Electroweak currents in chiral EFT

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Progress in Ab Initio Techniques in Nuclear Physics

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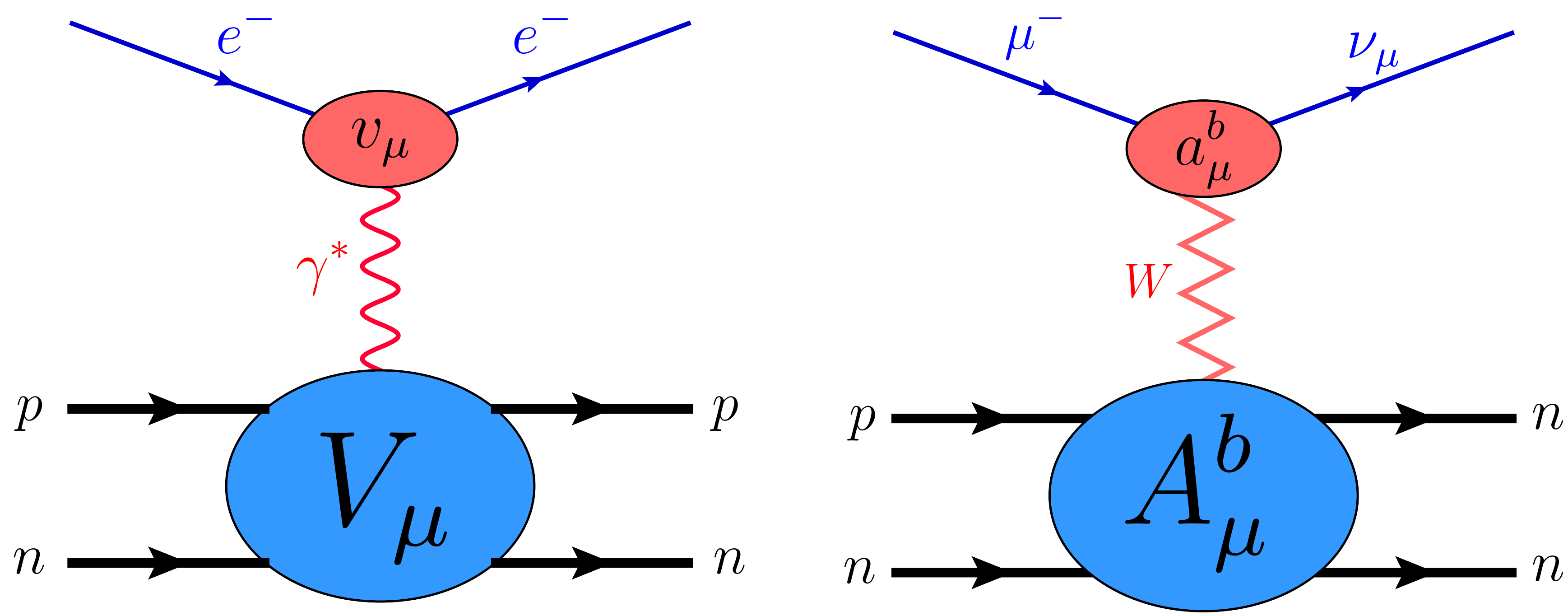
Outline

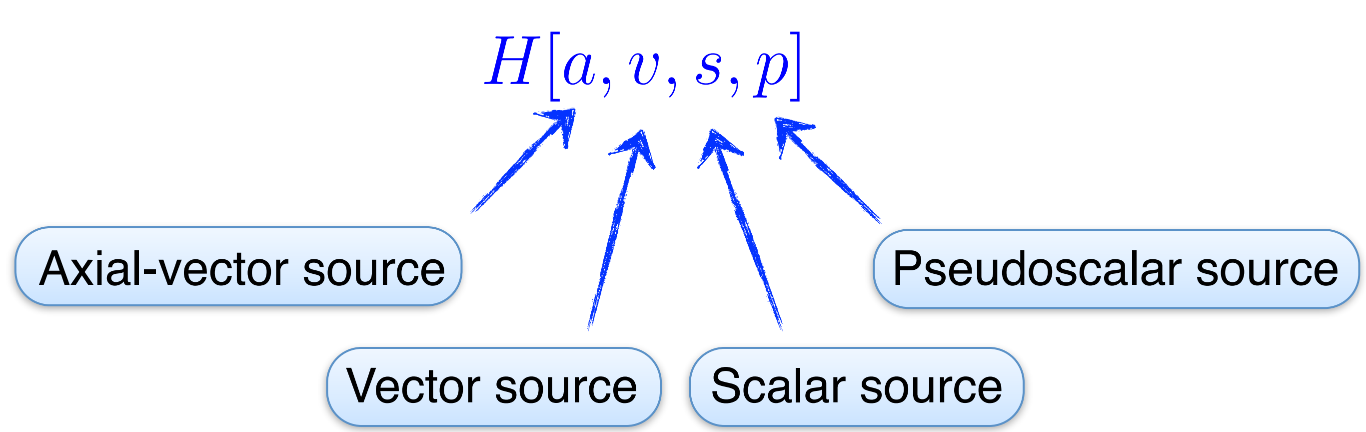
- Nuclear currents in chiral EFT
- Unitary transformations for currents
- Modified continuity equation
- Matching to nuclear forces
- Axial-vector current up to order Q

Nuclear currents in chiral EFT

Electroweak probes on nucleons and nuclei can be described by current formalism

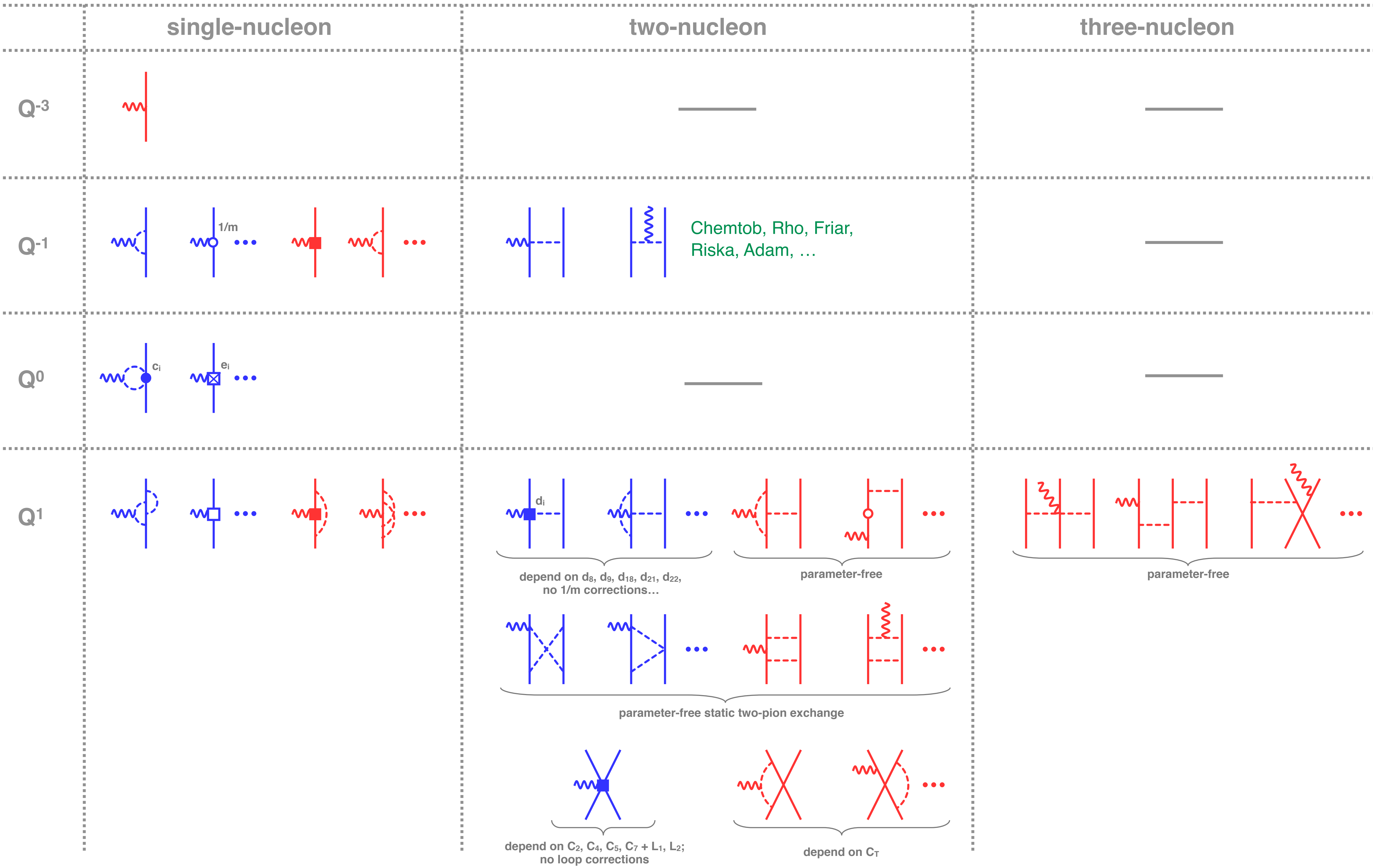


Chiral EFT Hamiltonian depends on external sources



Vector currents in chiral EFT

Chiral expansion of the electromagnetic **current** and **charge** operators



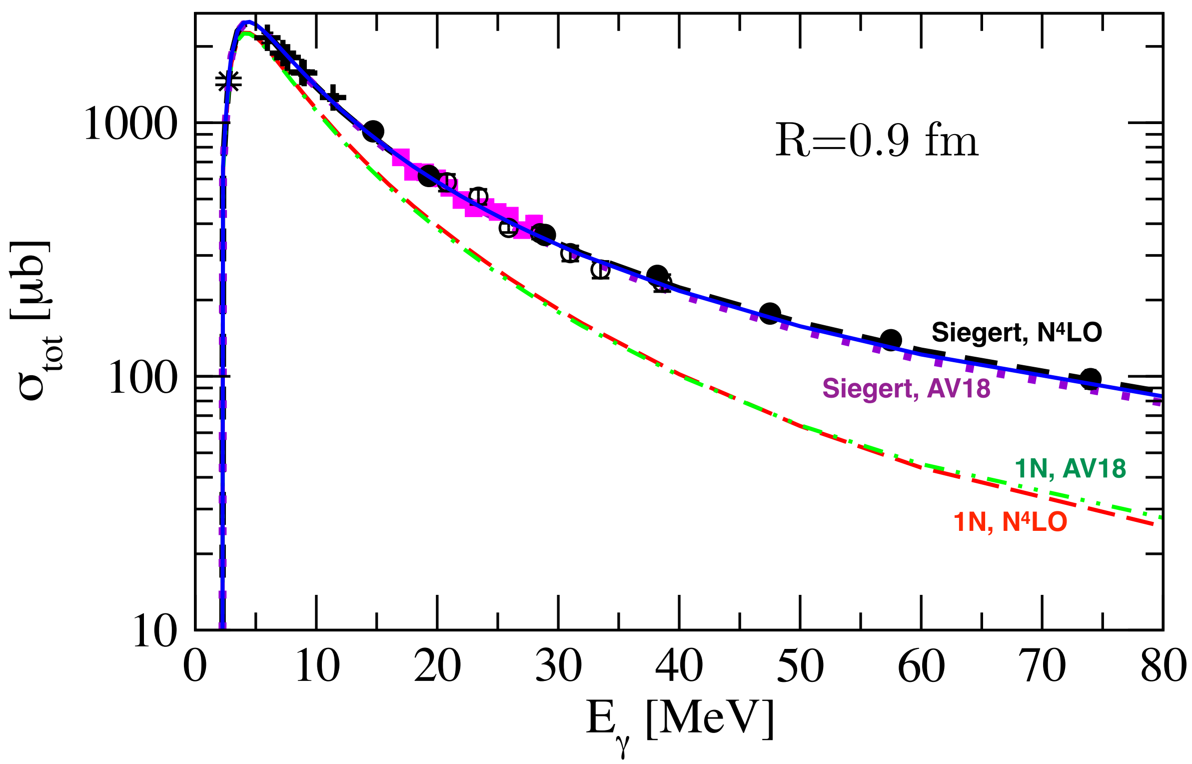
Park, Min, Rho, Kubodera, Song, Lazauskas (earlier works, incomplete, TOPT)
Pastore, Schiavilla et al. (TOPT), Kölling, Epelbaum, HK, Meißner (UT)

