Factorization and Universality in Nuclear Physics

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Ab Initio Techniques in Nuclear Physics
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האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

The Team

Ronen Weiss, Betzalel Bazak

Wine tasting, new year's eve Tzora (2014).

Short Range Correlations in a many-body system

Heavy Fermions

The Mara river, Kenya (2016).

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The short range wave function

Universality

We start with 2-body Schrodinger ...

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(\boldsymbol{r})\right]\psi = E\psi$$

Vanishing distance, $r \longrightarrow 0$

- The energy becomes negligible $E \ll \hbar^2/mr^2$
- The w.f. ψ assumes an asymptotic **energy independent** form φ

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi(\boldsymbol{r}) = 0 \qquad r\varphi(r) = 0|_{r=0}$$

- φ **is a universal function** (in a limited sense)

The short range wave function
Factorization

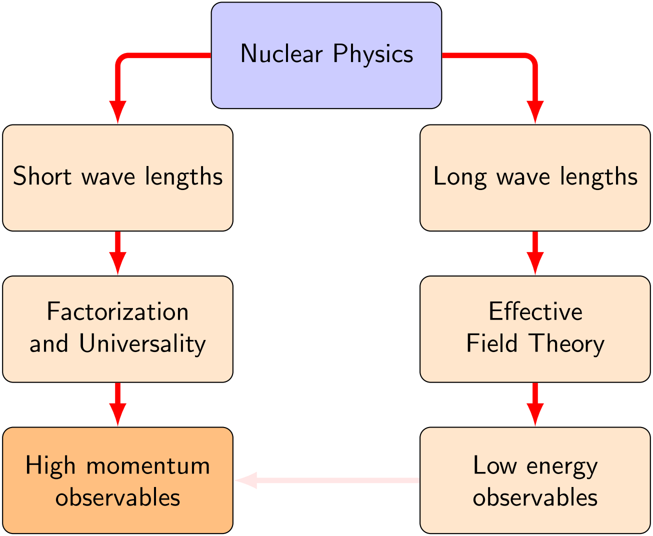
The 2-body system

$$\psi(\boldsymbol{r}) \longrightarrow \varphi(\boldsymbol{r})$$

The A-body system

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) \longrightarrow \varphi(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$

Short and Long



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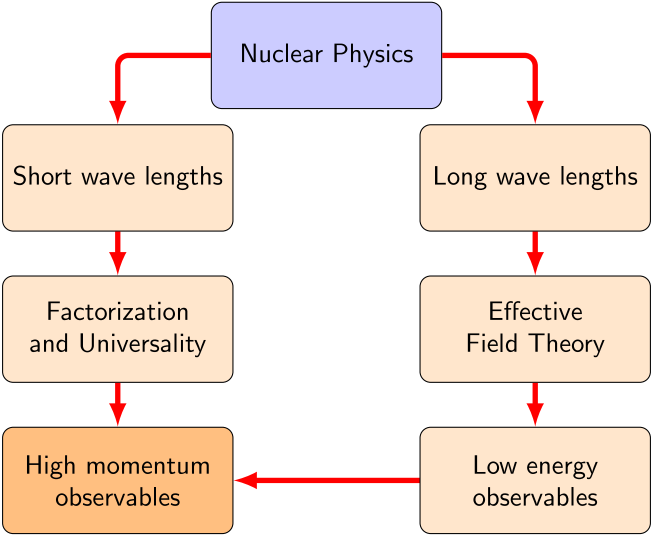

Theoretical develpoments in nuclear physics

1. **Levinger - Photoabsorption**
   J. S. Levinger, Phys. Rev. 84, 43 (1951).
2. **Amado, Woloshyn - Momentum Distribution**
   R. D. Amado, Phys. Rev. C 14, 1264 (1976).
   R. D. Amado and R. M. Woloshyn, Phys. Lett. B 62, 253 (1976).
3. **Ciofi degli Atti - Electron scattering**
   C. Ciofi degli Atti, Phys. Rep. 590, 1 (2015).
4. **Bogner, Roscher - Factorization**
   S. K. Bogner and D. Roscher, Phys. Rev. C **86**, 064304, (2012).
5. ...

