EFTs for Nuclei

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and

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Menu

1. IR extrapolations
   1. Using data that is not yet fully UV converged
   2. Quadrupole operators, radiative capture
2. Pion-less EFT for ^{16}O and ^{40}Ca
   1. DVR in the oscillator basis
   2. Bound nuclei at non-perturbative NLO
3. EFT for vibrations in odd-mass nuclei

Energy scales and relevant degrees of freedom

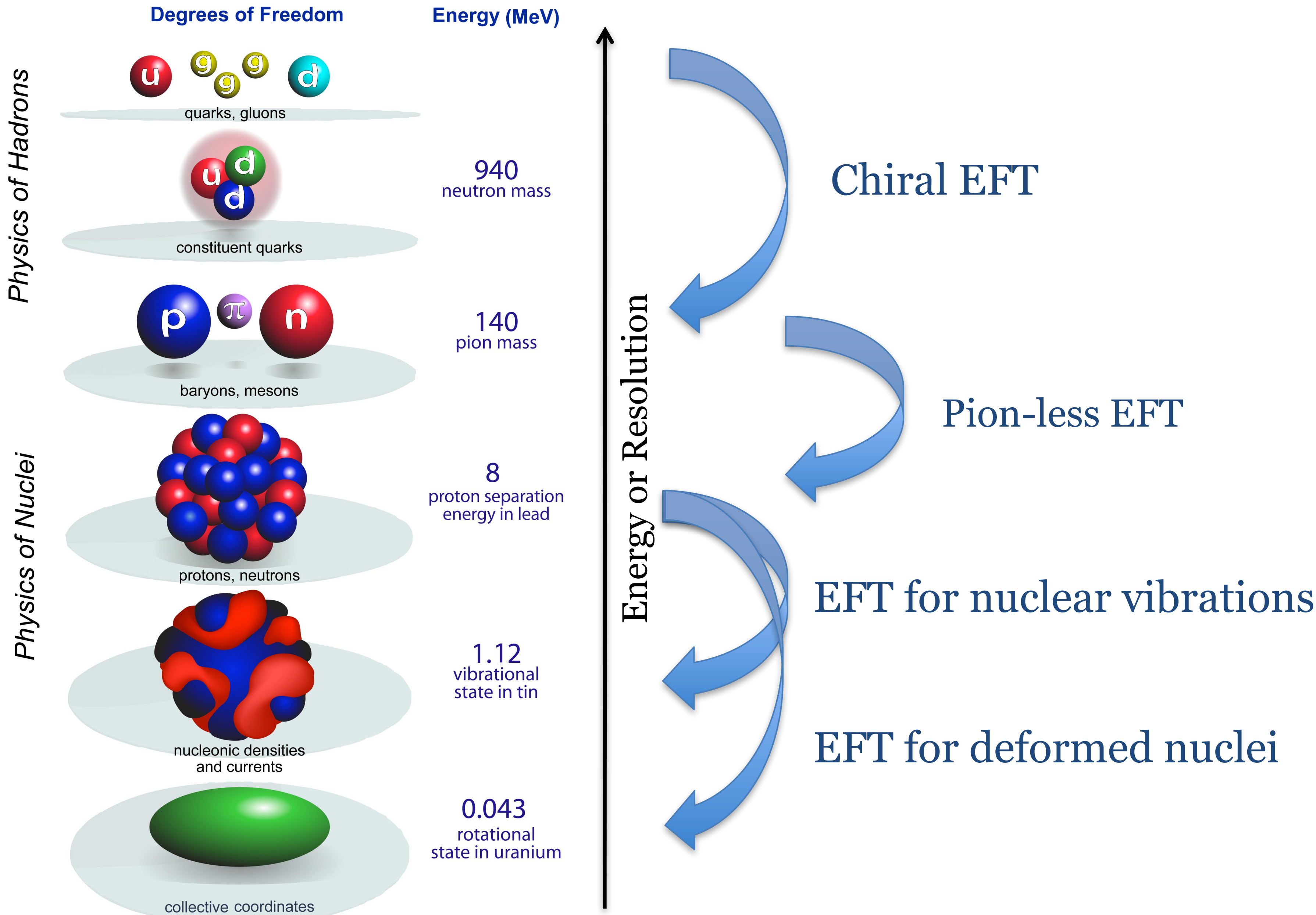
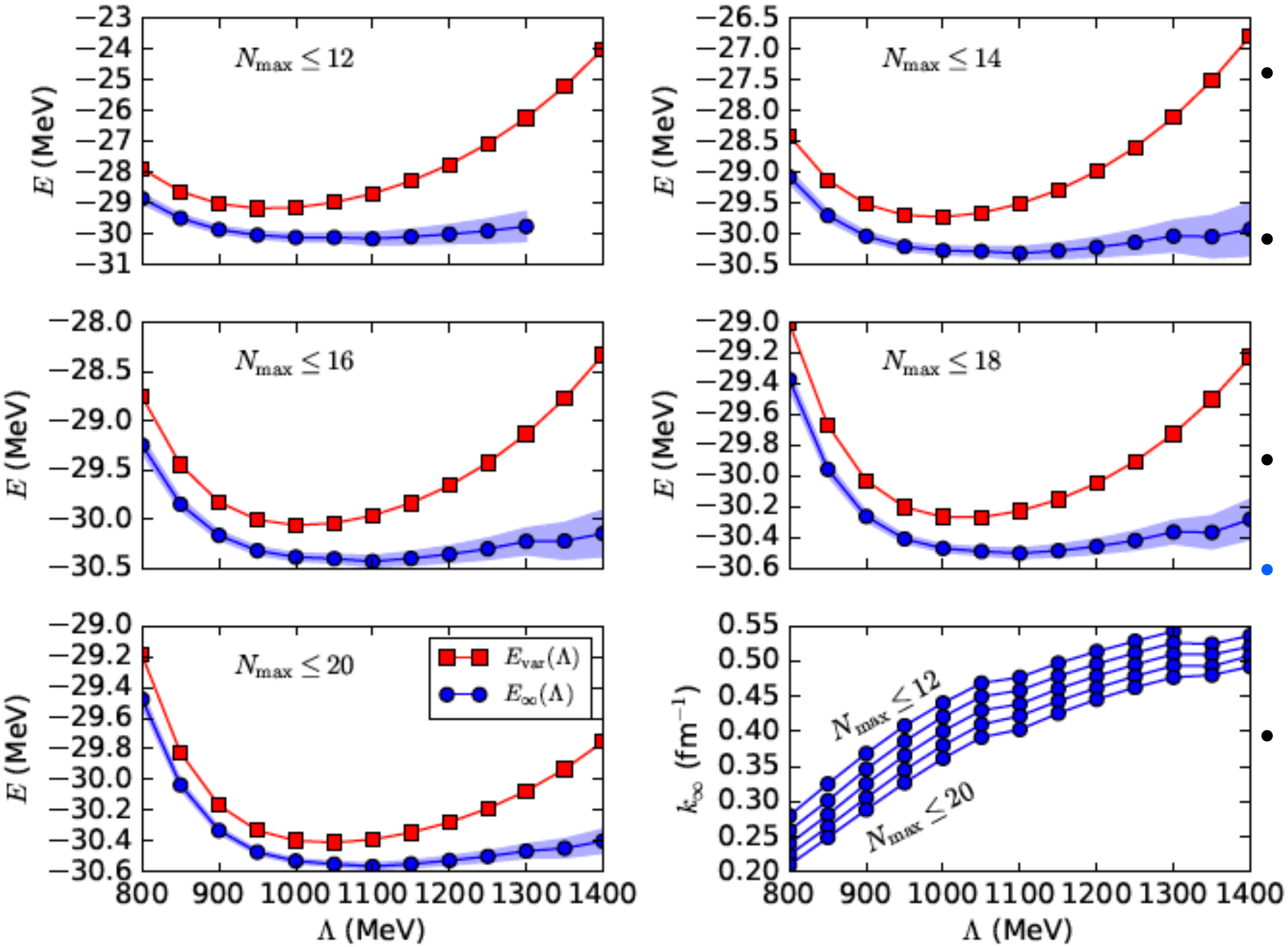


Fig.: Bertsch, Dean, Nazarewicz (2007)

IR extrapolation for ^{6}Li with NCSM

Use IR extrapolation formula [Coon et al. 2012 ; Furnstahl, Hagen & TP 2012] at fixed UV cutoff Λ

$$E(L,\Lambda) = E_\infty(\Lambda) + a(\Lambda)\exp\left[-2k_\infty(\Lambda)L\right]$$



Rationale:

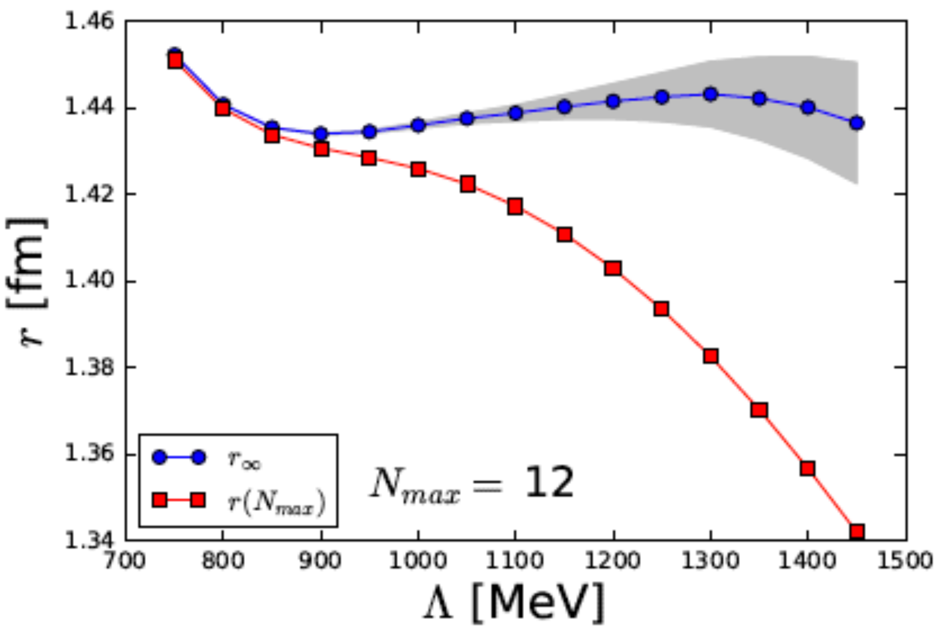
- At high Λ, UV converged, but NLO IR corrections enter
- IR extrapolations profitable as long as UV correction smaller than IR correction
- Improve on variational minimum
- Uncertainty estimates from IR extrapolation only
- k_∞ stabilizes

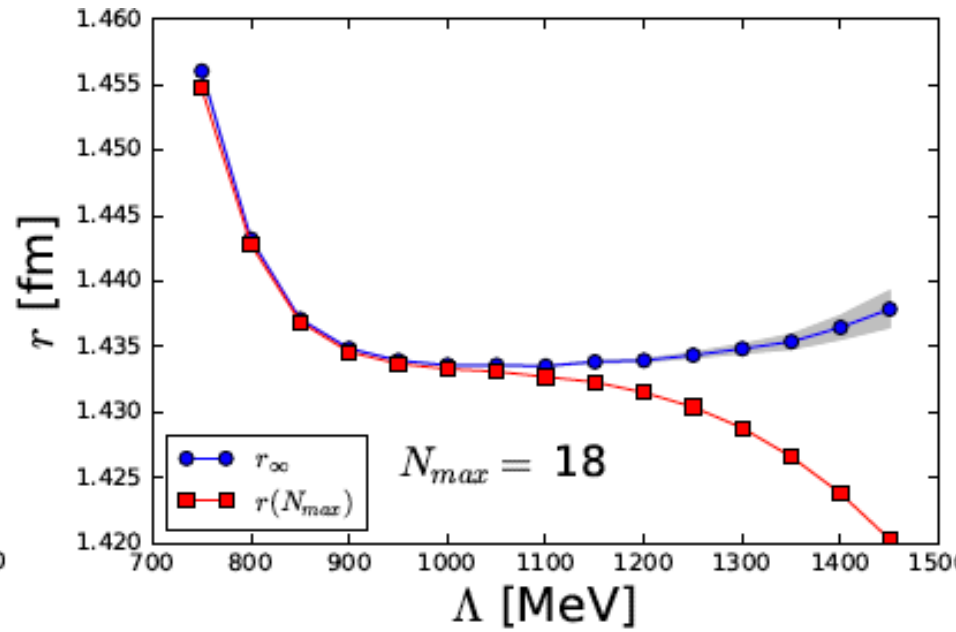
Forssén, Hagen, TP, Sääf (in preparation)

IR extrapolation for ^{6}Li with NCSM

Modified approach: Use IR extrapolation formula at fixed UV cutoff Λ

$$r^2(L,\Lambda) = r^2_\infty(\Lambda) - \alpha(\Lambda)\left[k_\infty(\Lambda)L\right]^3 \exp\left[-2k_\infty(\Lambda)L\right]$$





- Uncertainty estimates from IR extrapolation
- k_∞ stabilizes at higher cutoffs; consistent with energy extrapolation

Forssén, Hagen, TP, Sääf (in preparation)