Siegert approach + N^{4}LO

Skibinski et al. PRC93 (2016) no. 6, 064002

Generate longitudinal component of NN current by continuity equation

$$\left[H_{\mathrm{strong}}, \boldsymbol{\rho}\right] = \vec{k} \cdot \vec{\boldsymbol{J}} \leftarrow$$ regularized longitudinal current (Siegert approach)

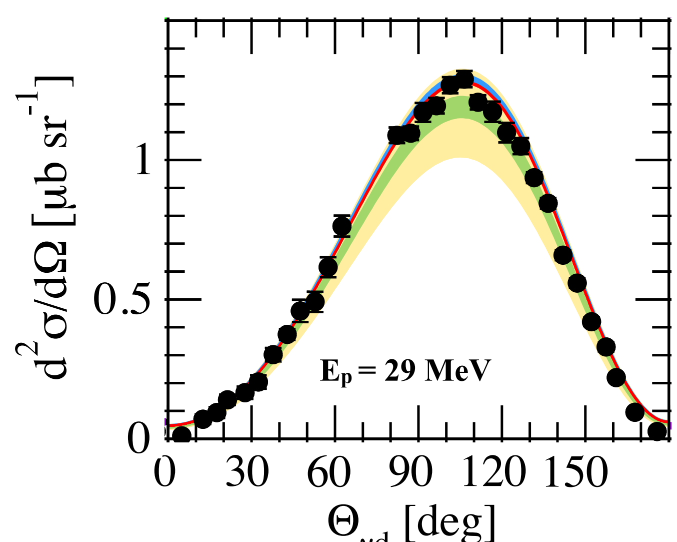


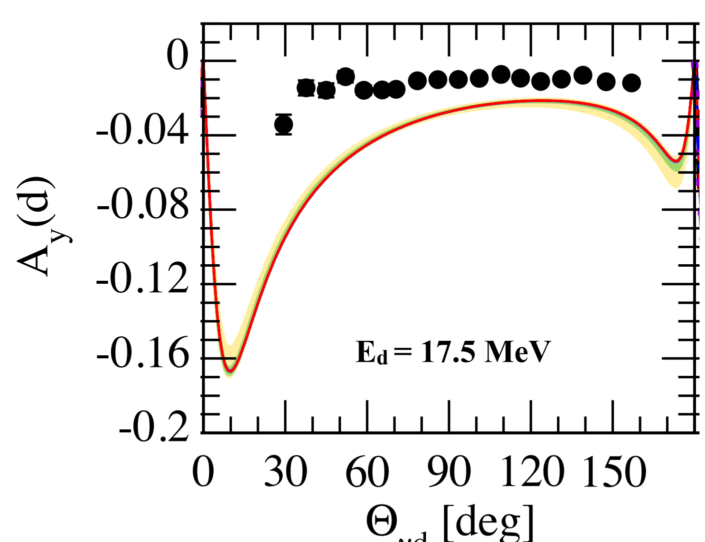
Deuteron photo-disintegration

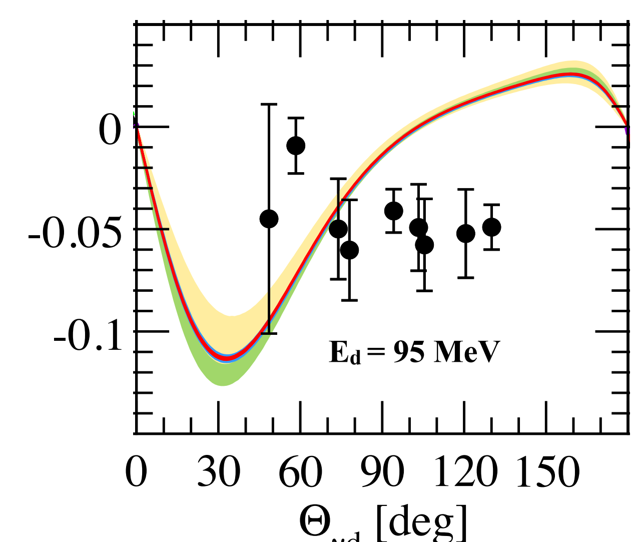
$$\gamma + d \rightarrow p + n$$

- consistent regularization via cont. eq.
- improvement by 1N+Siegert
- implementation of transverse part & exchange currents work in progress in collaboration with Arseniy Filin & Vadim Baru (local regularization)

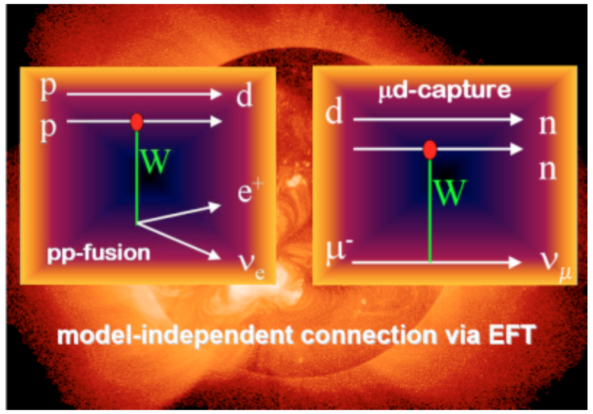
Nucleon-deuteron radiative capture: $p(n) + d \rightarrow {}^3\mathrm{H}({}^3\mathrm{He}) + \gamma$





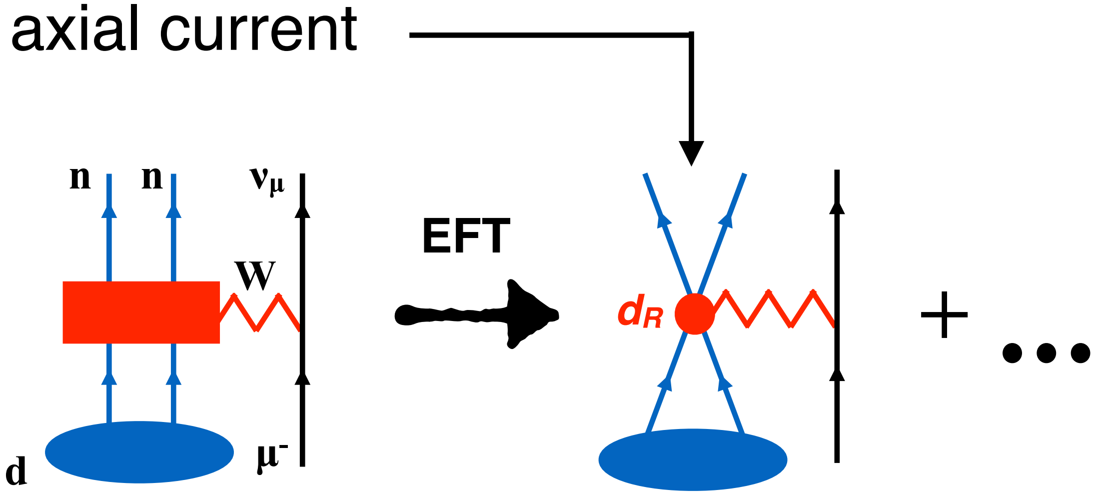


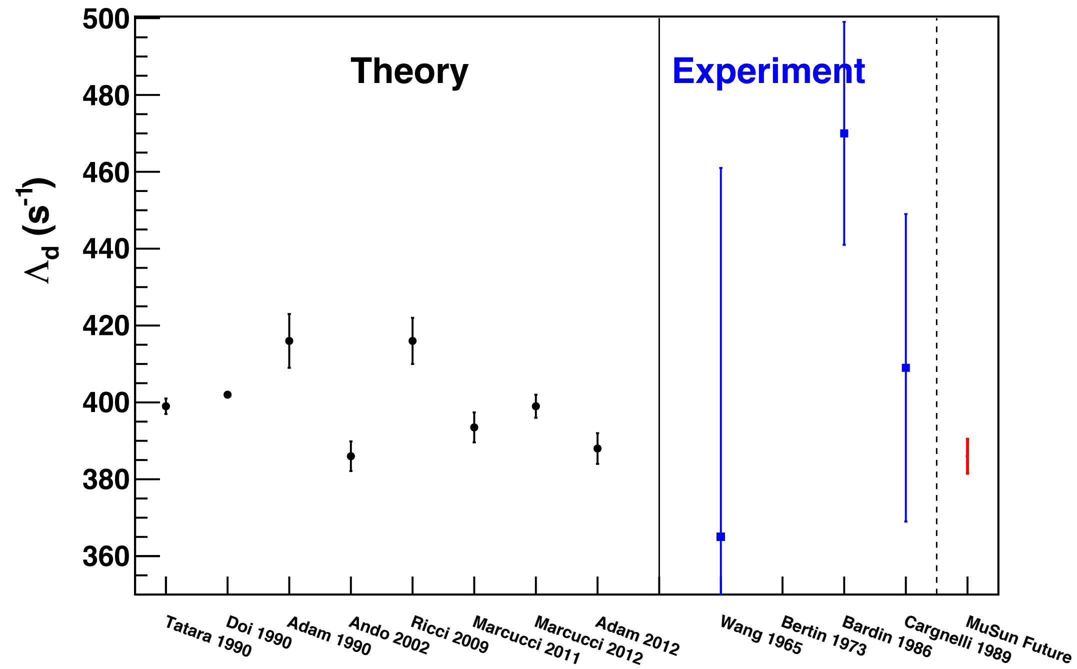
MuSun experiment at PSI



Main goal: measure the doublet capture rate Λ_d in
$\mu^- + d \to \nu_\mu + n + n$ with the accuracy of ~ 1.5%

This will strongly constrain the short-range axial current





The resulting axial exchange current can be used to make precision calculations for

- triton half life, $fT_{1/2} = 1129.6 \pm 3.0$ s, and the muon capture rate on ^{3}He, $\Lambda_0 = 1496 \pm 4\ s^{-1}$ → precision tests of the theory
- weak reactions of astrophysical interest such as e.g. the pp chain of the solar burning:
- d_R governs the leading 3NF

$$p + p \to d + e^+ + \nu_e$$
$$p + p + e^- \to d + \nu_e$$
$$p + {}^3He \to {}^4He + e^+ + \nu_e$$
$${}^7Be + e^- \to {}^7Li + \nu_e$$
$${}^8B \to {}^8Be^* + e^+ + \nu_e$$

Historical remarks

- Meson-exchange theory, Skyrme model, phenomenology, …
  Brown, Adam, Mosconi, Ricci, Truhlik, Nakamura, Sato, Ando, Kubidera, Riska, Sauer, Friar, …

- First derivation within chiral EFT to leading 1-loop order using TOPT
  Park, Min, Rho Phys. Rept. 233 (1993) 341; Park et al., Phys. Rev. C67 (2003) 055206

