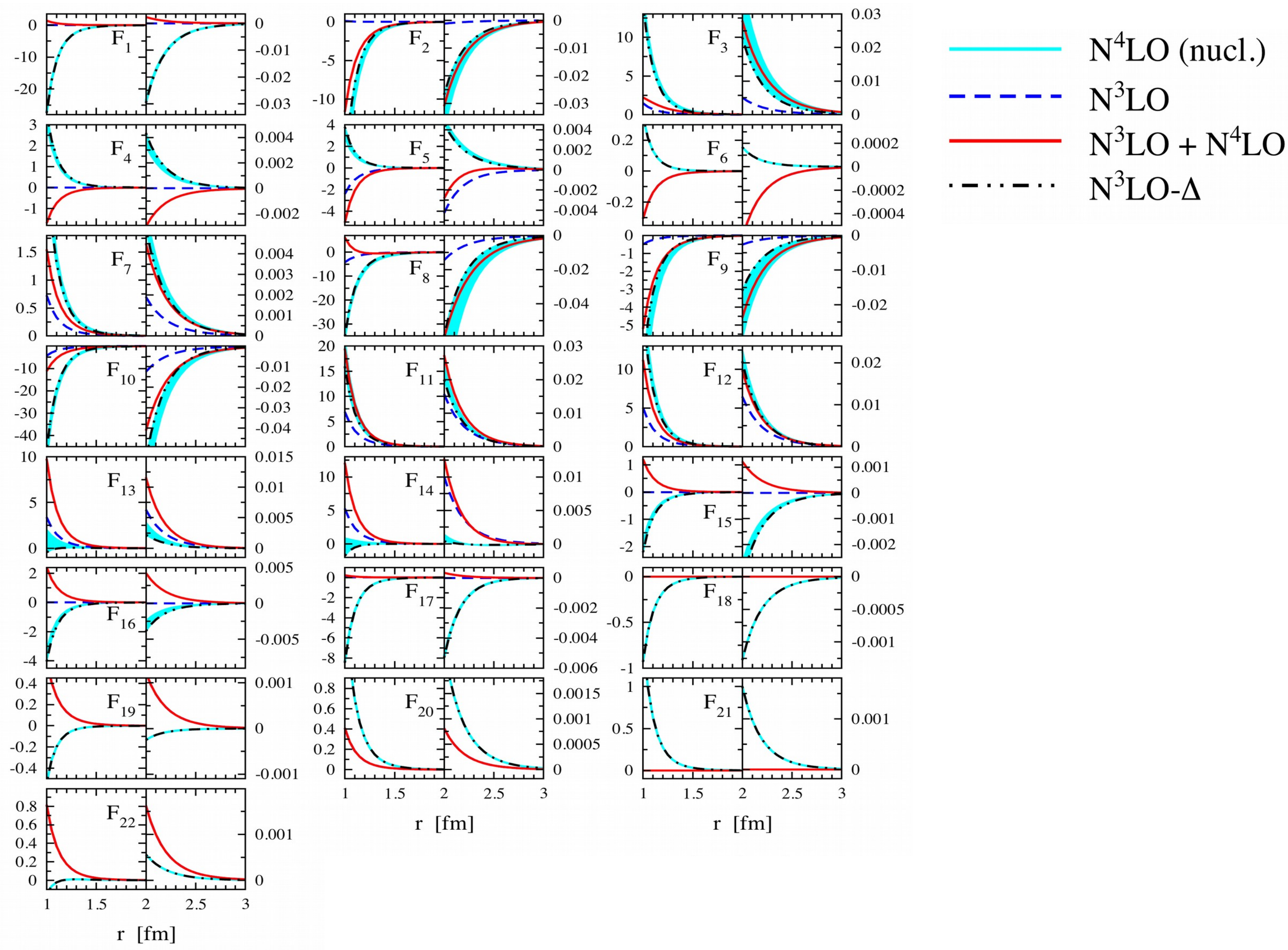
➔ The largest N^4LO contribution is saturated by Δ

Ring-topology 3NF in Δ–full and Δ–less approach (preliminary)