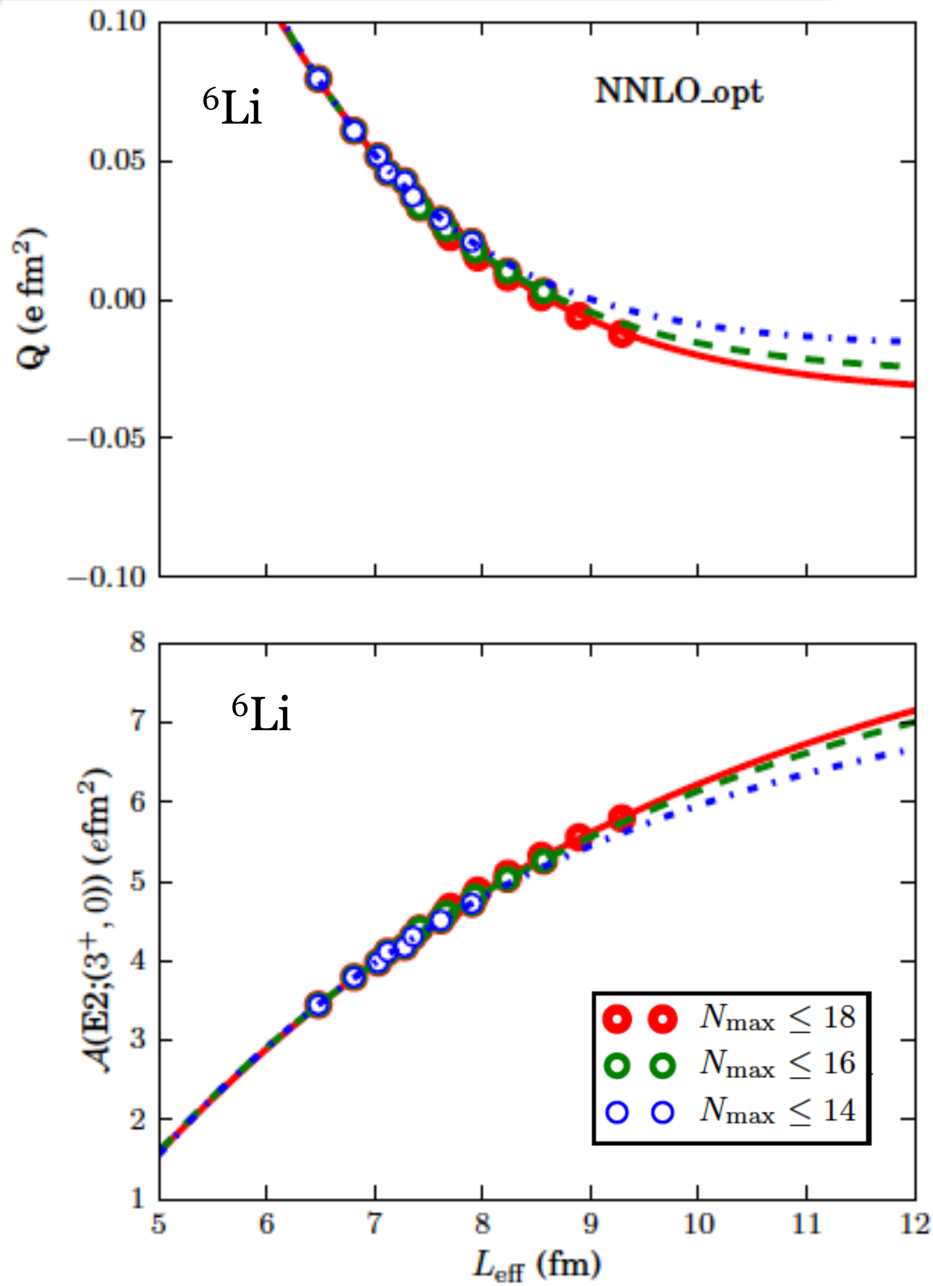
Extrapolations in finite Hilbert spaces

Quadrupole moment

$$Q_L = Q_\infty - a(k_\infty L)^3 e^{-2k_\infty L}$$

E2 transition amplitude

$$\mathcal{A}_L = \mathcal{A}_\infty + a_0 e^{-2k_< L}$$



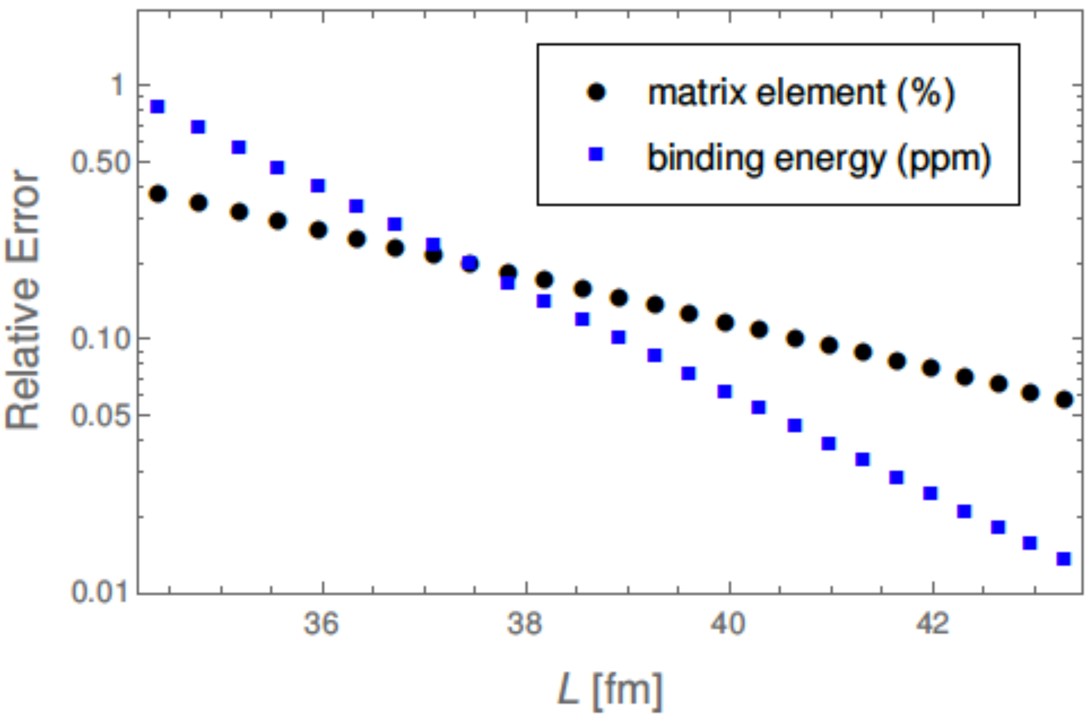
Derivation: Odell, TP, Platter, PRC (2016).
Application: Ik Jae Shin *et al.*, 1605.02819.

Extrapolations in finite Hilbert spaces

Radiative capture:

- from continuum to bound state
- Convergence depends on bound-state momentum and is slower than energy convergence

$$\Delta\mathcal{I}_\lambda(k;\eta;L) = \frac{2A_\infty\gamma_\infty}{\gamma_\infty^2+k^2}L^\lambda e^{-\gamma_\infty L}\sin\left(\delta_l+\sigma_l-\frac{\pi l}{2}+kL-\eta\log 2kL\right)$$



Corrections to capture cross sections ~ 1%, even when integrating out to 30 fm.
[Girlanda *et al.* PRL 2010]

Acharya *et al*, arXiv:1608.04699
→ PRC Rapid 2017

EFT in harmonic oscillator via discrete variable representation (DVR)

Motivation: optimize and generate interactions in basis of computation

- Formulate EFT directly in the oscillator basis [Haxton & Song (2000); Stetcu, Barrett & van Kolck (2007); Tölle, Hammer & Metsch (2011)]
- A finite harmonic oscillator basis exhibits IR and UV cutoffs [Stetcu, Barrett & van Kolck (2007); Coon *et al.* (2012); Furnstahl, Hagen & TP (2012)]
- Discrete momentum eigenstates from diagonalization of p^2 for DVR in oscillator basis [Binder *et al.*, PRC 93, 044332 (2016)]

Useful tool: Computation of scattering phase shifts directly in the finite oscillator basis [Heller & Yamani (1974); Bang *et al.* (2000); Shirokov *et al.* (2004)]

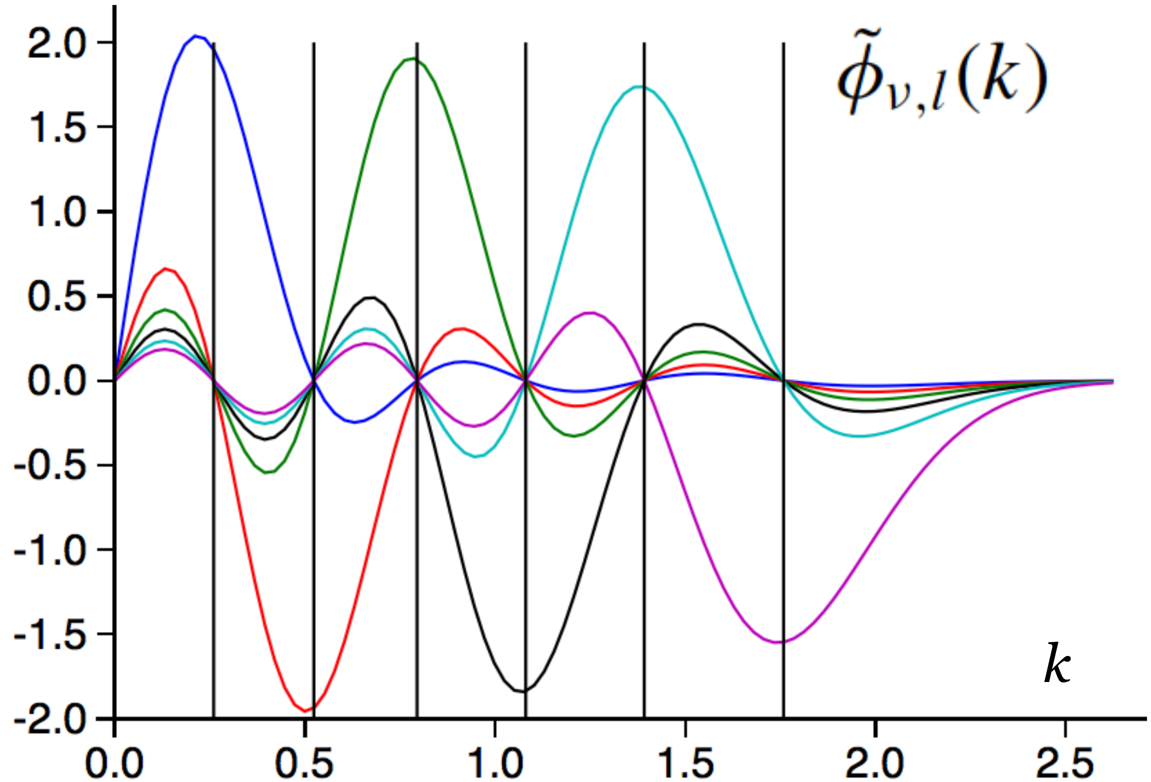
Eigenfunctions and eigenvalues of p^2

For a partial wave l in a Hilbert space with energies up to $(2N+l)\hbar\boldsymbol{\omega}$, the eigenvalues $k_{\mu l}^2$ of p^2 are the roots of the associated Laguerre polynomial $L_{N+1}^{l+1/2}$.

The eigenfunctions of p^2 are a DVR (discrete variable representation).

DVRs see: [Harris, Engerholm, & Gwinn (1965); Light, Hamilton, & Lill (1985); Baye & Heenen (1986); Littlejohn *et al*. (2002); Bulgac & McNeil Forbes (2013).]

Eigenfunctions of p^2 for l=0, N=10, and $\hbar\boldsymbol{\omega}$ = 10 MeV. Vertical lines are eigenvalues $k_{\mu l}$.



DVR yields simple matrix elements

1. Matrix elements in the DVR are just rescaled ME from continuum, evaluated at discrete momenta

$$\langle \phi_{\nu,l'} | \hat{V} | \phi_{\mu,l} \rangle = c_{\nu,l'} c_{\mu,l} \langle k_{\nu,l'}, l' | \hat{V} | k_{\mu,l}, l \rangle$$

2. Exact overlap between a Cartesian and a spherical oscillator state

$$\langle \phi_{\mathbf{a}} | nlm \rangle = c_{\mathbf{a}} \tilde{\psi}_{n,l}(|\mathbf{k_a}|) Y_{lm} \left(\widehat{\mathbf{k}}_{\mathbf{a}} \right)$$