Tan relations connects the contact C with:

1. **Tail of momentum distribution** $|a|^{-1} \ll k \ll r_0^{-1}$

$$\boxed{n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}}$$

2. **The energy relation**

$$\boxed{T + U = \sum_\sigma \int \frac{dk}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2}{4\pi m a} C}$$

3. **Adiabatic relation**

$$\boxed{\frac{dE}{d1/a} = -\frac{\hbar^2}{4\pi m} C}$$

4. ...

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Dilute low energy Fermi gas

A system of spin up - spin down fermions

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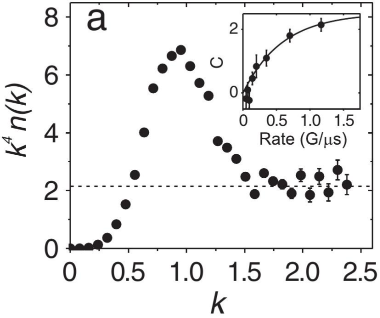
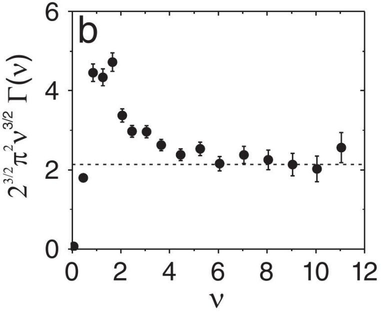
The Contact - Experimental Results

Momentum Distribution



RF line shape



`Verification of Universal Relations in a Strongly Interacting Fermi Gas`
`J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)`

The short range factorization
[Tan, Braatan & Platter, Werner & Castin,...]

- The interaction is represented through the boundary condition

$$\boxed{\left[\partial \log r_{ij}\Psi / \partial r_{ij}\right]_{r_{ij}=0} = -1/a}$$

- Thus, when two particles approach each other

$$\boxed{\Psi \xrightarrow[r_{ij}\to 0]{} (1/r_{ij} - 1/a) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})}$$

- The contact C represents the probability of finding an interacting pair within the system

$$\boxed{C \equiv 16\pi^2 \sum_{ij} \langle A_{ij} | A_{ij} \rangle}$$

- Where

$$\langle A_{ij} | A_{ij} \rangle = \int \prod_{k\neq i,j} d\boldsymbol{r}_k \, d\boldsymbol{R}_{ij} \, A^{\dagger}_{ij}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right) \cdot A_{ij}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

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The Nuclear Contact(s)

Nuclear Scales

- The pion mass $\mu_\pi^{-1} = \hbar / m_\pi c \approx 1.4$ fm
- Scattering lengths $a_t = 5.4$ fm , $a_s \approx 20$ fm, thus $\mu_\pi |a| \geq 3.8$
- The nuclear radius is $R \approx 1.2 A^{1/3}$ fm
- Interparticle distance $d \approx 2.4$ fm, thus $\mu_\pi d \approx 1.7$

Conclusions

- The Tan conditions are not strictly applicable in nuclear physics
- The interaction range is significant
- There could be different interaction channels - not only s-wave
- Therefore, we need replace the asymptotic form

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The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels

$$\alpha = (s, \ell)jm$$

- For each pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

  using the normalization

$$\int_{k_F} \frac{d\boldsymbol{k}}{(2\pi)^3} |\tilde{\varphi}_{\alpha}(\boldsymbol{k})|^2 = 1$$

- For $\ell = 0$ we need consider **4** contacts

$$\{C_{pp}^{S=0}, C_{nn}^{S=0}, C_{np}^{S=0}, C_{np}^{S=1}\}$$

- Adding isospin symmetry the number of contacts is **2**,

$$C_s = C_{np}^{S=0}, \ \ C_t = C_{np}^{S=1}$$

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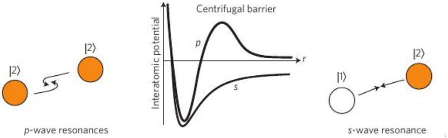
A comment

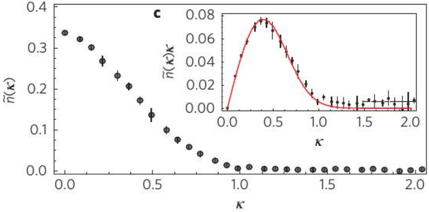
The contact matrix and $\ell \neq 0$ partial waves