  – only for the threshold kinematics

  – pion-pole diagrams ignored

  – box-type diagrams neglected

  – renormalization incomplete

- Leading one-loop expressions using TOPT including pion-pole terms for general kinematics (still incomplete, e.g. no 1/m corrections)

  Baroni, Girlanda, Pastore, Schiavilla, Viviani, PRC93 (2016) 015501, Erratum: PRC 93 (2016) 049902

Complete derivation to leading one-loop order using the method of UT

HK, Epelbaum, Meißner, Ann. Phys. 378 (2017) 317

Diagonalization via Okubo

- Decomposition of the Fock space $\mathcal{H}$

Projector operators: $\eta + \lambda = 1$

$$\mathcal{H}_M \overset{\eta}{\leftarrow} \mathcal{H} = \mathcal{H}_M \oplus \mathcal{H}_R \overset{\lambda}{\rightarrow} \mathcal{H}_R$$

Model-space including only pure nucleon states

Remainder-space including states with at least one pion

$$H|\Psi\rangle = (H_0 + H_I)|\Psi\rangle = E|\Psi\rangle \iff \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix}$$

- Block-diagonalization by applying unitary transformation

$$\tilde{H} = U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda H \lambda \end{pmatrix}$$

$$V_{\text{eff}} = \eta(\tilde{H} - H_0)\eta$$

V_{eff} is E - indep. ⟹ important for few-nucleon simulations

Possible parametrization by Okubo '54

$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}$$

With decoupling eq. $\lambda(H - [A, H] - AHA)\eta = 0$

Can be solved perturbatively within ChPT

Epelbaum, Glöckle, Meißner, ´98

Unitary transformations for currents

- Step 1: $\tilde{H} \to \tilde{H}[a,v,s,p] = U^\dagger H[a,v,s,p] U$

Okubo transf. or further strong unitary transf. are not enough to renormalize the currents

- Step 2: additional (time-dependendent) unitary transformations

$$i\frac{\partial}{\partial t}\Psi = H\Psi \quad \longrightarrow \quad i\frac{\partial}{\partial t}U(t)U^\dagger(t)\Psi = U(t)i\frac{\partial}{\partial t}U^\dagger(t)\Psi + \left(i\frac{\partial}{\partial t}U(t)\right)U^\dagger(t)\Psi = HU(t)U^\dagger(t)\Psi$$

$$\Psi' = U^\dagger(t)\Psi \quad \longrightarrow \quad i\frac{\partial}{\partial t}\Psi' = \left[U^\dagger(t)HU(t) - U^\dagger(t)\left(i\frac{\partial}{\partial t}U(t)\right)\right]\Psi'$$

Explicit time-dependence through source terms

$$\tilde{H}[a,v,s,p] \to \underbrace{U^\dagger[a,v]\tilde{H}[a,v,s,p]U[a,v] + \left(i\frac{\partial}{\partial t}U^\dagger[a,v]\right)U[a,v]}_{=:\, H_{\text{eff}}[a,\dot{a},v,\dot{v}]}$$

$$A^b_\mu(\vec{x},t) := \frac{\delta}{\delta a^{\mu,b}(\vec{x},t)} H_{\text{eff}}[a,\dot{a},v,\dot{v}]\Big|_{a=v=0}$$

Due to time-derivatives ($\dot{a}$,$\dot{v}$) the currents depend on energy transfer if transformed into momentum space

Chiral symmetry constraints

Chiral symmetry transformations on the path integral level

Gasser, Leutwyler Ann. Phys. (1984) 142: $v_\mu = \frac{1}{2}(r_\mu + l_\mu)$ and $a_\mu = \frac{1}{2}(r_\mu - l_\mu)$

$$\langle 0_{\rm out}|0_{\rm in}\rangle_{a,v,s,p} = \exp\left(i\,Z[a,v,s,p]\right) = \exp\left(i\,Z[a',v',s',p']\right) = \langle 0_{\rm out}|0_{\rm in}\rangle_{a',v',s',p'}$$

$$\begin{aligned} r_\mu &\to r'_\mu = R\,r_\mu R^\dagger + iR\,\partial_\mu R^\dagger \,, \\ l_\mu &\to l'_\mu = L\,l_\mu L^\dagger + iL\,\partial_\mu L^\dagger \,, \\ s + i\,p &\to s' + i\,p' = R(s + i\,p)L^\dagger \,, \\ s - i\,p &\to s' - i\,p' = L(s - i\,p)R^\dagger \,. \end{aligned}$$

Chiral $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ rotation does not change the generating functional → Ward identities

Chiral symmetry transformations on the Hamiltonian level

- There exists a unitary transformation $U(R,L)$ such that from Schrödinger eq.

$i\dfrac{\partial}{\partial t}\Psi = H_{\rm eff}[a,v,s,p]\Psi$ takes the form $i\dfrac{\partial}{\partial t}U^\dagger(R,L)\Psi = H_{\rm eff}[a',v',s',p']U^\dagger(R,L)\Psi$

Transformed Hamiltonian is unitary equivalent to the untransformed one

$$H_{\rm eff}[a',\dot a',v',\dot v',s',p'] = U^\dagger(R,L)H_{\rm eff}[a,\dot a,v,\dot v,s,p]U(R,L) + \left(i\frac{\partial}{\partial t}U^\dagger(R,L)\right)U(R,L)$$

Continuity equation

Infinitesimally we have $R = 1 + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}_R(x)$ and $L = 1 + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}_L(x)$

Expressed in $\boldsymbol{\epsilon}_V = \frac{1}{2}(\boldsymbol{\epsilon}_R + \boldsymbol{\epsilon}_L)$ and $\boldsymbol{\epsilon}_A = \frac{1}{2}(\boldsymbol{\epsilon}_R - \boldsymbol{\epsilon}_L)$ we have

$$\begin{aligned} \boldsymbol{v}_\mu &\to \boldsymbol{v}'_\mu = \boldsymbol{v}_\mu + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_V \\ \boldsymbol{a}_\mu &\to \boldsymbol{a}'_\mu = \boldsymbol{a}_\mu + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_A \end{aligned} \quad \longrightarrow \quad \begin{aligned} \dot{v}_\mu &\to \dot{v}'_\mu = \partial_\mu \dot{\epsilon}_V + \ldots \\ \dot{a}_\mu &\to \dot{a}'_\mu = \partial_\mu \dot{\epsilon}_A + \ldots \end{aligned}$$

$$H_{\text{eff}}[a', \dot{a}', v', \dot{v}', s', p'] = U^\dagger(R,L) H_{\text{eff}}[a, \dot{a}, v, \dot{v}, s, p] U(R,L) + \left(i \frac{\partial}{\partial t} U^\dagger(R,L)\right) U(R,L)$$

- $H_{\text{eff}}[a', \dot{a}', v', \dot{v}', s', p']$ is a function of $\boldsymbol{\epsilon}_V, \dot{\boldsymbol{\epsilon}}_V, \ddot{\boldsymbol{\epsilon}}_V, \boldsymbol{\epsilon}_A, \dot{\boldsymbol{\epsilon}}_A, \ddot{\boldsymbol{\epsilon}}_A$

$$\longrightarrow \quad U = \exp\left(i \int d^3x \left[\boldsymbol{R}_0^v(\vec{x}) \cdot \boldsymbol{\epsilon}_V(\vec{x},t) + \boldsymbol{R}_1^v(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_V(\vec{x},t) + \boldsymbol{R}_0^a(\vec{x}) \cdot \boldsymbol{\epsilon}_A(\vec{x},t) + \boldsymbol{R}_1^a(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_A(\vec{x},t)\right]\right)$$

Expanding both sides in $\vec{\epsilon}_V, \vec{\epsilon}_A$, comparing the coefficients and transforming to momentum space we get the continuity equation

$$\mathcal{C}(\vec{k}, k_0) \quad = \quad \left[H_{\text{strong}}, \boldsymbol{A}_0(\vec{k}, k_0)\right] - \vec{k} \cdot \vec{\boldsymbol{A}}(\vec{k}, k_0) + i\, m_q \boldsymbol{P}(\vec{k}, k_0)$$

$$\mathcal{C}(\vec{k}, 0) + \underbrace{\left[H_{\text{strong}}, \frac{\partial}{\partial k_0} \mathcal{C}(\vec{k}, k_0)\right]}_{\text{new term}} = 0$$

Unitary ambiguities

34 different unitary transformations are possible at the order Q

$$U_i(a) = \exp\left(S_i^{\text{ax}} - h.c.\right)$$

$$S_1^{ax} = \alpha_1^{ax} \eta A_{2,0}^{(0)} \eta H_{2,1}^{(1)} \lambda^1 \frac{1}{E_\pi^3} H_{2,1}^{(1)} \eta,$$

$$S_2^{ax} = \alpha_2^{ax} \eta H_{2,1}^{(1)} \lambda^1 \frac{1}{E_\pi^2} A_{2,0}^{(0)} \lambda^1 \frac{1}{E_\pi} H_{2,1}^{(1)} \eta$$

$$\dots$$

Vertices without axial source are denoted by $H_{n,p}^{(\kappa)}$

Vertices with one axial source are denoted by $A_{n,p}^{(\kappa)}$

n — number of nucleons

p — number of pions

a — number of axial sources

$\kappa = d + \frac{3}{2} n + p + a - 4$ ← inverse mass dimension

High unitary ambiguity is related to appearance of the axial-vector-one-pion interaction $A_{0,1}^{(-1)}$ (30 out of 34 transformations depend on it)

Reasonable constraints come from

- Perturbative renormalizability of the current

$$l_i = l_i^r(\mu) + \gamma_i \lambda =: \frac{1}{16\pi^2} \bar{l}_i + \gamma_i \lambda + \frac{\gamma_i}{16\pi^2} \ln\left(\frac{M_\pi}{\mu}\right),$$

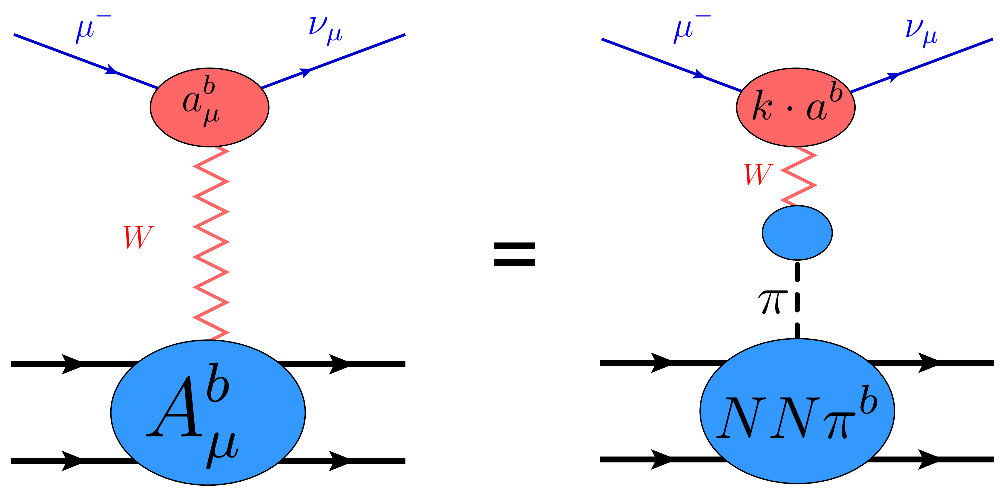
$$d_i = d_i^r(\mu) + \frac{\beta_i}{F^2} \lambda =: \bar{d}_i + \frac{\beta_i}{F^2} \lambda + \frac{\beta_i}{16\pi^2 F^2} \ln\left(\frac{M_\pi}{\mu}\right)$$

$$\begin{aligned} \gamma_3 &= -\frac{1}{2}, \\ \gamma_4 &= 2, \\ \beta_2 = -2\beta_5 = \frac{1}{2}\beta_6 &= -\frac{1}{12}(1 + 5g_A^2), \\ \beta_{15} = \beta_{18} = \beta_{22} = \beta_{23} &= 0, \\ \beta_{16} &= \frac{1}{2} g_A + g_A^3. \end{aligned}$$

