# Neutron matter based on consistently evolved chiral three-nucleon interactions

Kai Hebeler (OSU)

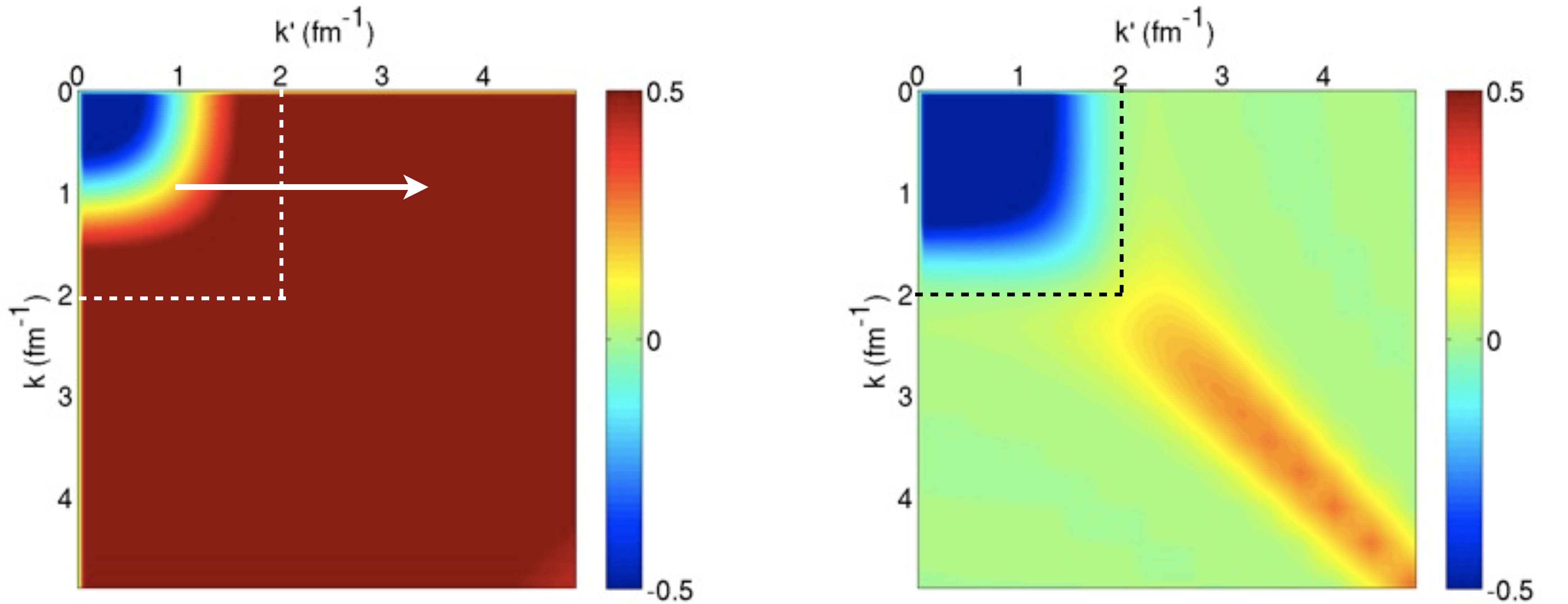
**Progress in Ab Initio Techniques in Nuclear Physics**

*in collaboration with Dick Furnstahl*

Vancouver, February 22, 2013

THE OHIO STATE UNIVERSITY

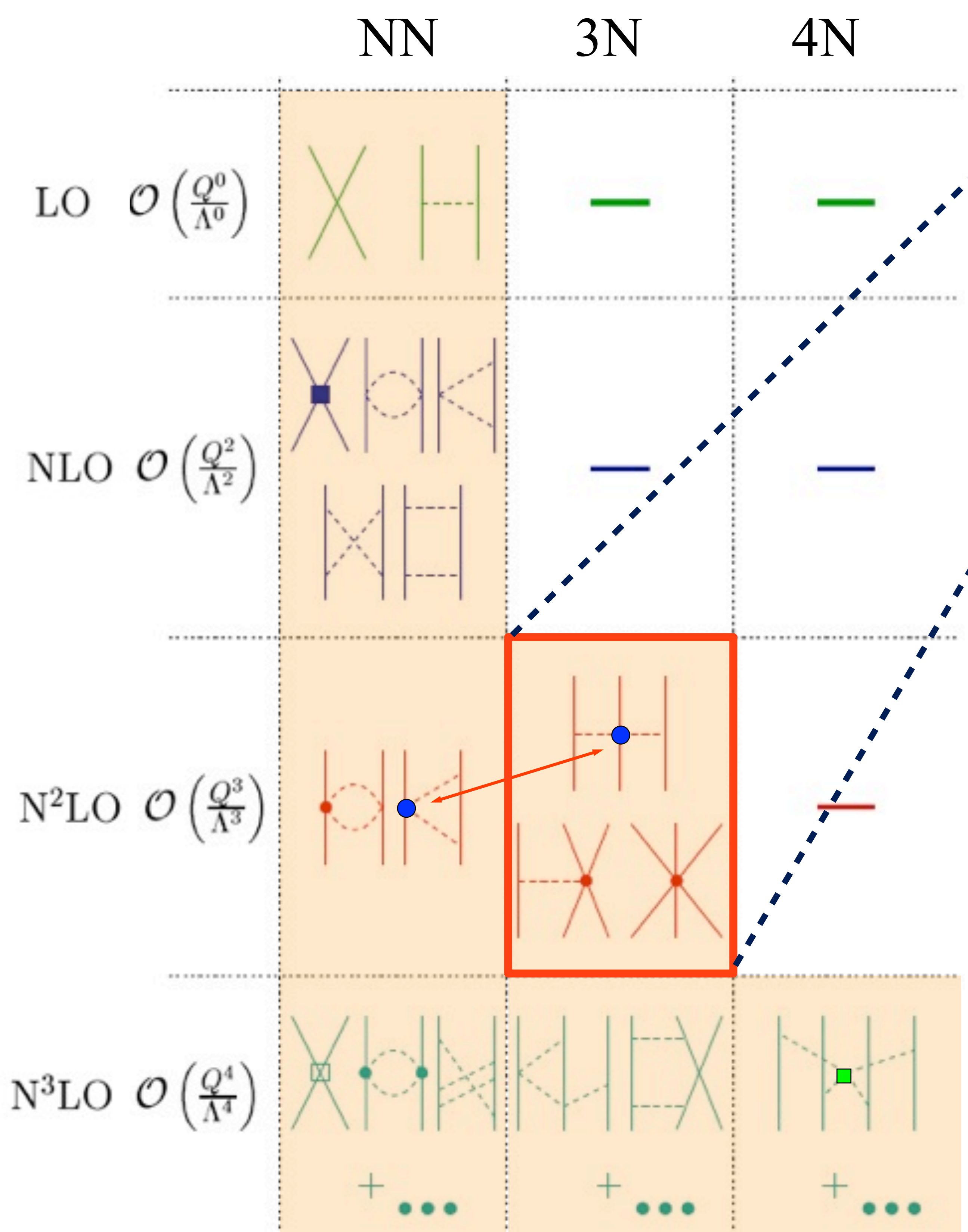
# Changing the resolution: The Similarity Renormalization Group

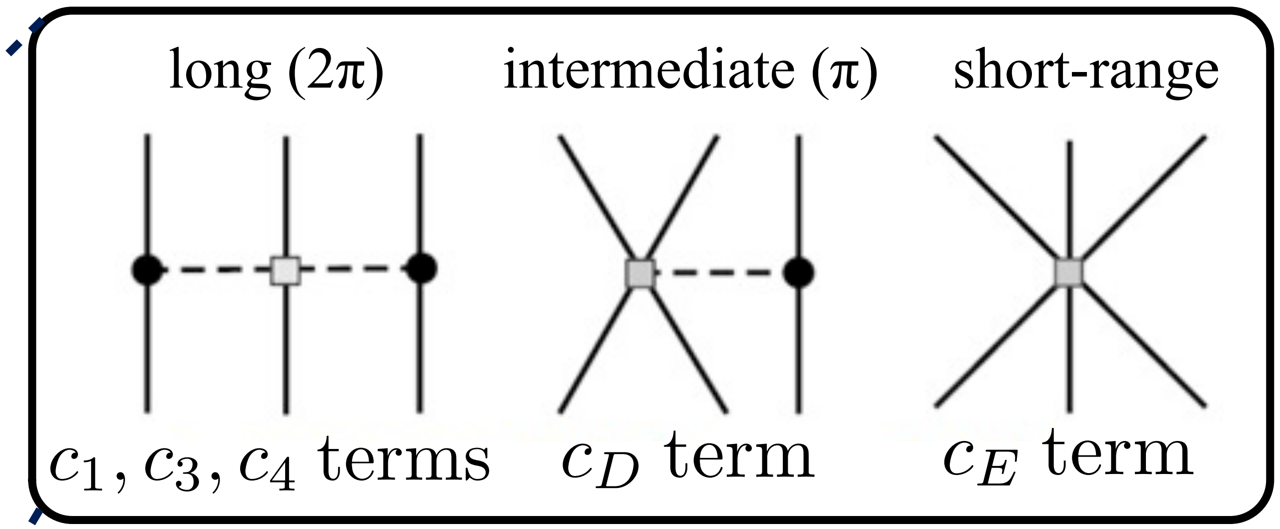



- elimination of coupling between low- and high momentum components, many-body problem more perturbative and tractable
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

# Chiral EFT for nuclear forces, leading order 3N forces



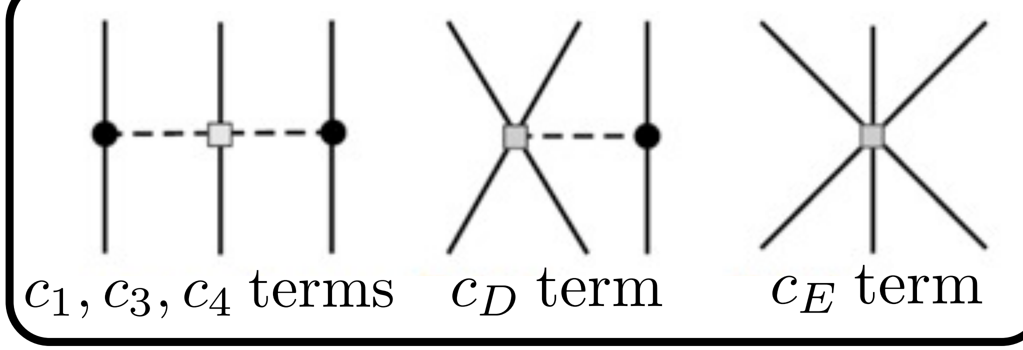

large uncertainties in coupling constants at present:

$$c_1 = -0.9^{+0.2}_{-0.5}\,,\; c_3 = -4.7^{+1.5}_{-1.0}\,,\; c_4 = 3.5^{+0.5}_{-0.2}$$

lead to theoretical uncertainties in many-body observables

use chiral interactions as input for RG evolution

# RG evolution of 3N interactions




- So far (in momentum basis):

intermediate ($c_D$) and short-range ($c_E$) 3NF couplings fitted to few-body systems at different resolution scales:

$E_{^3\text{H}} = -8.482\,\text{MeV}$ and $r_{^4\text{He}} = 1.464\,\text{fm}$

$\longrightarrow$ coupling constants of natural size

- in neutron matter contributions from $c_D$, $c_E$ and $c_4$ terms vanish
- long-range $2\pi$ contributions assumed to be invariant under RG evolution

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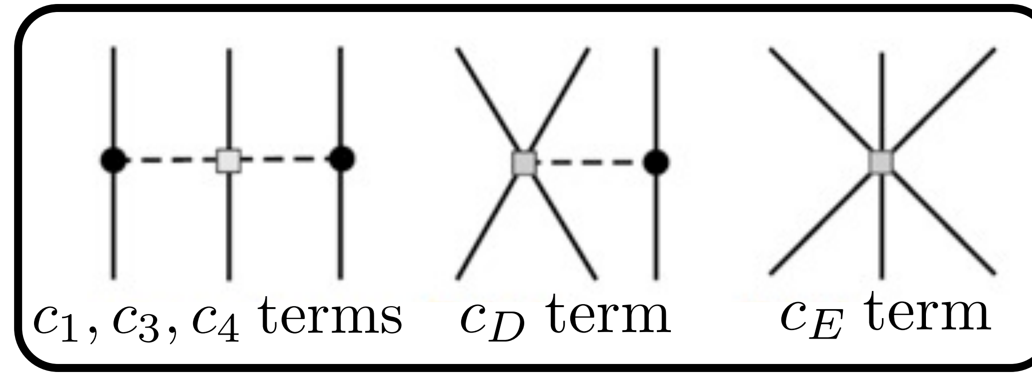


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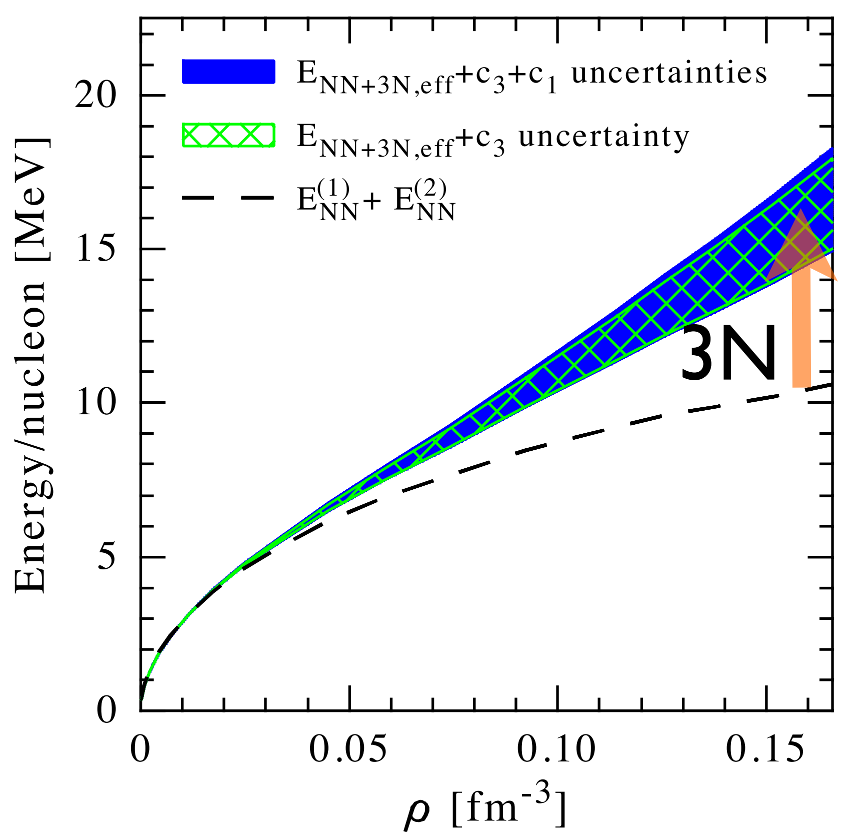
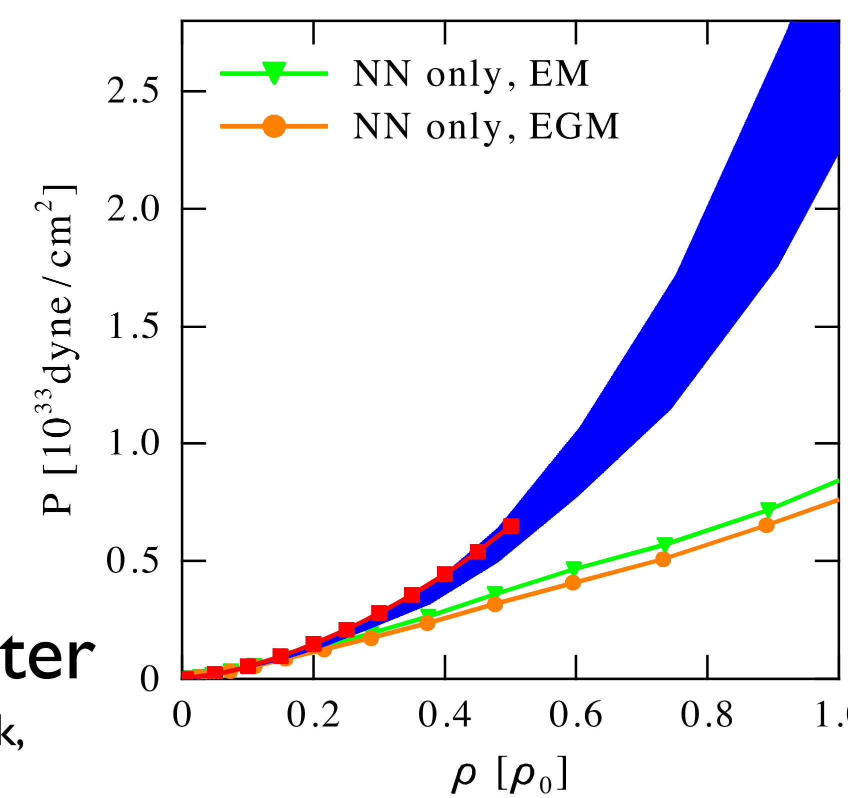



pure neutron matter

KH and Schwenk PRC 82, 014314 (2010)




neutron star matter

KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

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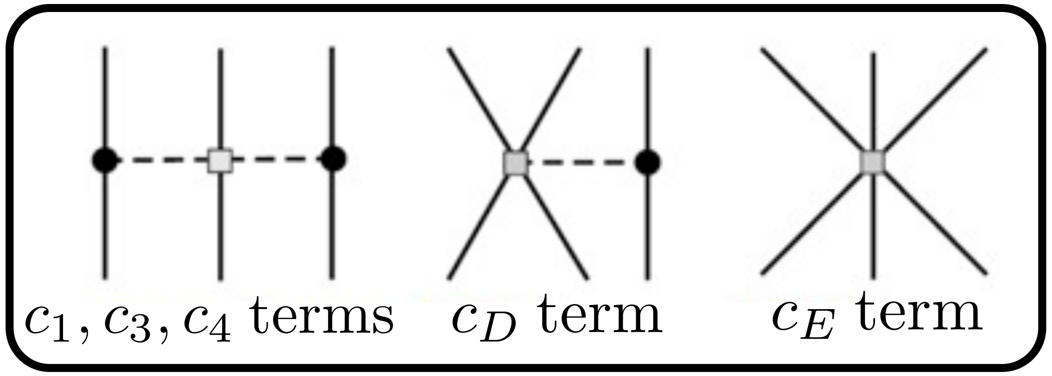


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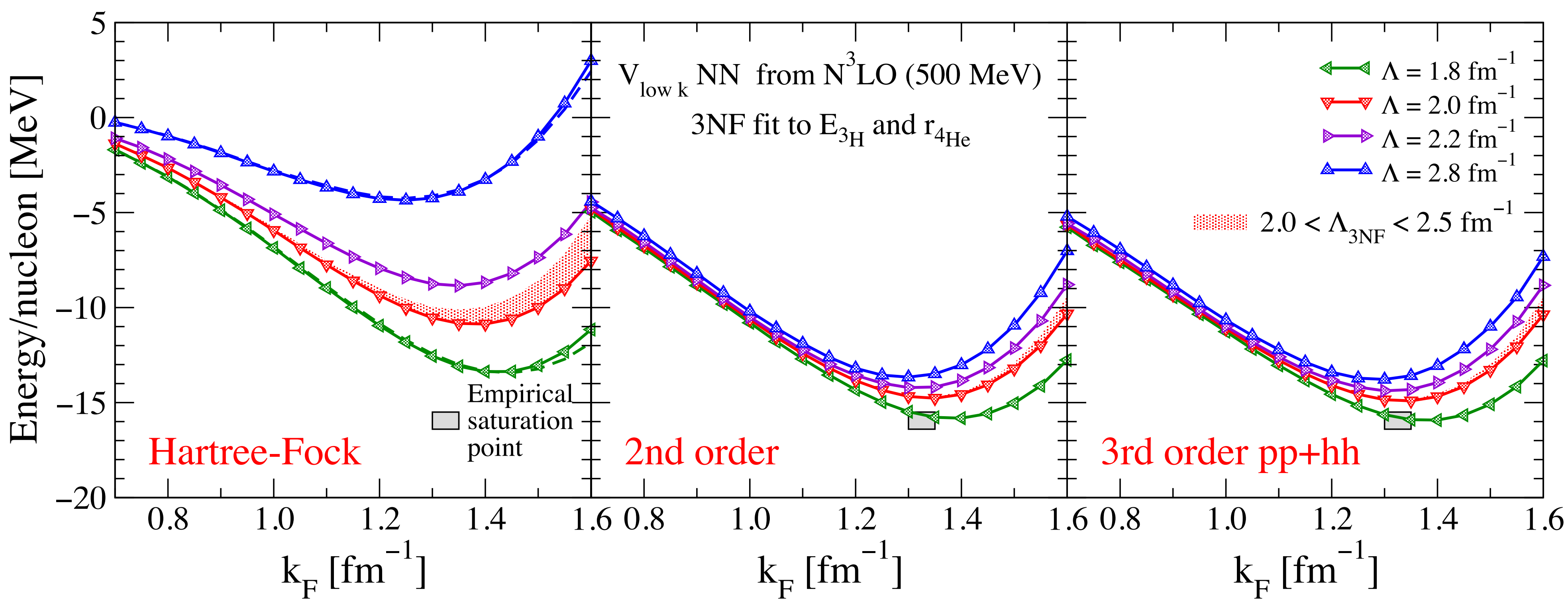
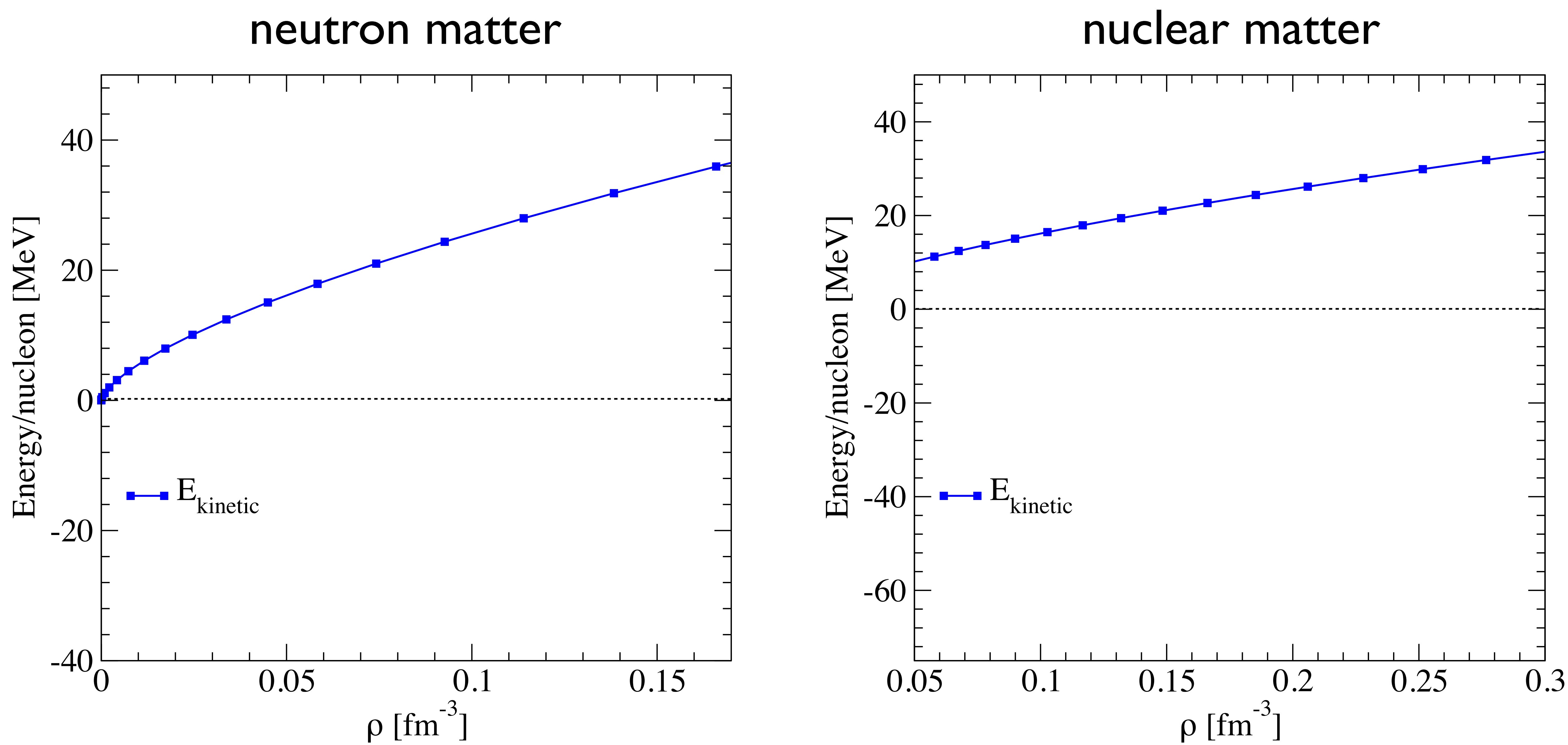