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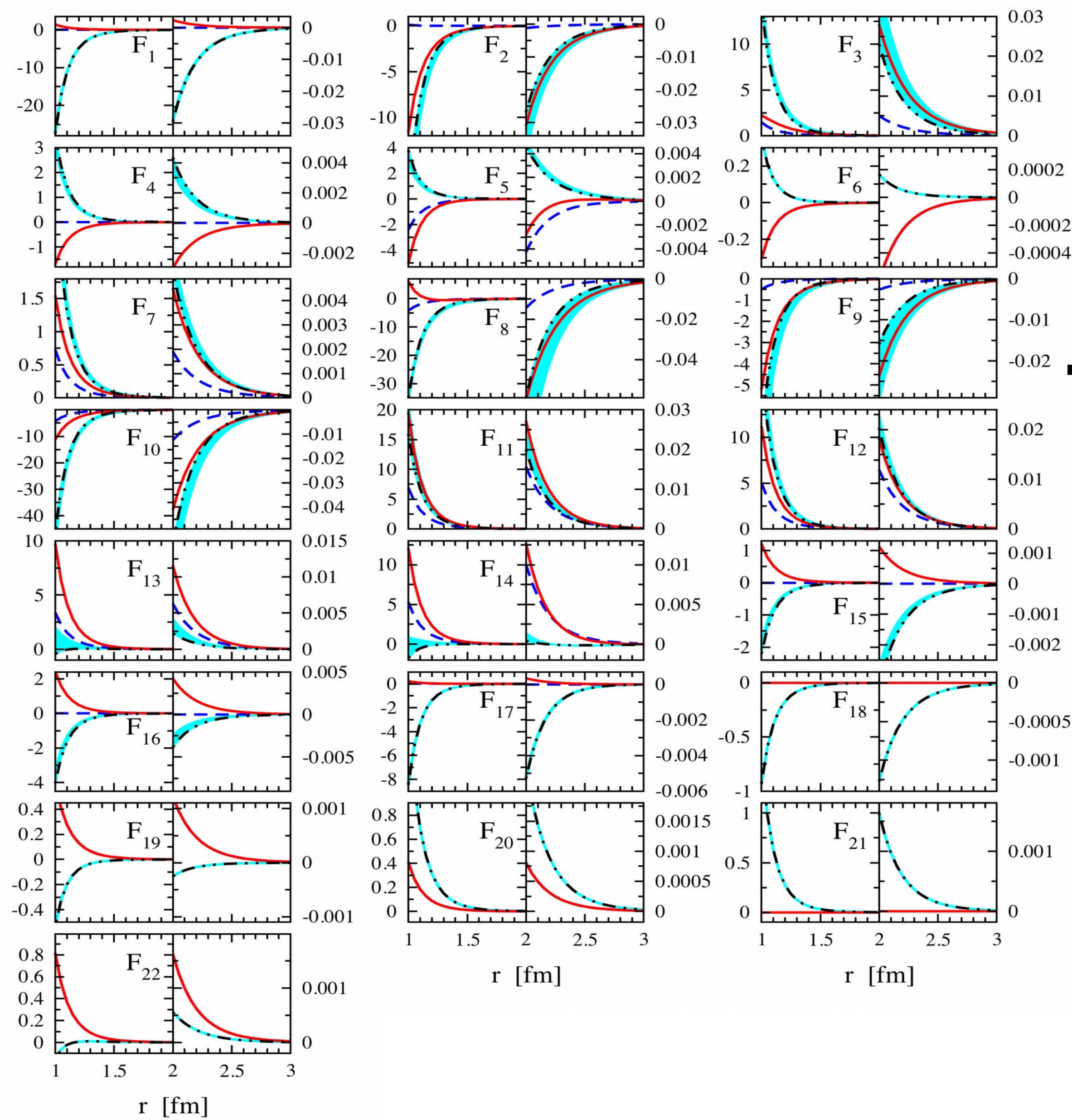

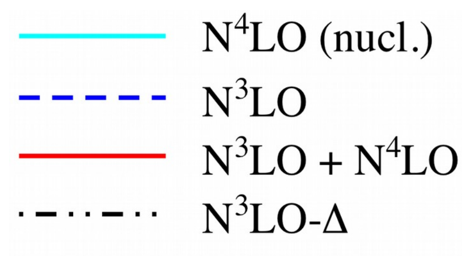

➔ Narrow bands: higher order contributions beyond Δ are small

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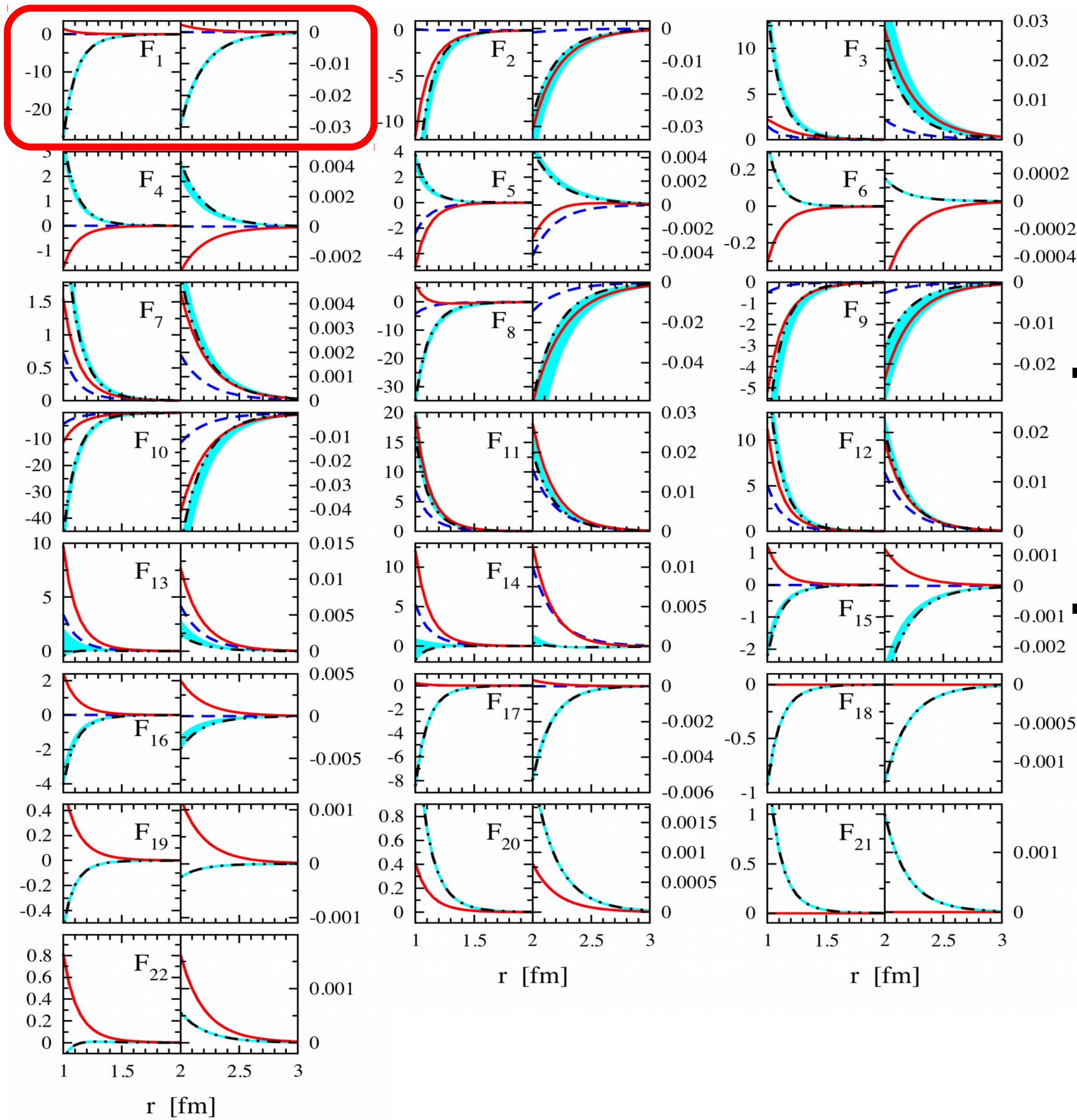