After renormalizing LECs l_i from $\mathcal{L}_\pi^{(4)}$ and d_i from $\mathcal{L}_{\pi N}^{(\bar{3})}$ and using well known β- and γ-functions (*Gasser et al. Eur. Phys. J. C26 (2002), 13*) we require the current to be finite

Matching to nuclear forces

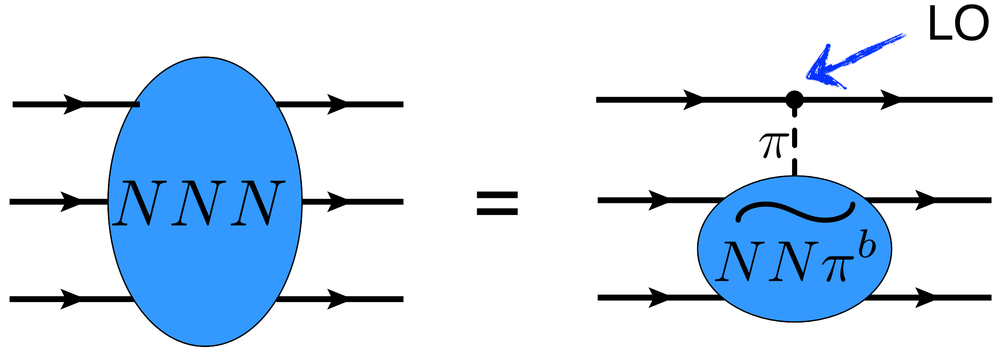
Dominance of the pion production operator at the pion-pole (axial-vector current)



+ Non-pole contributions

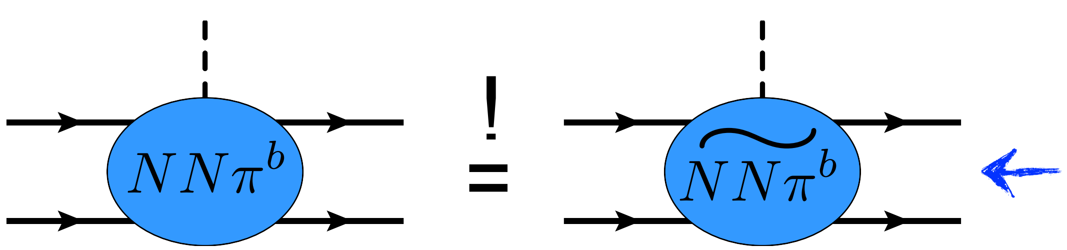
Dominance of the pion production operator at the pion-pole (three-nucleon force)

LO pion-nucleon vertex



+ Non-pole contributions

Consistent regularization of nuclear forces and currents calls for matching requirement between pion-production operators in different processes



Matching requirement is fulfilled only for particular choice of unitary phases

After renormalizability and matching requirement there are no further unitary ambiguities!

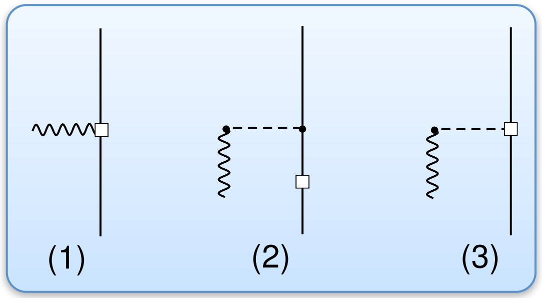
Single nucleon current up to order Q

Up to 1/m - corrections one can parametrize axial-vector current by form factors

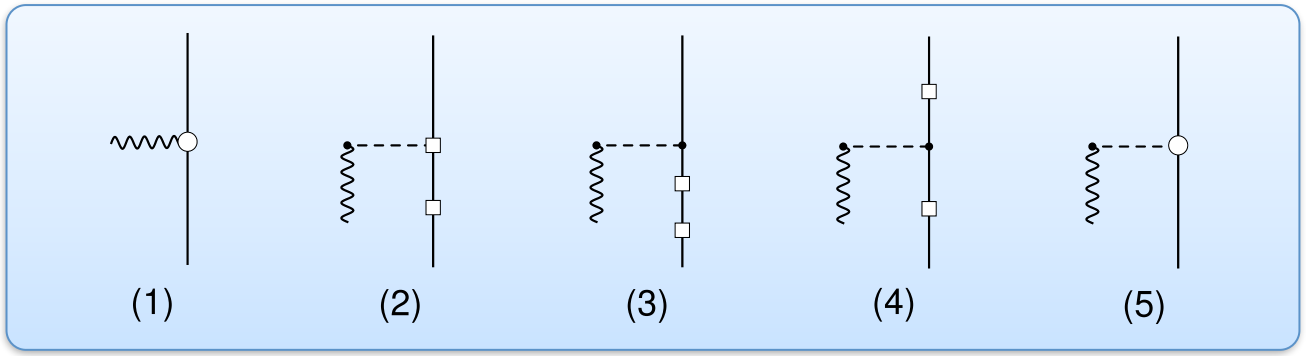
$$
\begin{aligned}
A^{0,a}_{1\mathrm{N}} &= -\frac{G_A(-k^2)}{2m}\tau_i^a \vec{k}_i \cdot \vec{\sigma}_i + \frac{G_P(-k^2)}{8m^2}\tau_i^a k_0\, \vec{k}\cdot\vec{\sigma}_i\,, \\
\vec{A}^{a}_{1\mathrm{N}} &= -\frac{G_A(-k^2)}{2}\tau_i^a \vec{\sigma}_i + \frac{G_P(-k^2)}{8m^2}\tau_i^a \vec{k}\,\vec{k}\cdot\vec{\sigma}_i + \vec{A}^{a\,(Q)}_{1\mathrm{N}:\,1/m,\mathrm{UT}'} + \vec{A}^{a\,(Q)}_{1\mathrm{N}:\,1/\mathrm{m}^2}
\end{aligned}
$$

- Axial and pseudoscalar formfactors are known up to two-loop order: *Kaiser PRC67 (2003) 027002*

$$
\vec{A}^{a\,(Q)}_{1\mathrm{N}:\,1/m,\mathrm{UT}'} = -\frac{g_A k_0}{8m}\frac{\vec{k}}{k^2+M_\pi^2}\tau_i^a\left(2(1+2\,\bar{\beta}_9)\vec{\sigma}_i\cdot\vec{k}_i - (1+2\,\bar{\beta}_8)\vec{k}\cdot\vec{\sigma}_i\frac{p_i'^2-p_i^2}{k^2+M_\pi^2}\right)
$$

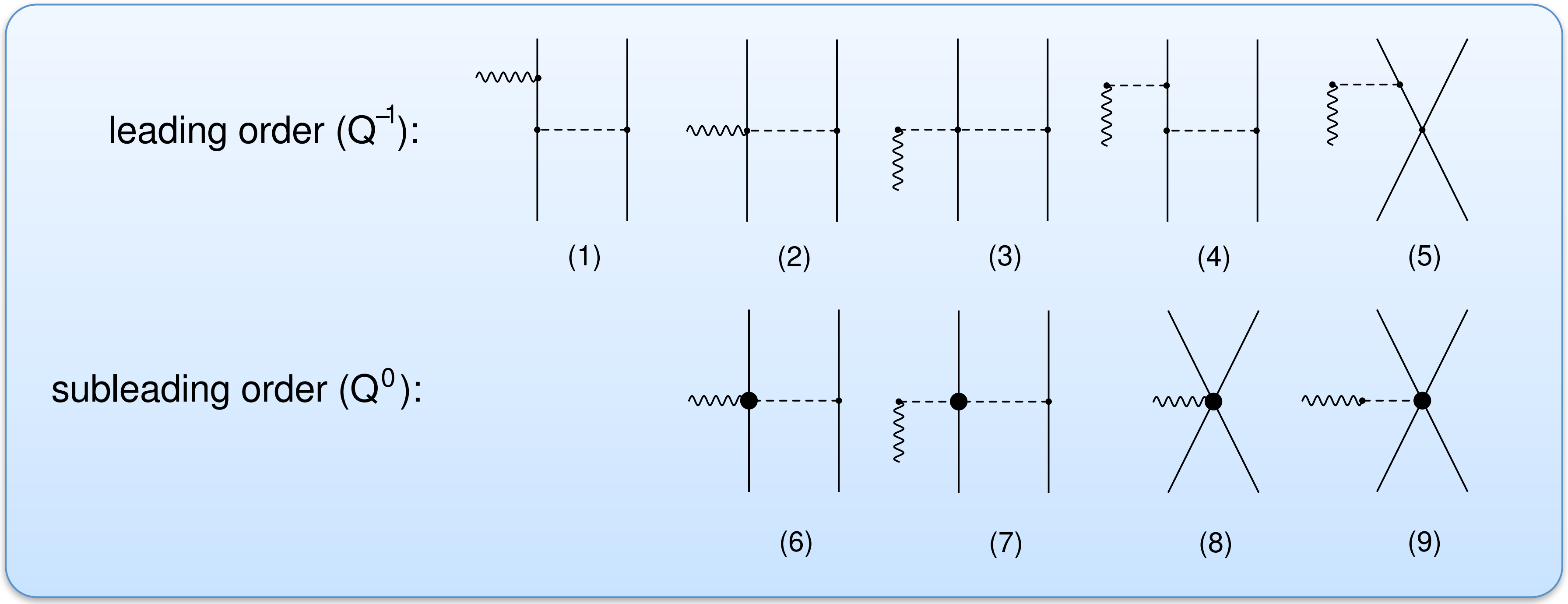


$k_0/m \sim Q^4/\Lambda_b^4$ due to adopted counting for 1/m-corrections



$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{1\mathrm{N}:\,1/\mathrm{m}^2} &= \frac{g_A}{16m^2}\tau_i^a\bigg(\vec{k}\,\vec{k}\cdot\vec{\sigma}_i(1-2\bar{\beta}_8)\frac{(p_i'^2-p_i^2)^2}{(k^2+M_\pi^2)^2} - 2\vec{k}\frac{(p_i'^2+p_i^2)\vec{k}\cdot\vec{\sigma}_i - 2\bar{\beta}_9(p_i'^2-p_i^2)\vec{k}_i\cdot\vec{\sigma}_i}{k^2+M_\pi^2} \\
&+ 2\,i\,[\vec{k}\times\vec{k}_i] + \vec{k}\,\vec{k}\cdot\vec{\sigma}_i - 4\,\vec{k}_i\,\vec{k}_i\cdot\vec{\sigma}_i + \vec{\sigma}_i\Big(2(p_i'^2+p_i^2)-k^2\Big)\bigg)\,.
\end{aligned}
$$