symmetric nuclear matter

KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

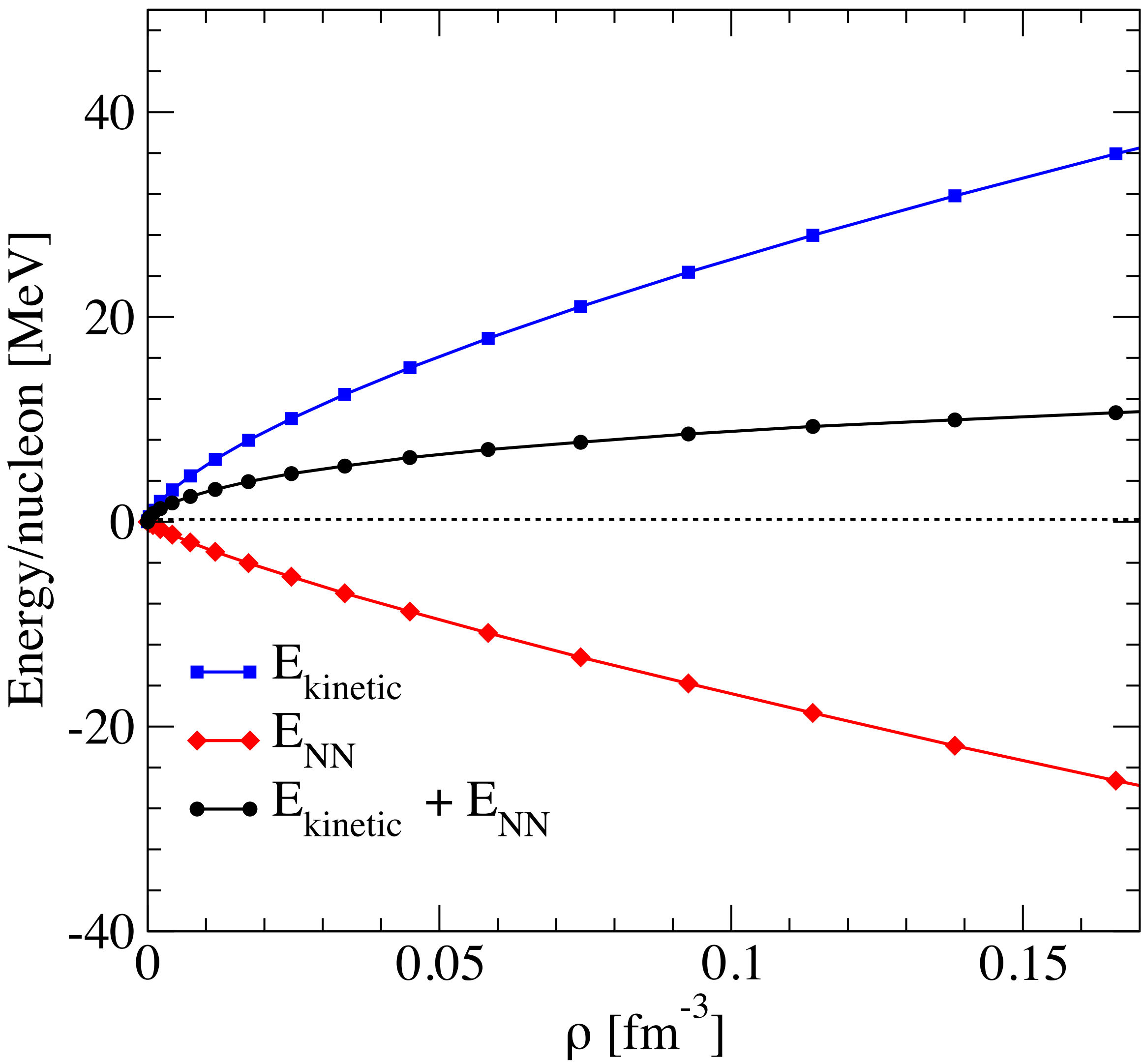
# Hierarchy of many-body contributions



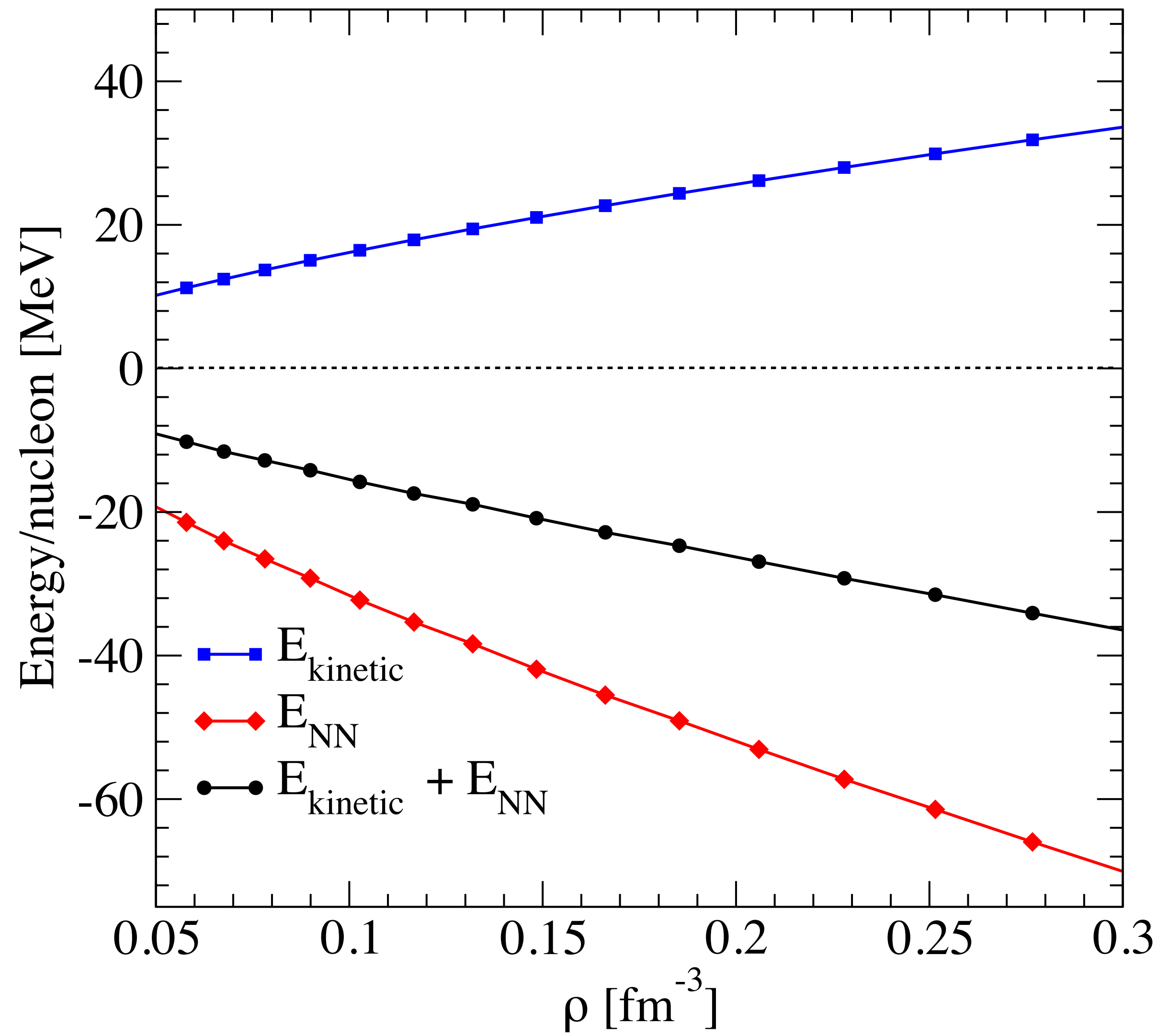

- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

# Hierarchy of many-body contributions

neutron matter




nuclear matter

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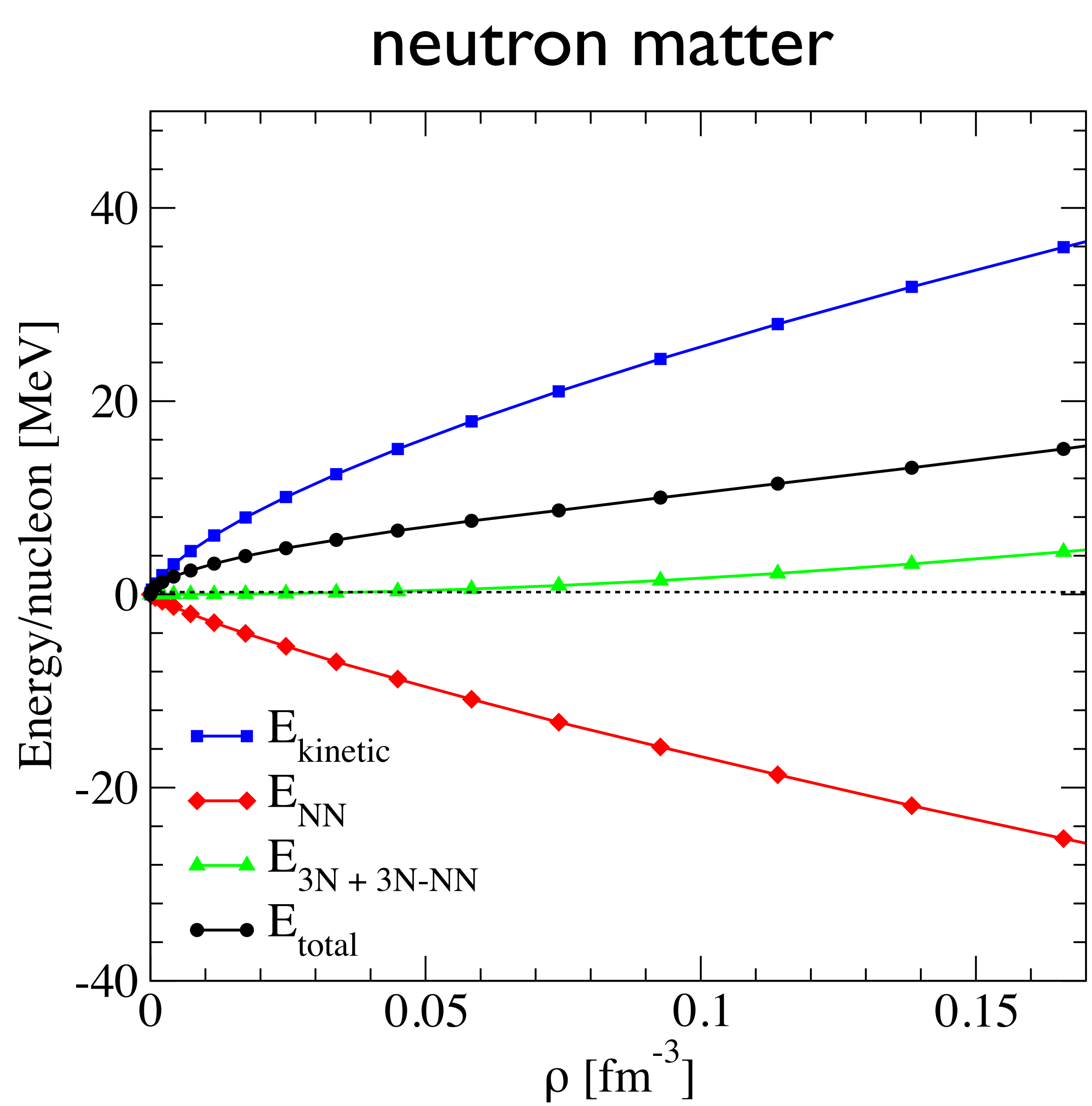


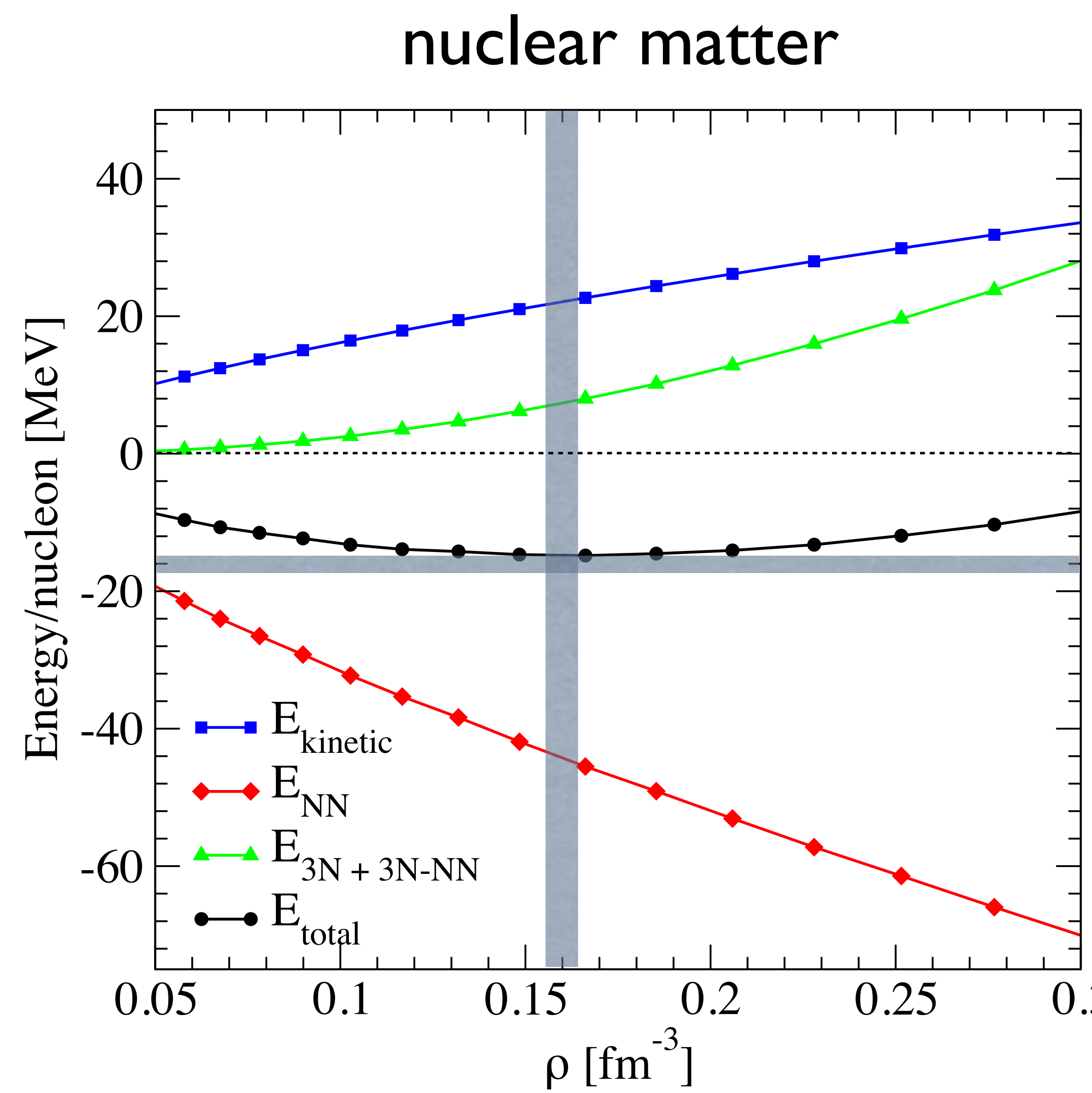


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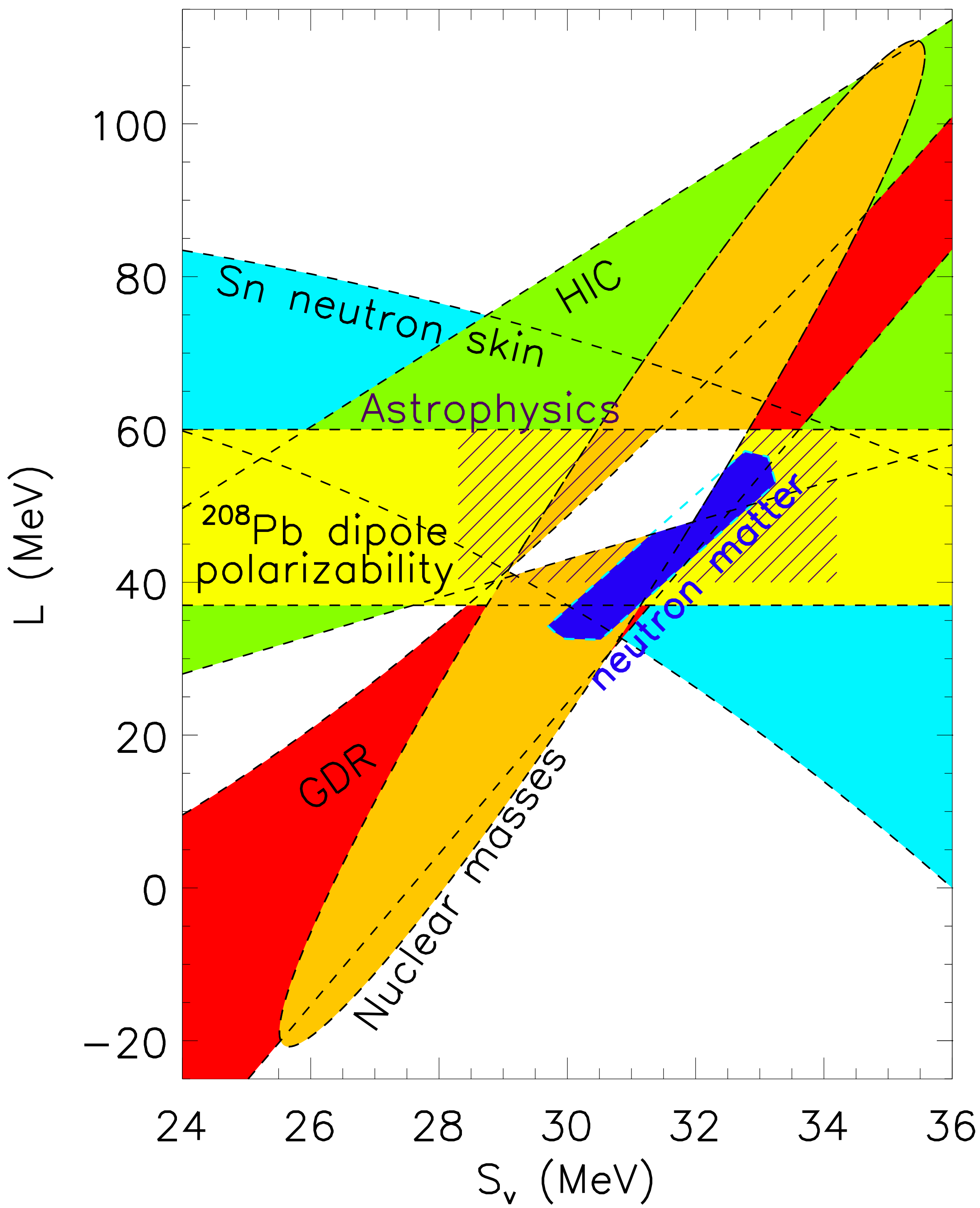
# Symmetry energy constraints




extend EOS to finite proton fractions $x$

and extract symmetry energy parameters

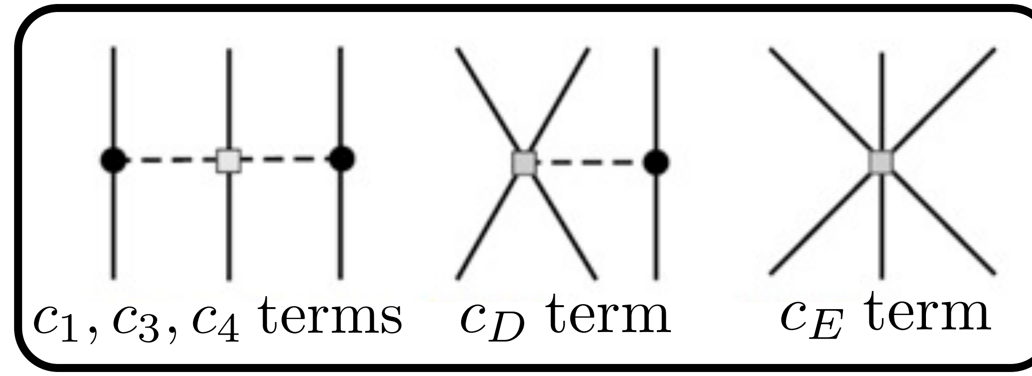
$$S_v = \left.\frac{\partial^2 E/N}{\partial^2 x}\right|_{\rho=\rho_0, x=1/2}$$

$$L = \frac{3}{8} \left.\frac{\partial^3 E/N}{\partial\rho\partial^2 x}\right|_{\rho=\rho_0, x=1/2}$$

KH, Lattimer, Pethick and Schwenk, in preparation

symmetry energy parameters consistent with other constraints

# RG evolution of 3N interactions

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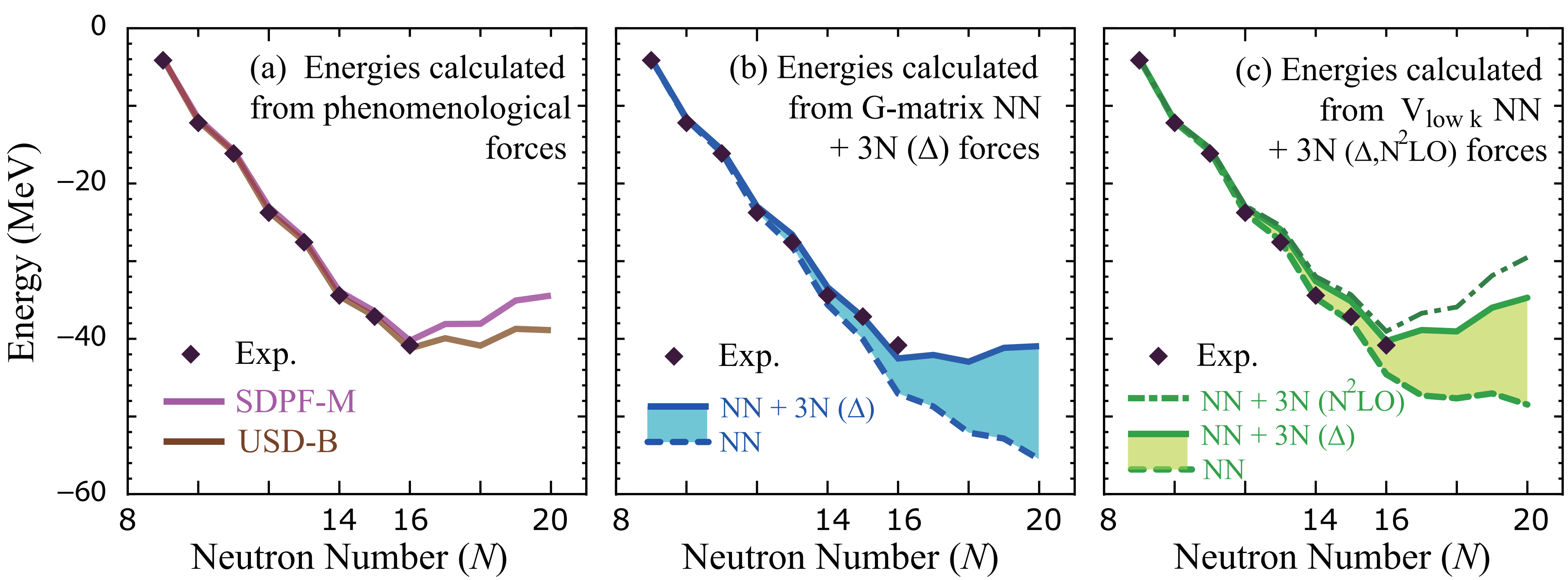


nuclear shell model

Otsuka et al., PRL 105, 032501 (2010)

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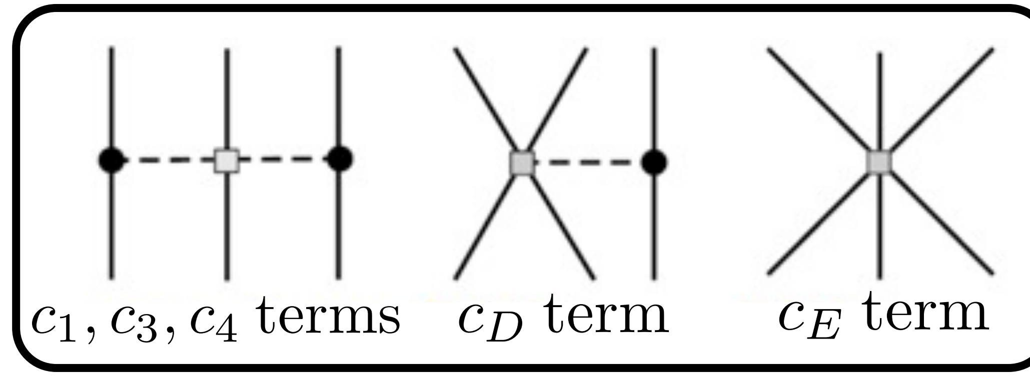