A system of one component fermion - p-**wave interaction**



The asymptotic momentum distribution takes the form

$$n(\boldsymbol{k}) = \frac{16\pi^2}{V} \sum_m Y_{1m}^2(\hat{\boldsymbol{k}}) \frac{C_p^m}{k^2}$$



C. Luciuk, *et al.*, Nature Phys. **12**, 599 (2016)

The Nuclear Contact

The nuclear contact relations/applications

1. **The nuclear photoabsorption cross-section - The quasi-deutron model**
   R. Weiss, B. Bazak, N. Barnea, PRL **114**, 012501 (2015)
2. **The 1-body and 2-body momentum distributions**
   R. Weiss, B. Bazak, N. Barnea, PRC **92**, 054311 (2015)
   M. Alvioli et al., arXiv:1607.04103 [nucl-th] (2016)
   R. Weiss, E. Pazy, N. Barnea, Few-Body syst. (2016)
3. **Generalized treatment of the photoabsorption cross-section**
   R. Weiss, B. Bazak, N. Barnea, EPJA (2016)
4. **Electron scattering**
   O. Hen et al., PRC **92**, 045205 (2015)
5. **Symmetry energy**
   B.J. Cai, B.A. Li, PRC **93**, 014619 (2016)
6. ...

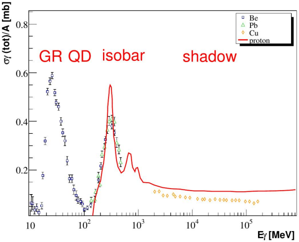
Photoabsorption of Nuclei

Up to $\hbar\omega \approx 200$ MeV the cross-section $\sigma_A(\omega)$ is dominated by the **dipole** operator

$$\sigma_A(\omega) = 4\pi^2 \alpha \omega R(\omega)$$

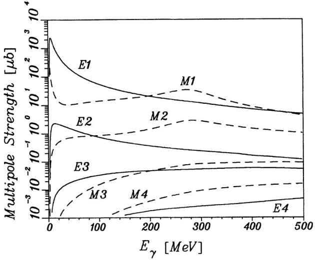
R is the response function

$$R(\omega) = \sum\!\!\!\!\!\!\int_f \left| \langle \Psi_f \left| \epsilon \cdot \hat{D} \right| \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$



R. Al Jebali, PhD Thesis, U. Glasgow (2013)

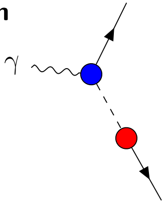
The Deuteron cross-section



H. Arenhovel, and M. Sanzone, Few-Body Syst. (1991).

The Quasi-Deuteron picture

J. S. Levinger
"The high energy nuclear photoeffect", Phys. Rev. **84**, 43 (1951).



- The photon carries **energy** but (almost) **no momentum**
- It is captured by a single **proton.**
- The proton is ejected without any FSI.
- Momentum conservation $\Rightarrow$ a nucleon with opposite momentum must be ejected $\boldsymbol{k} \approx -\boldsymbol{k}_p$.
- Dipole dominance $\Rightarrow$ this partner must be a **neutron**.
- $\hbar\omega \longrightarrow \infty \Rightarrow \sigma(\omega)$ depends on a **universal** short range pn wave-function.
- The resulting cross-section is given by

$$\sigma_A(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

- L is known as the Levinger Constant

The Quasi-Deuteron revisited

If the reaction take place when a **pn** pair are close together then

$$\Psi_0 \cong \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{pn}) A^\alpha_{pn}\left(\boldsymbol{R}_{pn}, \{\boldsymbol{r}_j\}_{j\neq p,n}\right)$$

$$\Psi^\alpha_f \cong \frac{4\pi}{\sqrt{C_\alpha}}\hat{\mathcal{A}}\left\{\frac{1}{\sqrt{\Omega}} e^{-i\mathbf{k}\cdot\mathbf{r}_{pn}} \chi_{s\mu_s} A^\alpha_{pn}(\boldsymbol{R}_{pn}, \{\boldsymbol{r}_j\}_{j\neq p,n})\right\}$$