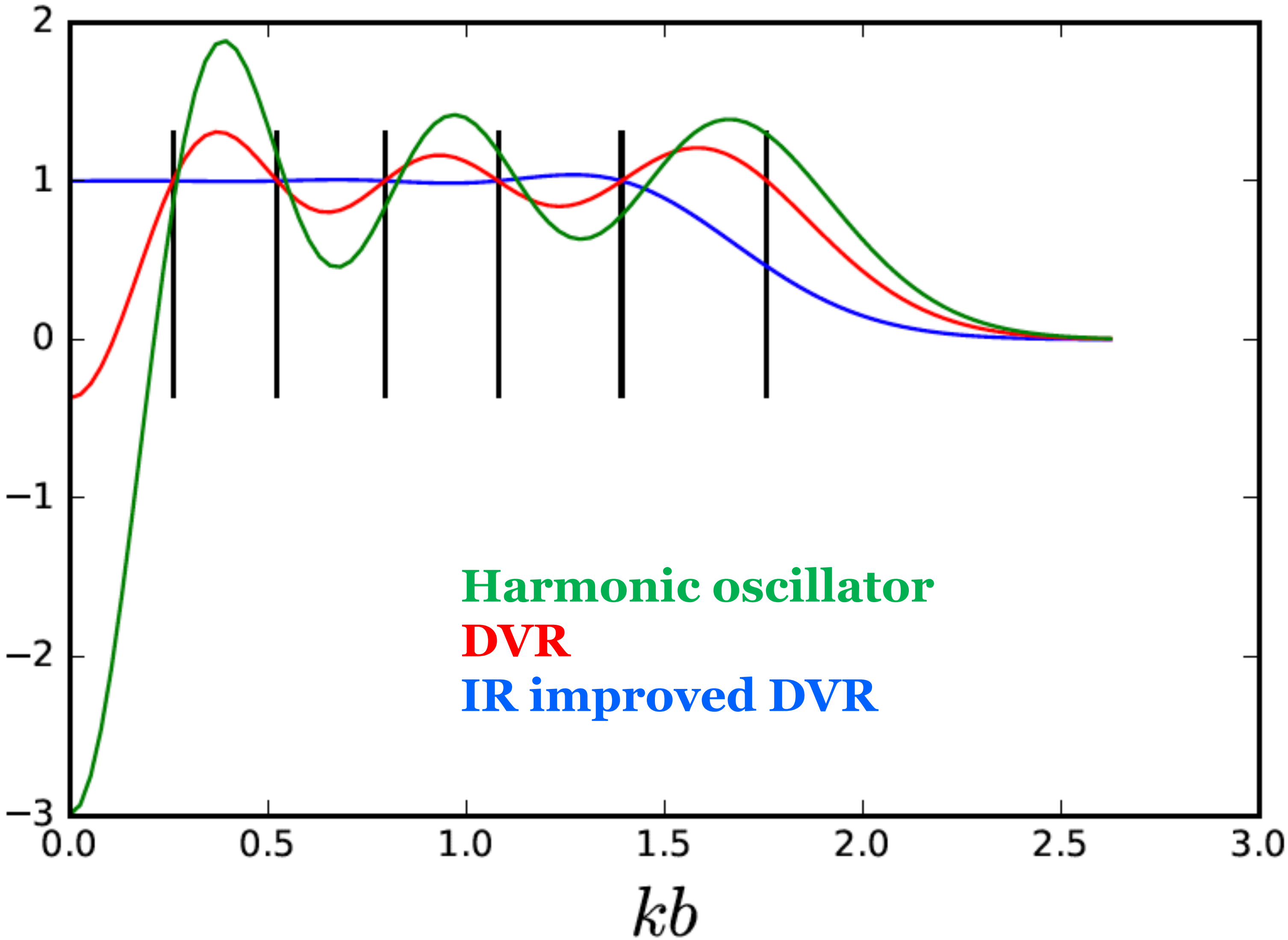
3. Partial-wave decomposition made easy:

$$\langle n'l'm' | V_M^{(L)} | nlm \rangle_o = C_{lmLM}^{l'm'} \frac{(-1)^l}{\sqrt{(2l'+1)(2L+1)}}$$

$$\times \sum_{\mathbf{ab}} c_{\mathbf{a}}^2 c_{\mathbf{b}}^2 \tilde{\psi}_{n',l'}(|\mathbf{k_a}|) \tilde{\psi}_{n,l}(|\mathbf{k_b}|) \ \langle \mathbf{k_a} | V^{(L)} | \mathbf{k_b} \rangle \cdot \left\{ Y_l \left(\widehat{\mathbf{k}}_{\mathbf{b}} \right) \times Y_{l'} \left(\widehat{\mathbf{k}}_{\mathbf{a}} \right) \right\}^{(L)}$$

Restoring spherical symmetry à la [Bing-Nan Lu et al., PRD 92, 014506 (2015)]

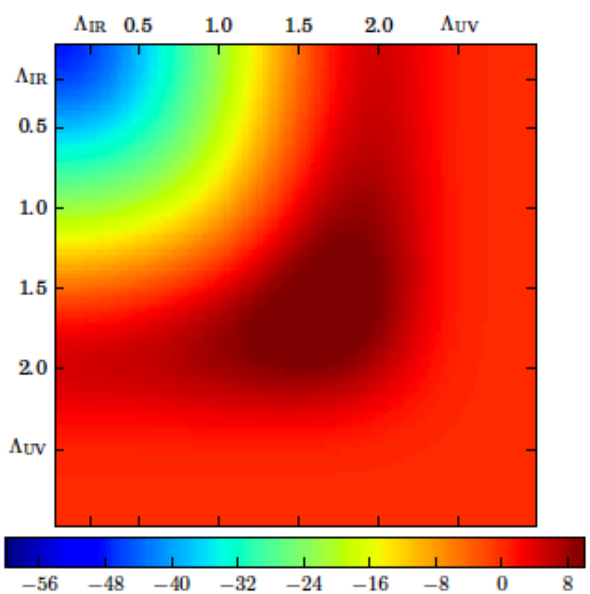
Contact interactions, transformed back to momentum space



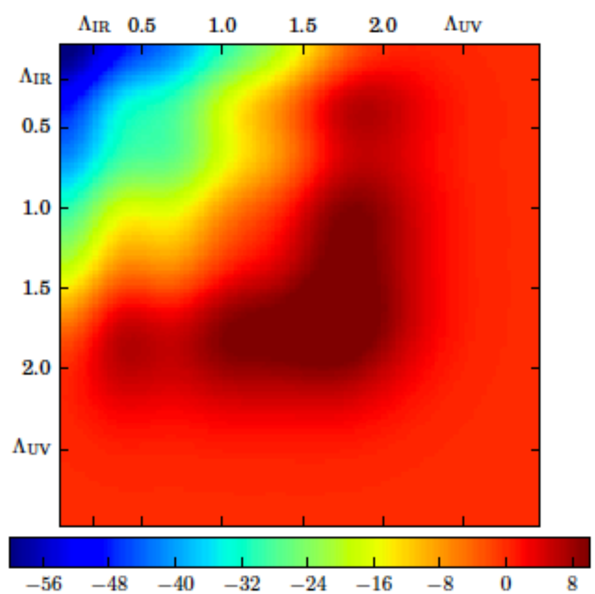
Matrix elements in a finite oscillator basis

Momentum-space matrix elements
$^1S_0$ channel of $NNLO_{sim}$ with Λ_χ = 400 MeV
E_{max}=10ħω in harmonic oscillator (6 *s* states)

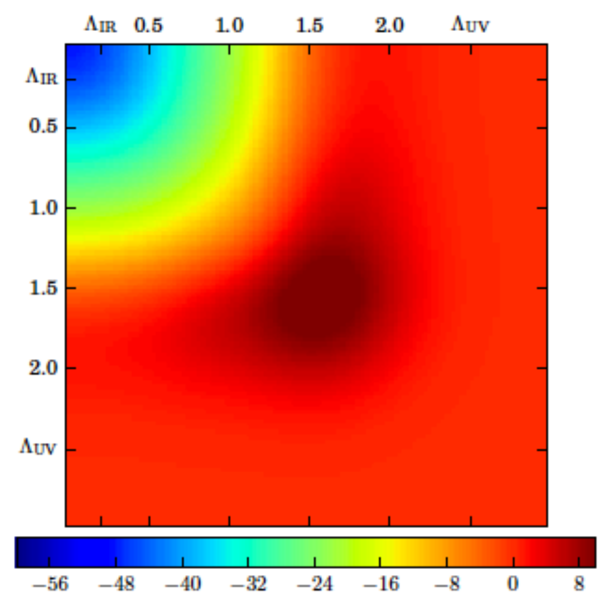
V(k’,k)



DVR: V_{HO}(k’,k)



IR improved V_{HO}(k’,k)



IR improvement in oscillator EFT:
Modify matrix elements at high discrete momenta to improve low-momentum physics