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  - has been achieved in oscillator basis (Jurgenson, Roth)
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  - puzzling effects in heavier nuclei (higher-body forces?)
  - not immediately applicable to infinite systems
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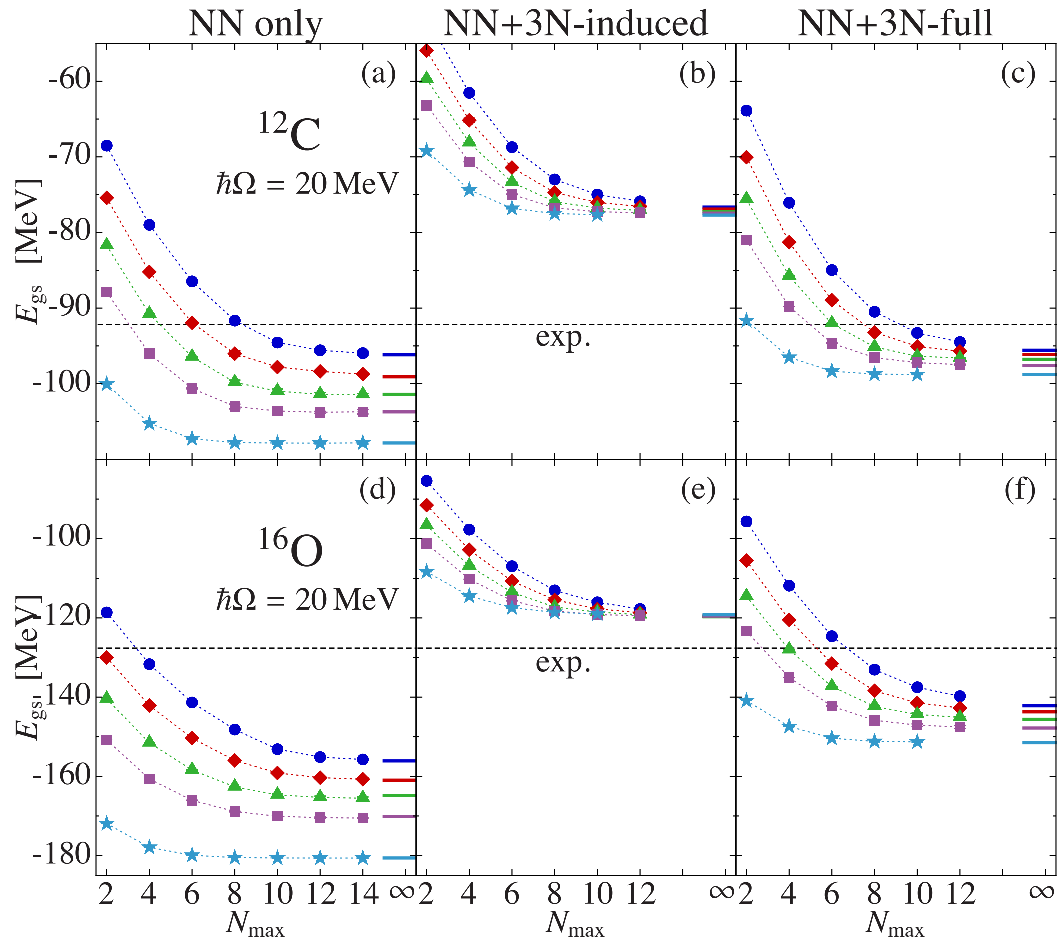


Roth et al. PRL 107, 072501 (2011)

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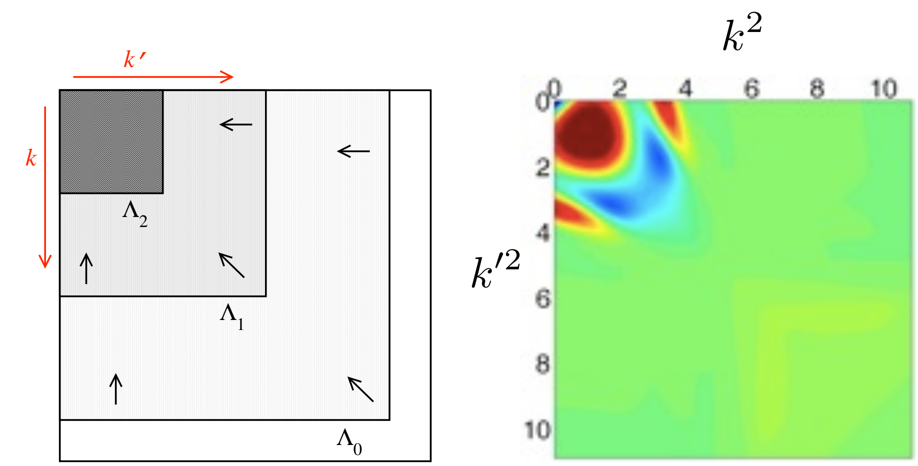
- study of various generators
  - different decoupling patterns (e.g. $V_{\text{low k}}$)
  - improved efficiency of evolution
  - suppression of many-body forces?




Anderson et al. , PRC 77, 037001 (2008)

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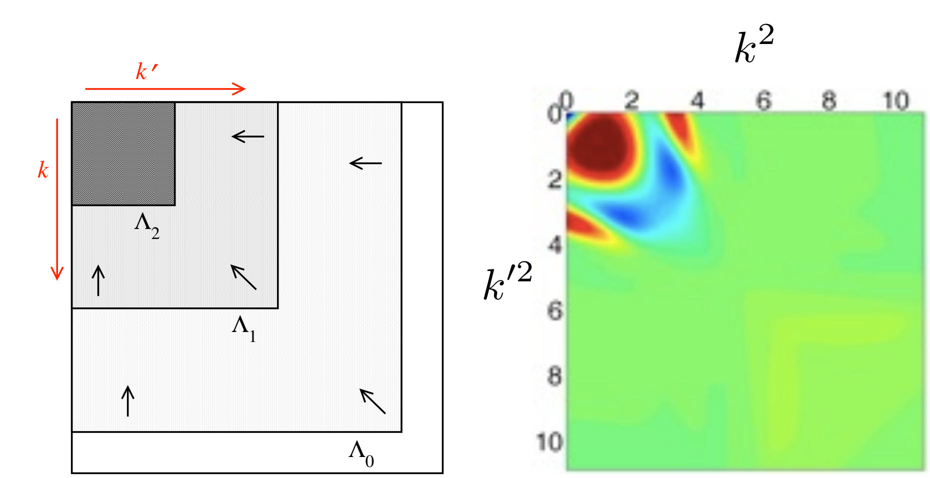


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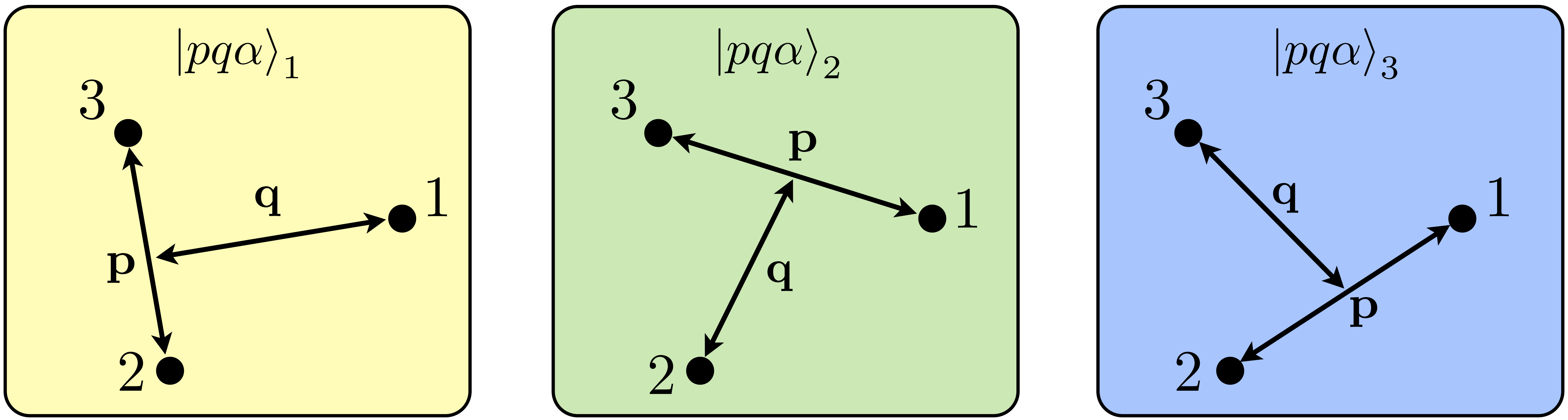
- explicit calculation of unitary 3N transformation
  - RG evolution of operators
  - study of correlations in nuclear systems ⟶ factorization

# RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$$




$${}_i\langle pq\alpha|P|p'q'\alpha'\rangle_i = {}_i\langle pq\alpha|p'q'\alpha'\rangle_j$$

Faddeev bound-state equation:

$$|\psi_i\rangle = G_0\left[2t_iP + (1+t_iG_0)\,V_{3N}^i(1+2P)\right]|\psi_i\rangle$$

# SRG flow equations of NN and 3N forces in momentum basis

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [T_{\rm rel}, H_s]$$

$$H = T + V_{12} + V_{13} + V_{23} + V_{123}$$

- spectators correspond to delta functions, matrix representation of $H_s$ ill-defined
- solution: explicit separation of NN and 3N flow equations

$$\begin{aligned}
\frac{dV_{ij}}{ds} &= \left[\left[T_{ij}, V_{ij}\right], T_{ij} + V_{ij}\right], \\
\frac{dV_{123}}{ds} &= \left[\left[T_{12}, V_{12}\right], V_{13} + V_{23} + V_{123}\right] \\
&+ \left[\left[T_{13}, V_{13}\right], V_{12} + V_{23} + V_{123}\right] \\
&+ \left[\left[T_{23}, V_{23}\right], V_{12} + V_{13} + V_{123}\right] \\
&+ \left[\left[T_{\rm rel}, V_{123}\right], H_s\right]
\end{aligned}$$

- only connected terms remain in $\dfrac{dV_{123}}{ds}$, 'dangerous' delta functions cancel

see Bogner, Furnstahl, Perry PRC 75, 061001(R) (2007)

# SRG evolution in momentum space

- evolve the antisymmetrized 3N interaction

*special thanks to*
*J. Golak, R. Skibinski, K. Topolnicki*

$$\overline{V}_{123} = {}_i\langle pq\alpha | \, (1 + P_{123} + P_{132}) V_{123}^{(i)} (1 + P_{123} + P_{132}) \, | p'q'\alpha' \rangle_i$$

- embed NN interaction in 3N basis:

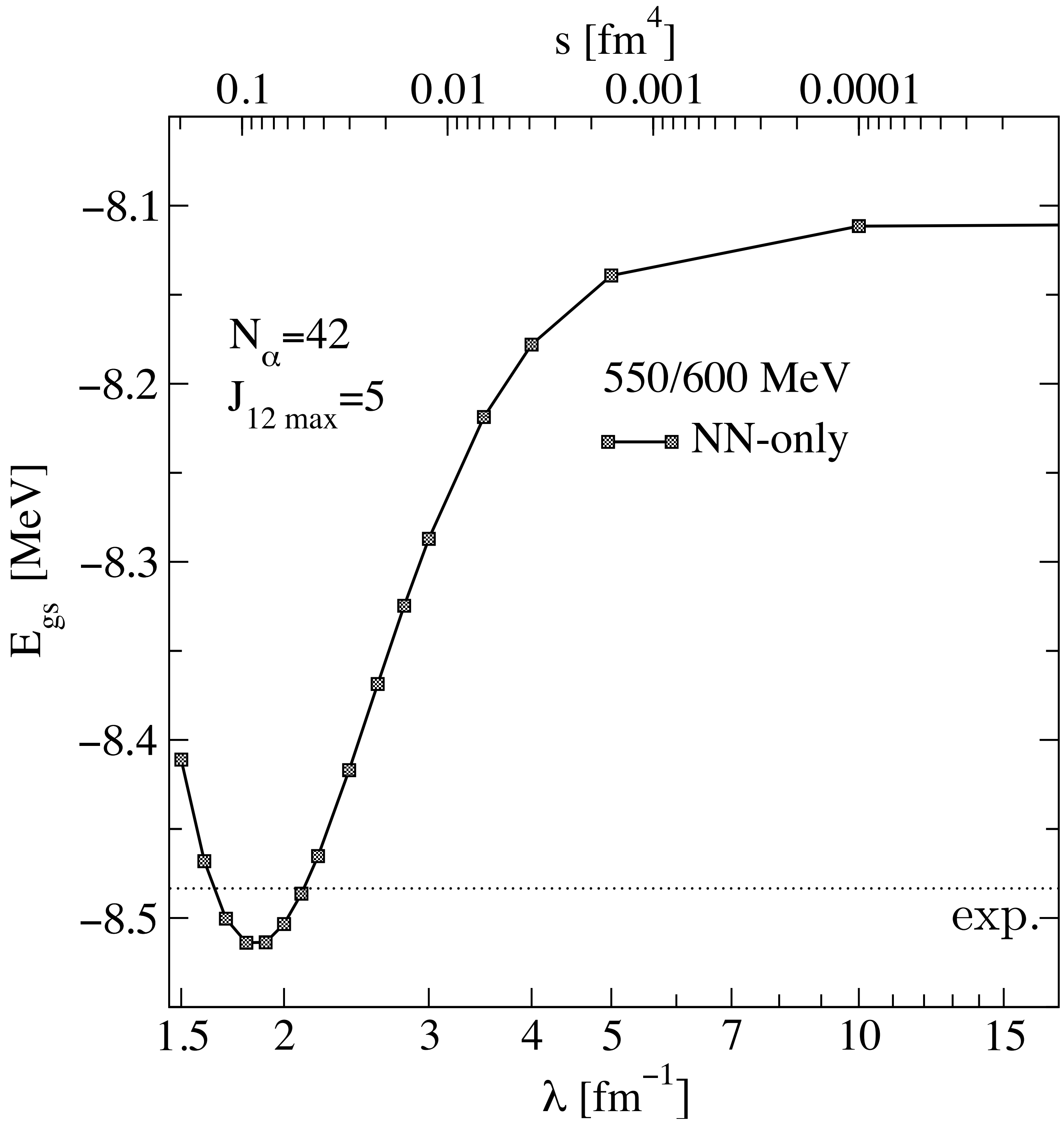
$$V_{13} = P_{123} V_{12} P_{132}, \quad V_{23} = P_{132} V_{12} P_{123}$$

with $${}_3\langle pq\alpha | V_{12} | p'q'\alpha' \rangle_3 = \langle p\tilde{\alpha} | V_{\rm NN} | p'\tilde{\alpha}' \rangle \, \delta(q - q')/q^2$$

- use $P_{123} \overline{V}_{123} = P_{132} \overline{V}_{123} = \overline{V}_{123}$

$$\Rightarrow \quad d\overline{V}_{123}/ds = C_1(s, T, V_{\rm NN}, P) + C_2(s, T, V_{\rm NN}, \overline{V}_{123}, P) + C_3(s, T, \overline{V}_{123})$$

# SRG evolution of 3N interactions in momentum space: Results for the Triton




Hebeler PRC(R) 85, 021002 (2012)

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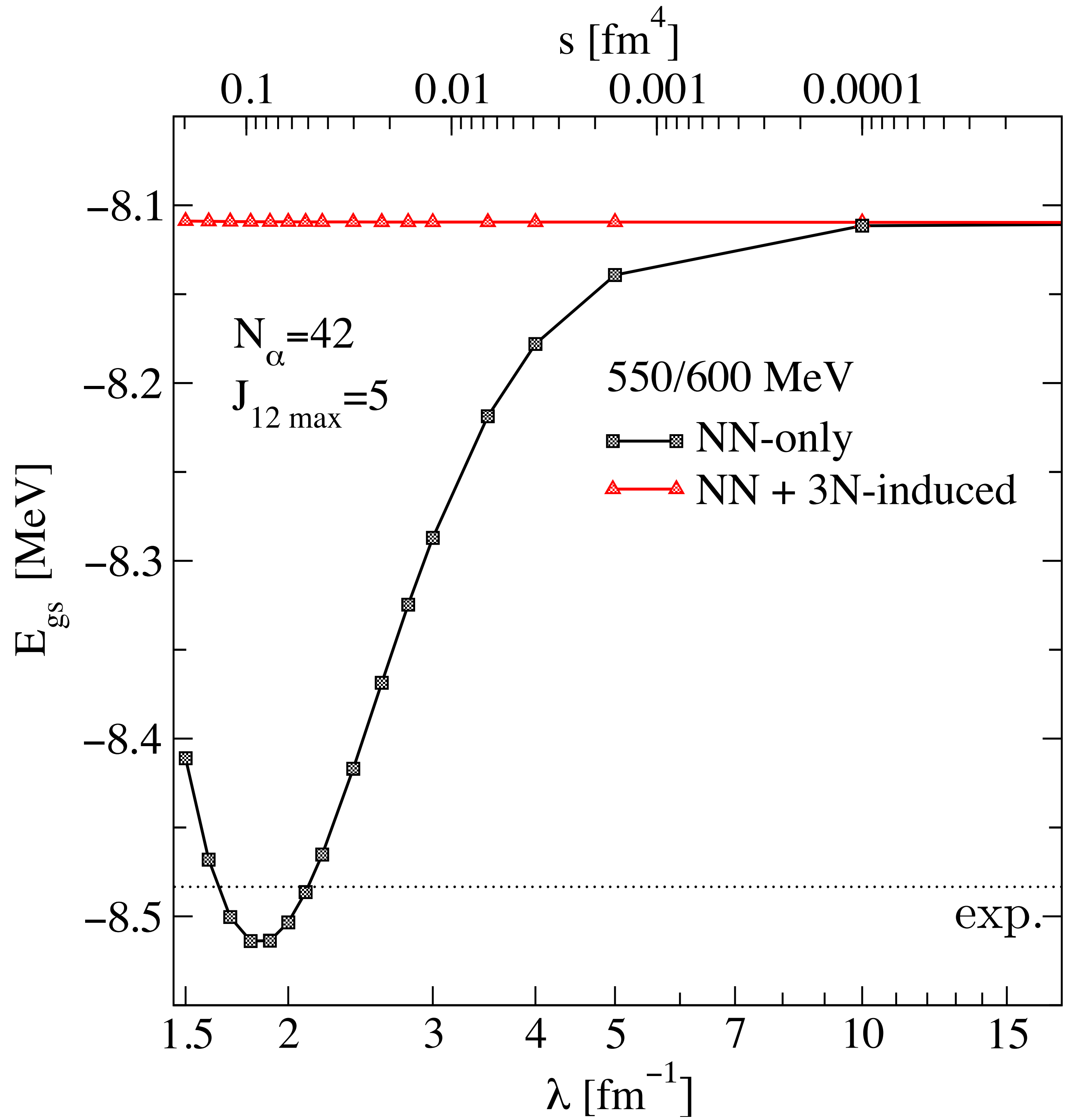


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It works:

Invariance of $E_{\rm gs}^{^3H}$ within $\leq 1\,\mathrm{eV}$ for consistent chiral interactions at $\mathrm{N^2LO}$

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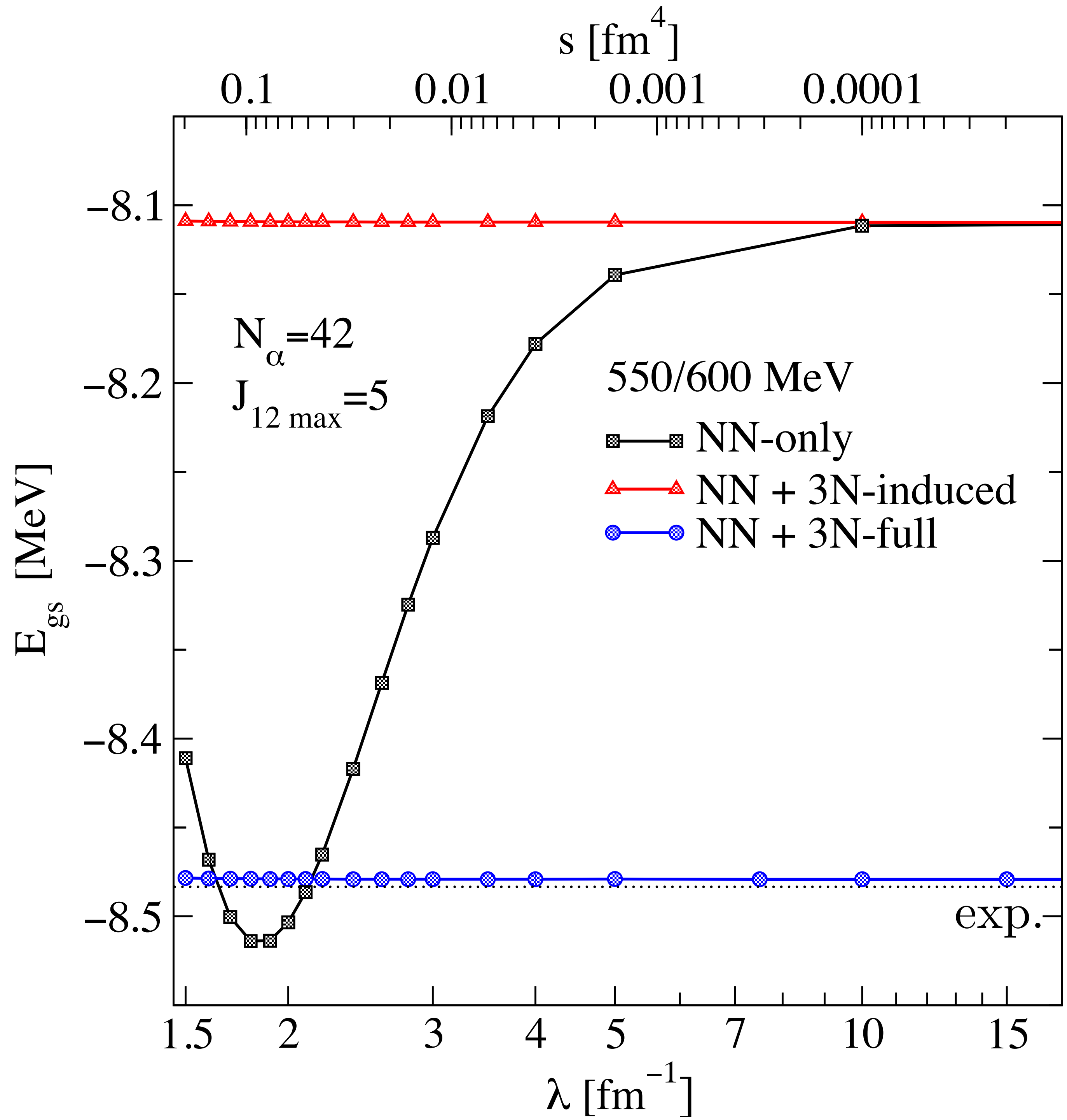


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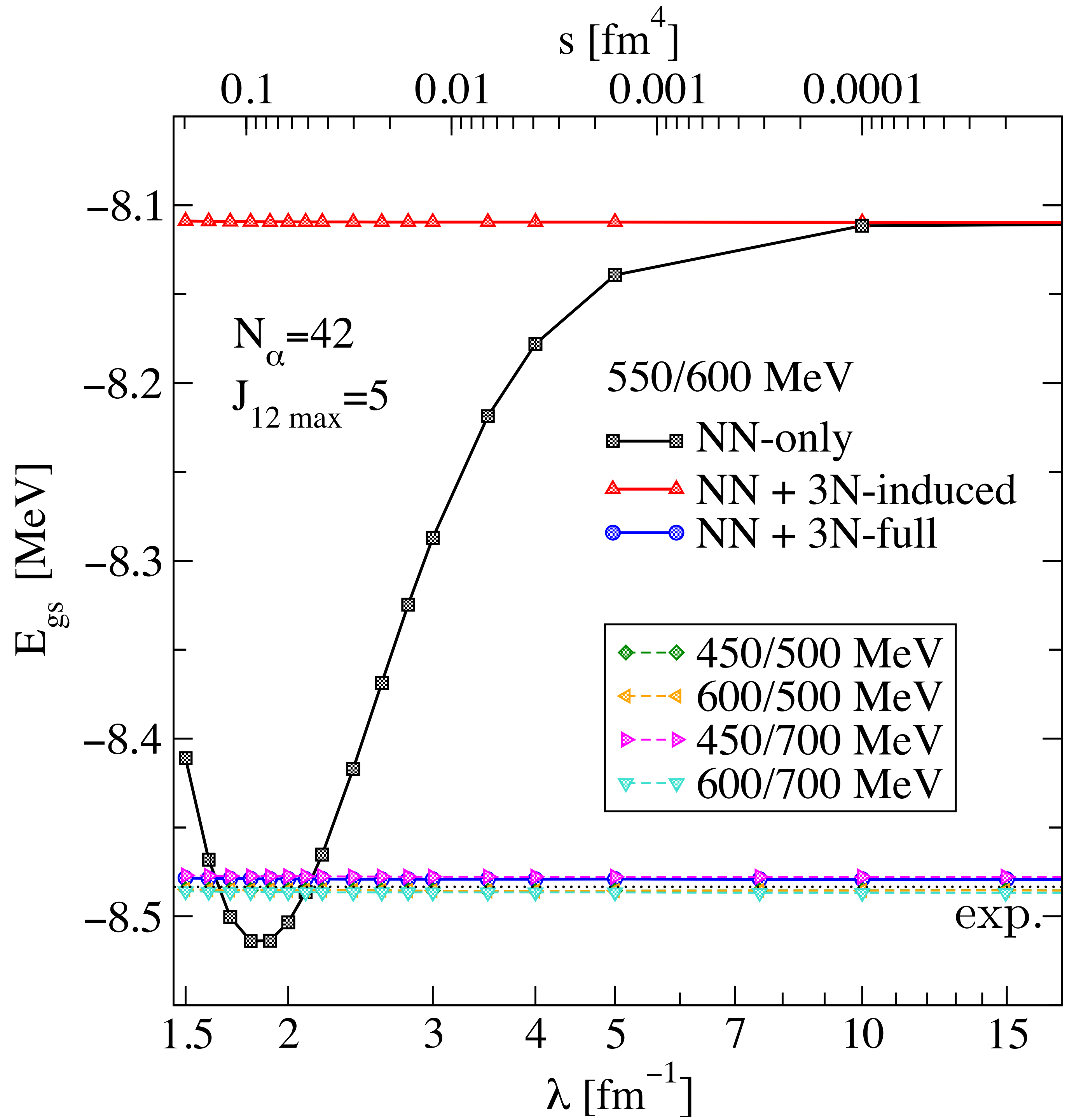