NN current at order Q^{-1} & Q^0

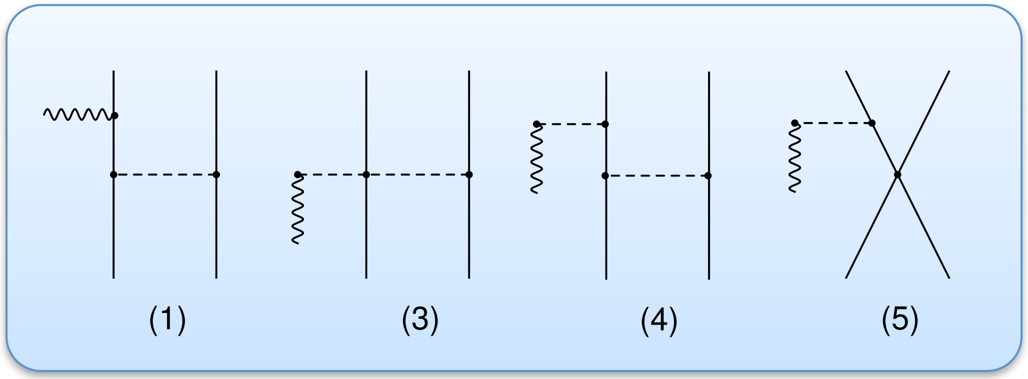


Well known results for axial NN current at Q^{-1} and Q^0 - order

Ando et al. PLB533 (2002) 25; Hoferichter et al. PLB746 (2015) 410

$$
\begin{aligned}
A^{0,a\,(Q^{-1})}_{2\mathrm{N}:\,1\pi} &= -\frac{i g_A \vec{q}_1\cdot\vec{\sigma}_1[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a}{4F_\pi^2\,(q_1^2+M_\pi^2)} \;+\; 1\leftrightarrow 2\,,\\
\vec{A}^{a\,(Q^{-1})}_{2\mathrm{N}:\,1\pi} &= 0,\\
\vec{A}^{a\,(Q^{0})}_{2\mathrm{N}:\,1\pi} &= \frac{g_A}{2F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\bigg\{\tau_1^a\bigg[-4c_1M_\pi^2\frac{\vec{k}}{k^2+M_\pi^2}+2c_3\bigg(\vec{q}_1-\frac{\vec{k}\,\vec{k}\cdot\vec{q}_1}{k^2+M_\pi^2}\bigg)\bigg]+c_4[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\bigg(\vec{q}_1\times\vec{\sigma}_2-\frac{\vec{k}\,\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2}{k^2+M_\pi^2}\bigg)\\
&\quad-\frac{\kappa_v}{4m}[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\vec{k}\times\vec{\sigma}_2\bigg\}\;+\;1\leftrightarrow 2\,,\\
\vec{A}^{a\,(Q^{0})}_{2\mathrm{N}:\,\mathrm{cont}} &= -\frac{1}{4}D\,\tau_1^a\bigg(\vec{\sigma}_1-\frac{\vec{k}\,\vec{\sigma}_1\cdot\vec{k}}{k^2+M_\pi^2}\bigg)\;+\;1\leftrightarrow 2\,,
\end{aligned}
$$

NN current at order Q



Tree-level diagrams contribute to energy-transfer dependent contributions

$$A^{0,a\,(Q)}_{2\mathrm{N}:\,1\pi,\mathrm{UT}'} = 0\,,$$

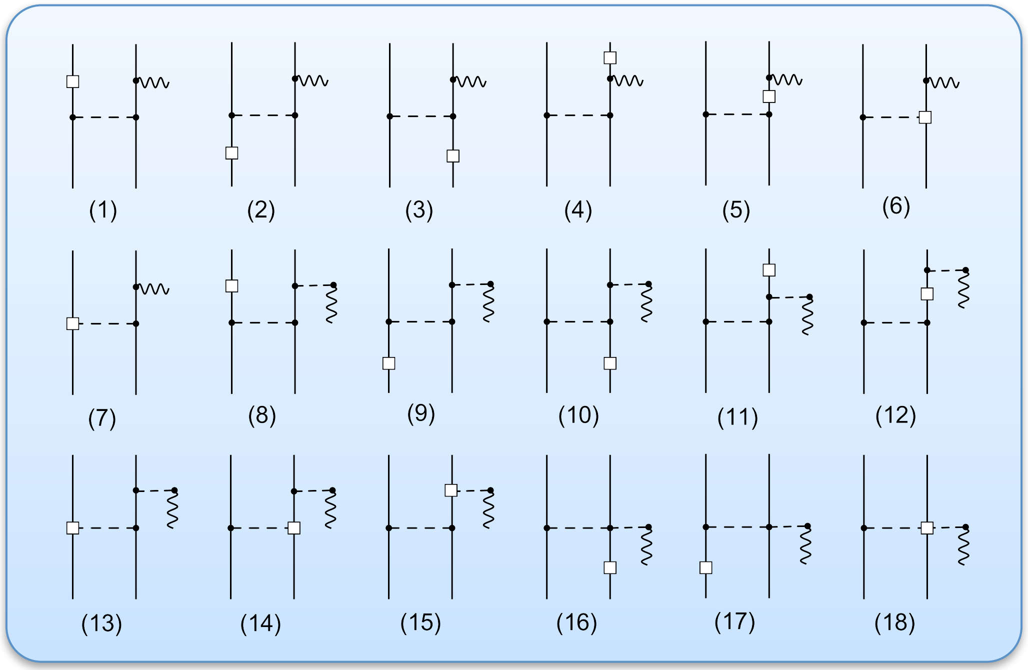
$$\vec{A}^{a\,(Q)}_{2\mathrm{N}:\,1\pi,\mathrm{UT}'} = -i\frac{g_A}{8F_\pi^2}\frac{k_0\,\vec{k}\,\vec{q}_1\cdot\vec{\sigma}_1}{(k^2+M_\pi^2)(q_1^2+M_\pi^2)}\left([\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\left(1-\frac{2g_A^2\vec{k}\cdot\vec{q}_1}{k^2+M_\pi^2}\right)-\frac{2g_A^2\tau_1^a\vec{k}\cdot[\vec{q}_1\times\vec{\sigma}_2]}{k^2+M_\pi^2}\right)+\;1\leftrightarrow 2\,.$$

$$A^{0,a\,(Q)}_{2\mathrm{N}:\,\mathrm{cont},\,\mathrm{UT}'} = 0,$$

$$\vec{A}^{a\,(Q)}_{2\mathrm{N}:\,\mathrm{cont},\,\mathrm{UT}'} = -i\,k_0\vec{k}\frac{g_A C_T\vec{k}\cdot\vec{\sigma}_1[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a}{(k^2+M_\pi^2)^2}\;+\;1\leftrightarrow 2\,.$$

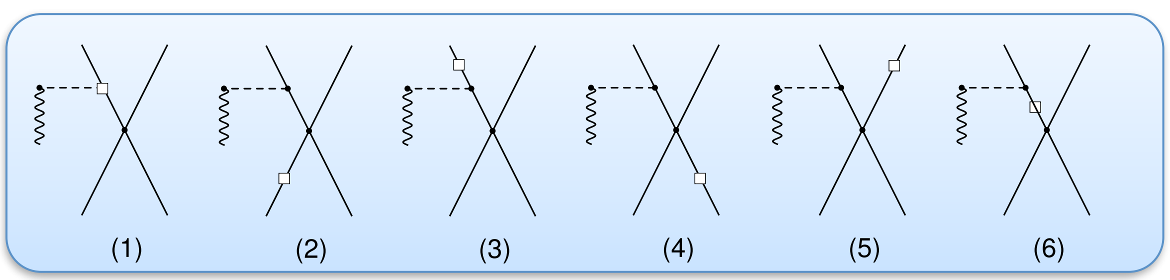
Off-shell effects proportional to energy transfer are important for frame-independent investigations and also for checking the continuity equation

1/m-corrections to axial NN current



$$
\begin{aligned}
\vec{B}_1 &= g_A^2 \vec{q}_1\cdot\vec{\sigma}_1[-2(1+2\bar{\beta}_8)\vec{q}_1\,\vec{k}_1\cdot\vec{q}_1-(1-2\bar{\beta}_8)(2\vec{q}_1\,\vec{k}_2\cdot\vec{q}_1-i\,\vec{q}_1\times\vec{\sigma}_2\,\vec{k}\cdot\vec{q}_1],\\
\vec{B}_2 &= (1-2\bar{\beta}_8)g_A^2\vec{k}\,\vec{k}\cdot\vec{q}_1\vec{q}_1\cdot\vec{\sigma}_1[2\vec{k}\cdot\vec{k}_2-i\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2],\\
\vec{B}_3 &= 2\vec{k}\Big[-g_A^2((1+2\bar{\beta}_9)\vec{k}\cdot\vec{q}_1\vec{k}_1\cdot\vec{\sigma}_1+(1-2\bar{\beta}_9)\vec{q}_1\cdot\vec{\sigma}_1(\vec{k}\cdot\vec{k}_2+\vec{k}_2\cdot\vec{q}_1))\\
&+ \vec{q}_1\cdot\vec{\sigma}_1(\vec{k}\cdot\vec{k}_2+i\,\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2-\vec{k}_1\cdot\vec{q}_1+\vec{k}_2\cdot\vec{q}_1)\Big],\\
\vec{B}_4 &= g_A^2[2(1+2\bar{\beta}_9)\vec{q}_1\vec{k}_1\cdot\vec{\sigma}_1+(1-2\bar{\beta}_9)\vec{q}_1\cdot\vec{\sigma}_1(2\vec{k}_2-i\vec{k}\times\vec{\sigma}_2)]-2\vec{q}_1\cdot\vec{\sigma}_1(i\vec{q}_1\times\vec{\sigma}_2-i\vec{k}\times\vec{\sigma}_2+2\vec{k}_2),\\
\vec{B}_5 &= g_A^2\vec{q}_1\cdot\vec{\sigma}_1\Big[(1-2\bar{\beta}_8)(\vec{q}_1\,\vec{k}\cdot\vec{q}_1-2i\,\vec{q}_1\times\vec{\sigma}_2\vec{k}_2\cdot\vec{q}_1)-2i(1+2\bar{\beta}_8)\vec{q}_1\times\vec{\sigma}_2\,\vec{k}_1\cdot\vec{q}_1\Big],\\
\vec{B}_6 &= -(1-2\bar{\beta}_8)g_A^2\vec{k}\,\vec{q}_1\cdot\vec{\sigma}_1[(\vec{k}\cdot\vec{q}_1)^2-2i\,\vec{k}\cdot\vec{k}_2\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2],\\
\vec{B}_7 &= g_A^2\vec{k}\Big[(1-2\bar{\beta}_9)\vec{q}_1\cdot\vec{\sigma}_1(-2i(\vec{k}\cdot\vec{k}_2\times\vec{\sigma}_2+\vec{k}_2\cdot\vec{q}_1\times\vec{\sigma}_2)+k^2+q_1^2)-2i(1+2\bar{\beta}_9)\vec{k}_1\cdot\vec{\sigma}_1\vec{k}\cdot\vec{q}_1\times\vec{\sigma}_2\Big],\\
\vec{B}_8 &= -g_A^2[(1-2\bar{\beta}_9)\vec{q}_1\cdot\vec{\sigma}_1(\vec{k}-2i\,\vec{k}_2\times\vec{\sigma}_2)-2i(1+2\bar{\beta}_9)\vec{q}_1\times\vec{\sigma}_2\,\vec{k}_1\cdot\vec{\sigma}_1].
\end{aligned}
$$