With these wave functions it is easy to get the **"universal"** tail of the nuclear photoabsorption **dipole** response function

$$R(\omega) = \sum_{\alpha,\beta} C^{\alpha\beta}_{pn} R_{\alpha\beta}(\omega)$$

where

$$R_{\alpha\beta}(\omega) = \sum_{s,\mu_s}\int \frac{d\hat{k}}{(2\pi)^3}\langle k s\mu_s|\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{D}}_{pn}|\alpha\rangle^*\langle k s\mu_s|\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{D}}_{pn}|\beta\rangle$$

are **"universal"** 2-body channel response functions

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The **Levinger** quasi-deutron model is recoverd if we assume **quasi-deuteron dominance**

$$\sigma_A(\omega) = 4\pi^2 \alpha\omega \sum_{\alpha,\beta} C_{pn}^{\alpha\beta} R_{\alpha\beta}(\omega) \approx 4\pi^2 \alpha\omega C_t R_t(\omega)$$

The cross-section of **any** nucleus is therefore proportional to the dueteron cross-section $\sigma_d(\omega)$

$$\sigma_A(\omega) = \frac{C_t}{C_t(^2\mathrm{H})}\sigma_d(\omega) \xrightarrow[\mathrm{zero-range}]{} \frac{a_t}{4\pi}\bar{C}_{pn}\sigma_d(\omega)$$

Comparing to Levinger's formula

$$\sigma_A(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

We see that the Levinger constant L is a close relative of the nuclear contacts,

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The Cross-Section

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Momentum distributions

1-body neutron and proton momentum distributions

$$n_n(\boldsymbol{k}),\ \ n_p(\boldsymbol{k})$$

2-body nn, np, pp momentum distributions

$$F_{nn}(\boldsymbol{k}),\ \ F_{pn}(\boldsymbol{k}),\ \ F_{pp}(\boldsymbol{k})$$

Momentum distributions

The proton momentum distribution

$$n_p^{JM}(\boldsymbol{k}) = Z \int \prod_{l \neq p} \frac{d^3 k_l}{(2\pi)^3} \left| \tilde{\Psi}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_p = \boldsymbol{k}, \ldots, \boldsymbol{k}_A) \right|^2$$

Using the asymptotic wave-function

$$\Psi \xrightarrow[r_{ij} \to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$

we get

$$n_p(\boldsymbol{k}) = \frac{1}{2J+1} \sum_{\alpha,\beta} \tilde{\varphi}_{pp}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k}) Z(Z-1) \langle A_{pp}^\alpha | A_{pp}^\beta \rangle + \frac{1}{2J+1} \sum_{\alpha,\beta} \tilde{\varphi}_{pn}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k}) NZ \langle A_{pn}^\alpha | A_{pn}^\beta \rangle$$

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$$n_p(\boldsymbol{k}) = \sum_{\alpha,\beta} \underbrace{\tilde{\varphi}_{pp}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k})}_{\text{universal 2b}} 2C_{pp}^{\alpha\beta} + \sum_{\alpha,\beta} \underbrace{\tilde{\varphi}_{pn}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k})}_{\text{universal 2b}} C_{pn}^{\alpha\beta}$$

Momentum distributions

Similarly

$$F_{ij}(\boldsymbol{k}) = \sum_{\alpha,\beta} \tilde{\varphi}_{ij}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{ij}^{\beta}(\boldsymbol{k}) C_{ij}^{\alpha\beta}$$

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$$n_p(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{pp}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$

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These are **model independent** relations, that hold regardless of the specific form of φ_α and without any assumptions on $\{\alpha\}$

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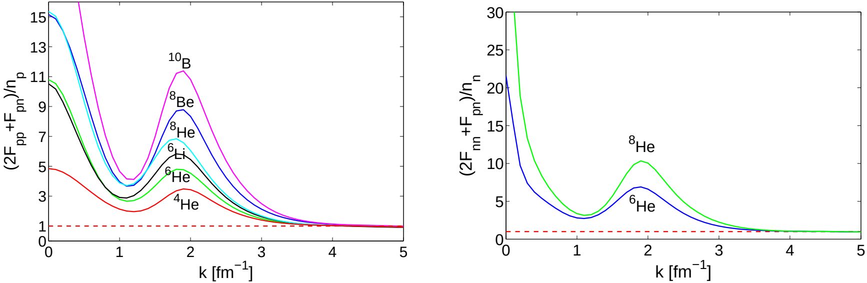
Numerical verification of the momentum relations