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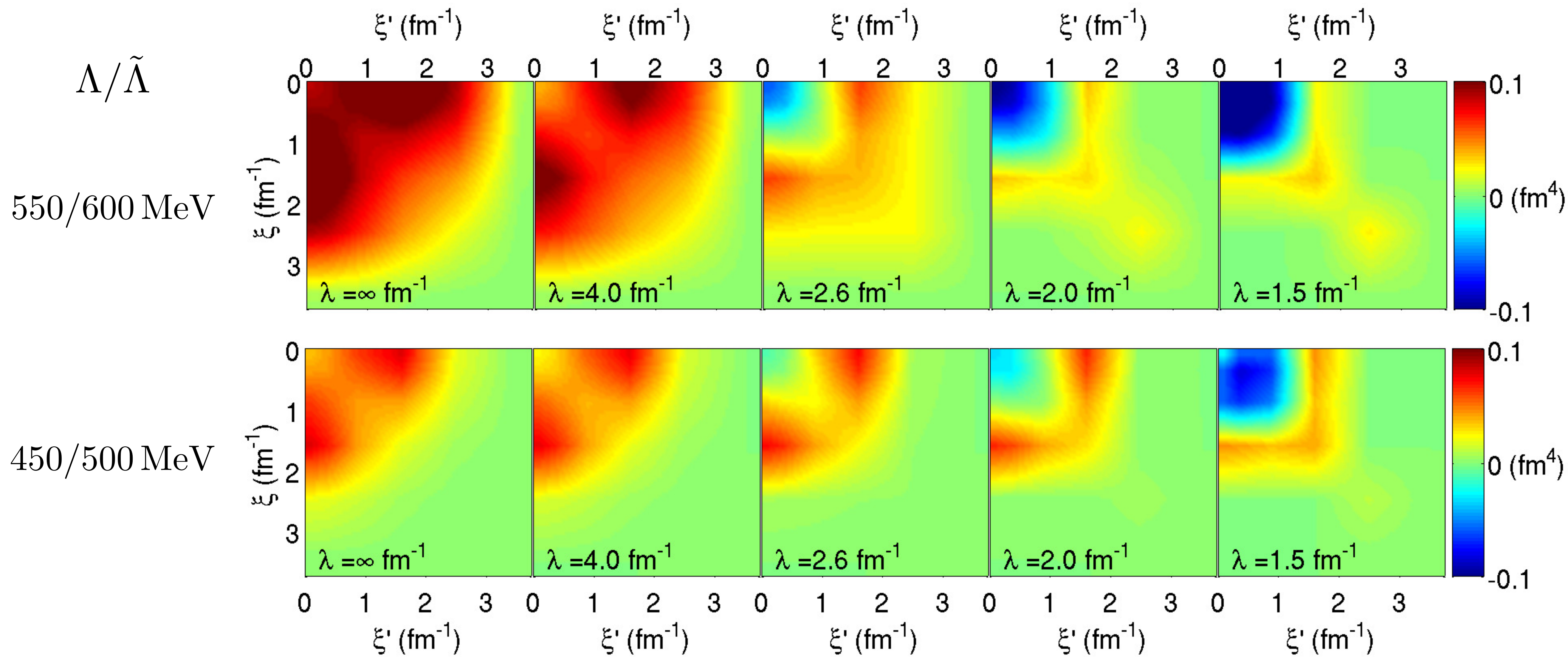
# Decoupling of matrix elements

$$\theta = \frac{\pi}{12} \qquad \mathcal{T} = \mathcal{J} = \frac{1}{2}$$




KH, PRC(R) 85, 021002 (2012)

hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$ hyperangle: $\tan\theta = \frac{2\,p}{\sqrt{3}\,q}$

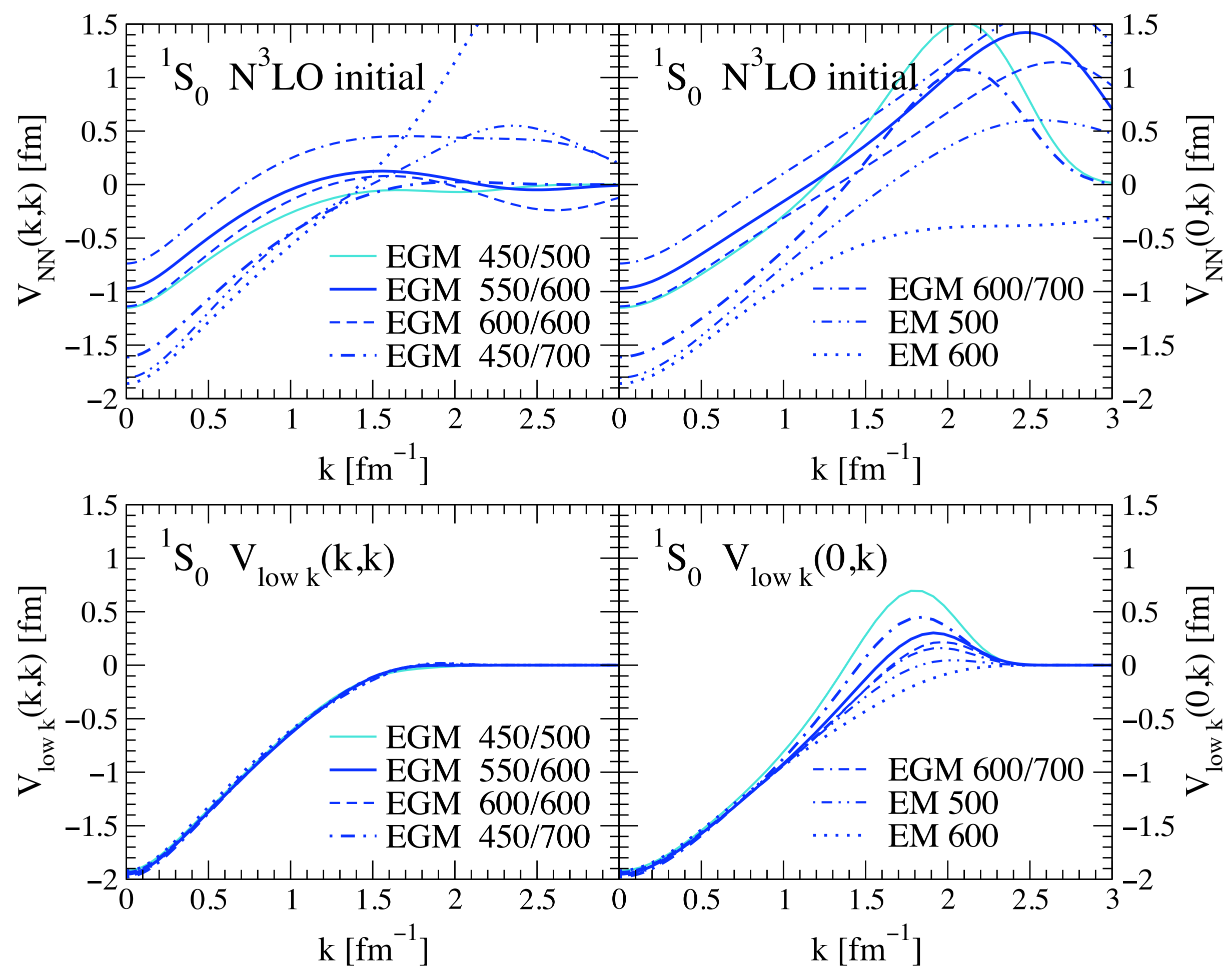
same decoupling patterns like in NN interactions

# Universality in 3N interactions at low resolution

phase-shift equivalence

common long-range physics

(approximate) universality of low-resolution NN interactions




## To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants $c_D$ and $c_E$
- 3N interactions give only subleading contributions to observables

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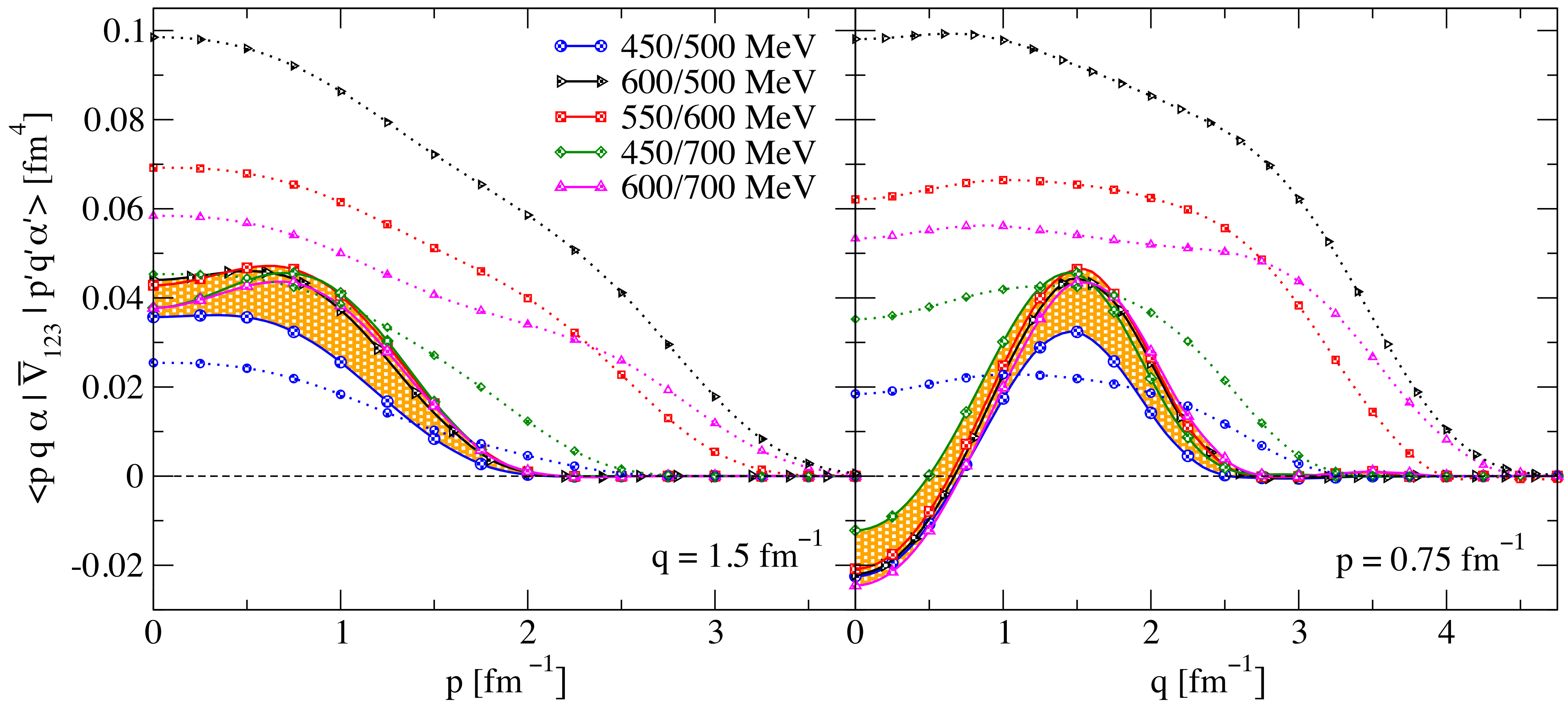


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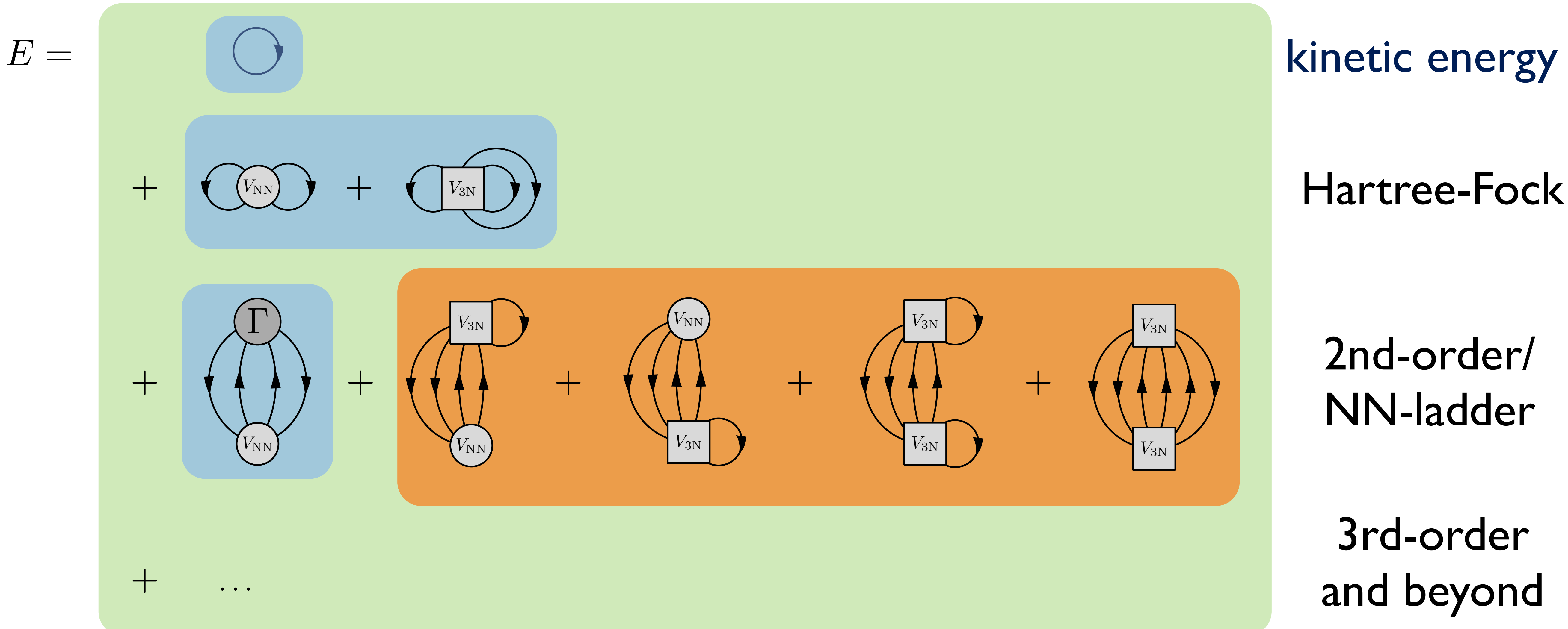
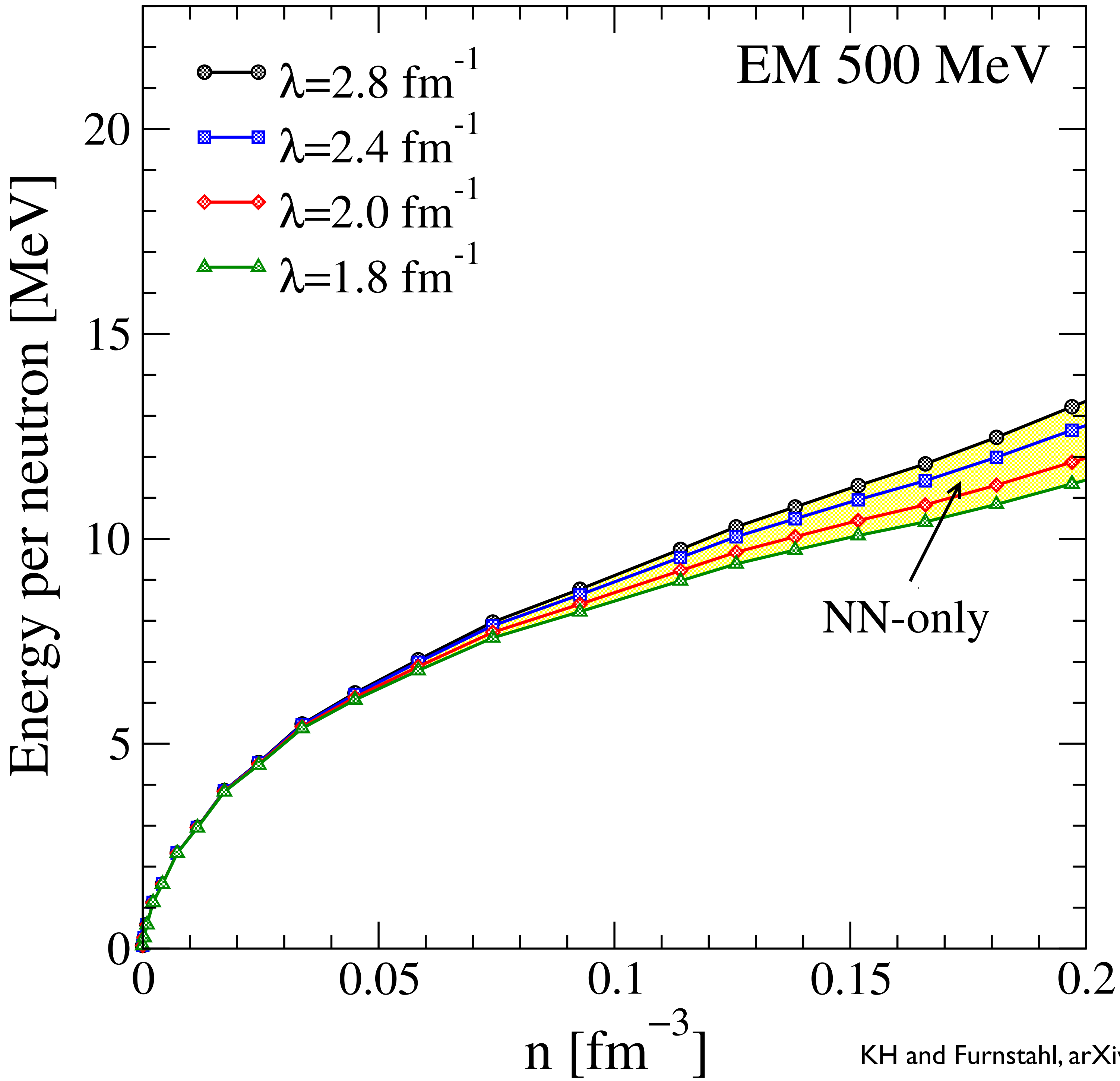
- remarkably reduced scheme dependence for typical momenta $\sim 1\,\mathrm{fm}^{-1}$, matrix elements with significant phase space well constrained at low resolution
- new momentum structures induced at low resolution
- study based on $\mathrm{N^2LO}$ chiral interactions, improved universality at $\mathrm{N^3LO}$ ?

# First application to neutron matter: Equation of state




- evolve consistently NN + 3NF in the isospin $\mathcal{T} = 3/2$ channel
- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

# First results for neutron matter




KH and Furnstahl, arXiv:1301.7467

- all partial waves included up to $\mathcal{J} = 9/2$ in SRG evolution and EOS calculation
- consistent 3NF with $c_1 = -0.81\,\mathrm{GeV}^{-1}$ and $c_3 = -3.2\,\mathrm{GeV}^{-1}$

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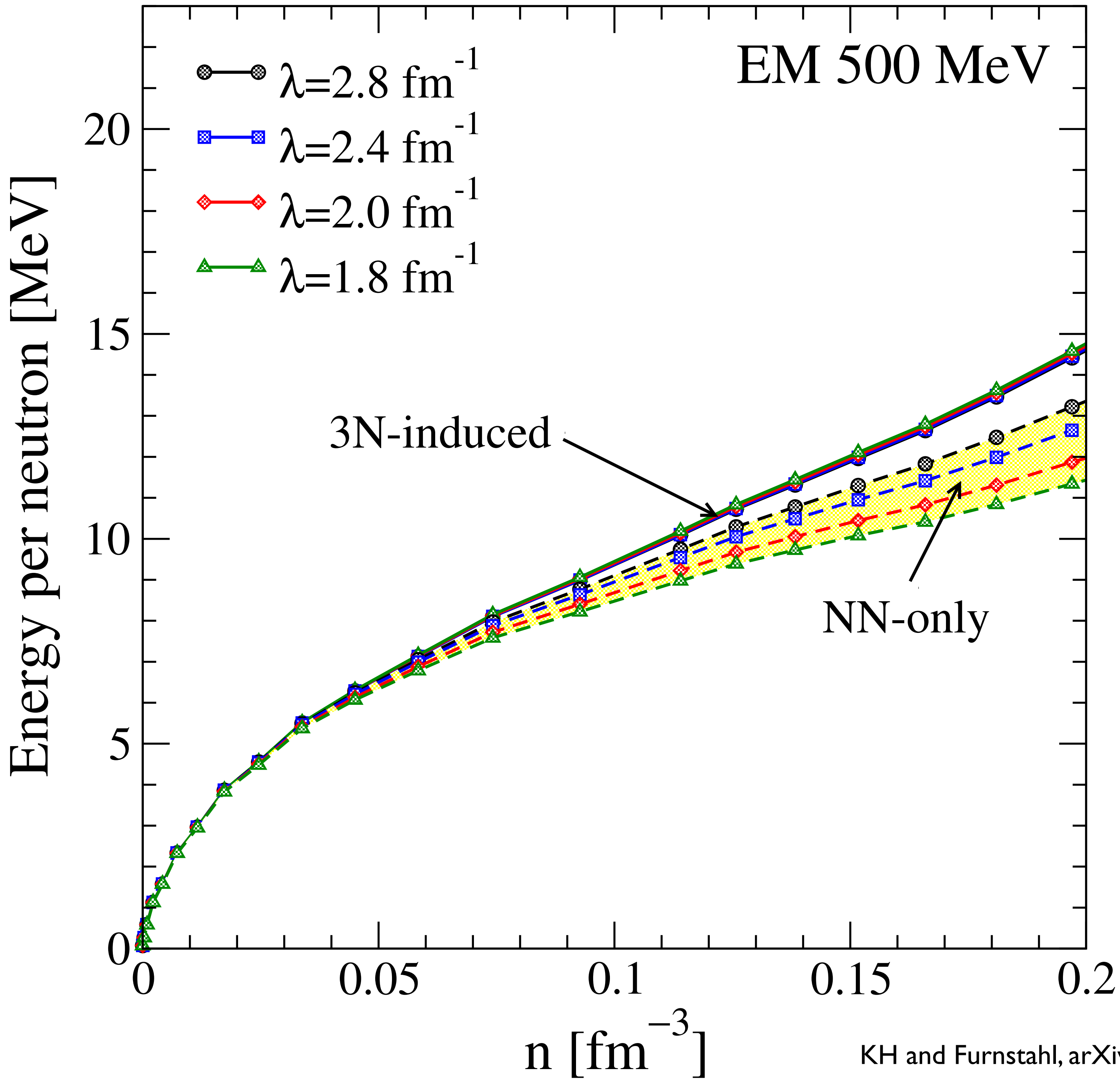


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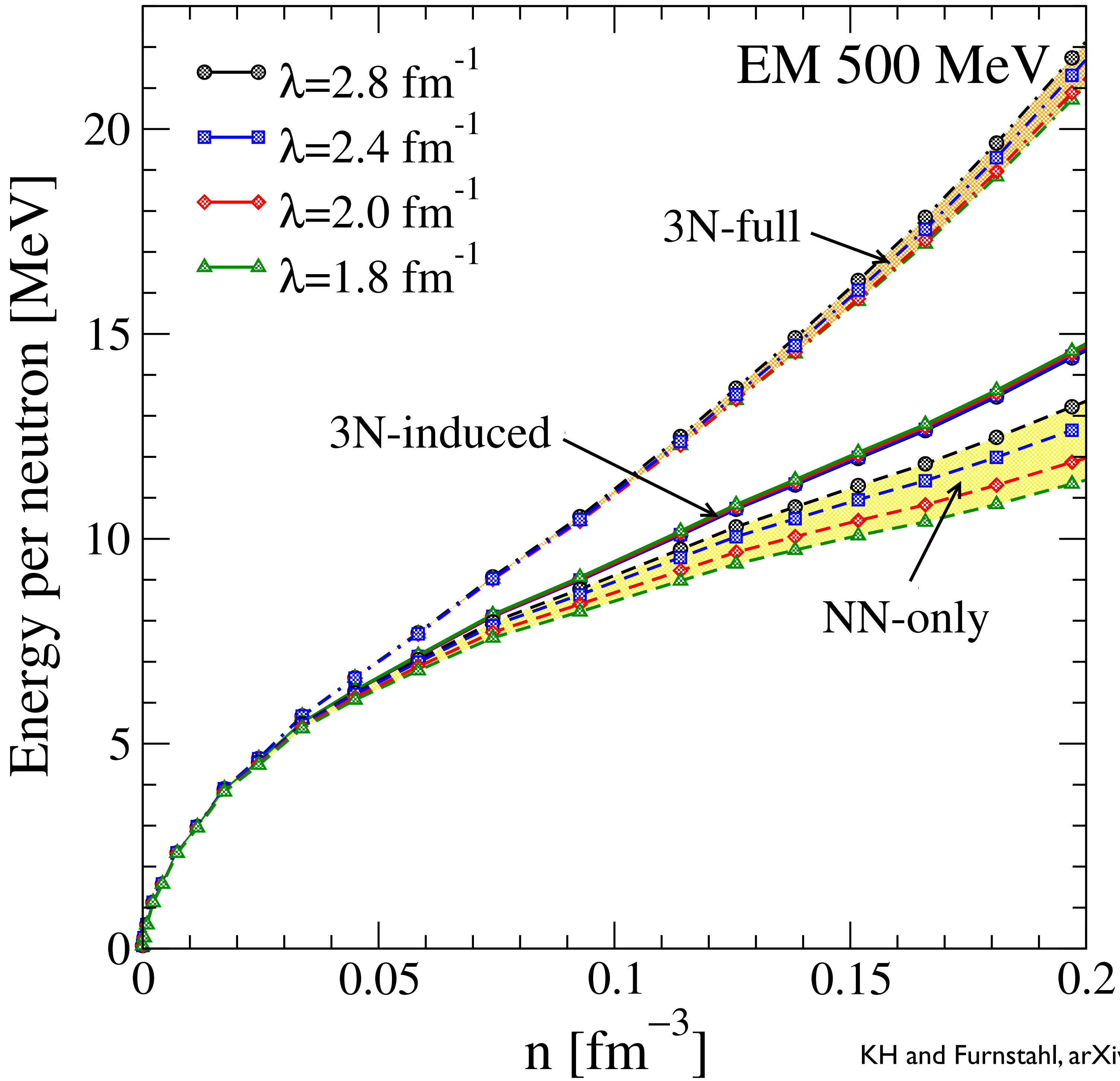