$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{2\mathrm{N}:\,1\pi,\,1/m} &= \frac{g_A}{16F_\pi^2 m}\bigg\{i[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\bigg[\frac{1}{(q_1^2+M_\pi^2)^2}\bigg(\vec{B}_1-\frac{\vec{k}\,\vec{k}\cdot\vec{B}_1}{k^2+M_\pi^2}\bigg)+\frac{1}{q_1^2+M_\pi^2}\bigg(\frac{\vec{B}_2}{(k^2+M_\pi^2)^2}+\frac{\vec{B}_3}{k^2+M_\pi^2}+\vec{B}_4\bigg)\bigg]\\
&+ \tau_1^a\bigg[\frac{1}{(q_1^2+M_\pi^2)^2}\bigg(\vec{B}_5-\frac{\vec{k}\,\vec{k}\cdot\vec{B}_5}{k^2+M_\pi^2}\bigg)+\frac{1}{q_1^2+M_\pi^2}\bigg(\frac{\vec{B}_6}{(k^2+M_\pi^2)^2}+\frac{\vec{B}_7}{k^2+M_\pi^2}+\vec{B}_8\bigg)\bigg]\bigg\}+1\;\leftrightarrow\;2
\end{aligned}
$$

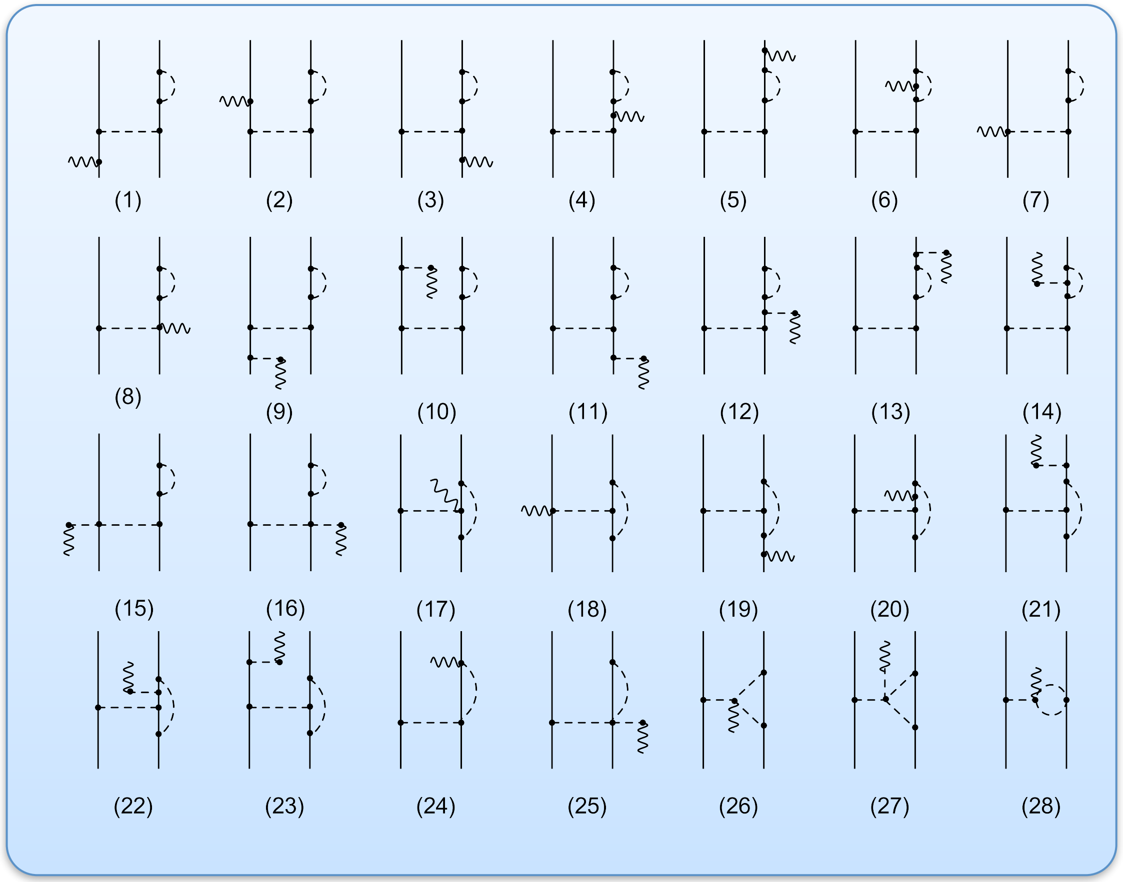


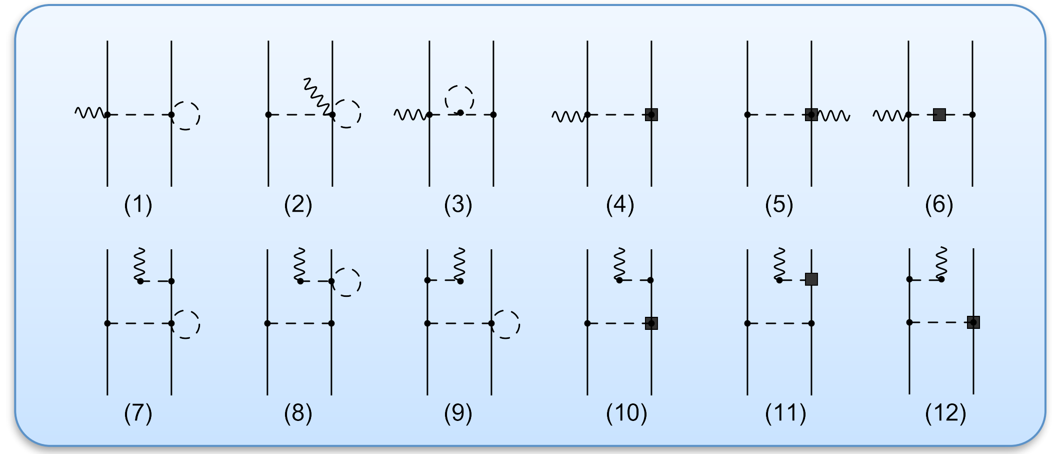
$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{2\mathrm{N}:\,\mathrm{cont},\,1/m} &= -\frac{g_A}{4m}\frac{\vec{k}}{k^2+M_\pi^2}\tau_1^a\bigg\{(1-2\bar{\beta}_9)\Big(C_S\vec{q}_2\cdot\vec{\sigma}_1+C_T(\vec{q}_2\cdot\vec{\sigma}_2+2i\,\vec{k}_1\cdot\vec{\sigma}_1\times\vec{\sigma}_2)\Big)\\
&- \frac{1-2\bar{\beta}_8}{k^2+M_\pi^2}\Big(C_S\vec{k}\cdot\vec{q}_2\vec{k}\cdot\vec{\sigma}_1+C_T(\vec{k}\cdot\vec{q}_2\vec{k}\cdot\vec{\sigma}_2+2i\,\vec{k}\cdot\vec{k}_1\vec{k}\cdot\vec{\sigma}_1\times\vec{\sigma}_2)\Big)\bigg\}\;+\;1\leftrightarrow 2\,.
\end{aligned}
$$

No relativistic corrections to the axial NN charge

NN current at order Q

One-pion exchange contributions match to $2\pi-$ exchange 3NF at N³LO





$$
\begin{aligned}
h_1(q_2) &= -\frac{g_A^6 M_\pi}{128\pi F_\pi^6}, \\
h_2(q_2) &= \frac{g_A^4 M_\pi}{256\pi F_\pi^6} + \frac{g_A^4 A(q_2)\left(4M_\pi^2+q_2^2\right)}{256\pi F_\pi^6}, \\
h_3(q_2) &= \frac{g_A^4\left(g_A^2+1\right)M_\pi}{128\pi F_\pi^6} + \frac{g_A^4 A(q_2)\left(2M_\pi^2+q_2^2\right)}{128\pi F_\pi^6}, \\
h_4(q_2) &= \frac{g_A^4}{256\pi F_\pi^6}\left(A(q_2)\left(2M_\pi^4+5M_\pi^2 q_2^2+2q_2^4\right)+\left(4g_A^2+1\right)M_\pi^3+2\left(g_A^2+1\right)M_\pi q_2^2\right), \\
h_5(q_2) &= -\frac{g_A^4}{256\pi F_\pi^6}\left(A(q_2)\left(4M_\pi^2+q_2^2\right)+\left(2g_A^2+1\right)M_\pi\right), \\
h_6(q_2) &= \frac{g_A^2\left(3\left(64+128g_A^2\right)M_\pi^2+8\left(19g_A^2+5\right)q_2^2\right)}{36864\pi^2 F_\pi^6} - \frac{g_A^2}{768\pi^2 F_\pi^6}L(q_2)\left(\left(8g_A^2+4\right)M_\pi^2+\left(5g_A^2+1\right)q_2^2\right) \\
&+ \frac{\bar{d}_{18} g_A M_\pi^2}{8F_\pi^4} - \frac{g_A^2(2\bar{d}_2+\bar{d}_6)\left(M_\pi^2+q_2^2\right)}{16F_\pi^4} - \frac{\bar{d}_5 g_A^2 M_\pi^2}{2F_\pi^4}, \\
h_7(q_2) &= \frac{g_A^2(2\bar{d}_2-\bar{d}_6)}{16F_\pi^4}, \\
h_8(q_2) &= -\frac{g_A^2(\bar{d}_{15}-2\bar{d}_{23})}{8F_\pi^4}.
\end{aligned}
$$

- h_i are related to TPE 3NF functions *A* & *B*

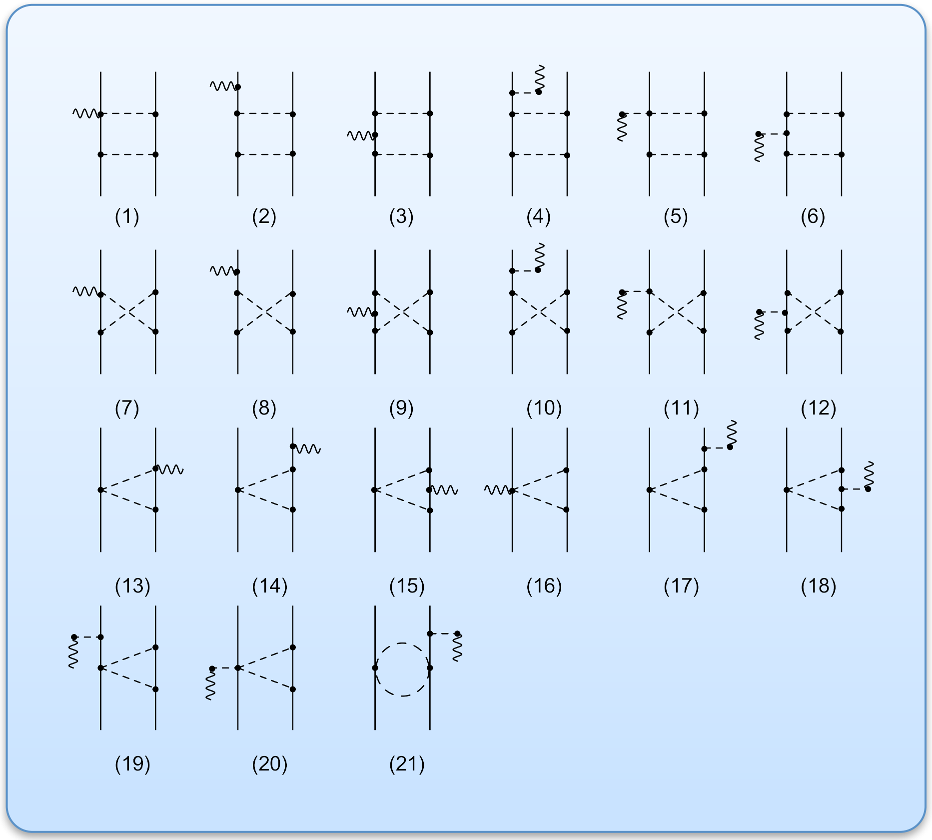
$$
h_4(q_2) = \mathcal{A}^{(4)}(q_2), \quad h_5(q_2) = \mathcal{B}^{(4)}(q_2)
$$