VMC calculations of light nuclei

- Wiringa et. al. published a series of 1-body, 2-body momentum distributions
  R. B. Wiringa, *et al.*, PRC **89**, 024305 (2014)
- The data is available for nuclei in the range $2 \leq A \leq 10$.
- The calculations were done with the VMC method
- For symmetric nuclei $n_p = n_n$

The momentum relations holds for $4\ \mathrm{fm}^{-1} \leq k \leq 5\ \mathrm{fm}^{-1}$

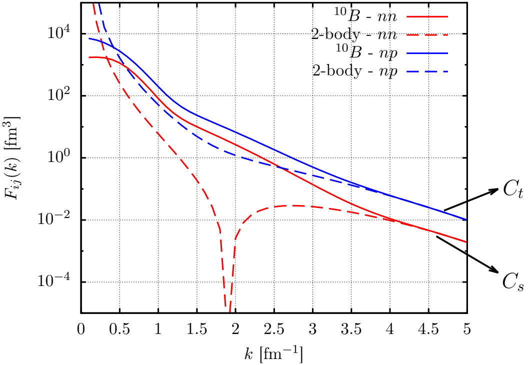
Extracting the leading contacts

We can extract the **leading** contacts using the asymptotic 2-body momentum distributions

$$F_{ij}(\boldsymbol{k}) = \sum_{\alpha,\beta} \underbrace{\tilde{\varphi}_{ij}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{ij}^{\beta}(\boldsymbol{k})}_{\text{universal 2b}} C_{ij}^{\alpha\beta} \longrightarrow \underbrace{|\tilde{\varphi}_{ij}^{\alpha_0}(\boldsymbol{k})|^2}_{\text{universal 2b}} C_{ij}^{\alpha_0\alpha_0}$$

For non-deuteron channels the 2-body functions are $E = 0$ scattering w.f.

Example - VMC calculations of ^{10}B

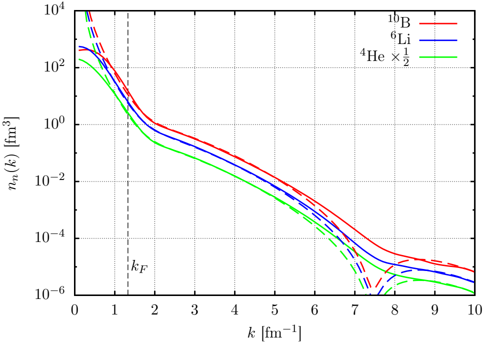


Further numerical verifications

The resulting **asymptotic** 1-body momentum distribution is given by

$$n_n^\infty(\boldsymbol{k}) \cong |\tilde{\varphi}_{np}^t(\boldsymbol{k})|^2 C_t + 2|\tilde{\varphi}_{nn}^s(\boldsymbol{k})|^2 C_s$$

Comparing with the VMC data



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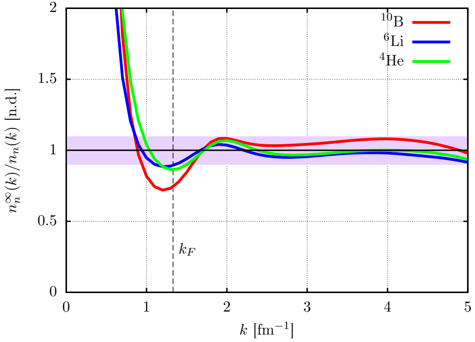

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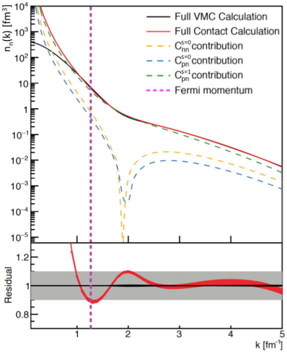
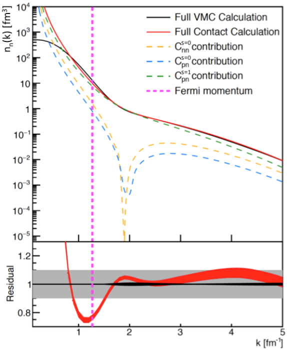
The 1-body momentum distribution