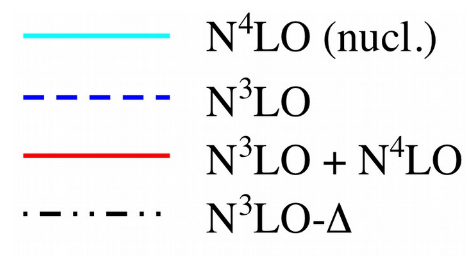

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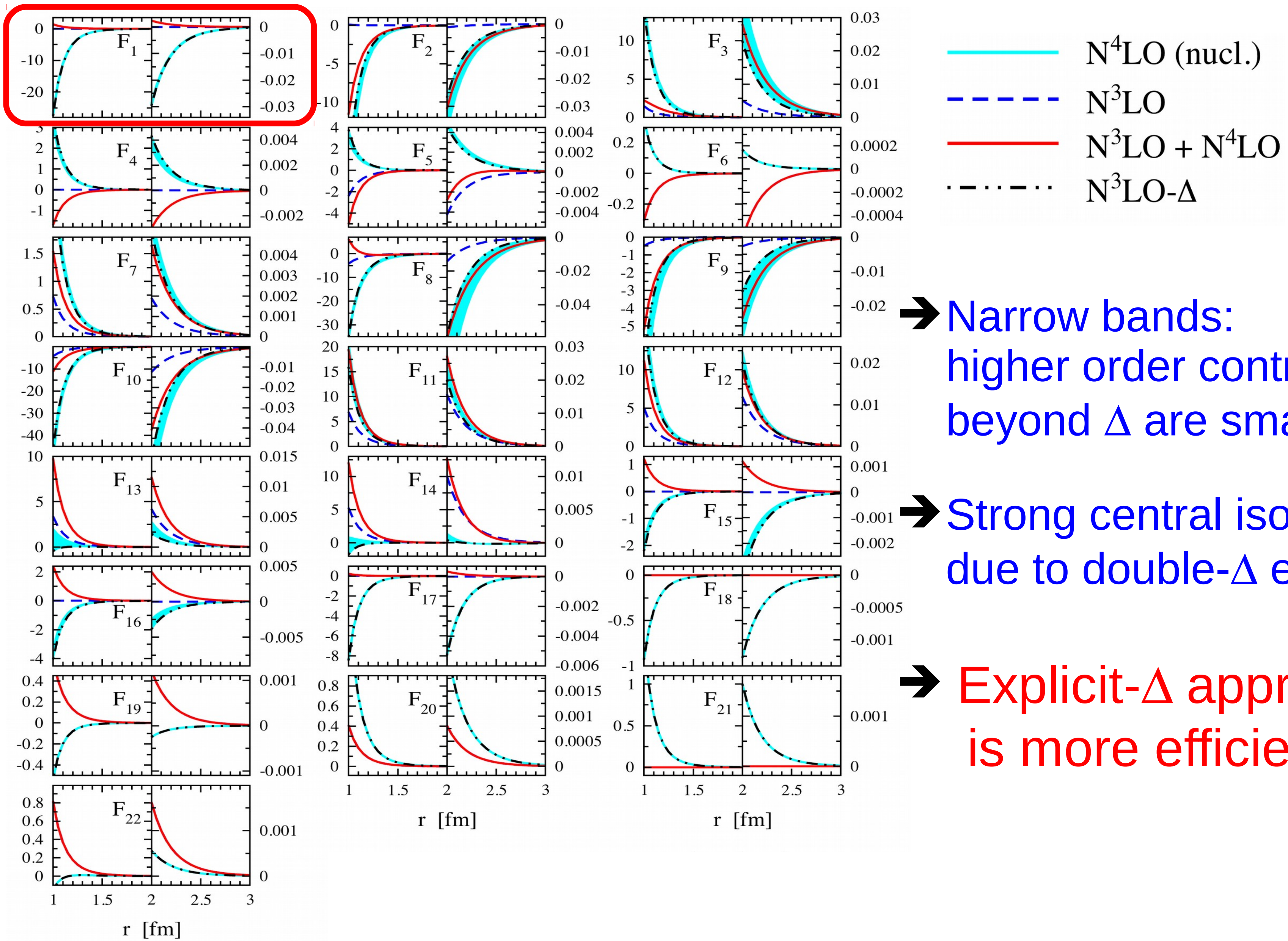