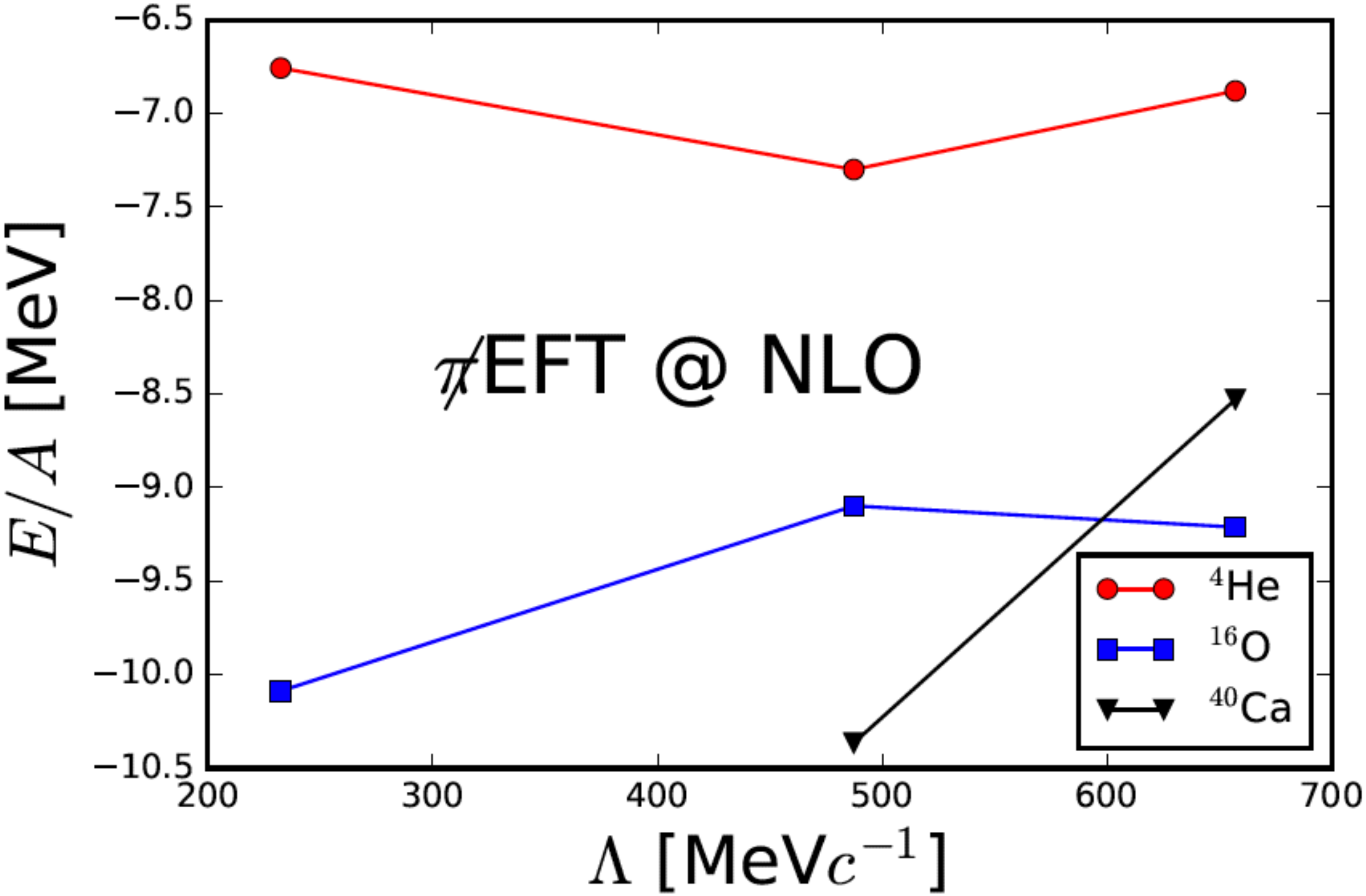
Pionless EFT for A>4

- A>4 nuclei explored only very recently [Kirscher *et al*. 2010; Lensky, Birse & Walet 2016; Contessi *et al*. 2017]
- Used as tool to compute finite nuclei from lattice QCD input (at unphysical pion masses, though)
- At LO, ^{16}O is not bound with respect to decay into four α particles [Contessi *et al*. 2017]

Here:

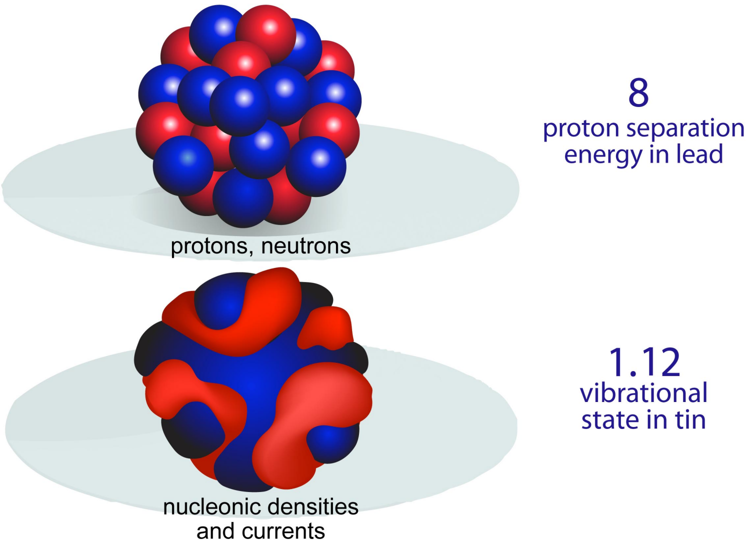
- Nonperturbative NLO
- Interaction as IR improved DVR in N=8 shells
- Increase kinetic energy until convergence is reached
- Compute ^{4}He, ^{16}O, and ^{40}Ca

Preliminary results



Bansal, Binder, Ekström, Hagen, and TP (in preparation)

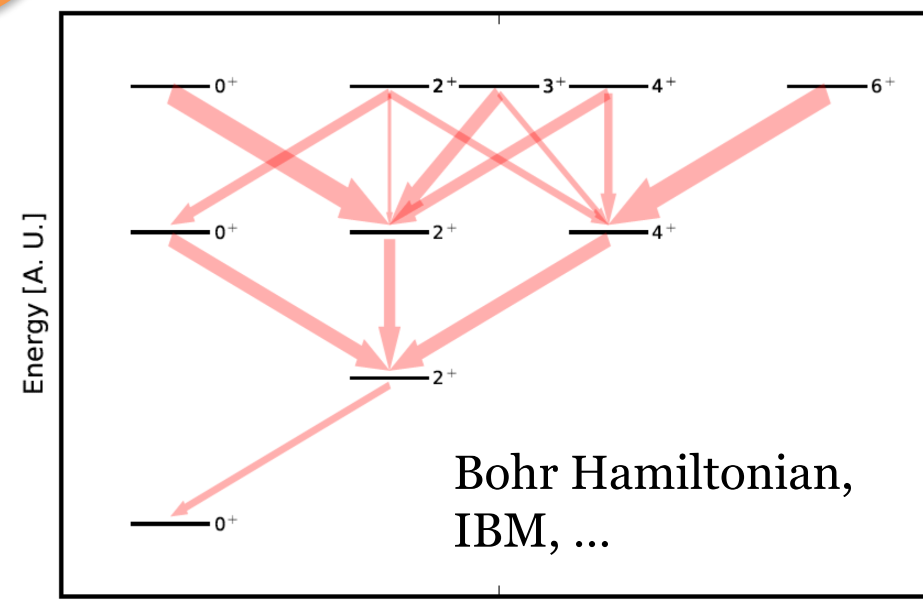
EFT for nuclear vibrations



EFT for nuclear vibrations
[Coello Pérez & TP 2015, 2016]

While spectra of certain nuclei appear to be harmonic, B(E2) transitions do not.

Garrett & Wood (2010): “Where are the quadrupole vibrations in atomic nuclei?”

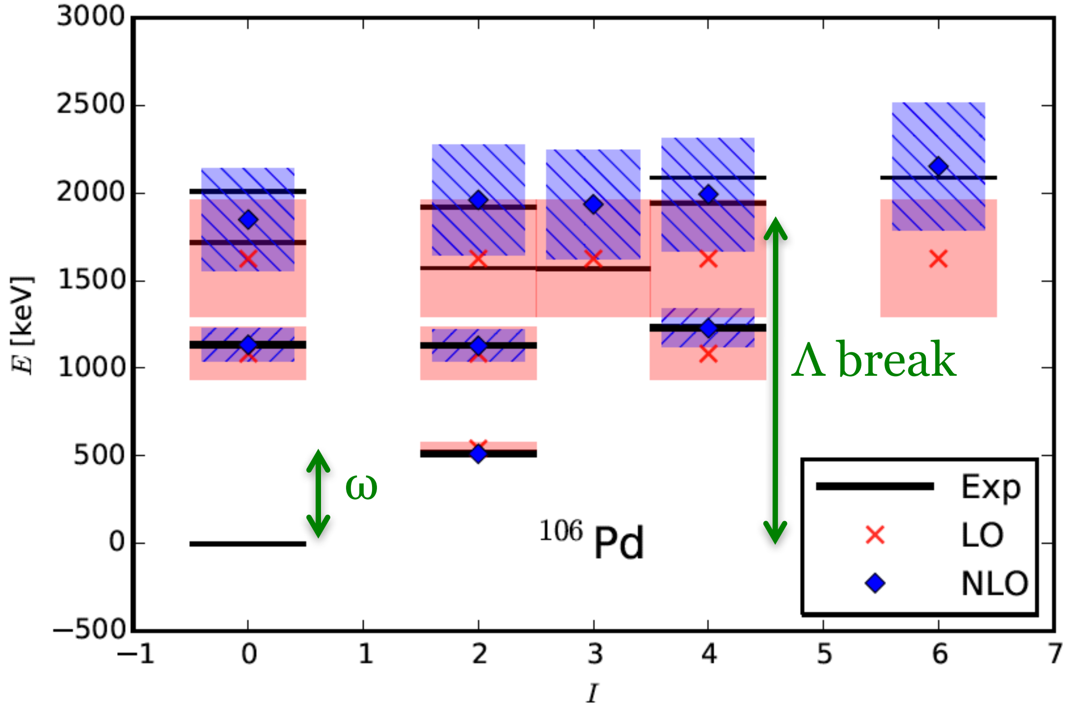


Spectrum and B(E2) transitions of the *harmonic* quadrupole oscillator

EFT for nuclear vibrations

EFT ingredients:

- quadrupole degrees of freedom
- breakdown scale around three-phonon levels
- “small” expansion parameter: ratio of vibrational energy to breakdown scale: $\omega/\Lambda \approx 1/3$



- Uncertainties show 68% DOB intervals from truncating higher EFT orders [Cacciari & Houdeau (2011); Bagnaschi et al (2015); Furnstahl, Klco, Phillips & Wesolowski (2015)]
    - Expand observables according to power counting
    - Employ “naturalness” assumptions as log-normal priors in Bayes’ theorem
    - Compute distribution function of uncertainties due to EFT truncation
    - Compute degree-of-believe (DOB) intervals.

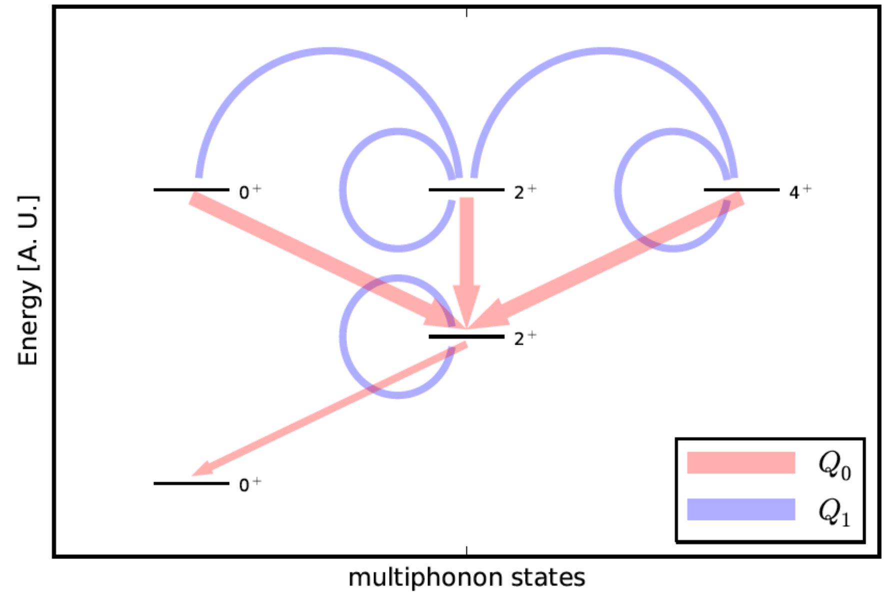
EFT result: sizeable quadrupole matrix elements are natural in size

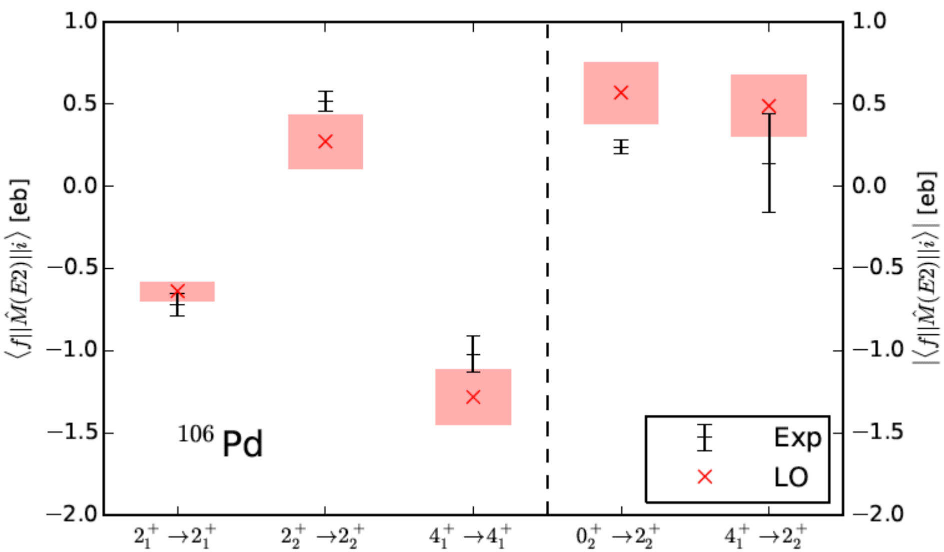
In the EFT, the quadrupole operator is also expanded:

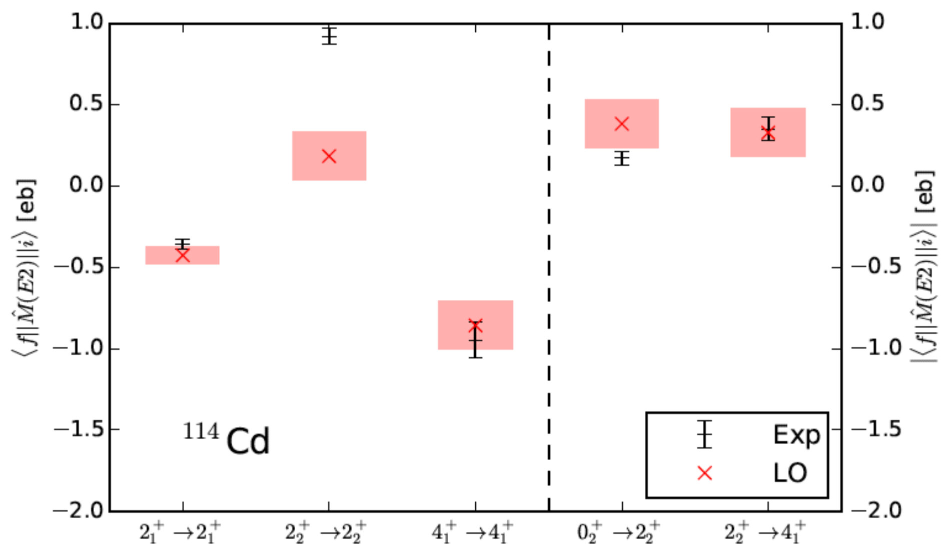
$$\hat{Q}_\mu = Q_0 \left(d^\dagger_\mu + \tilde{d}_\mu \right) + Q_1 \left(d^\dagger \times d^\dagger + \tilde{d} \times \tilde{d} + 2 d^\dagger \times \tilde{d} \right)^{(2)}_\mu$$

Subleading corrections are sizable:

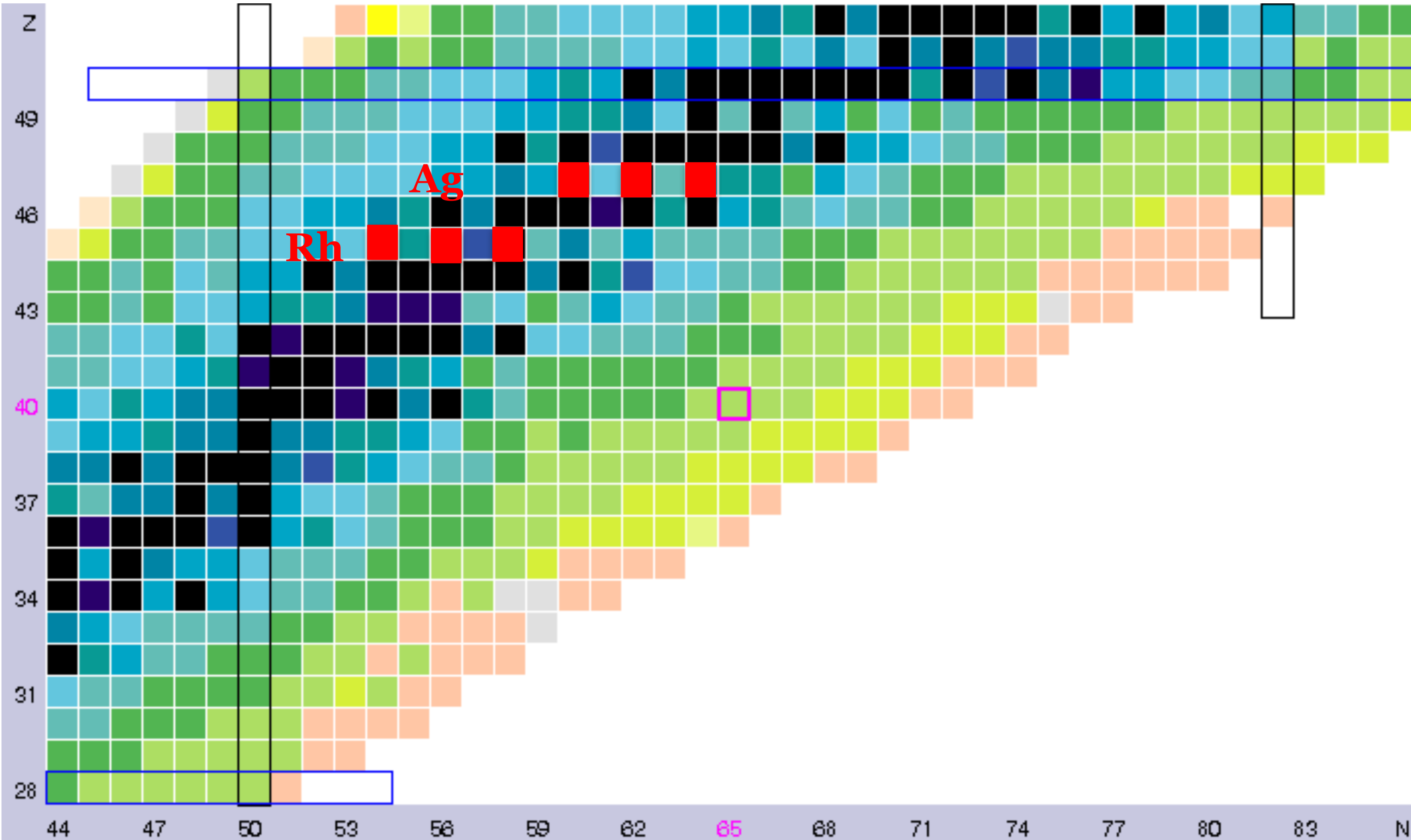
$$Q_1 \sim \left(\frac{\omega}{\Lambda} \right)^{1/2} Q_0$$