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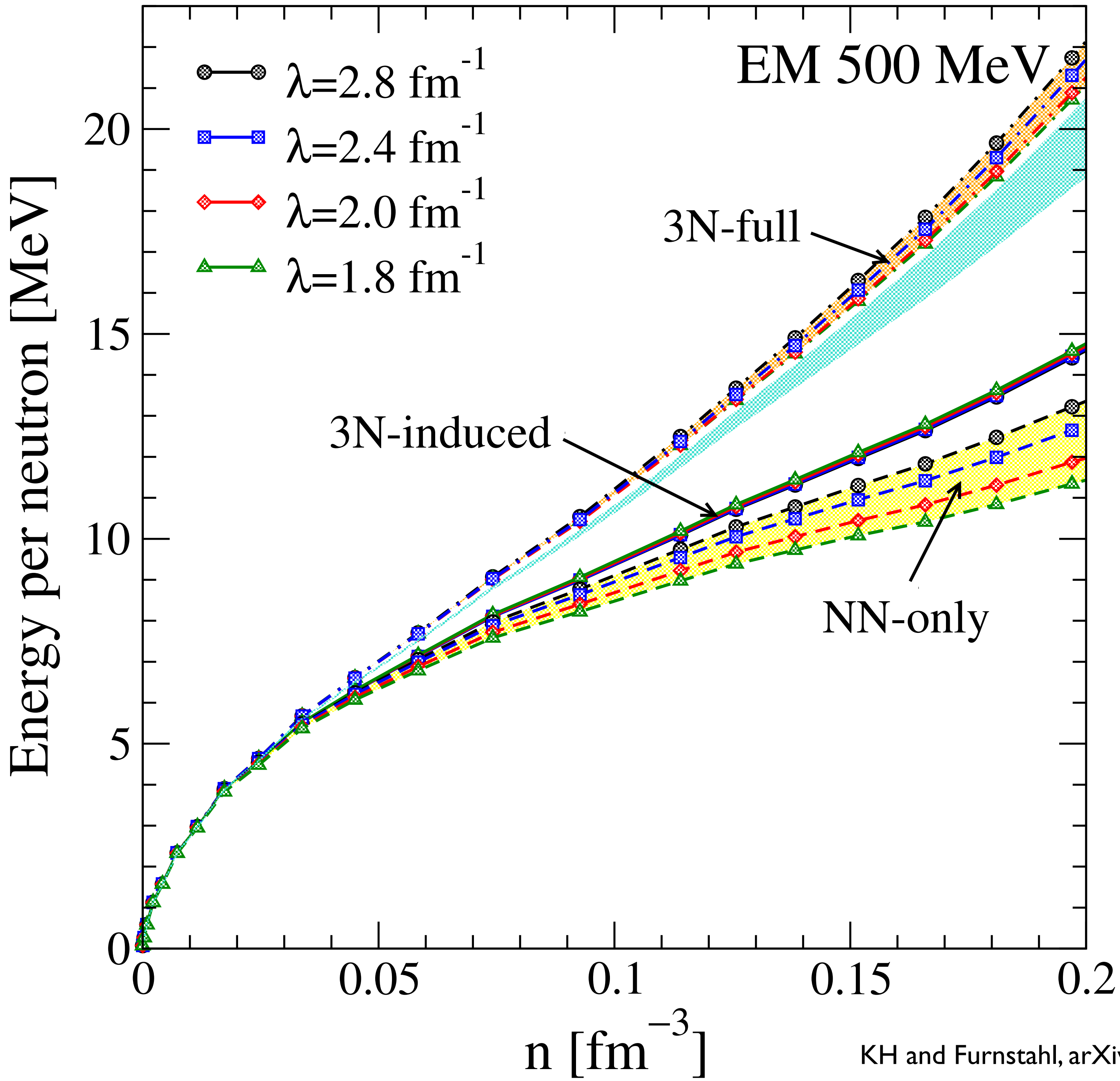


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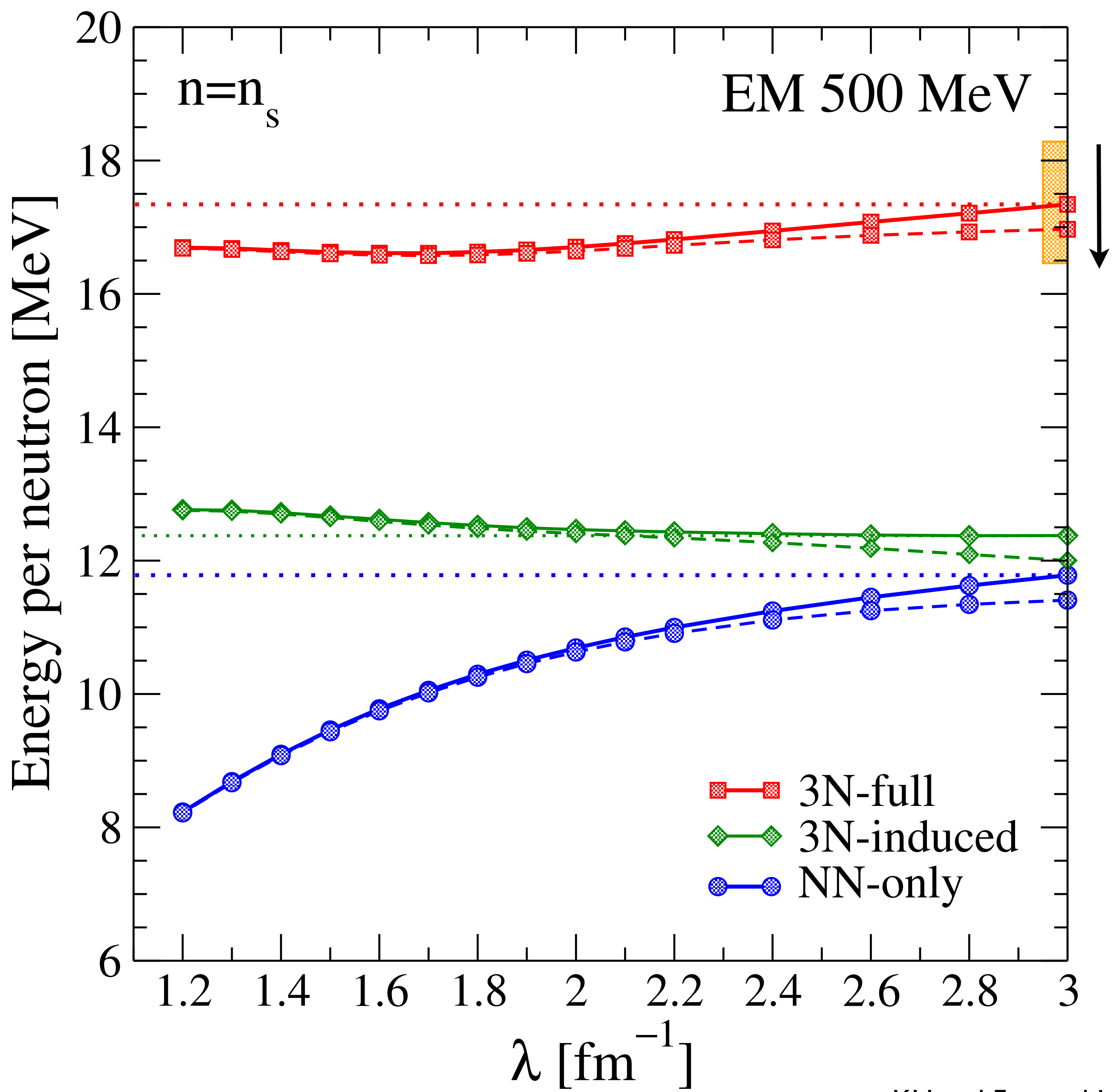


2nd order NN-3N contributions at $\lambda = \infty$

KH and Furnstahl, arXiv:1301.7467

- solid lines: NN resummed, dashed lines: 2nd order
- variations: NN-only 3.6 MeV, 3N-induced: 390 keV, 3N-full: 650 keV
- indications for 4N forces at small $\lambda$?

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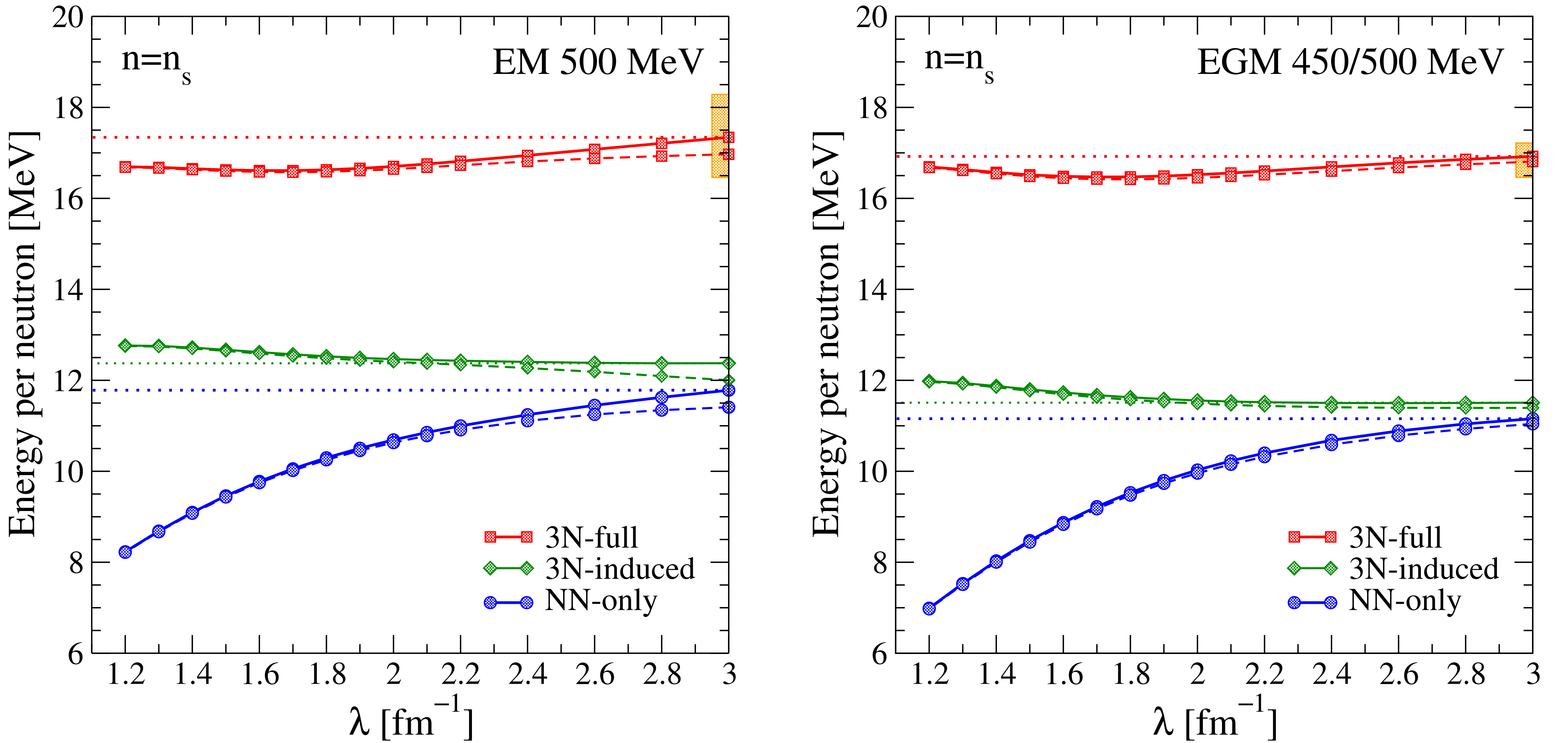


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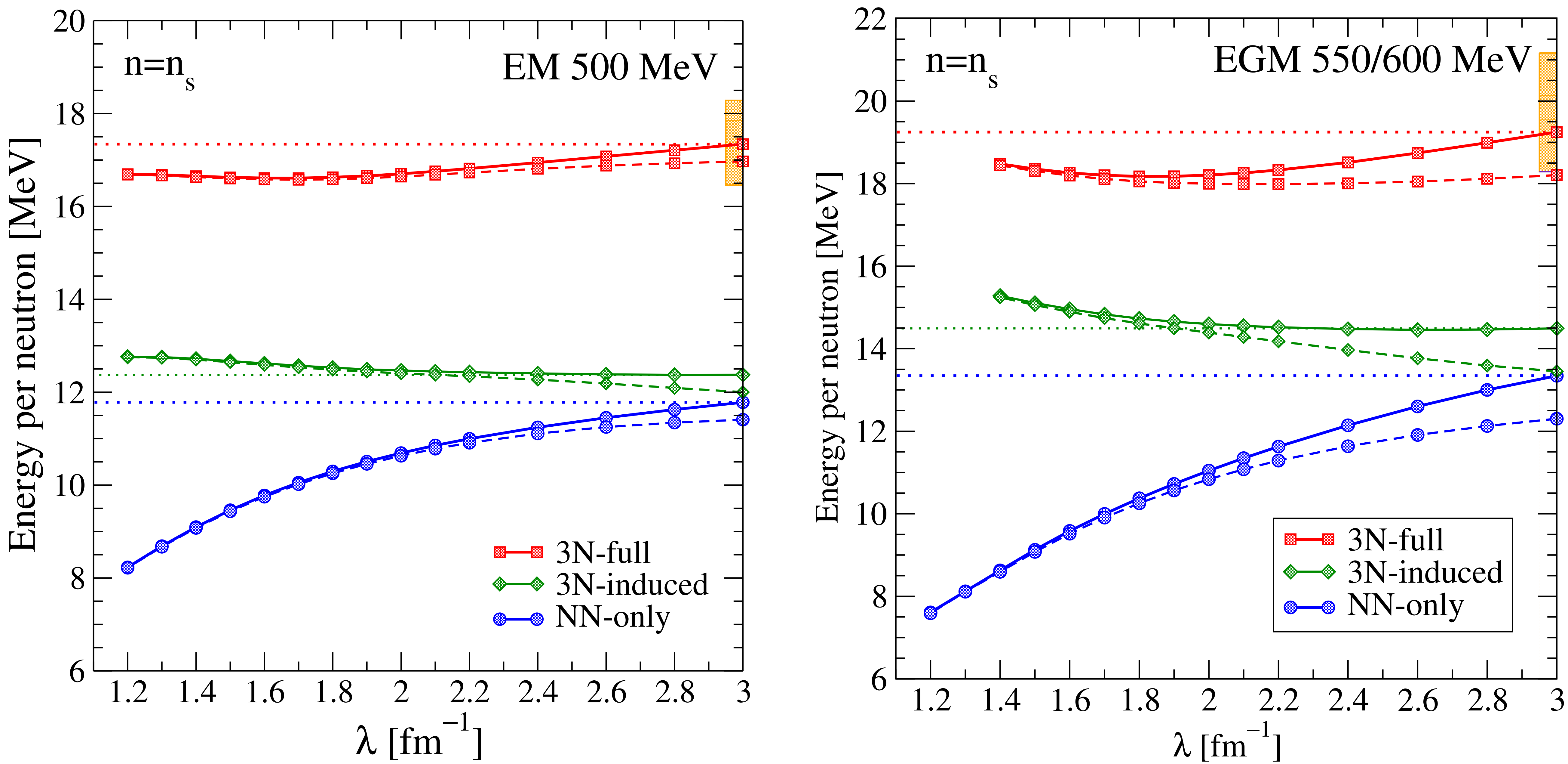
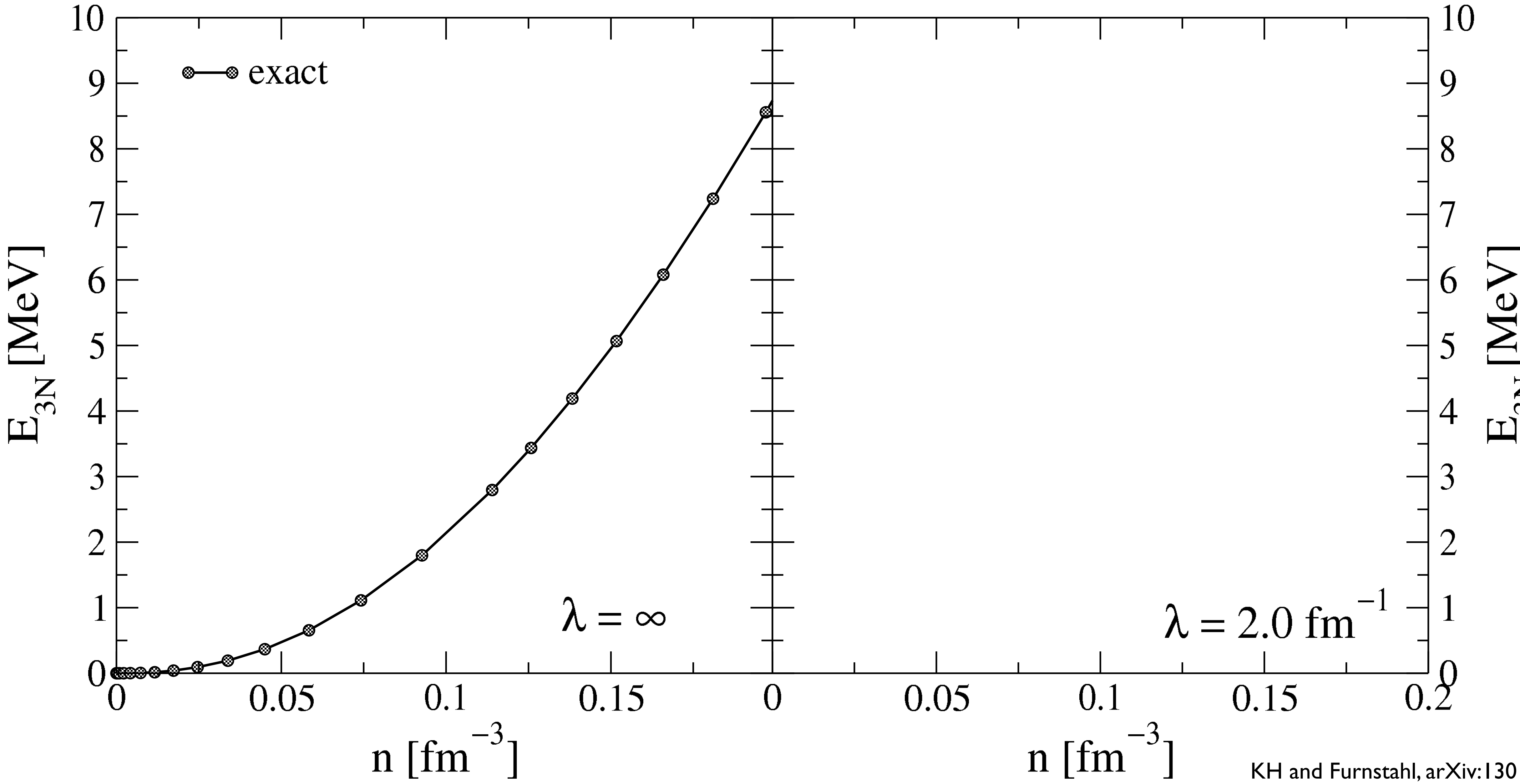
# Resolution-scale dependence at saturation density




KH and Furnstahl, arXiv:1301.7467

- solid lines: NN resummed, dashed lines: NN 2nd order
- indications for 4N forces at small $\lambda$?

# Partial-wave convergence of 3NF contributions




KH and Furnstahl, arXiv:1301.7467

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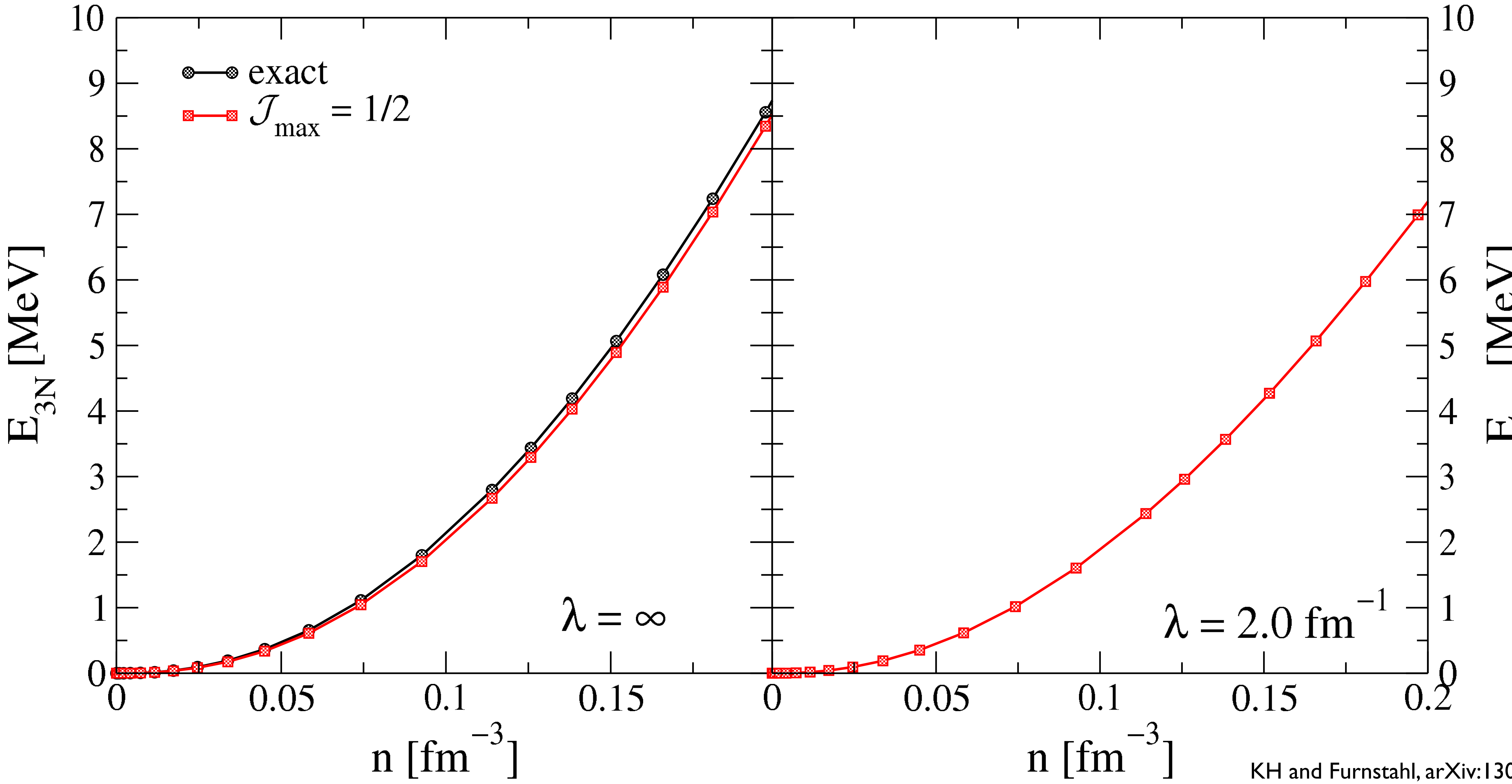
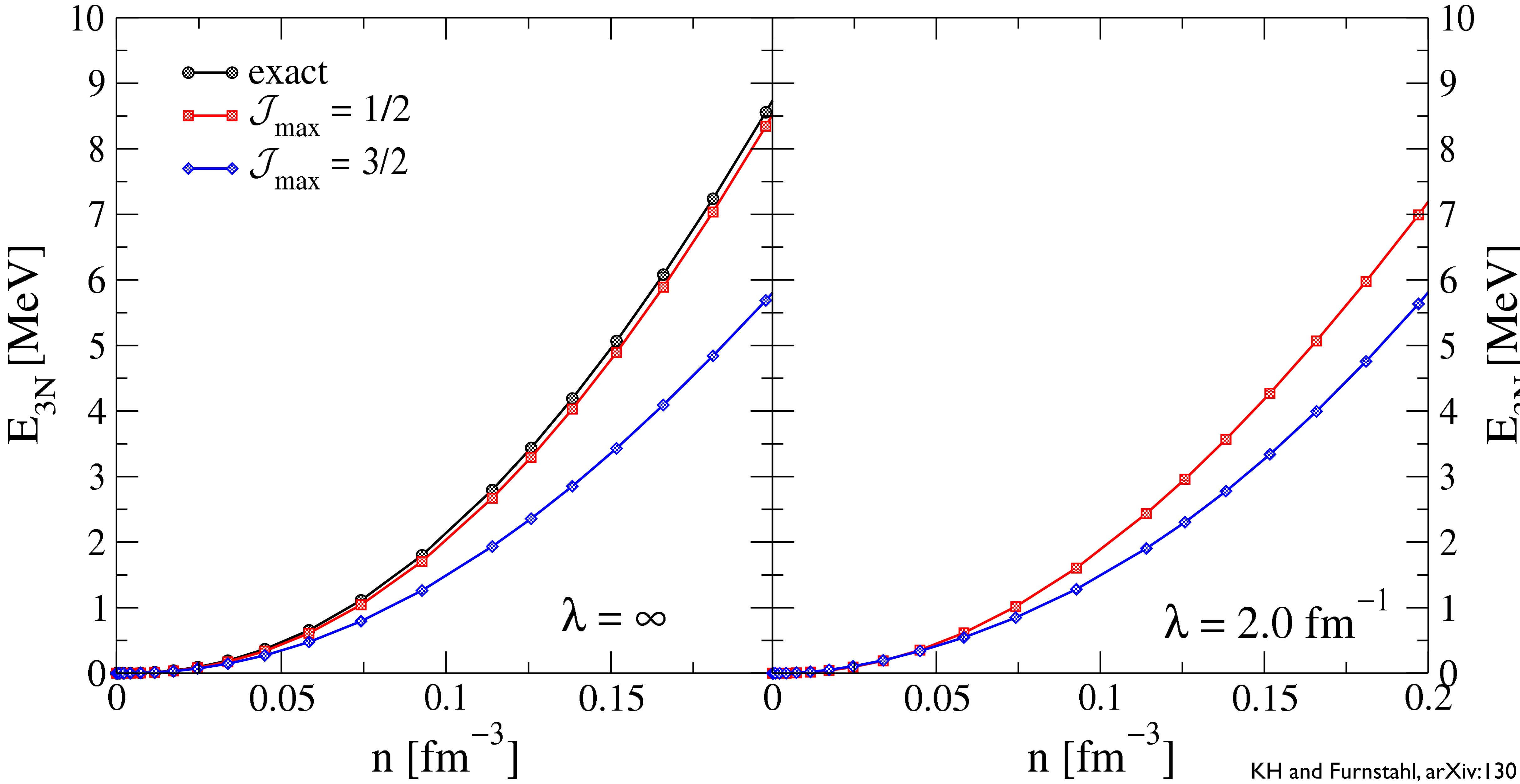