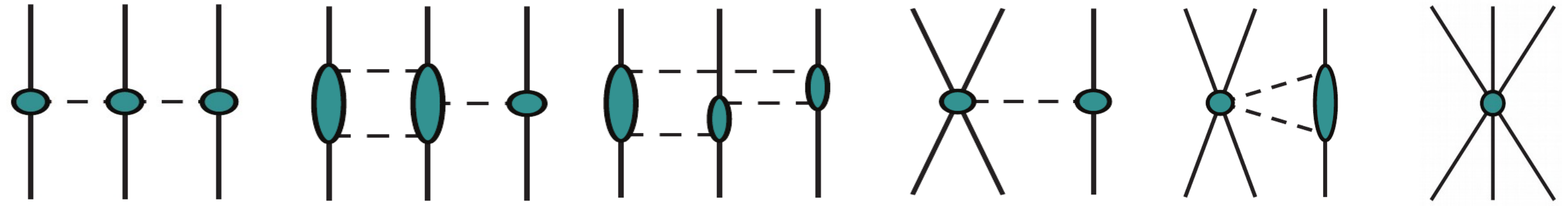
➔ Narrow bands: higher order contributions beyond Δ are small

➔ Strong central isoscalar 3NF due to double-Δ excitation

➔ Explicit-Δ approach is more efficient !

πN input for 3-Nucleon Forces

- ➔ Longest-range contributions
- ➔ Intermediate-range contributions
- ➔ Short-range contributions



2π–exchange   2π–1π–exchange   ring

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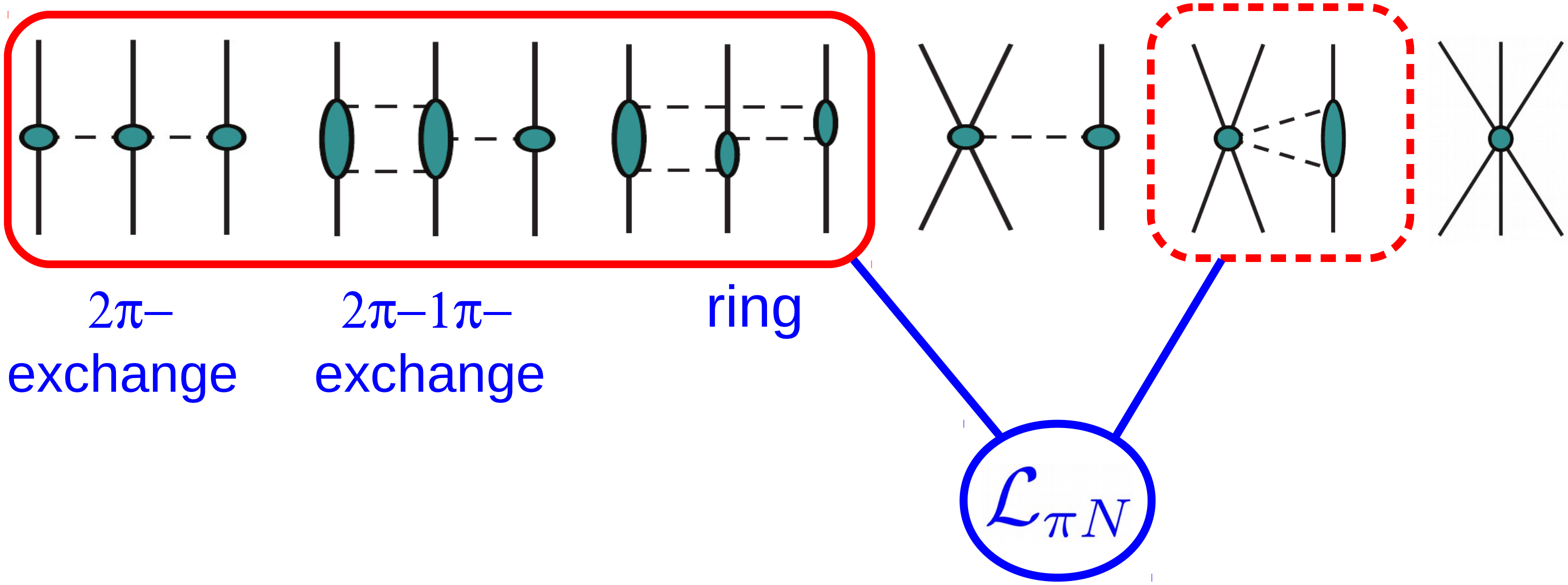
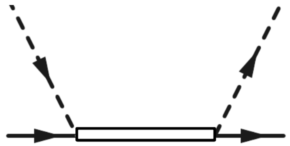