$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{2\text{N}:\,1\pi} &= \frac{4F_\pi^2}{g_A}\frac{\vec{q}_1\cdot\vec{\sigma}_1}{q_1^2+M_\pi^2}\Big\{[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\Big([\vec{q}_1\times\vec{\sigma}_2]\,h_1(q_2)+[\vec{q}_2\times\vec{\sigma}_2]\,h_2(q_2)\Big)+\boldsymbol{\tau}_1^a\big(\vec{q}_1-\vec{q}_2\big)h_3(q_2)\Big\} \\
&+ \frac{4F_\pi^2}{g_A}\frac{\vec{q}_1\cdot\vec{\sigma}_1\,\vec{k}}{(k^2+M_\pi^2)(q_1^2+M_\pi^2)}\Big\{\boldsymbol{\tau}_1^a h_4(q_2)+[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\vec{q}_1\cdot[\vec{q}_2\times\vec{\sigma}_2]h_5(q_2)\Big\} \;+\; 1\leftrightarrow 2,
\end{aligned}
$$

$$
A^{0,a\,(Q)}_{2\text{N}:\,1\pi} = i\frac{4F_\pi^2}{g_A}\frac{\vec{q}_1\cdot\vec{\sigma}_1}{q_1^2+M_\pi^2}\Big\{[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\left(h_6(q_2)+k^2 h_7(q_2)\right)+\tau_1^a\,\vec{q}_1\cdot[\vec{q}_2\times\vec{\sigma}_2]\,h_8(q_2)\Big\} \;+\; 1\leftrightarrow 2\,,
$$

NN current at order Q

Two-pion exchange contributions match to $2\pi - 1\pi$ 3NF at N³LO



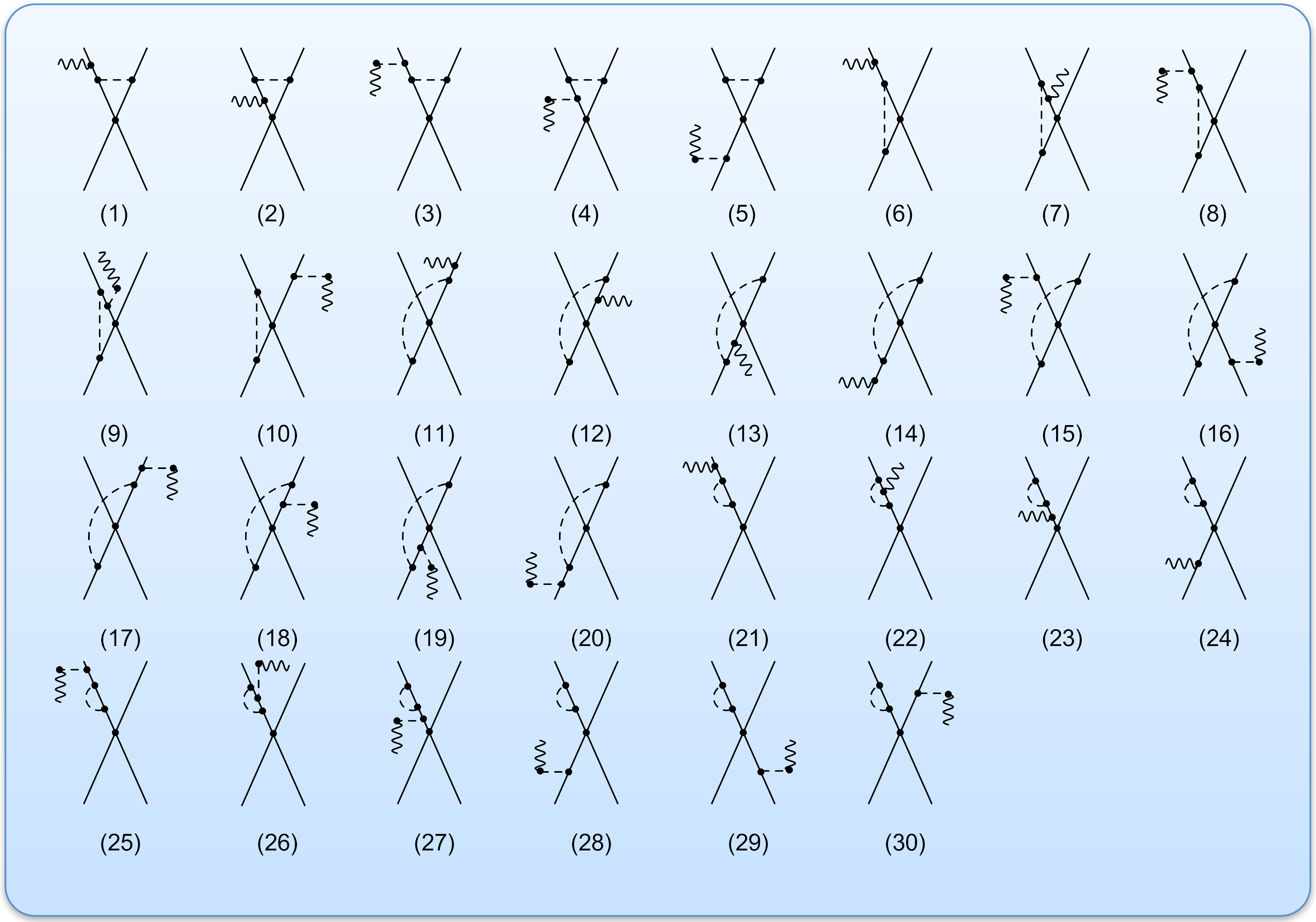
$$
\begin{aligned}
g_1(q_1) &= \frac{g_A^4 A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(g_A^2+1\right)q_1^2\right)}{256\pi F_\pi^6 q_1^2} - \frac{g_A^4 M_\pi\left(\left(8g_A^2-4\right)M_\pi^2+\left(3g_A^2-1\right)q_1^2\right)}{256\pi F_\pi^6 q_1^2\left(4M_\pi^2+q_1^2\right)} \\
g_2(q_1) &= \frac{g_A^4 A(q_1)\left(2M_\pi^2+q_1^2\right)}{128\pi F_\pi^6} + \frac{g_A^4 M_\pi}{128\pi F_\pi^6}, \\
g_3(q_1) &= -\frac{g_A^4 A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(3g_A^2-1\right)q_1^2\right)}{256\pi F_\pi^6} - \frac{\left(3g_A^2-1\right)g_A^4 M_\pi}{256\pi F_\pi^6}, \\
g_4(q_1) &= -\frac{g_A^6 A(q_1)}{128\pi F_\pi^6}, \\
g_5(q_1) &= -q_1^2\, g_4(q_1), \\
g_6(q_1) &= g_8(q_1) = g_{10}(q_1) = g_{12}(q_1) = 0, \\
g_7(q_1) &= \frac{g_A^4 A(q_1)\left(2M_\pi^2+q_1^2\right)}{128\pi F_\pi^6} + \frac{\left(2g_A^2+1\right)g_A^4 M_\pi}{128\pi F_\pi^6}, \\
g_9(q_1) &= \frac{g_A^6 M_\pi}{64\pi F_\pi^6}, \\
g_{11}(q_1) &= -\frac{g_A^4 A(q_1)\left(4M_\pi^2+q_1^2\right)}{512\pi F_\pi^6} - \frac{g_A^4 M_\pi}{512\pi F_\pi^6}, \\
g_{13}(q_1) &= -\frac{g_A^6 A(q_1)}{128\pi F_\pi^6}, \\
g_{14}(q_1) &= \frac{g_A^4 A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(g_A^2+1\right)q_1^2\right)}{256\pi F_\pi^6 q_1^2} + \frac{g_A^4 M_\pi\left(\left(4-8g_A^2\right)M_\pi^2+\left(1-3g_A^2\right)q_1^2\right)}{256\pi F_\pi^6 q_1^2\left(4M_\pi^2+q_1^2\right)} \\
g_{15}(q_1) &= \frac{g_A^4 A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(3g_A^2-1\right)q_1^2\right)}{256\pi F_\pi^6} + \frac{\left(3g_A^2-1\right)g_A^4 M_\pi}{256\pi F_\pi^6}, \\
g_{16}(q_1) &= \frac{g_A^4 A(q_1)\left(2M_\pi^2+q_1^2\right)}{64\pi F_\pi^6} + \frac{g_A^4 M_\pi}{64\pi F_\pi^6}, \\
g_{17}(q_1) &= -\frac{g_A^6 q_1^2 A(q_1)}{128\pi F_\pi^6}, \\
g_{18}(q_1) &= \frac{g_A^2 L(q_1)\left(\left(4-8g_A^2\right)M_\pi^2+\left(1-3g_A^2\right)q_1^2\right)}{128\pi^2 F_\pi^6\left(4M_\pi^2+q_1^2\right)}, \\
g_{19}(q_1) &= \frac{g_A^4 L(q_1)}{32\pi^2 F_\pi^6}.
\end{aligned}
$$

$$
\begin{aligned}
\vec{A}^{a\,(Q)}_{2\mathrm{N}:\,2\pi} &= \frac{2F_\pi^2}{g_A}\frac{\vec{k}}{k^2+M_\pi^2}\bigg\{\tau_1^a\Big(-\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_1\cdot\vec{k}\,g_1(q_1)+\vec{q}_1\cdot\vec{\sigma}_2\,g_2(q_1)-\vec{k}\cdot\vec{\sigma}_2\,g_3(q_1)\Big) + \tau_2^a\Big(-\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_1\cdot\vec{k}\,g_4(q_1) \\
&- \vec{k}\cdot\vec{\sigma}_1\,g_5(q_1)-\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_1\cdot\vec{k}\,g_6(q_1)+\vec{q}_1\cdot\vec{\sigma}_2\,g_7(q_1)+\vec{k}\cdot\vec{\sigma}_2\,\vec{q}_1\cdot\vec{k}\,g_8(q_1)-\vec{k}\cdot\vec{\sigma}_2\,g_9(q_1)\Big) \\
&+ [\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\Big(-\vec{q}_1\cdot[\vec{\sigma}_1\times\vec{\sigma}_2]\,\vec{q}_1\cdot\vec{k}\,g_{10}(q_1)+\vec{q}_1\cdot[\vec{\sigma}_1\times\vec{\sigma}_2]\,g_{11}(q_1)-\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_1\cdot[\vec{q}_2\times\vec{\sigma}_1]\,g_{12}(q_1)\Big)\bigg\} \\
&+ \frac{2F_\pi^2}{g_A}\bigg\{\vec{q}_1\Big(\tau_2^a\,\vec{q}_1\cdot\vec{\sigma}_1\,g_{13}(q_1)+\tau_1^a\,\vec{q}_1\cdot\vec{\sigma}_2\,g_{14}(q_1)\Big)-\tau_1^a\,\vec{\sigma}_2\,g_{15}(q_1)-\tau_2^a\,\vec{\sigma}_2\,g_{16}(q_1)-\tau_2^a\,\vec{\sigma}_1\,g_{17}(q_1)\bigg\} + 1\leftrightarrow 2\,,
\end{aligned}
$$