KH and Furnstahl, arXiv:1301.7467

# Partial-wave convergence of 3NF contributions




KH and Furnstahl, arXiv:1301.7467

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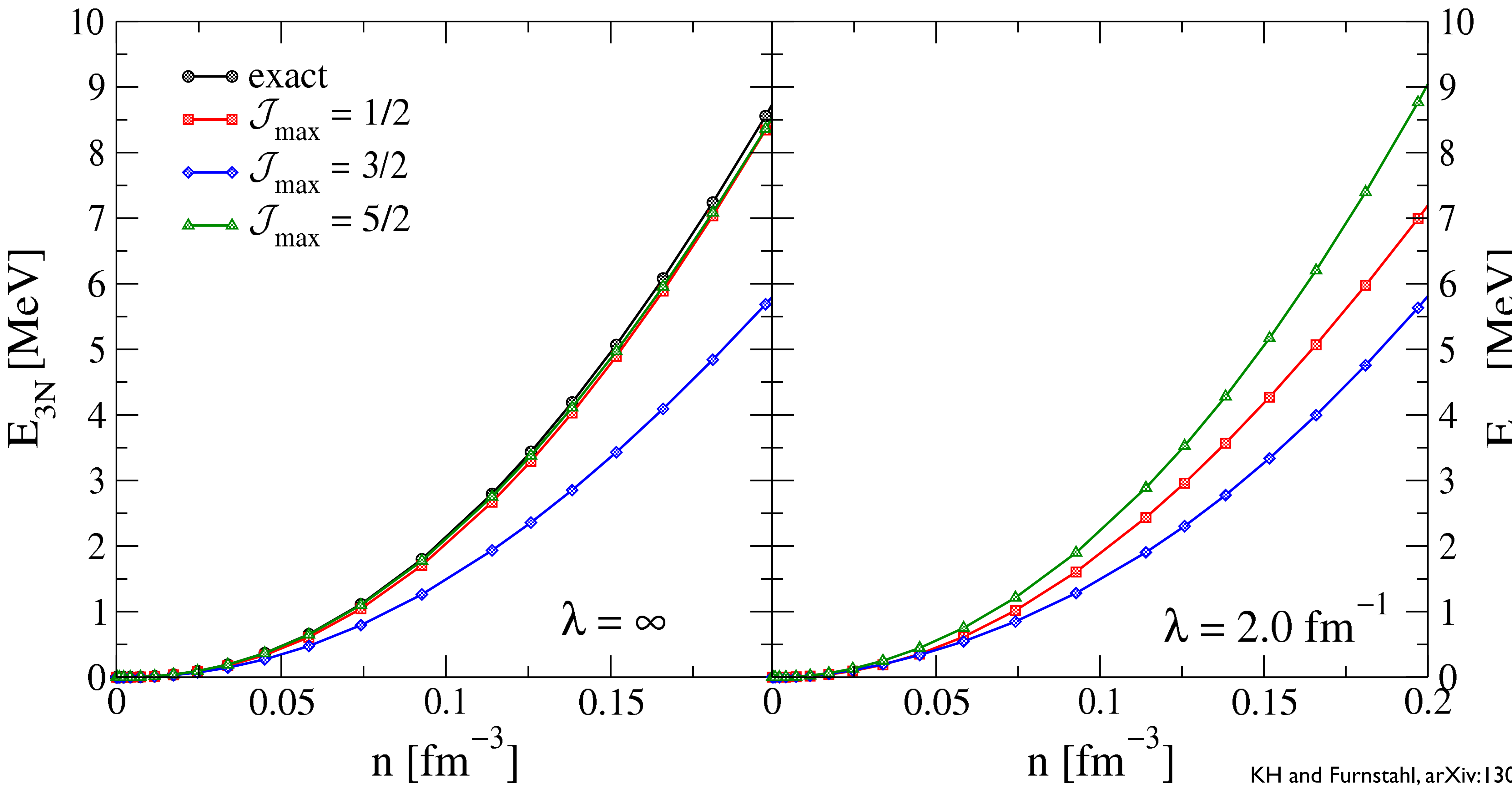


KH and Furnstahl, arXiv:1301.7467

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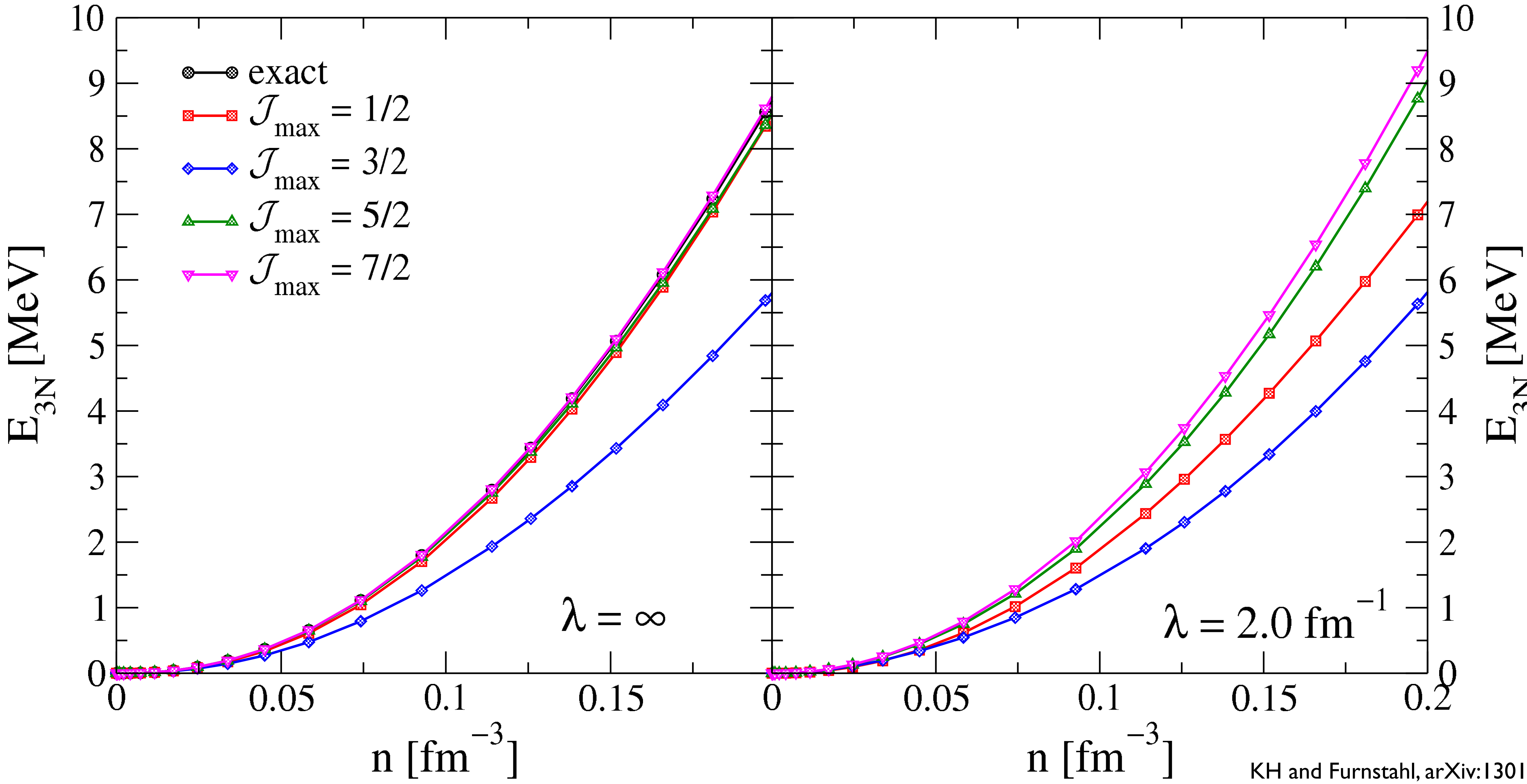


KH and Furnstahl, arXiv:1301.7467

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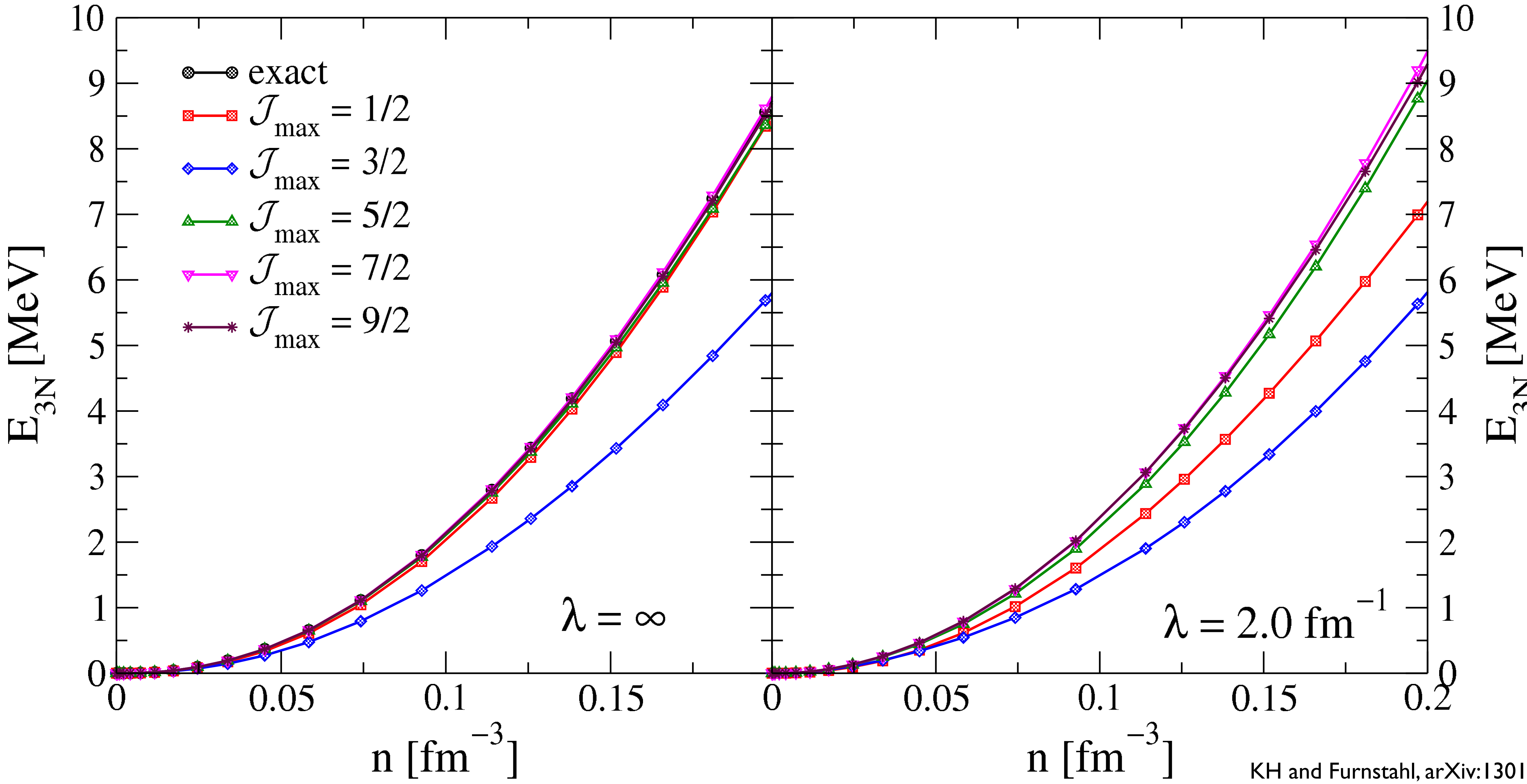
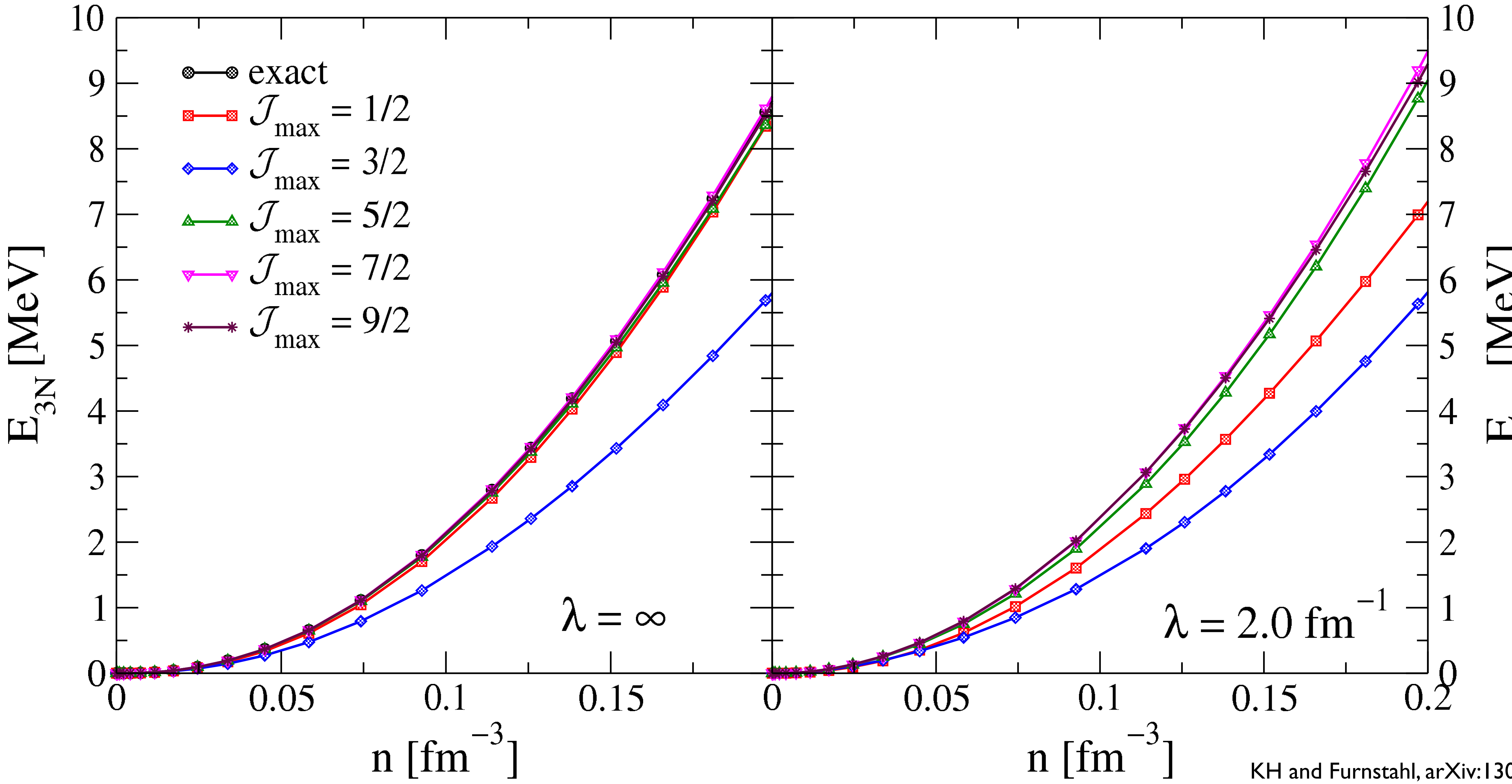



KH and Furnstahl, arXiv:1301.7467

# Partial-wave convergence of 3NF contributions




KH and Furnstahl, arXiv:1301.7467

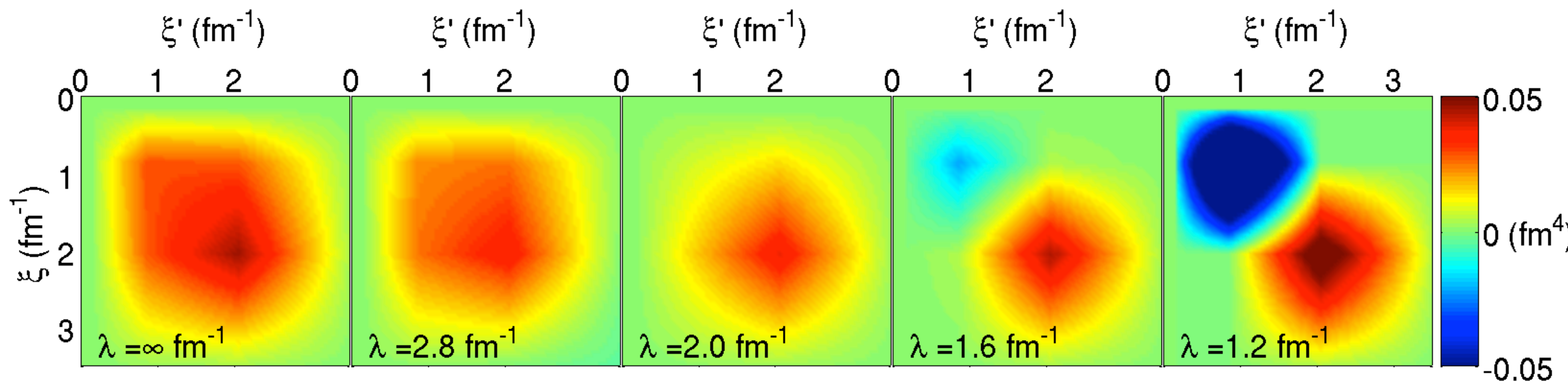
- $E_{3N}$ agrees within 0.4 % with the exact result at saturation density
- $E_{3N}$ converged in partial waves at both scales, $\lambda = \infty$ and $\lambda = 2.0\ \mathrm{fm}^{-1}$

# Matrix elements of evolved 3-neutron interactions

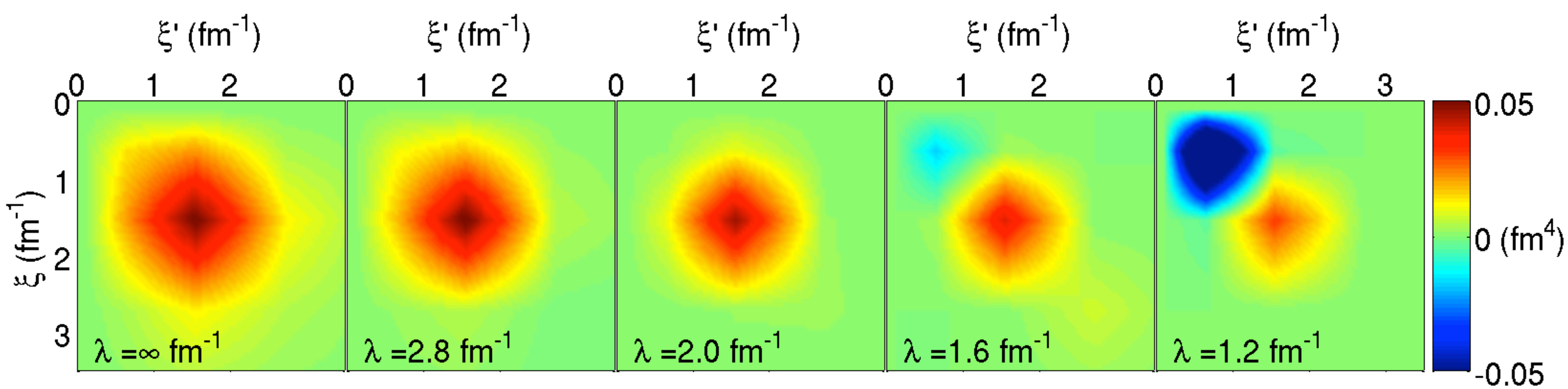
$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \frac{\pi}{20}$




$\theta = \frac{\pi}{15}$




- strong renormalization effects of long-range two-pion exchange
- moderate effects in range $\lambda = \infty$ to $\lambda = 2.0\,\mathrm{fm}^{-1}$

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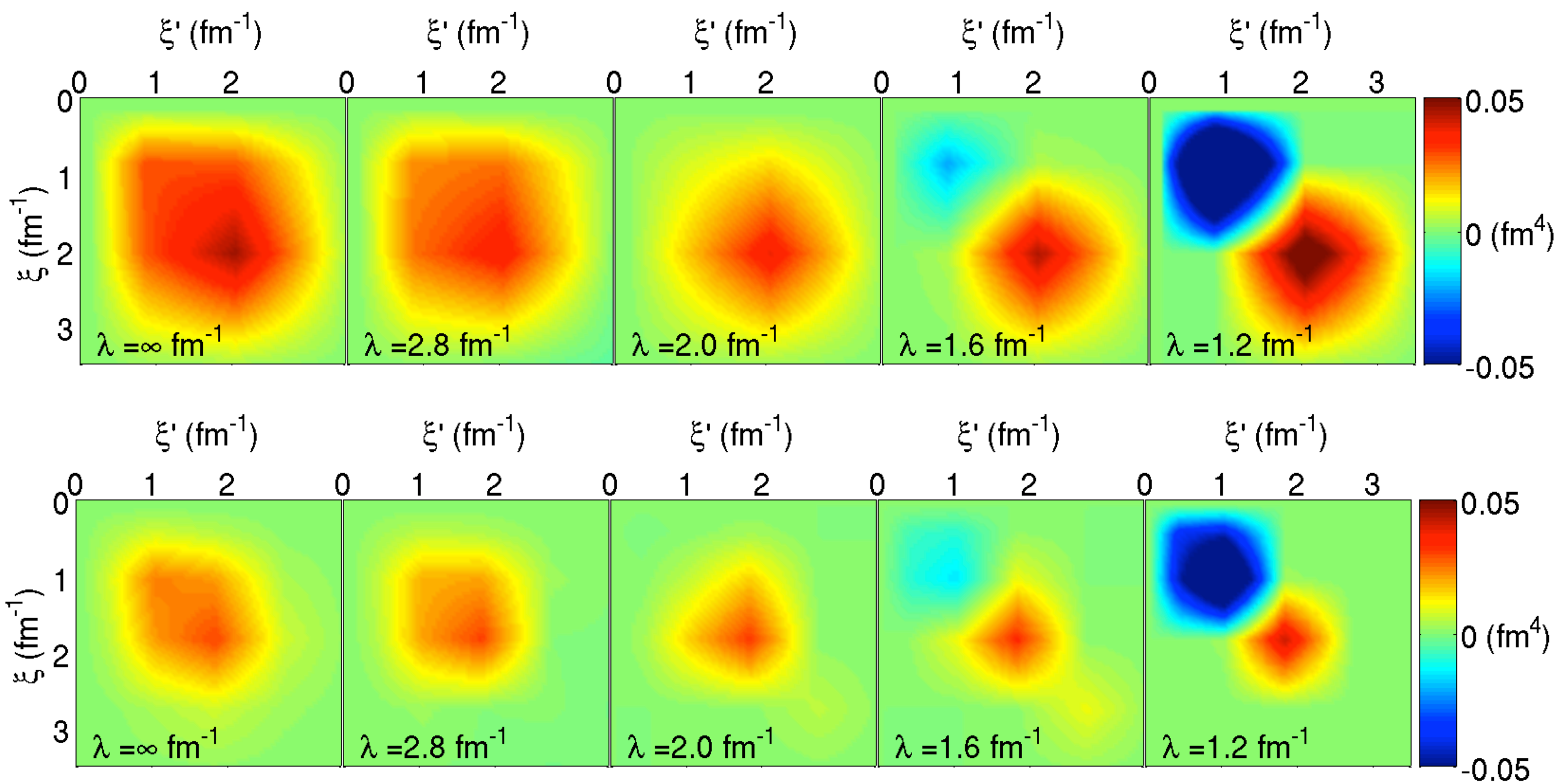
$\theta = \frac{\pi}{10}$