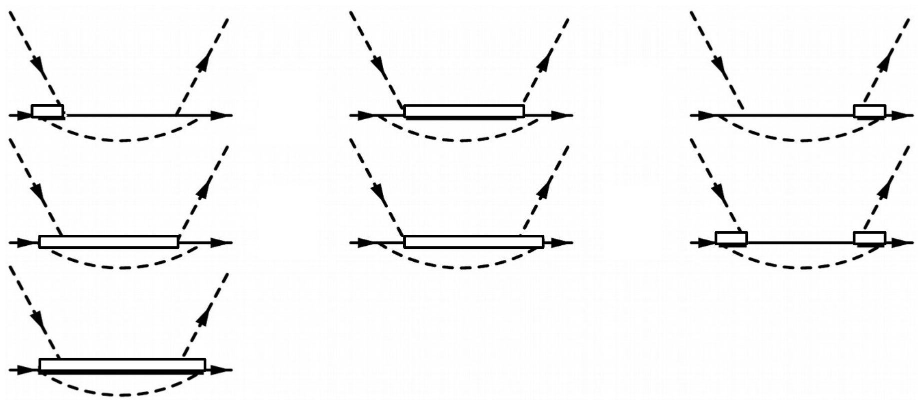
πN scattering up to ε^4

Siemens et al. In preparation

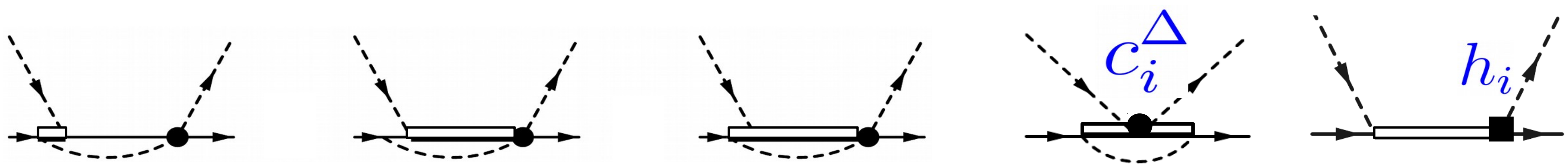
ε^1



ε^3



ε^4



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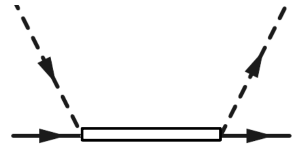
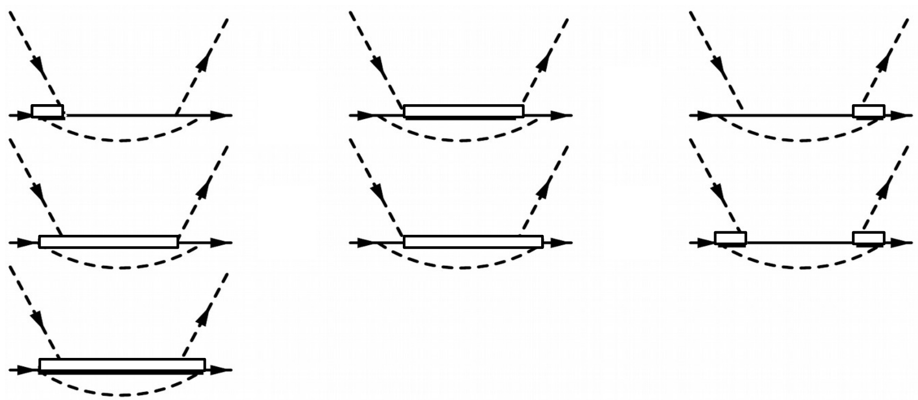
Siemens et al. In preparation