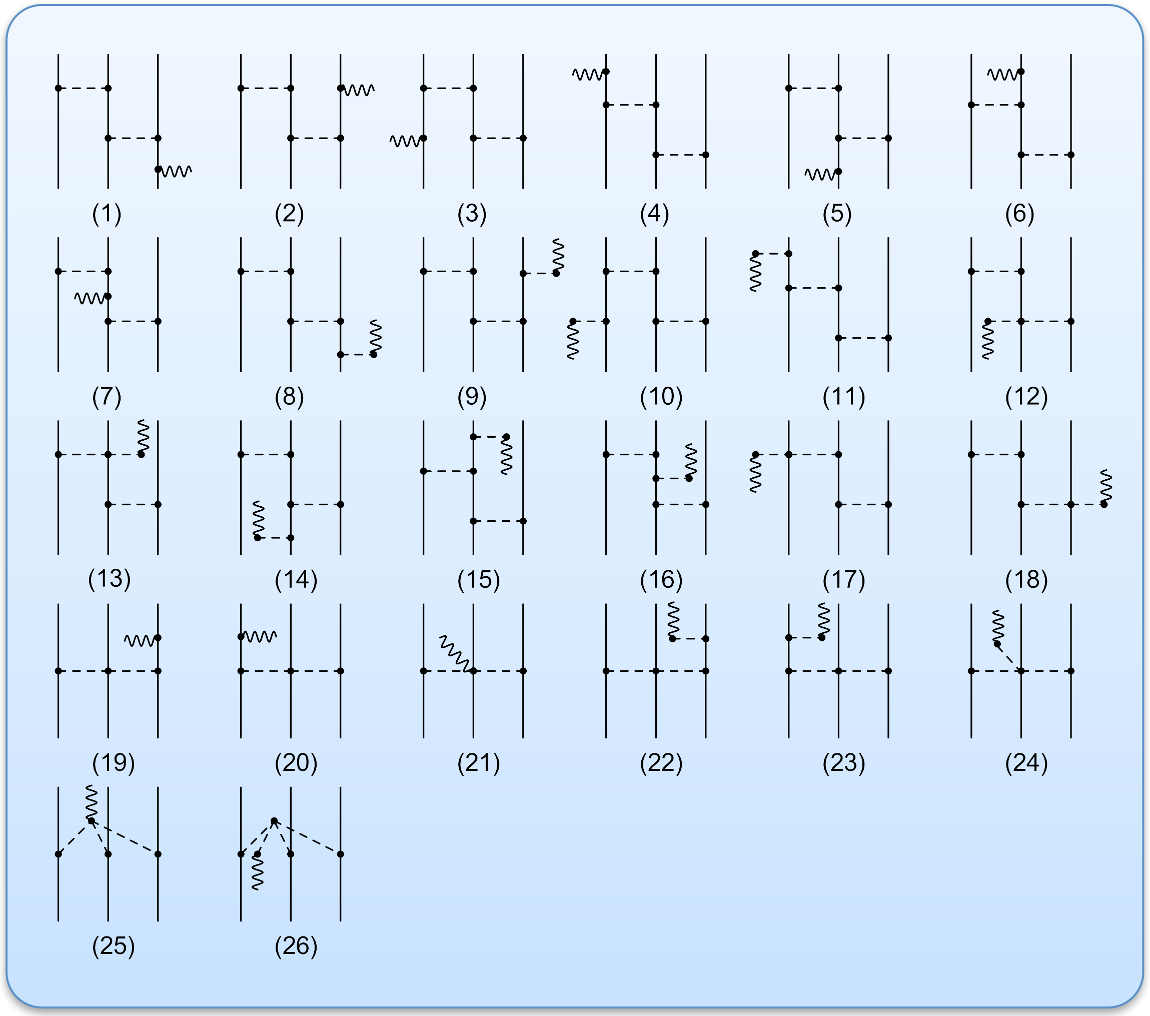
$$
A^{0,a\,(Q)}_{2\mathrm{N}:\,2\pi} = i\frac{2F_\pi^2}{g_A}\bigg\{[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a\vec{q}_1\cdot\vec{\sigma}_2\,g_{18}(q_1)+\tau_2^a\vec{q}_1\cdot[\vec{\sigma}_1\times\vec{\sigma}_2]g_{19}(q_1)\bigg\} + 1\leftrightarrow 2\,,
$$

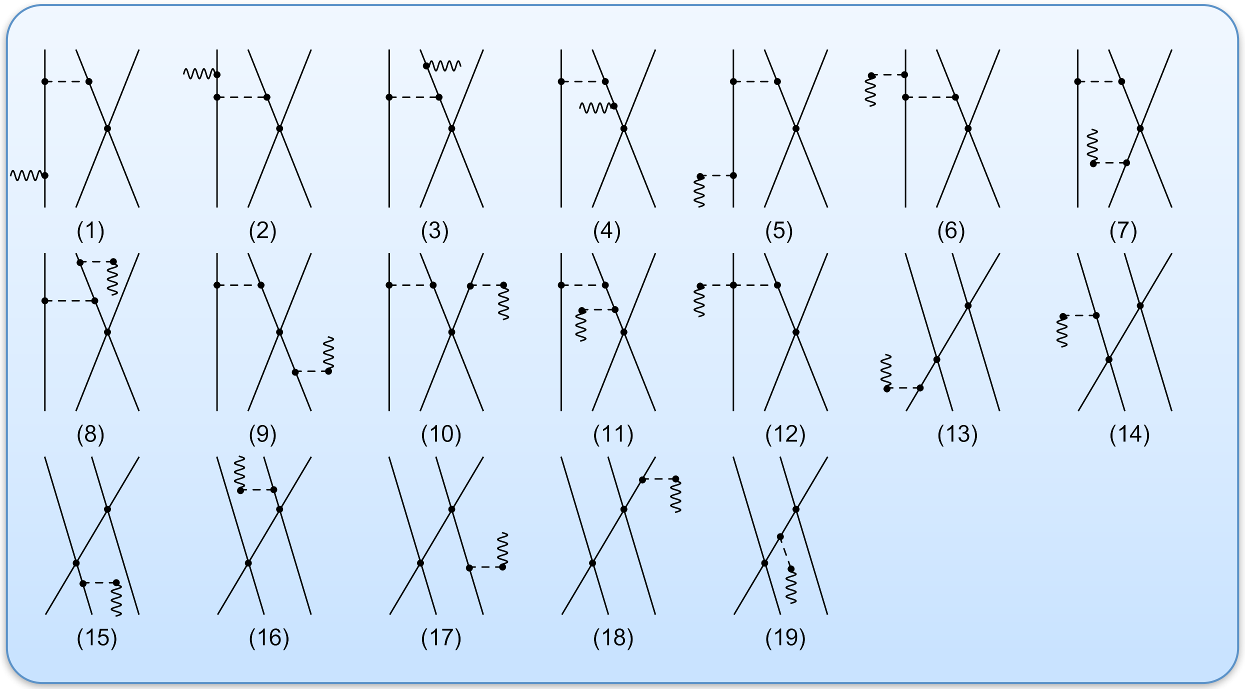
NN current at order Q

Vanishing short-range contributions for the current, after antisymmetrization



Three-nucleon current





- First complete calculation of axial 3N currents
  - Lengthy expression for current: *HK, Epelbaum, Meißner, Ann. Phys. (2017) in press; arXiv:1610.03569*
  - Vanishing charge operator
- Pion-pole terms match to 4NF

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-3})	$\vec{A}^a_{1\text{N}:\,\text{static}}$,	—	—
NLO (Q^{-1})	$\vec{A}^a_{1\text{N}:\,\text{static}}$,	—	—
N^{2}LO (Q^0)	—	$\vec{A}^a_{2\text{N}:\,1\pi}$, ✓ $+\ \vec{A}^a_{2\text{N}:\,\text{cont}}$, ✓	—
N^{3}LO (Q)	$\vec{A}^a_{1\text{N}:\,\text{static}}$, $+\ \vec{A}^a_{1\text{N}:\,1/m,\text{UT}'}$, $+\ \vec{A}^a_{1\text{N}:\,1/\text{m}^2}$,	$\vec{A}^a_{2\text{N}:\,1\pi}$, $+\ \vec{A}^a_{2\text{N}:\,1\pi,\text{UT}'}$, ✗ $+\ \vec{A}^a_{2\text{N}:\,1\pi,\,1/m}$, ✗ $+\ \vec{A}^a_{2\text{N}:\,2\pi}$, $+\ \vec{A}^a_{2\text{N}:\,\text{cont},\,\text{UT}'}$, ✗ $+\ \vec{A}^a_{2\text{N}:\,\text{cont},\,1/\text{m}}$, ✗	$\vec{A}^a_{3\text{N}:\,\pi}$, $+\ \vec{A}^a_{3\text{N}:\,\text{cont}}$, ✗ Baroni et al. considered only irr. diagrams of 3N current

✗ terms not discussed by Baroni et al. ´16 ✓ terms on which we agree with Baroni et al. ´16

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-3})	—	—	—
NLO (Q^{-1})	$A^{0,a}_{1\text{N}:\,\text{UT}'}$, $+\ A^{0,a}_{1\text{N}:\,1/m}$,	$A^{0,a}_{2\text{N}:\,1\pi}$, ✓	—
N^{2}LO (Q^0)	—	—	—
N^{3}LO (Q)	$A^{0,a}_{1\text{N}:\,\text{static},\,\text{UT}'}$, $+\ A^{0,a}_{1\text{N}:\,1/m}$,	$A^{0,a}_{2\text{N}:\,1\pi}$, $+\ A^{0,a}_{2\text{N}:\,2\pi}$, ✓ $+\ A^{0,a}_{2\text{N}:\,\text{cont}}$, ✓	—

Pseudoscalar current

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-4})	$P^a_{\rm 1N:\,static}$,	—	—
NLO (Q^{-2})	$P^a_{\rm 1N:\,static}$,	—	—
N^{2}LO (Q^{-1})	—	$P^a_{\rm 2N:\,1\pi}$, $+ P^a_{\rm 2N:\,cont}$,	—
N^{3}LO (Q^{0})	$P^a_{\rm 1N:\,static}$, $+ P^a_{{\rm 1N:}\,1/m,{\rm UT}'}$, $+ P^a_{\rm 1N:\,1/m^2}$,	$P^a_{\rm 2N:\,1\pi}$, $+ P^a_{\rm 2N:\,1\pi,UT'}$, $+ P^a_{{\rm 2N:}\,1\pi,\,1/m}$, $+ P^a_{\rm 2N:\,2\pi}$, $+ P^a_{\rm 2N:\,cont,\,UT'}$, $+ P^a_{\rm 2N:\,cont,\,1/m}$,	$P^a_{{\rm 3N:}\,\pi}$, $+ P^a_{\rm 3N:\,cont}$,

Continuity equations are verified (perturbatively) for all currents

Call for consistent regularization

Extraction of d_R at N^{2}LO level from ^{3}H β - decay

Gårdestig, Phillips, PRL96 (2006) 232301; Gazit, Quaglioni, Navrátil, PRL103 (2009) 102502;

Klos et al. arXiv:1612.08010

Strong dependence of d_R on the regulator shape and the cutoff

→ Consistent regularization of forces and currents is called for

Symmetry constraints on the consistent regularization of the current

- Chiral symmetry requires direct relation between d_R and C_D

  At N^{2}LO level → $\left[H_{\rm strong}, \boldsymbol{A}_0(\vec{k},0)\right] - \vec{k}\cdot\vec{\boldsymbol{A}}(\vec{k},0) + i\, m_q \boldsymbol{P}(\vec{k},0) = 0$

- Continuity equation is always satisfied perturbatively mod higher order effects

  Higher order corrections are only small after *explicit renormalization* of LECs

- Due to implicit renormalization of LECs we require

  Exact validity of continuity equation for regularized forces and currents

Summary

- Axial-vector current is analyzed up to order Q
- There is a high degree of unitary ambiguity
- Modified continuity equation
- Renormalizability and matching to nuclear forces conditions lead to unique current
- Differences in long range part between our results and Baroni et al.

Outlook

- Regularization and PWD of the currents
- Axial-vector current up to order Q^2