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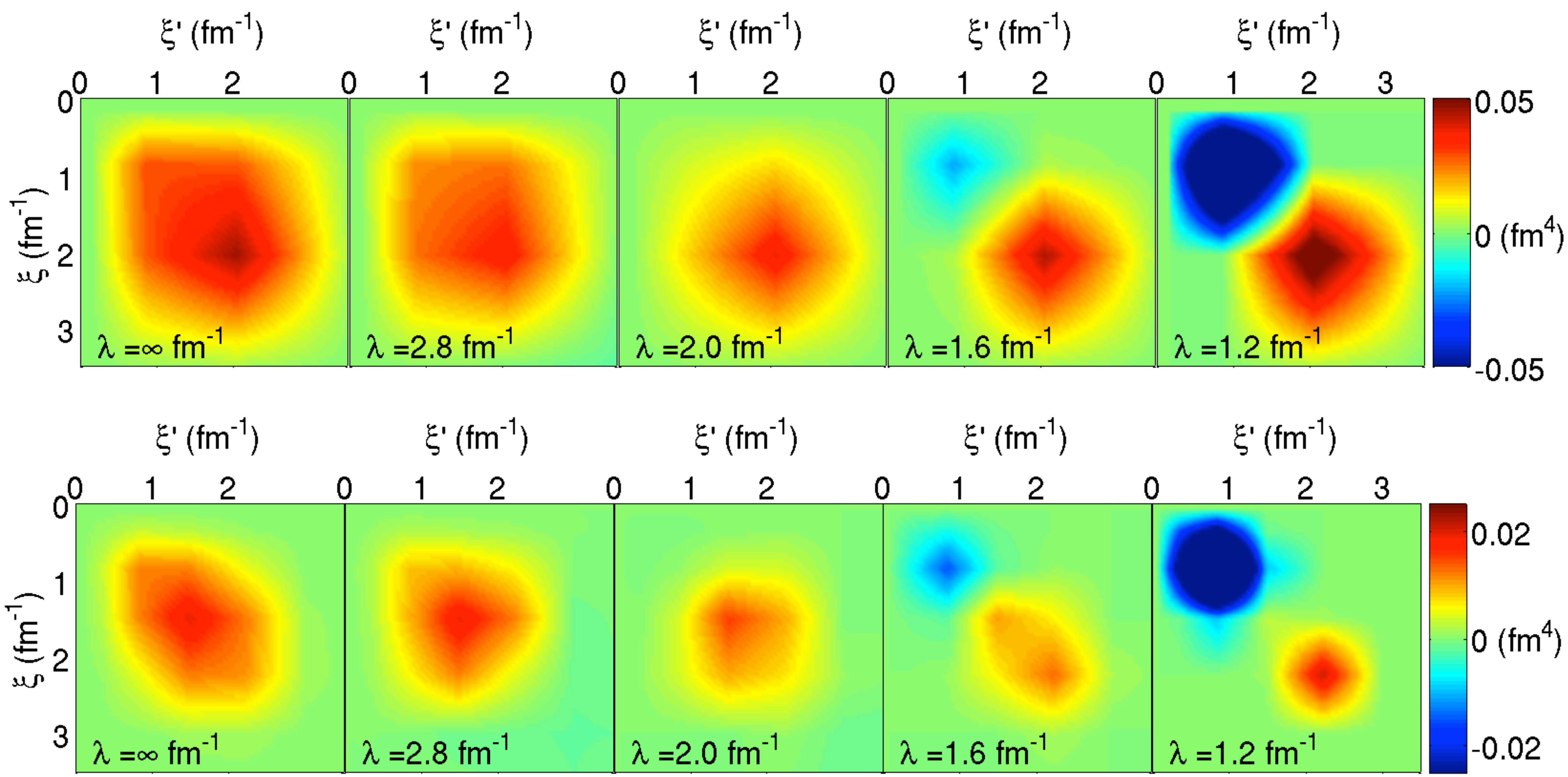
show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \frac{\pi}{20}$

$\theta = \frac{\pi}{8}$

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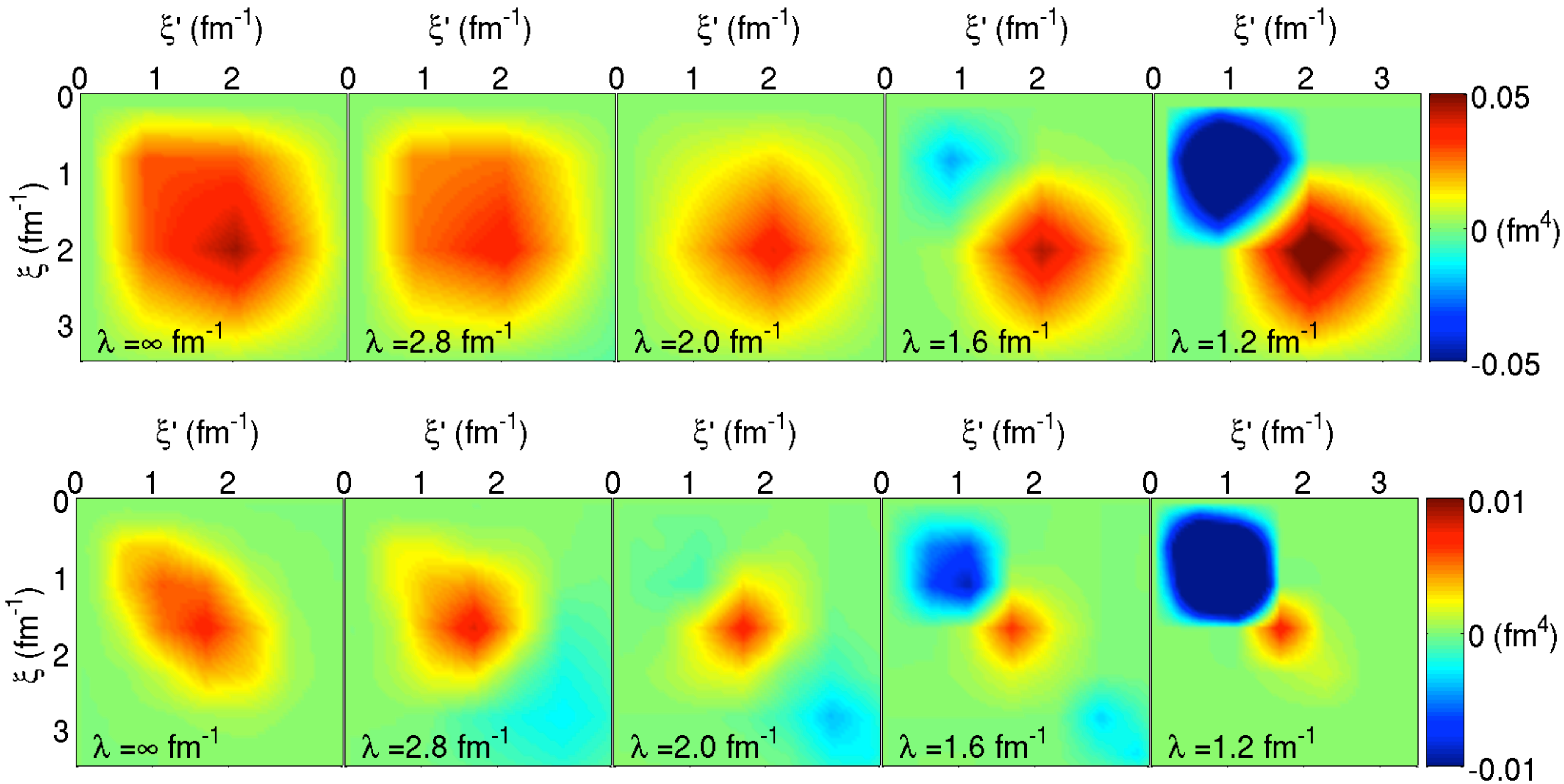
$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \frac{\pi}{20}$

$\theta = \frac{\pi}{6}$

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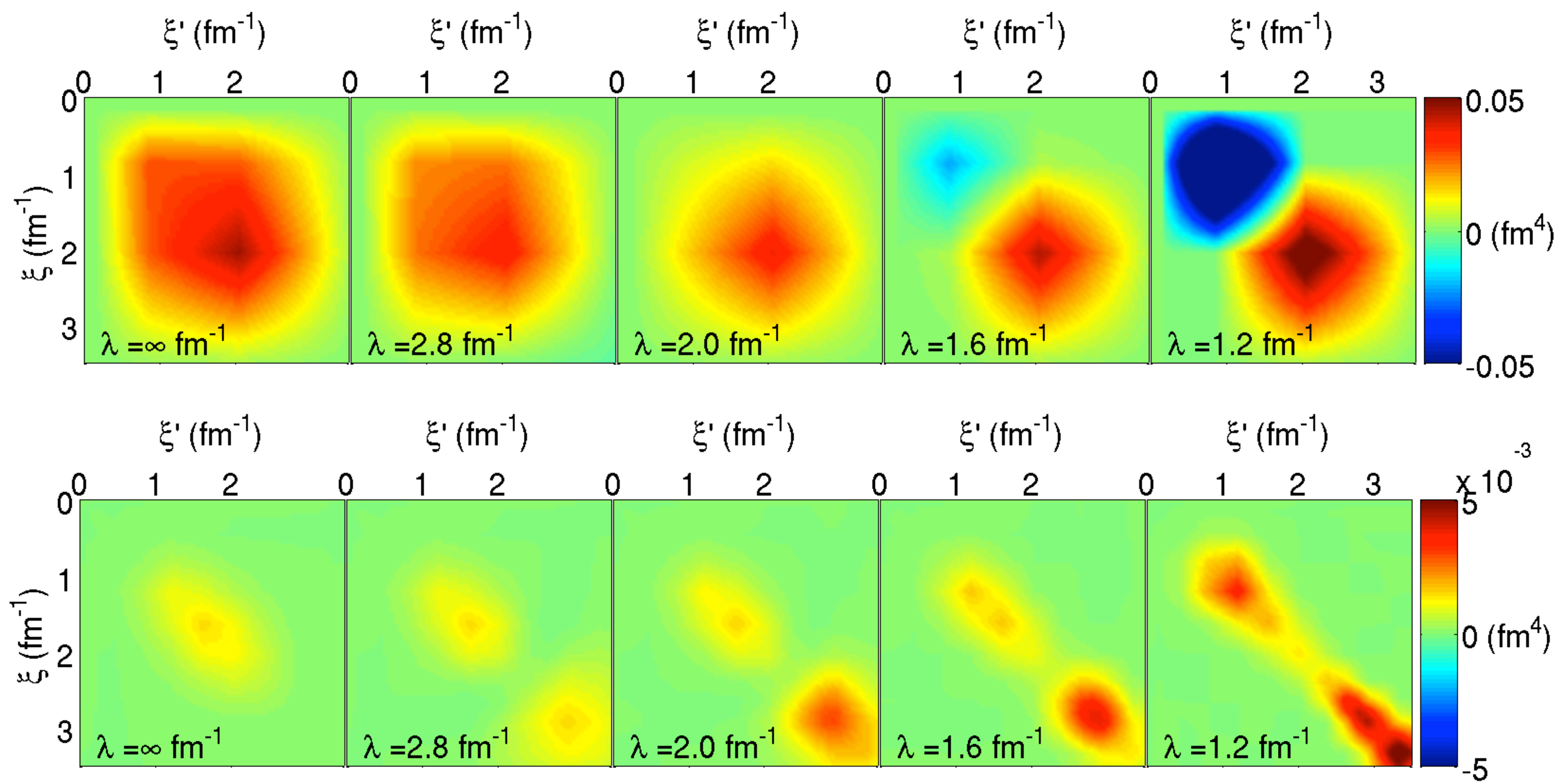
$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \frac{\pi}{20}$

$\theta = \frac{\pi}{4}$

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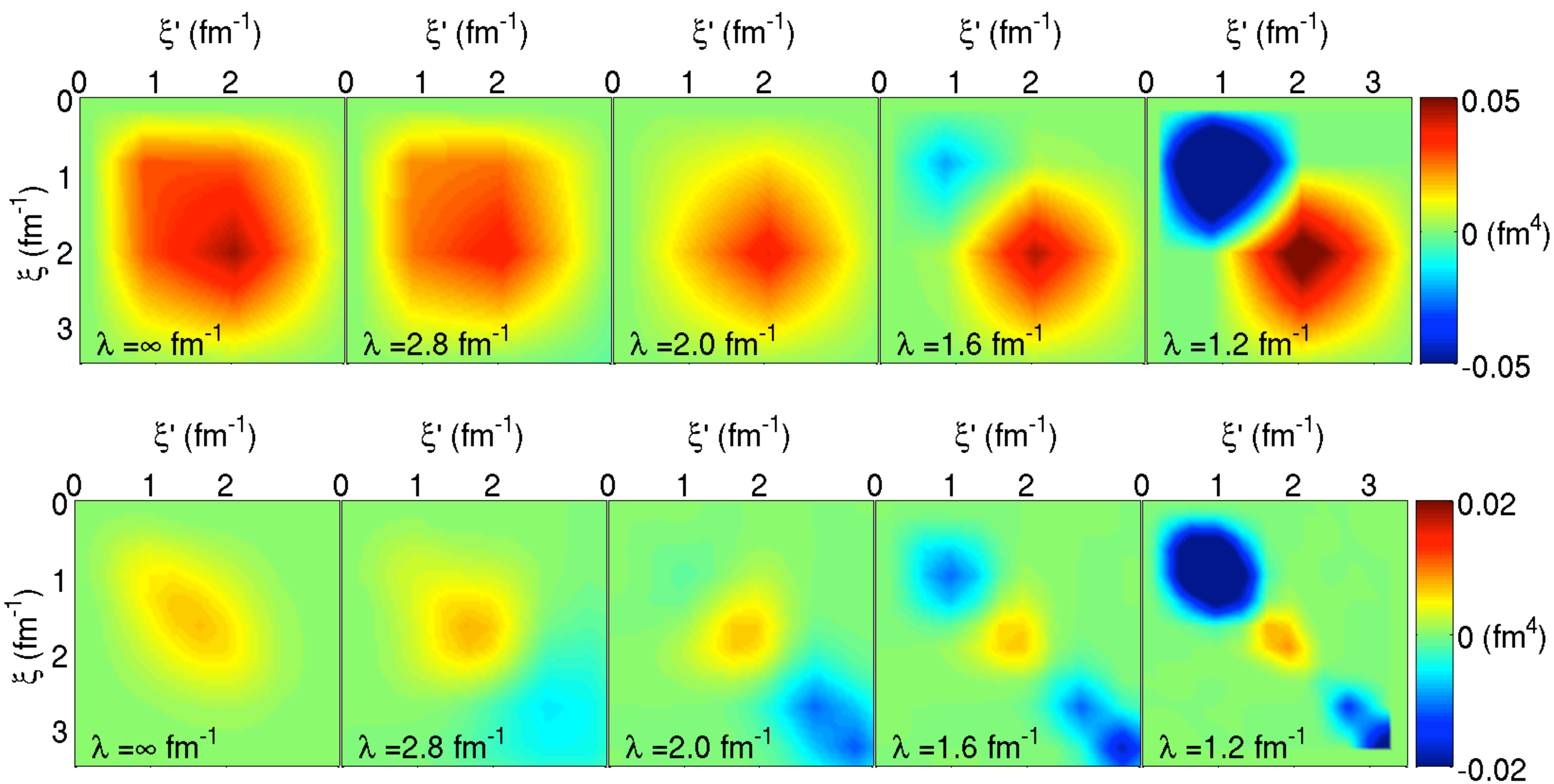
$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \frac{\pi}{20}$

$\theta = \frac{\pi}{3}$

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$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

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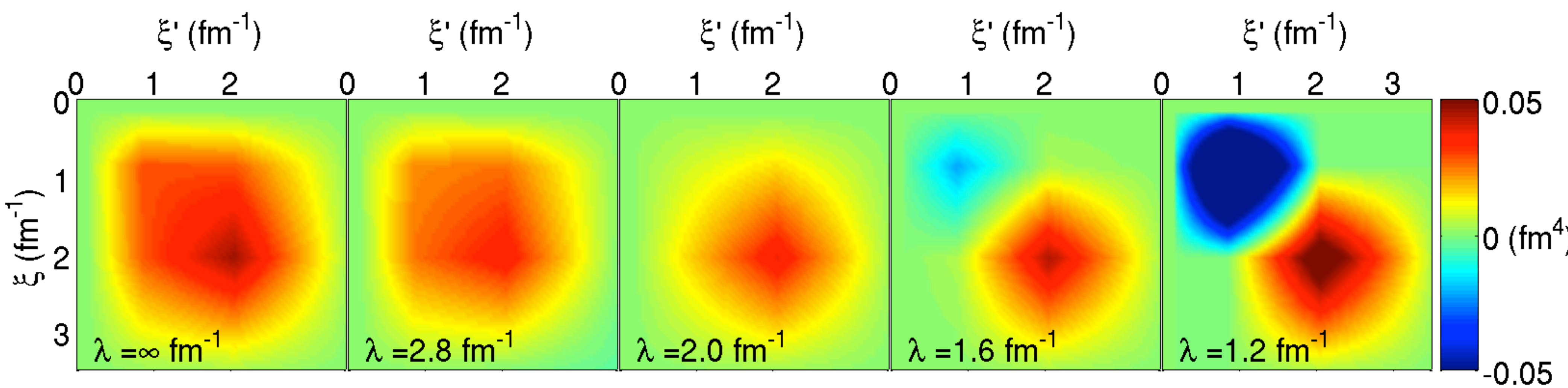
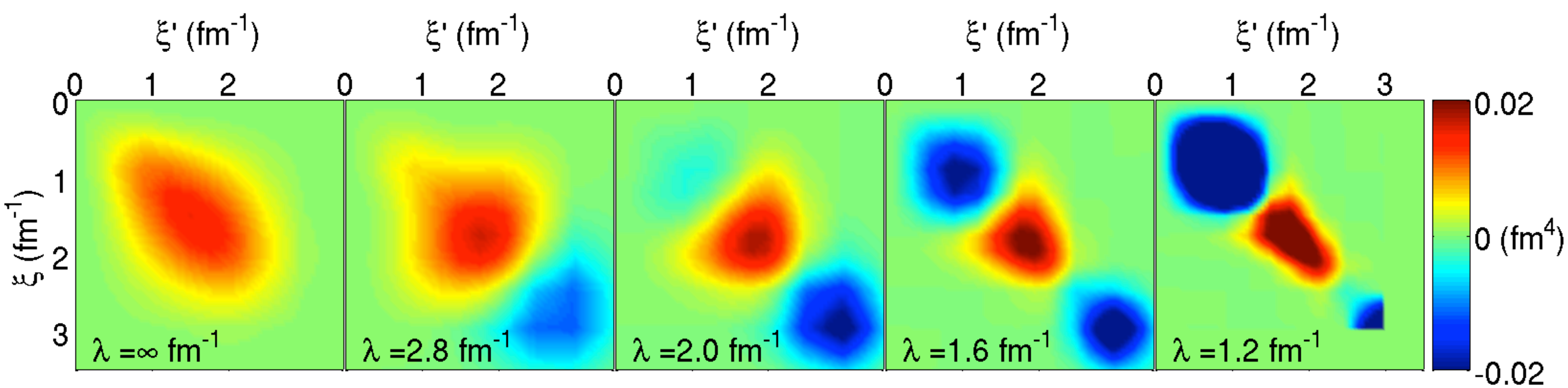



$\theta = \frac{\pi}{2.5}$

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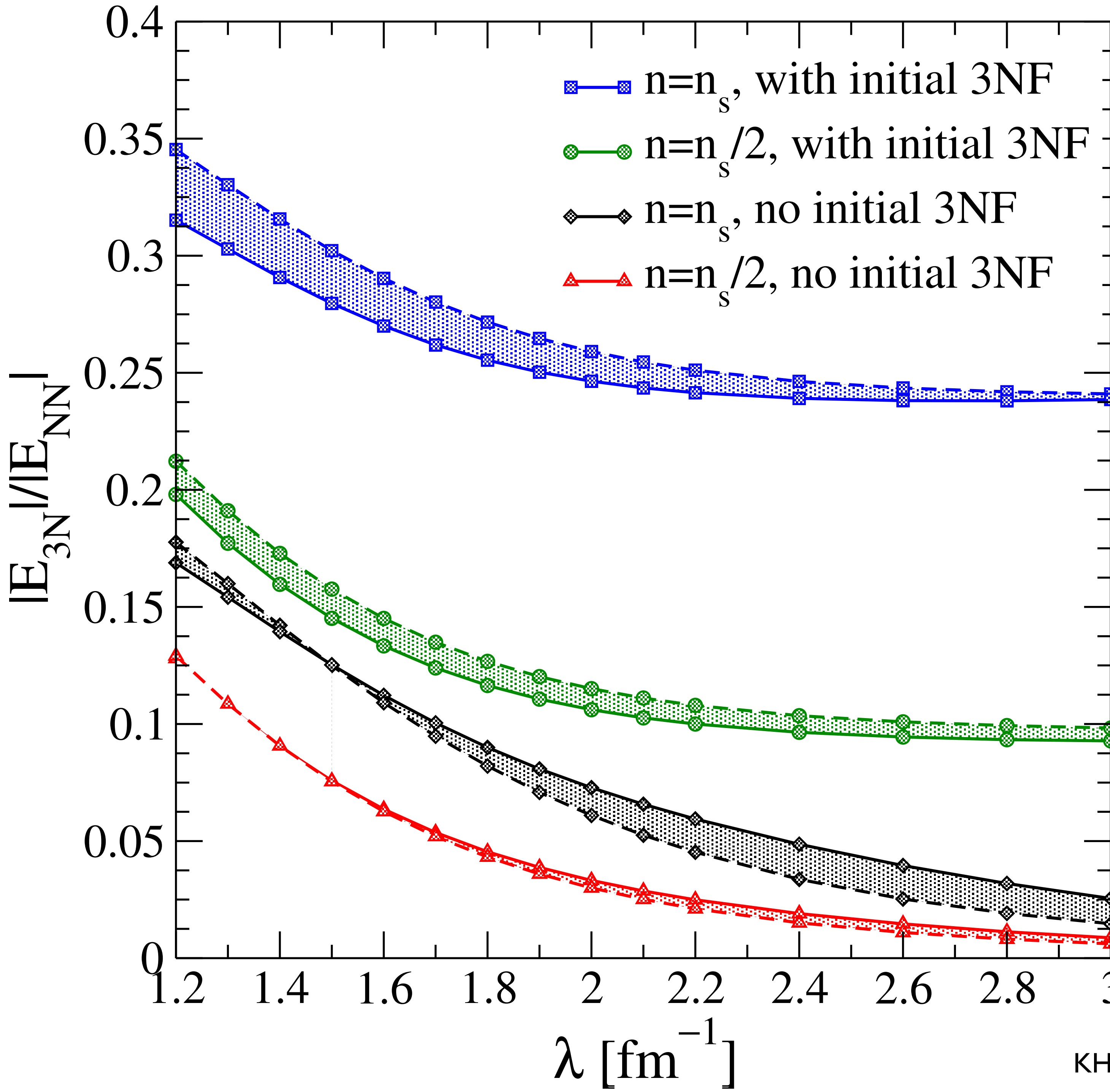
- moderate effects in range $\lambda = \infty$ to $\lambda = 2.0\,\mathrm{fm}^{-1}$

# Scaling of three-body contributions




KH and Furnstahl, arXiv:1301.7467

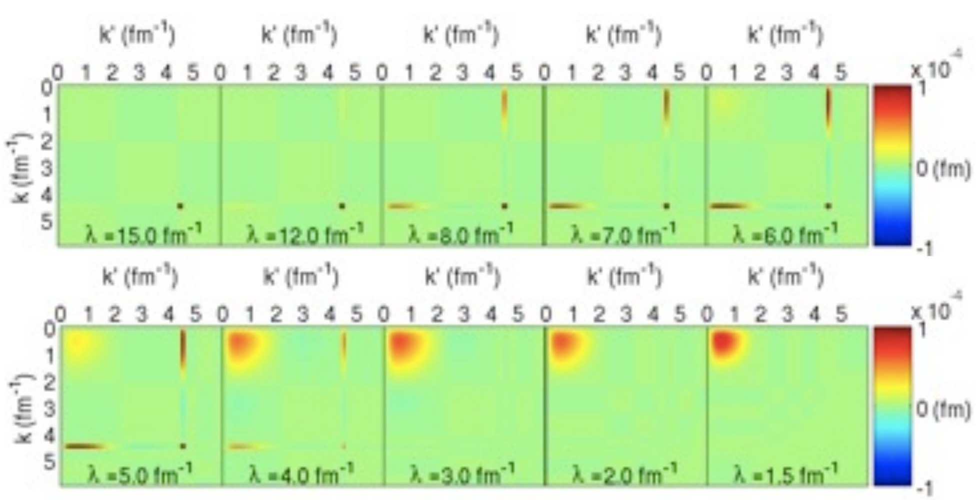
- relative size of 3N contribution grows systematically towards smaller $\lambda$
- no obvious trend with density (may be obscured by cancellations among contributions)