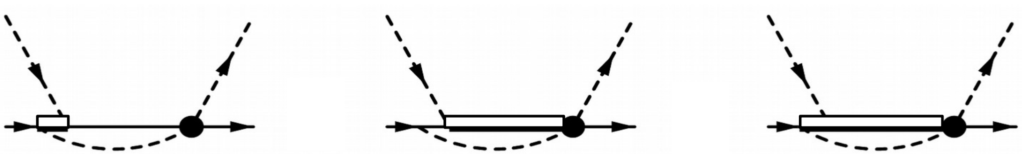
ε^1

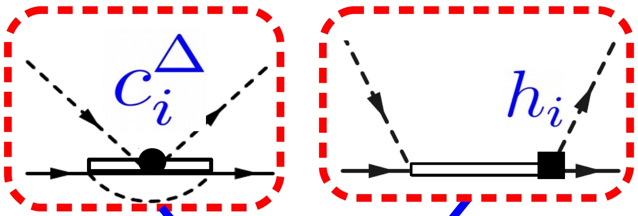


ε^3



ε^4



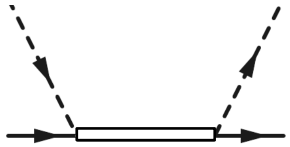


redundant, can be absorbed
by redefining other LEC's

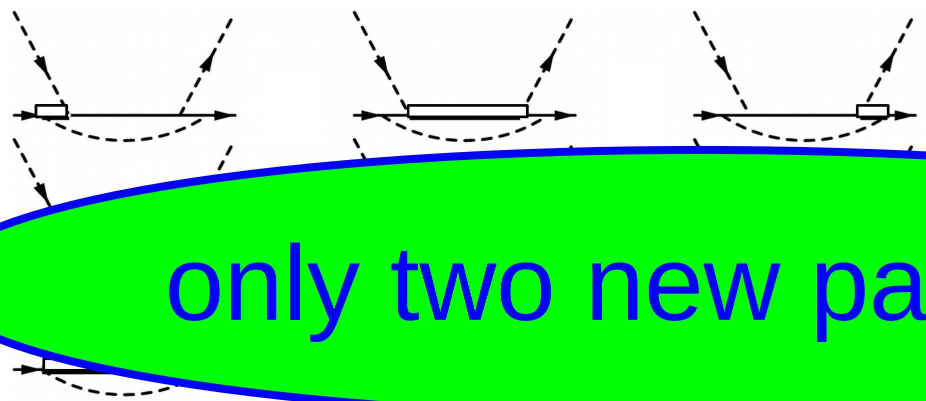
πN scattering up to ε^4

Siemens et al. In preparation

ε^1

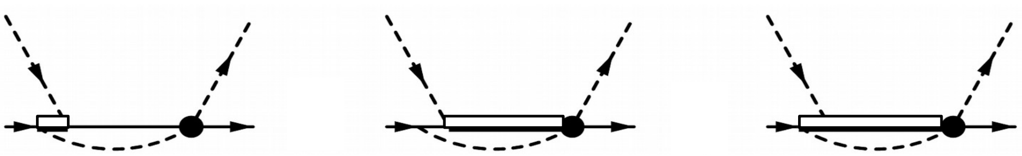


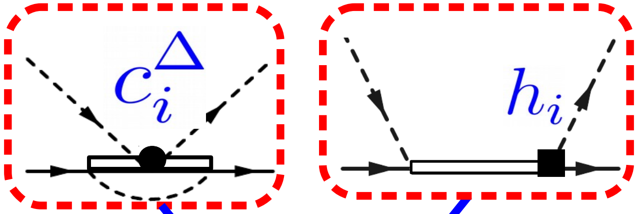
ε^3



only two new parameters