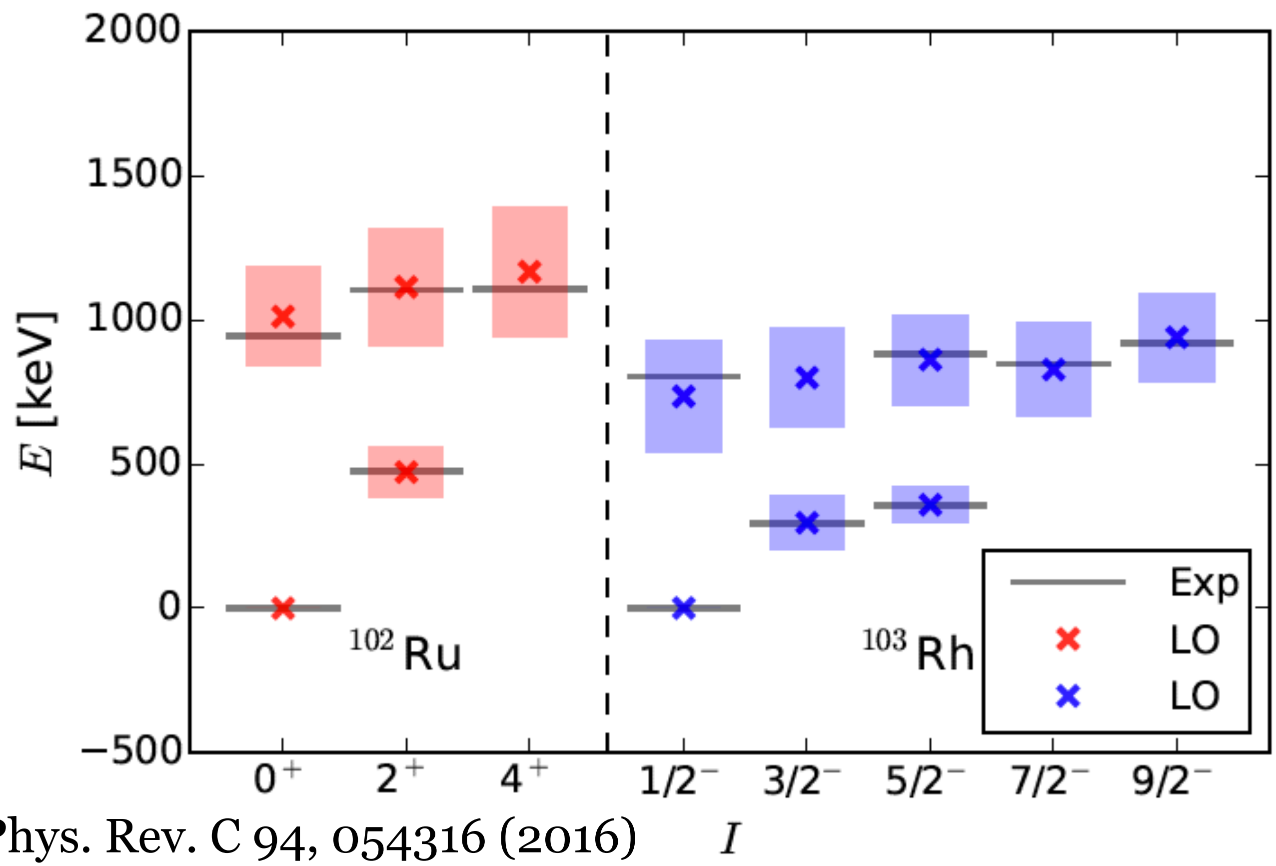
^{99,101,103}Rh and ^{105,107,109,111}Ag as a proton coupled to ^{98,100,102}Ru and ^{104,106,108,110}Pd, respectively, or ^{107,109,111}Ag as a proton-hole in ^{108,110,112}Cd



Fermion coupled to vibrating nucleus

Approach: Follow Halo EFT [Bertulani, Hammer, van Kolck (2002); Higa, Hammer, van Kolck (2008); Hammer & Phillips (2011); Ryberg et al. (2014)], and couple a fermion to describe odd-mass neighbors; particle-vibrator models [de Shalit (1961); Iachello & Scholten (1981); Vervier (1982);...]

Two new LECs enter at LO



E. A. Coello Pérez & TP, Phys. Rev. C 94, 054316 (2016)

Static E2 moments (in eb)

Nucleus	I_i^π	Q_{exp}	Q_{EFT}	Nucleus	I_i^π	Q_{exp}	Q_{EFT}
^{102}Ru	2_1^+	$-0.63(3)$	$-0.41(6)$	^{108}Pd	2_1^+	$-0.56(3)$	$-0.57(7)$
	2_2^+		$0.18(18)$		2_2^+	$0.73(9)$	$0.24(20)$
	4_1^+		$-0.82(14)$		4_1^+	$-0.78(11)$	$-1.14(17)$
^{103}Rh	$\frac{3}{2}{}_1^-$	$-0.3(2)$	$-0.29(7)$	^{109}Ag	$\frac{3}{2}{}_1^-$	$-0.7(3)$	$-0.40(8)$
	$\frac{5}{2}{}_1^-$	$-0.4(2)$	$-0.41(6)$		$\frac{5}{2}{}_1^-$	$-0.3(3)$	$-0.57(6)$
^{106}Pd	2_1^+	$-0.54(4)$	$-0.50(7)$	^{110}Cd	2_1^+	$-0.39(3)$	$-0.57(7)$
	2_2^+	$0.39(6)$	$0.21(20)$		2_2^+		$0.24(17)$
	4_1^+	$-0.79(11)$	$-1.00(17)$		4_1^+		$-1.12(14)$
^{107}Ag	$\frac{3}{2}{}_1^-$		$-0.35(8)$	^{109}Ag	$\frac{3}{2}{}_1^-$	$-0.7(3)$	$-0.39(6)$
	$\frac{5}{2}{}_1^-$		$-0.50(7)$		$\frac{5}{2}{}_1^-$	$-0.3(3)$	$-0.56(6)$

Single LEC fit to all data with EFT weighting.