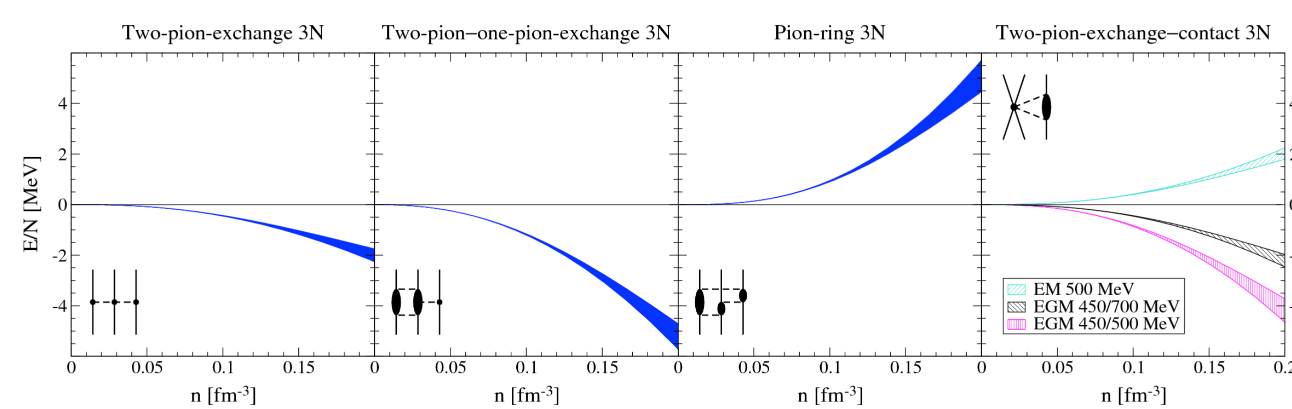
# Summary

- demonstrated the feasibility of SRG evolution of NN+3NF in momentum space
- first results of neutron matter based on consistently evolved NN+3NF interactions
- strong renormalization effects of chiral two-pion exch. interaction in neutron matter
- no indications of significant contributions from 4N forces down to $\lambda = 1.2\,\mathrm{fm}^{-1}$ in neutron matter

# Outlook

- inclusion of 3NF N3LO contributions in RG evolution
- extend RG evolution to $\mathcal{T} = 1/2$ channels, application to nuclear matter
- transformation to HO basis, application to finite nuclei (CC, NCSM)
- RG evolution of operators: nuclear scaling and correlations in nuclear systems



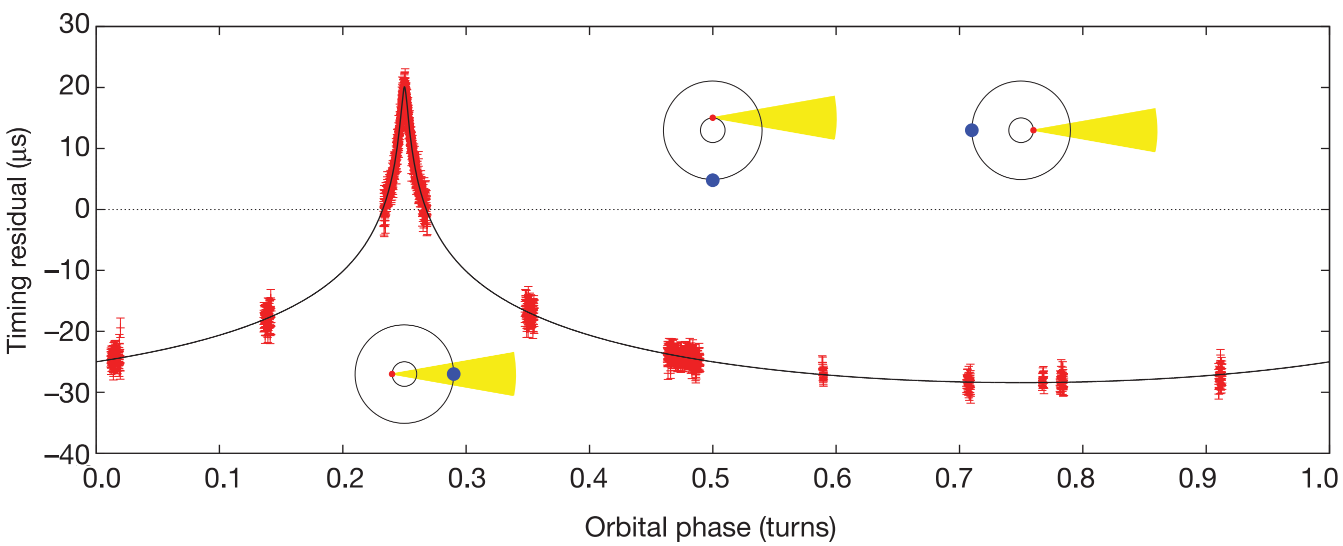



# Constraints on the nuclear equation of state (EOS)

nature

**A two-solar-mass neutron star measured using Shapiro delay**

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

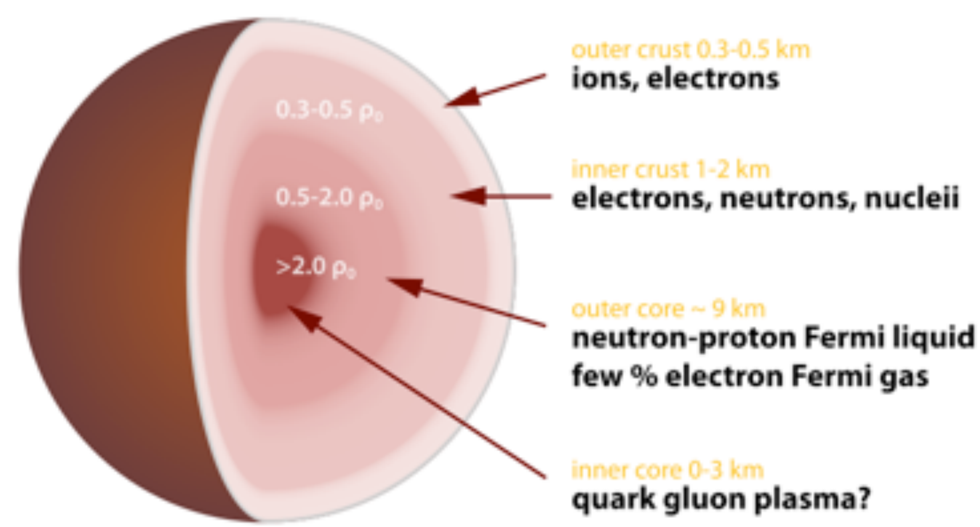



Demorest et al., Nature 467, 1081 (2010)

$$M_{\rm max} = 1.65 M_{\odot} \rightarrow 1.97 \pm 0.04\, M_{\odot}$$

Credit: NASA/Dana Berry

Calculation of neutron star properties requires EOS up to high densities.




Strategy:

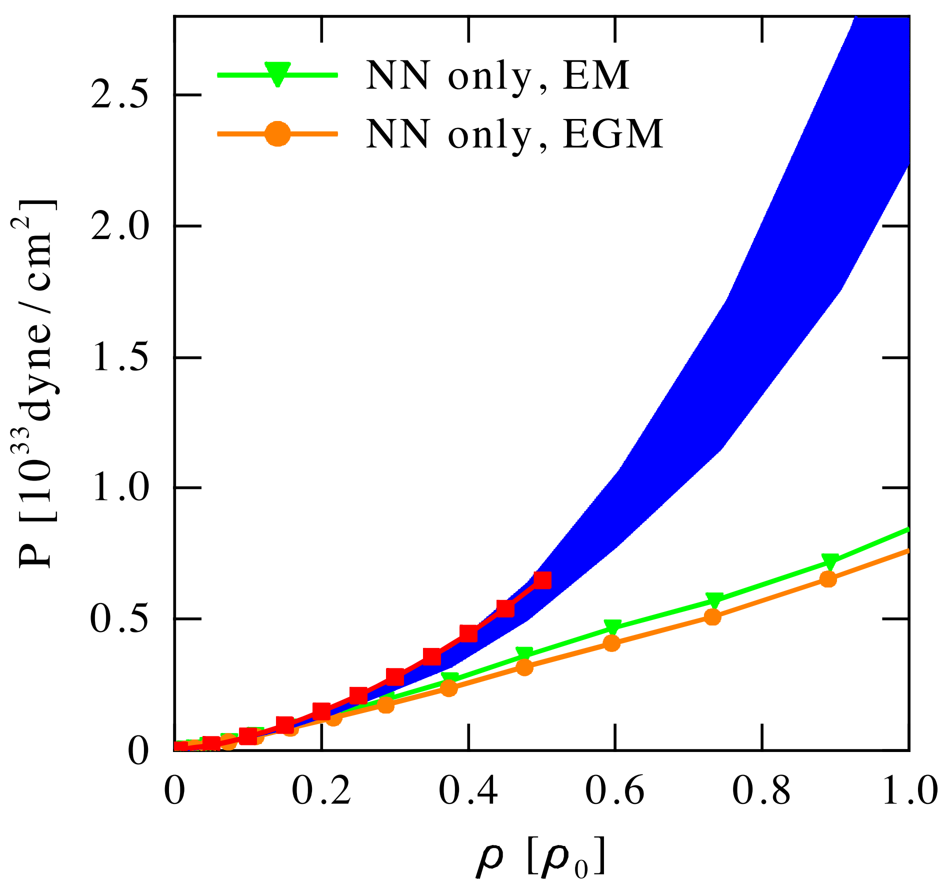
Use observations to constrain the high-density part of the nuclear EOS.

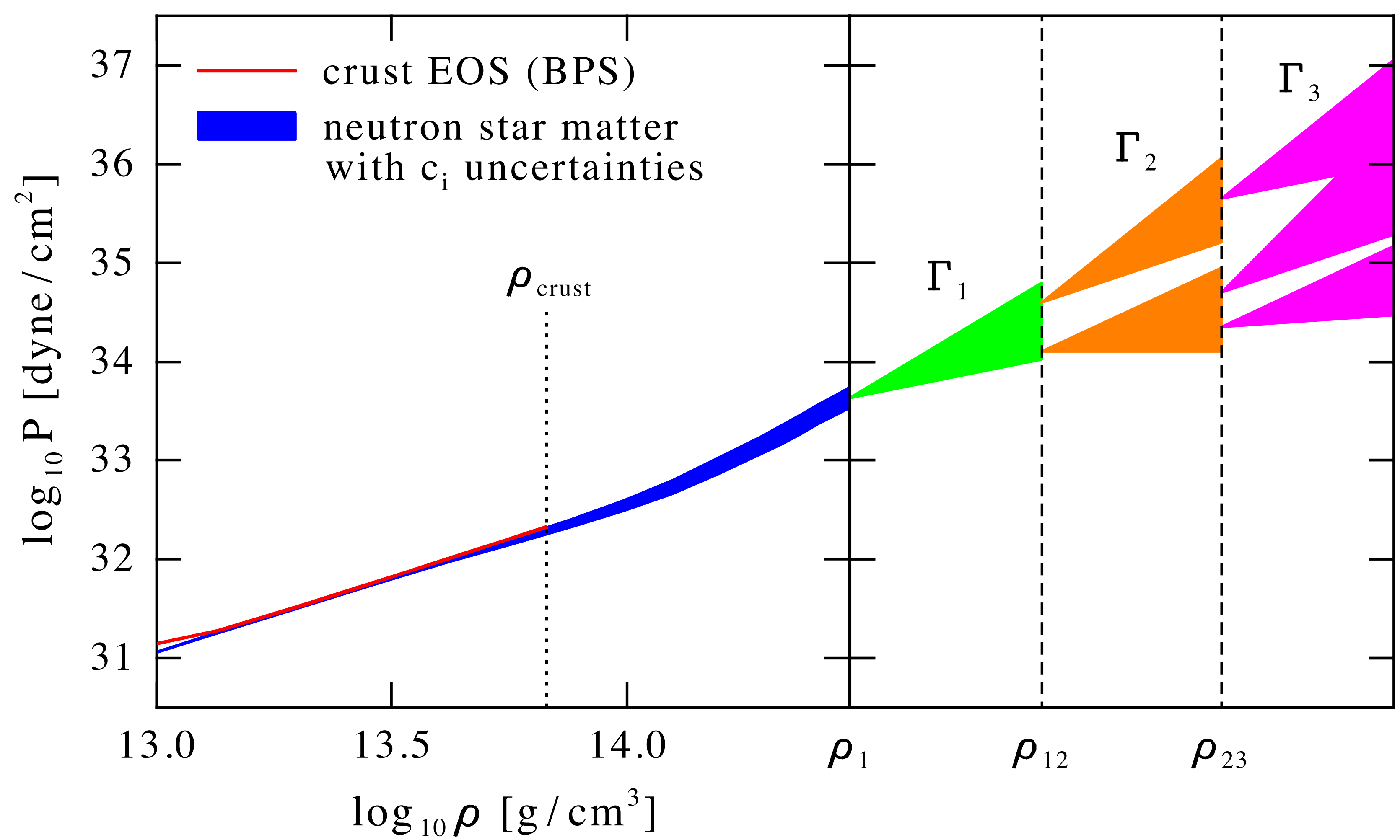
# Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter $\longrightarrow$ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics!




KH, Lattimer, Pethick, Schwenk, in preparation
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

# Constraints on the nuclear equation of state

use the constraints:

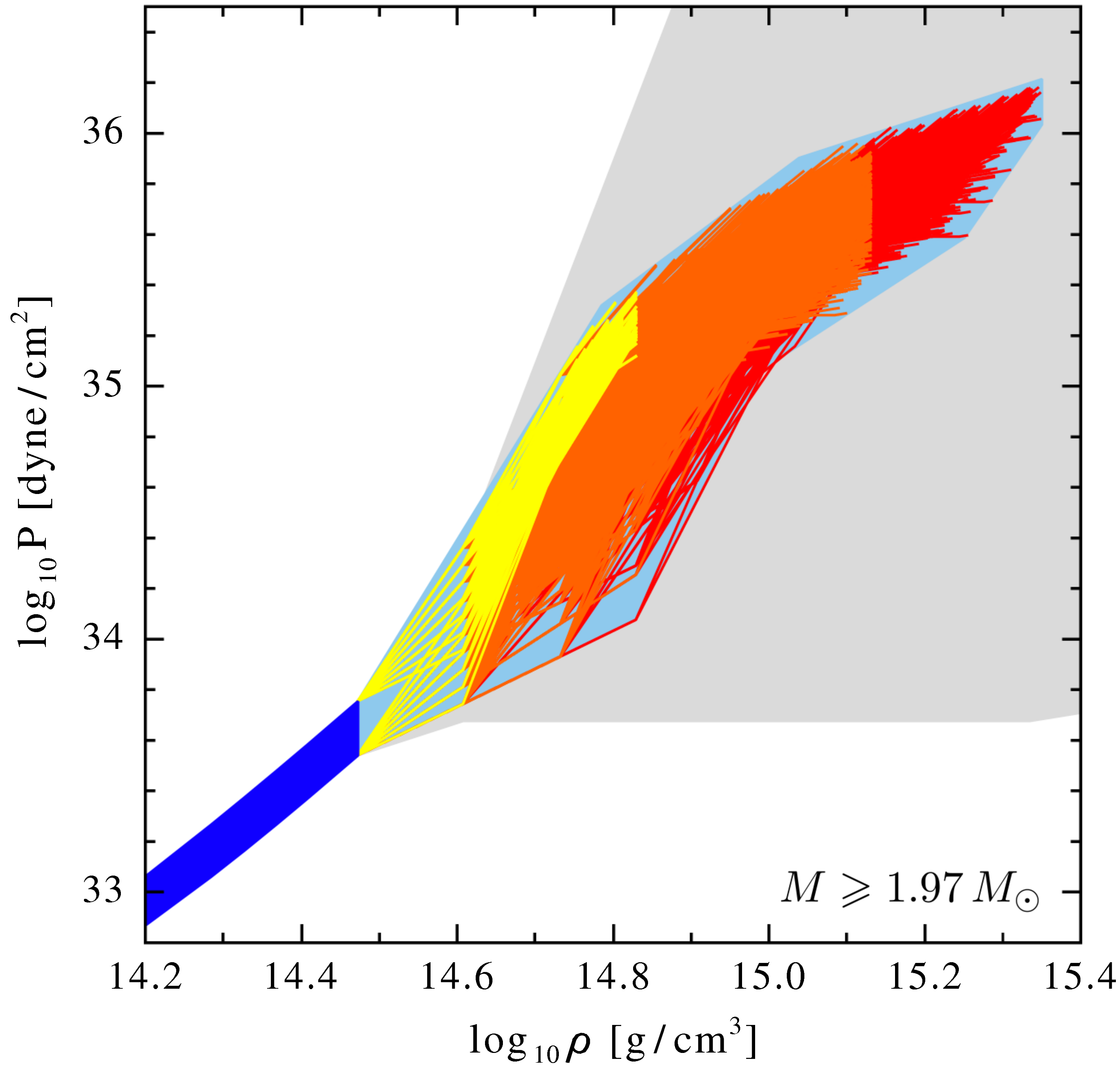
recent NS observation

$M_{\rm max} > 1.97\, M_\odot$

causality

$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$

KH, Lattimer, Pethick, Schwenk, in preparation




significant reduction of uncertainty band

# Constraints on the nuclear equation of state

use the constraints:

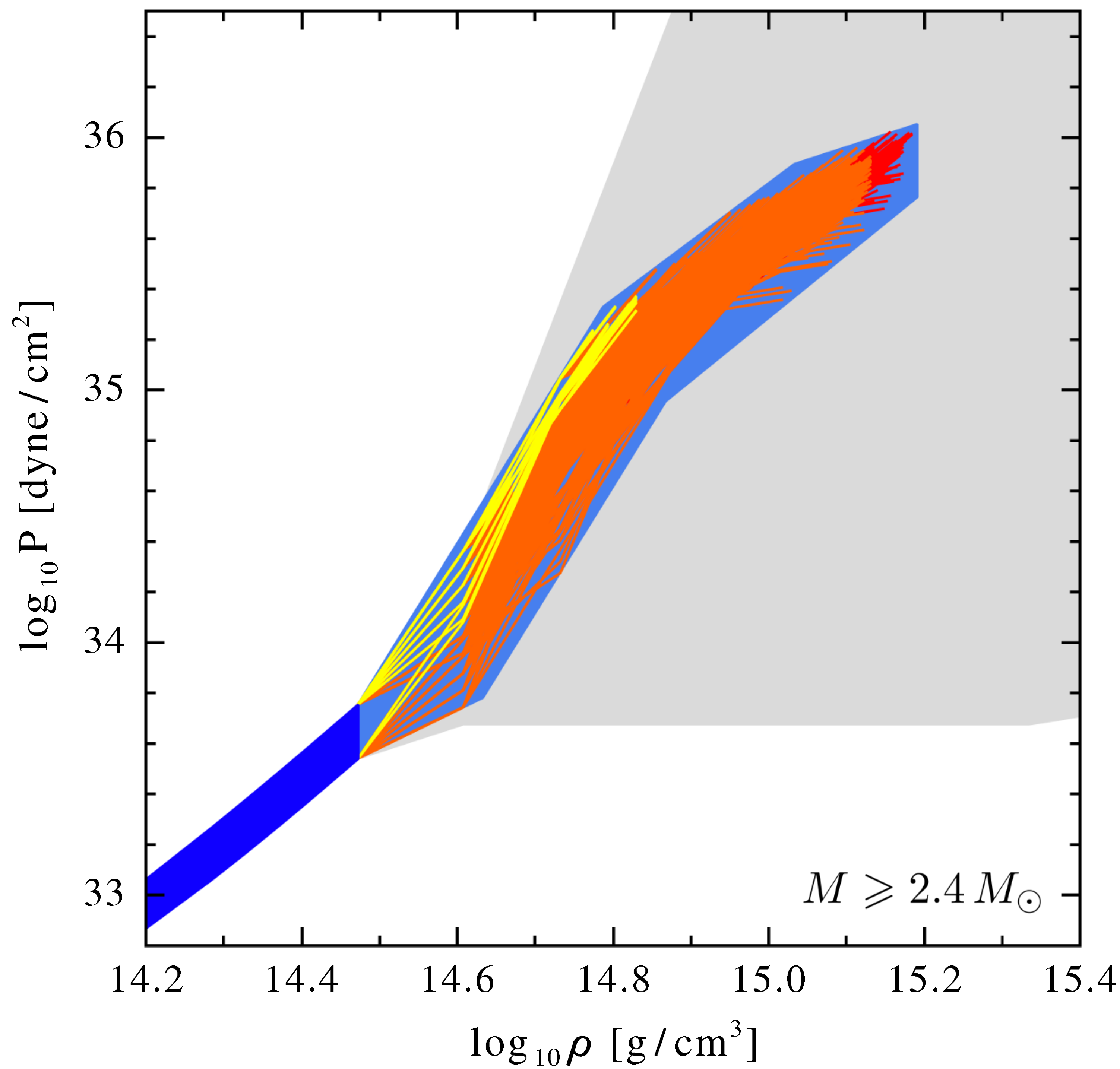
NS mass

$$M_{\text{max}} > 2.4\, M_\odot$$

causality

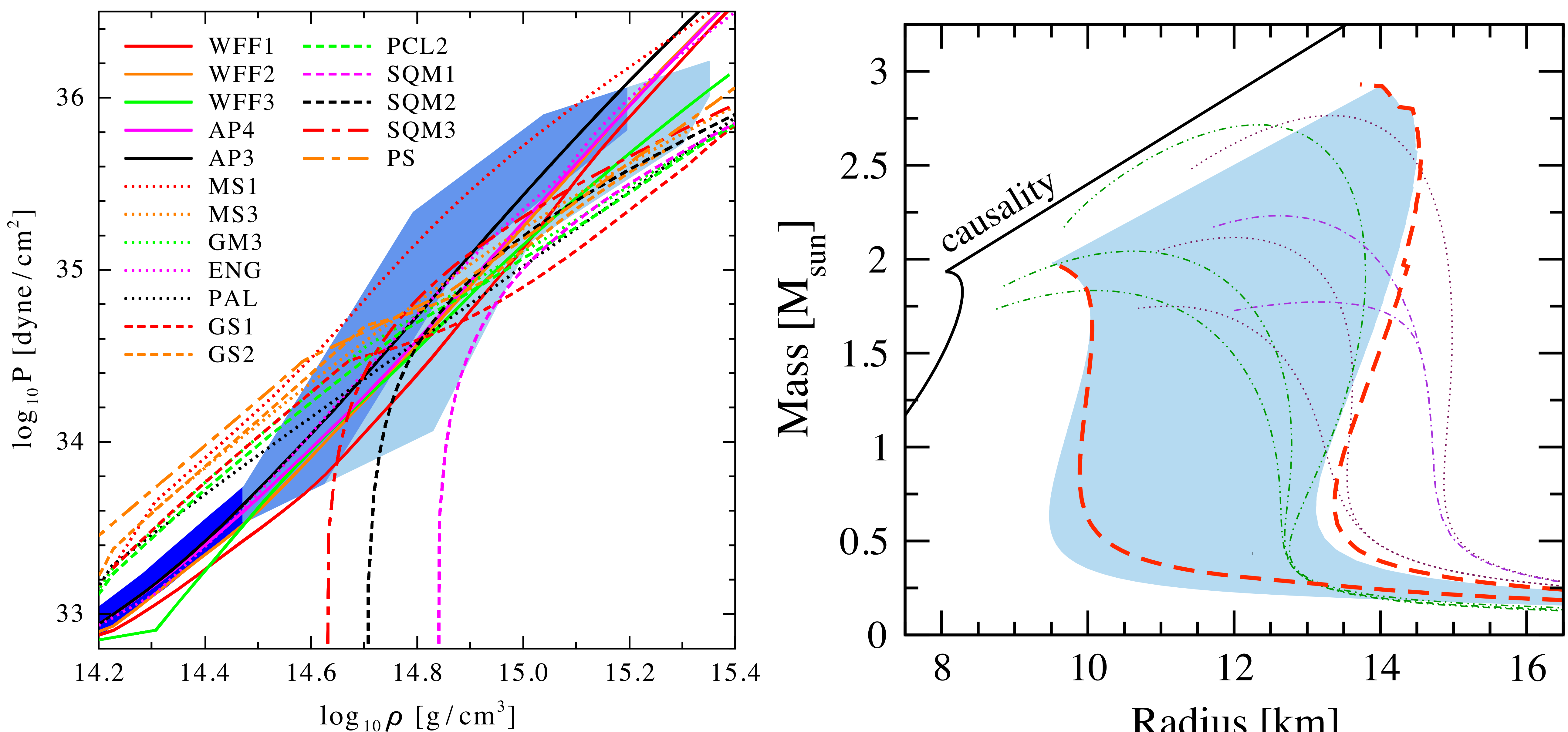
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, in preparation




increased $M_{\text{max}}$ systematically reduces width of band

# Constraints on neutron star radii




KH, Lattimer, Pethick, Schwenk, in preparation
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint for typical $1.4\,M_\odot$ neutron star: $9.8 - 13.4\,\mathrm{km}$