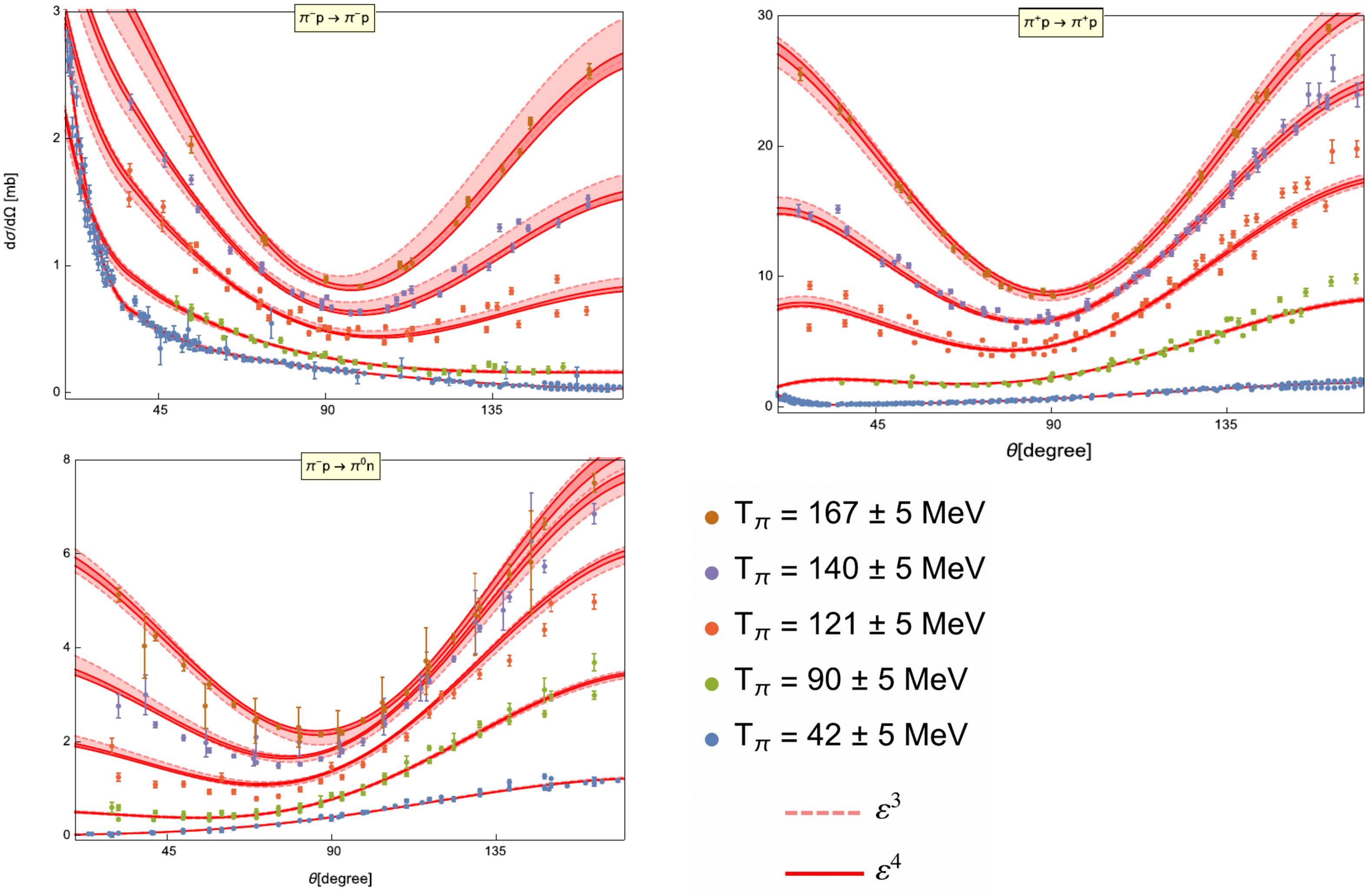
ε^4



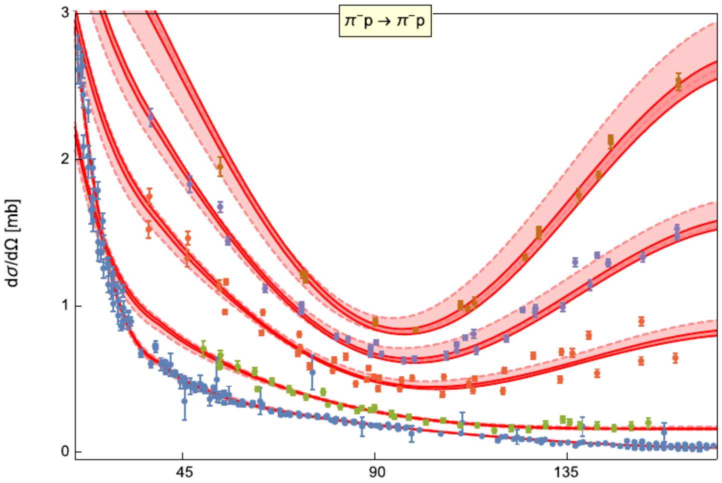


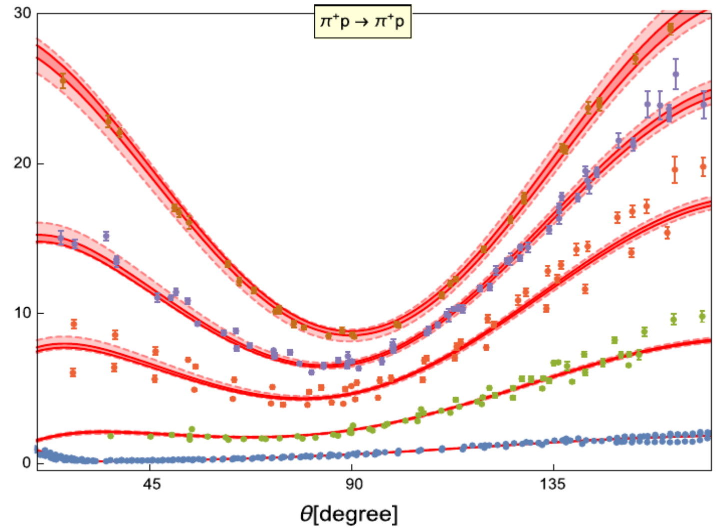
redundant, can be absorbed by redefining other LEC's

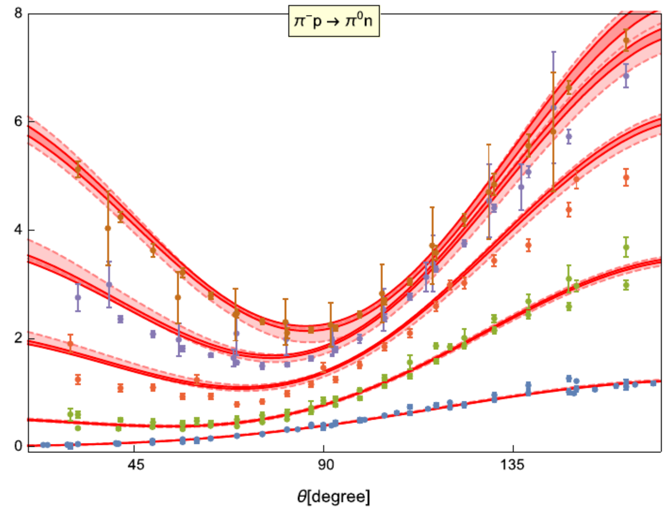
πN differential cross section



πN differential cross section







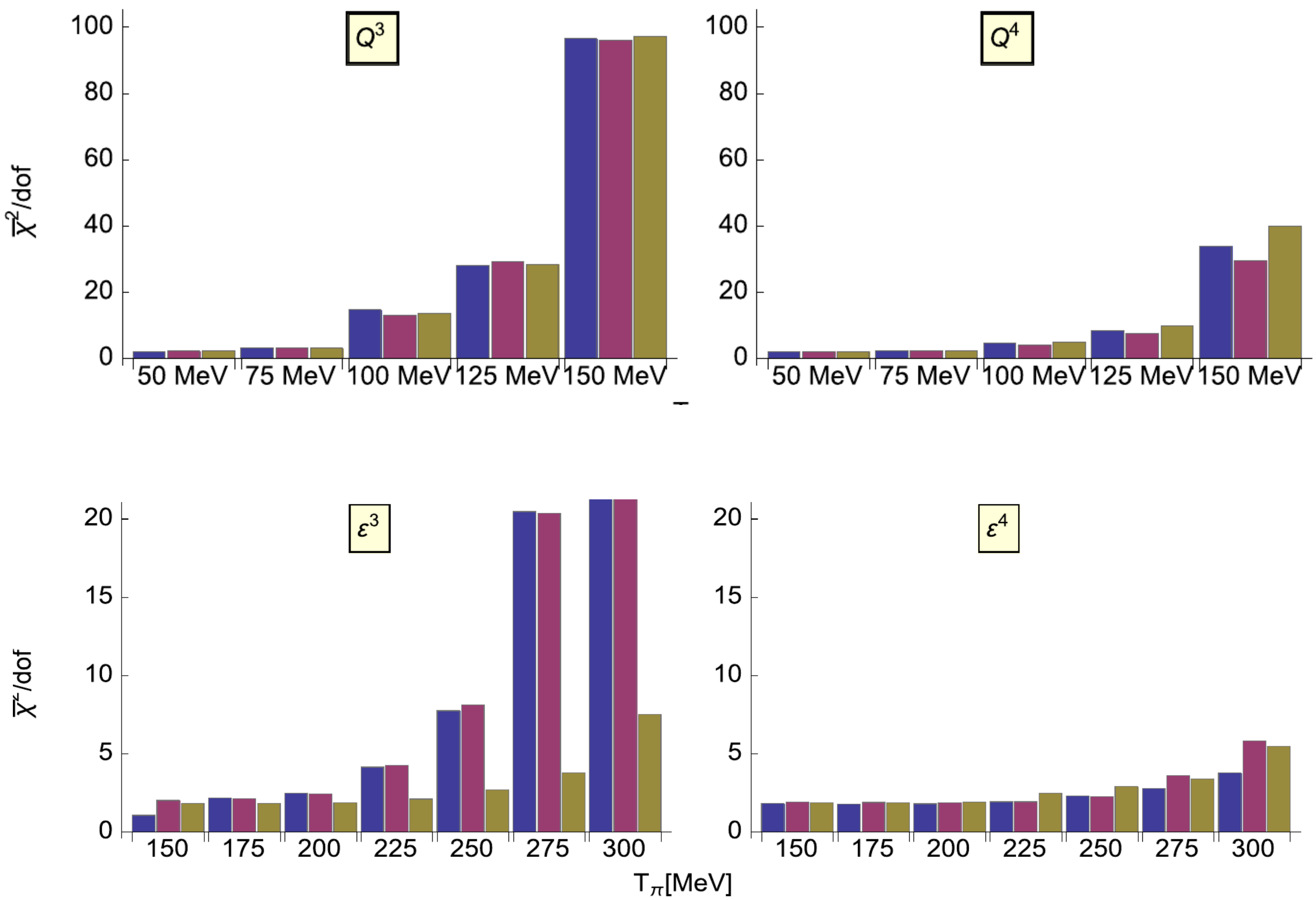
- T_π = 167 ± 5 MeV
- T_π = 140 ± 5 MeV
- T_π = 121 ± 5 MeV
- T_π = 90 ± 5 MeV
- T_π = 42 ± 5 MeV