E. A. Coello Pérez & TP, Phys. Rev. C 94, 054316 (2016)

Magnetic moments: Relations between even-even and even-odd nuclei

Nucleus	I_i^π	$\mu_{\mathrm{exp}}(I_i^\pi)$	$\mu_{\mathrm{EFT}}(I_i^\pi)$
^{102}Ru	2_1^+	$0.85(3)^*$	$0.85(5)$
	2_2^+		$0.85(10)$
	4_1^+		$1.70(8)$
^{103}Rh	$\frac{1}{2}_1$	-0.088^*	-0.088
	$\frac{3}{2}_1$	$0.77(7)$	$0.81(5)$
	$\frac{5}{2}_1$	$1.08(4)$	$0.76(5)$
	$\frac{7}{2}_1$	$2.0(6)$	$1.7(1)$
	$\frac{9}{2}_1$	$2.8(5)$	$1.6(1)$

Nucleus	I_i^π	$\mu_{\mathrm{exp}}(I_i^\pi)$	$\mu_{\mathrm{EFT}}(I_i^\pi)$
^{106}Pd	2_1^+	$0.79(2)^*$	$0.79(5)$
	2_2^+	$0.71(10)$	$0.79(10)$
	4_1^+	$1.8(4)$	$1.58(8)$
^{107}Ag	$\frac{1}{2}_1$	-0.11^*	-0.11
	$\frac{3}{2}_1$	$0.98(9)$	$0.78(5)$
	$\frac{5}{2}_1$	$1.02(9)$	$0.68(4)$
	$\frac{7}{2}_1$		$1.6(1)$
	$\frac{9}{2}_1$		$1.5(1)$

At LO, one new LEC enters to describe the magnetic moments in the odd-mass neighbor

E. A. Coello Pérez & TP, Phys. Rev. C 94, 054316 (2016)

Summary

- IR extrapolations
  - Lüscher-like formulas for energies, radii, quadrupole moments, transitions, radiative capture reactions
  - Compute at fixed Λ and IR extrapolate on all data.
- DVR in HO basis economical implementation of EFT
  - DVR in momentum space
  - fast convergence of many-body calculations
- Pion-less EFT
  - bound ^{16}O, ^{40}Ca at nonperturbative NLO for cutoffs $200 < \Lambda < 700$ MeV
- EFT for nuclear vibrations
  - Picture of anharmonic vibrations consistent with data within uncertainties

Hamiltonian

LO Hamiltonian $\hat{H}_{\mathrm{LO}} = \omega \hat{N}$

NLO correction $\hat{h}_{\mathrm{NLO}} = g_N \hat{N}^2 + g_v \hat{\Lambda}^2 + g_I \hat{I}^2$

with
$$\hat{N}^2 = (d^\dagger \cdot \tilde{d})^2,$$
$$\hat{\Lambda}^2 = -(d^\dagger \cdot d^\dagger)(\tilde{d} \cdot \tilde{d}) + \hat{N}^2 - 3\hat{N},$$
$$\hat{I}^2 = 10(d^\dagger \otimes \tilde{d})^{(1)} \cdot (d^\dagger \otimes \tilde{d})^{(1)}.$$

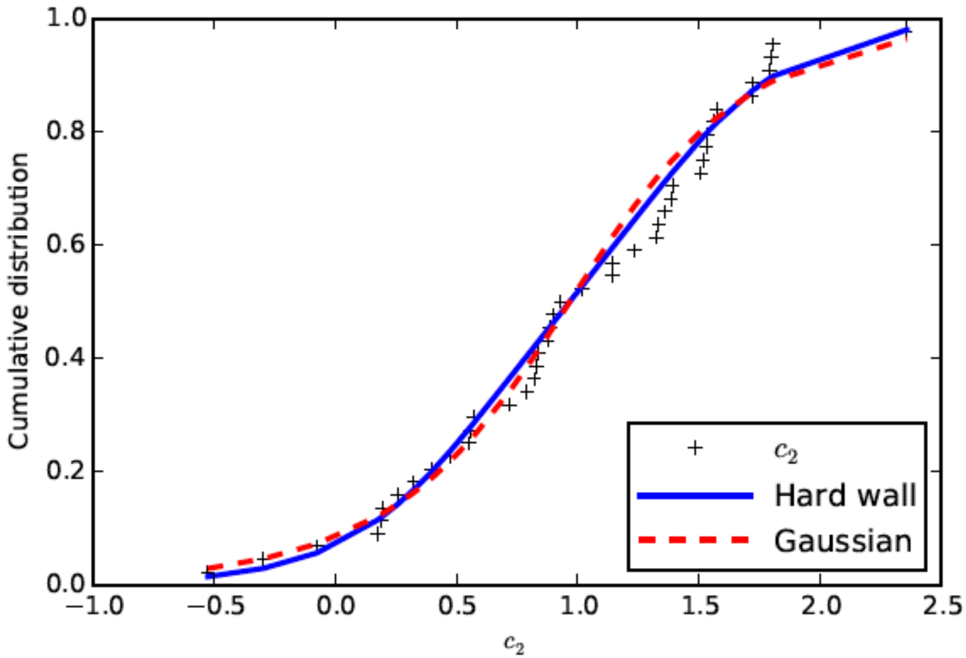
"Small" expansion parameter $\varepsilon \equiv (N\omega/\Lambda)$

Uncertainty quantification

$$E_{\mathrm{NLO}} = \omega N + g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)$$

$$c_2 \equiv c_2(N, v, I) = \frac{g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)}{\varepsilon^2 \omega}$$

$$X = X_0 \sum_{n=0}^{\infty} c_n \varepsilon^n$$



Linear combinations of LECs enter observables. LECs are random, but with EFT expectations, i.e. log-normal distributed. Making assumptions about these distributions then allows one to quantify uncertainties. The assumptions can be tested.

$$\Delta_k^{(M)} = \sum_{n=k+1}^{k+M} c_n \varepsilon^n$$

$$p_M(\Delta|c_0, \ldots, c_k) = \frac{\int_0^\infty dc\ \mathrm{pr}(c) p_M(\Delta|c) \prod_{m=0}^{k} \mathrm{pr}(c_m|c)}{\int_0^\infty dc\ \mathrm{pr}(c) \prod_{m=0}^{k} \mathrm{pr}(c_m|c)}$$

Lüscher formulas for many-body systems

Key idea: compute eigenvalues of kinetic energy and compare with *corresponding* (hyper)spherical cavity to find L.

What is the corresponding cavity?

Single particle	A particles (product space)	A particles in No-core shell model
Diagonalize $T_{kin}=p^2$	Diagonalize A-body T_{kin}	Diagonalize A-body T_{kin}
3D spherical cavity	A fermions in 3D cavity	3(A-1) hyper-radial cavity

$$L_2 = \sqrt{2(N+3/2+2)}b$$

$$L_{\rm eff} = \left(\frac{\sum_{nl}\nu_{nl}a_{l,n}^2}{\sum_{nl}\nu_{nl}\kappa_{l,n}^2}\right)^{1/2}$$

$$L_{\rm eff} = b\frac{X_{1,\mathcal{L}}}{\sqrt{T_{1,\mathcal{L}}(N_{\rm max}^{\rm tot})}}$$

More, Ekström, Furnstahl, Hagen, TP, PRC 87, 044326 (2013)

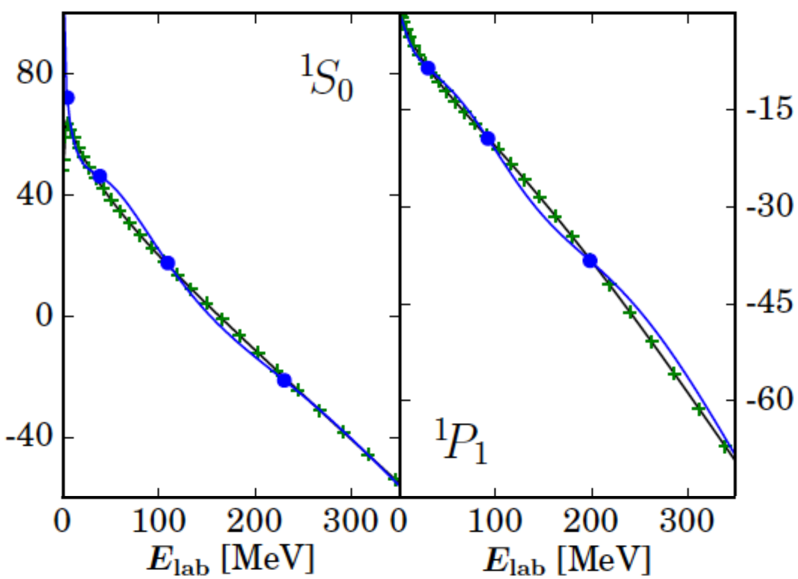
Furnstahl, Hagen, TP, Wendt, J. Phys. G 42, 034032 (2015)

Wendt, Forssén, TP, Sääf, PRC 91, 061301(R) (2015)

Chiral interaction at NLO in the oscillator basis

- Construct and optimize interaction in oscillator basis
- UV convergence by construction
- NLO interaction constructed with E_{max}=10ħω at ħω=22 MeV
- Rapid convergence of ground-state energies even for heavy nuclei

Phase shifts compared to NLO_{sim}



Convergence of ground-state energies

N_{max}	^{4}He	^{16}O	^{40}Ca	^{90}Zr	^{132}Sn
	E_{CCSD} [MeV]				
10	-31.57	-142.89	-402.0	-918.4	-1230.0
12	-31.57	-142.92	-402.4	-923.1	-1249.3
14	-31.57	-142.93	-402.5	-924.6	-1255.6
16	-31.57	-142.93	-402.5	-925.1	-1258.3
∞	-31.57	-142.93	-402.5	-925.4	-1260.1
exp	-28.30	-127.62	-342.1	-783.9	-1102.9

Binder *et al.* PRC 93, 044332 (2016); see also Yang PRC 94, 064004 (2016)