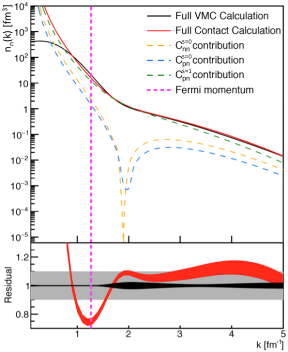
^{4}**He**



^{7}**Li**



^{10}**B**



R. Weiss, R. Cruz-Torres, *et al.*, arXiv:1612.00923 (2016)

The Levinger Constant again

Theory and Experiment

Assuming deuteron channel dominance $C_t \gg C_s$, we can derive the relations

$$\frac{F_{pn}(^AX)}{n_p(^2H)} \cong \frac{C_t(^AX)}{C_t(^2H)} \cong L\frac{NZ}{A}$$

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Experiment - Photoabsorption

QD Model

$L = 5.50 \pm 0.21$

Theory - AV18+UBIX

Momentum distributions $A \leq 40$

$L = 5.6 \pm 0.6$

The Levinger Constant again

Theory and Experiment

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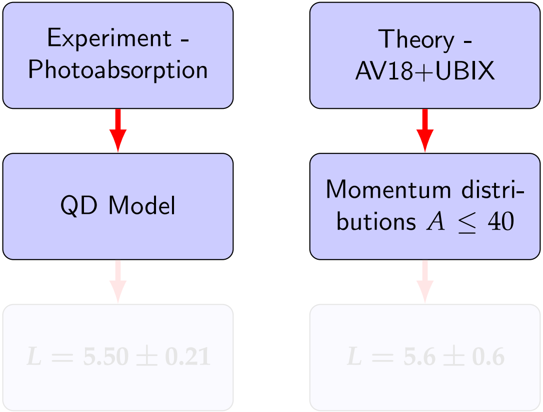

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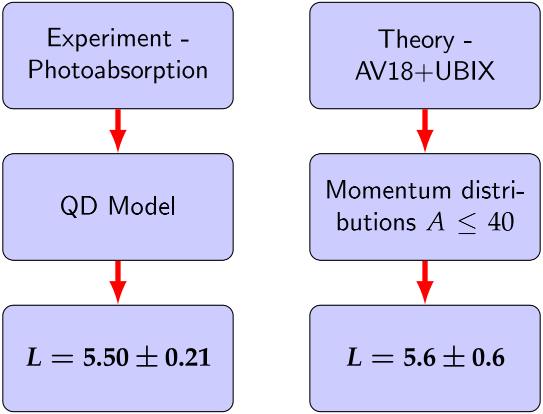
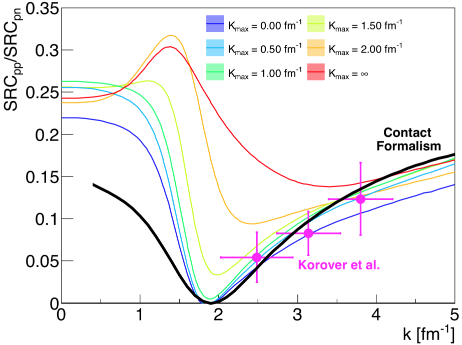


Two-body knockout reactions

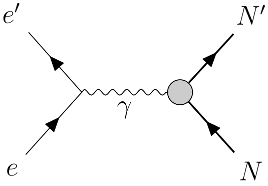
Electron scattering

The ratio of short range pp and np pairs is given by

$$\frac{SRC_{pp}(k)}{SRC_{pn}(k)} = \frac{F_{pp}(k)}{F_{pn}(k)} = \frac{C_{pp}^{s=0}|\tilde{\varphi}_{pp(nn)}^{s=0}(k)|^2}{C_{pn}^{s=0}|\tilde{\varphi}_{pn}^{s=0}(k)|^2 + C_{pn}^{s=1}|\tilde{\varphi}_{pn}^{s=1}(k)|^2}$$



The Coulomb sum rule



$$CSR(\boldsymbol{q}) \equiv \int_{0^+} d\omega R_L(\omega, \boldsymbol{q})$$

Assuming point-like particles

$$CSR(\boldsymbol{q}) = \langle\Psi|\hat{\rho}_c^\dagger(\boldsymbol{q})\hat{\rho}_c(\boldsymbol{q})|\Psi\rangle - |\langle\Psi|\hat{\rho}_c(\boldsymbol{q})|\Psi\rangle|^2$$

where

$$\hat{\rho}_c(\boldsymbol{q}) = \sum_{j=1}^{A} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_j}\frac{1-\tau_z^i}{2} = \sum_{p=1}^{Z} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_p}$$