ε^3

ε^4 ➔ Theoretical error-bands are narrower

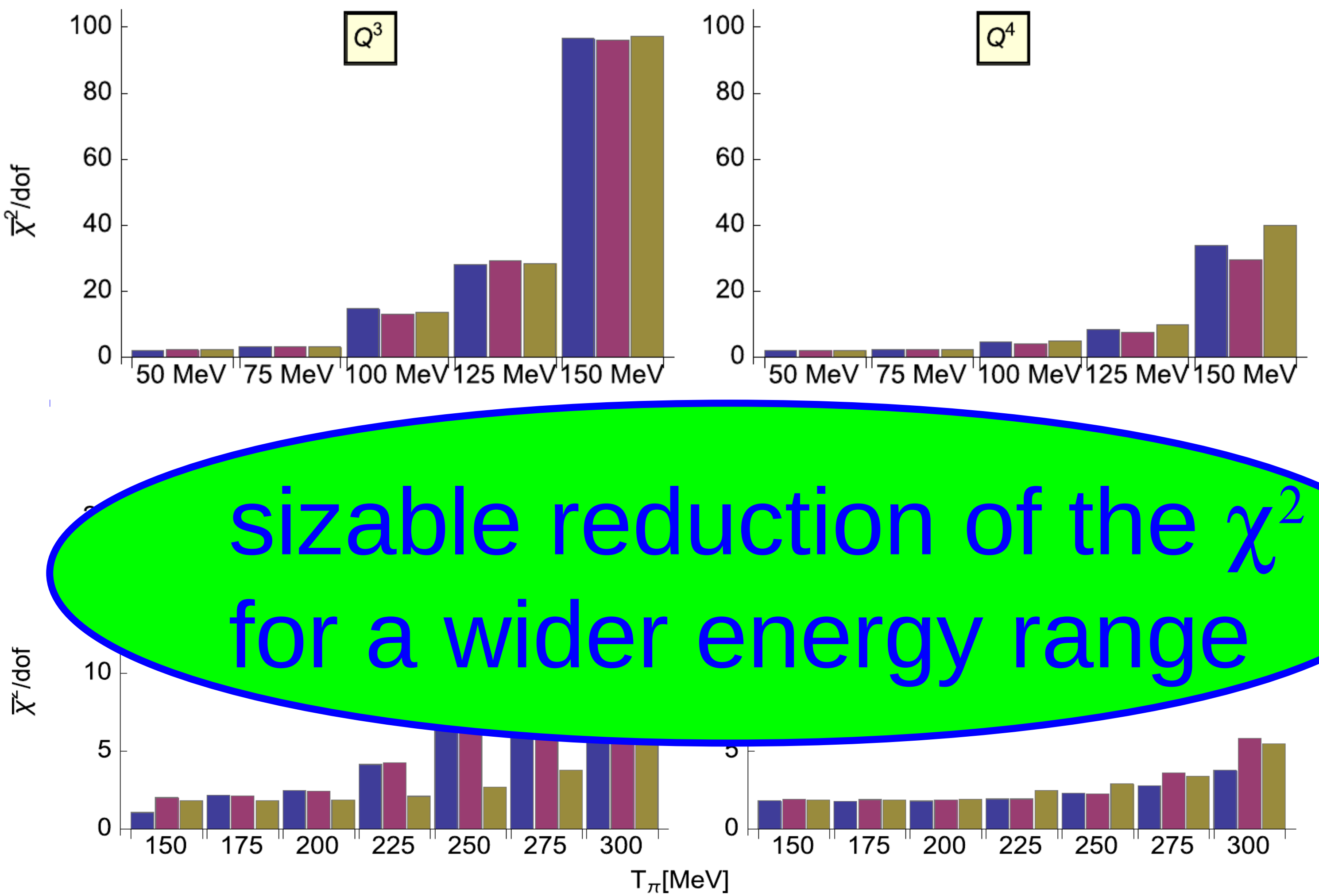
Quality of the fit to πN data in the Δ–less and Δ–full χPT(without theoretical errors)

HB-NN HB-πN covariant



Quality of the fit to πN data in the Δ–less and Δ–full χPT(without theoretical errors)

HB-NN HB-πN covariant



sizable reduction of the χ^2
for a wider energy range

Summary

➔ Preliminary results for Δ-full chiral 2-nucleon and 3-nucleon forces at N^3LO are presented

➔ 2-nucleon forces (peripheral phases): significant improvement compared to the Δ-less case

➔ 3-nucleon forces: indication of a better convergence; sizable Δ-contributions missing in Δ-less N^4LO 3NF ~$O(1/\Delta^2)$

➔ New results for πN scattering at order ε^4: much better fit to data

Outlook

➔ Completing construction of Δ-full chiral 2N and 3N forces at N^3LO and moving forward to even more precise nuclear forces.