Thus

$$\langle\Psi|\hat{\rho}_c^\dagger(\boldsymbol{q})\hat{\rho}_c(\boldsymbol{q})|\Psi\rangle = Z + \langle\Psi|\sum_{p'\neq p} e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_p-\boldsymbol{r}_{p'})}|\Psi\rangle$$

The $q \longrightarrow \infty$ limit

$$\langle\Psi|\hat{\rho}_c^\dagger(\boldsymbol{q})\hat{\rho}_c(\boldsymbol{q})|\Psi\rangle = Z + \langle\Psi|\sum_{p\neq p'} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{p'p}}|\Psi\rangle$$

In this limit we can replace the wave-function by its asymptotic form

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

therefore

$$\langle\Psi|\hat{\rho}_c^\dagger(\boldsymbol{q})\hat{\rho}_c(\boldsymbol{q})|\Psi\rangle = Z + \sum_{\alpha\beta} Z(Z-1)\langle A_{pp}^{\alpha\dagger}|A_{pp}^\beta\rangle \underbrace{h_{pp}^{\alpha\beta}(\boldsymbol{q})}_{\text{universal 2b}}$$

where

$$h_{pp}^{\alpha\beta}(\boldsymbol{q}) = \int d\boldsymbol{r}\varphi_{pp}^{\alpha\dagger}(\boldsymbol{r})e^{i\boldsymbol{q}\cdot\boldsymbol{r}}\varphi_{pp}^\beta(\boldsymbol{r})$$

Summing up, for $q \to \infty$

$$CSR(\boldsymbol{q}) = Z + \sum_{\alpha\beta} 2C_{pp}^{\alpha\beta}h_{pp}^{\alpha\beta}(\boldsymbol{q}) - \rho_c^2(\boldsymbol{q})$$

The CSR - Numerical examples

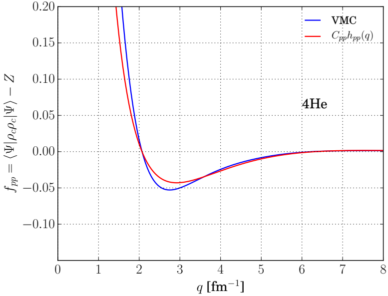
Comparison with VMC calculations

$$f_{pp}(\boldsymbol{q}) = \langle \Psi | \rho_c(\boldsymbol{q}) \rho_c(\boldsymbol{q}) | \Psi \rangle - Z$$

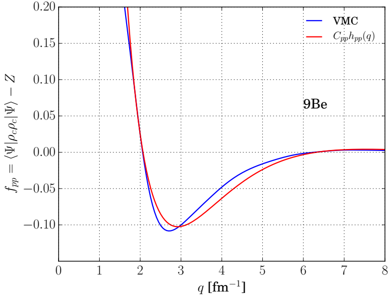
The asymptotic result

$$f_{pp}(\boldsymbol{q}) \longrightarrow 2C_{pp}^{S=0} h_{pp}^{S=0}(\boldsymbol{q})$$

^{4}He



^{9}Be



Summary and Conclusions

Factorization and universality and nuclear physics

- Rederived Levinger's Quasi-Deuteron model utilizing the factorization ansatz
- The Levinger constant and the nuclear contacts are close relatives
- Derived momentum relations for nuclear physics

$$\boxed{\begin{aligned} n_p(\boldsymbol{k}) &\xrightarrow[k\to\infty]{} 2F_{pp}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k}) \\ n_n(\boldsymbol{k}) &\xrightarrow[k\to\infty]{} 2F_{nn}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k}) \end{aligned}}$$

- 3-body generalization is under way
- The 1-body momentum distribution seems to be dominated (upto 10%) by 2-body correlations, from k_F up
- CSR

Summary and Conclusions (II)

Outlook

- Electron scattering
- Neutrino scattering
- ...
- ...
- ...

We have only started to explore the usefulness of the contact formalism in nuclear physics !

Thank you !