Ab initio NCSM/RGM for three-body cluster systems and application to ^{4}He+n+n

Carolina Romero-Redondo
Petr Navrátil and Sofia Quaglioni

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Outline

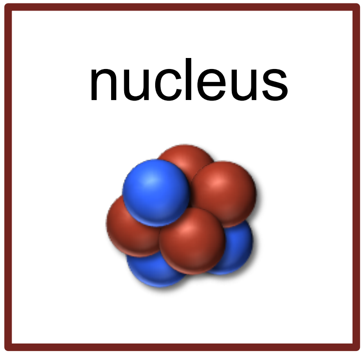
Introduction: ab initio NCSM/RGM

Extension of the method to
Three-body cluster states

Preliminary results: ^{4}He+n+n

Summary and outlook

Introduction



Ab initio in nuclear physics

Assumes nucleons as the effective degrees of freedom

Uses realistic interactions

The goal is to achieve a predictive theory for light nuclear systems to study:

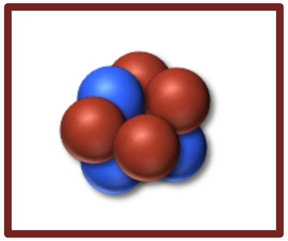
- Exotic nuclei
- Reactions important in nuclear astrophysics
- Reactions important for energy production projects

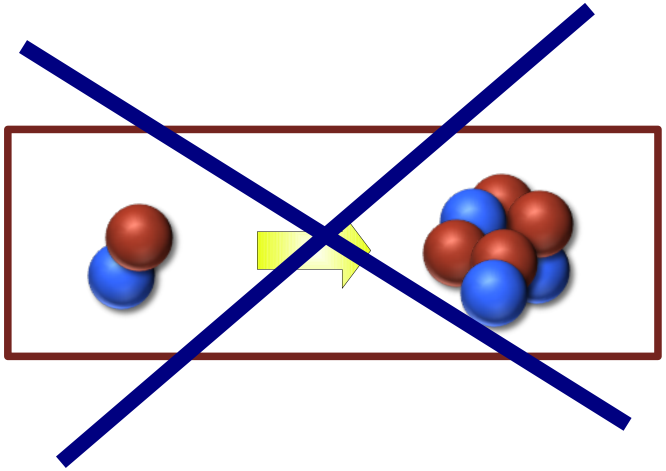
NCSM/RGM

No-core shell model (NCSM)

Is an ***ab initio*** method capable of studying light bound nuclei from an accurate Hamiltonian.

Is not able to deal with continuum states and therefore is not applicable to reactions.





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Resonating group method (RGM)

Microscopic cluster approach.

Permits studying the scattering of clusters

Non-realistic Hamiltonian

NCSM/RGM

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Resonating group method (RGM)

Microscopic cluster approach.

Permits studying the scattering of clusters

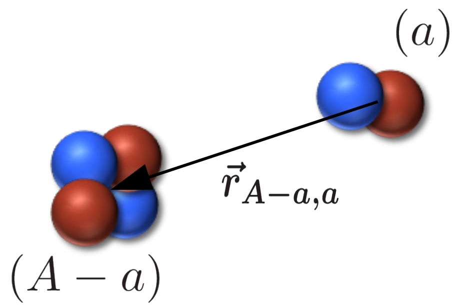
Non-realistic Hamiltonian

NCSM/RGM

Combines NCSM and RGM to obtain an *ab initio* formalism which uses an accurate nuclear Hamiltonian and is capable of studying both structure and scattering problems in light nuclear systems

NCSM/RGM

Binary clusters



***A*-nucleon system**

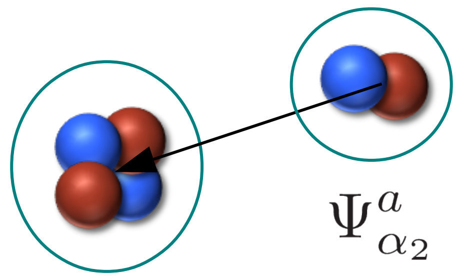
S. Quaglioni and P. Navrátil

- PRL 101, 092501 (2008)

- PRC 79, 044606 (2009)

NCSM/RGM

Binary clusters



A-nucleon system

$\Psi^{a}_{\alpha_2}$

$\Psi^{A-a}_{\alpha_1}$

NCSM wave functions

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NCSM/RGM

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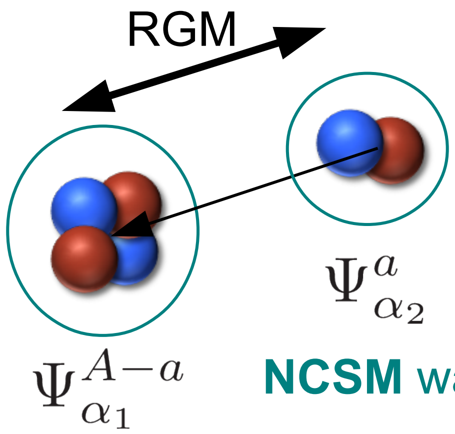

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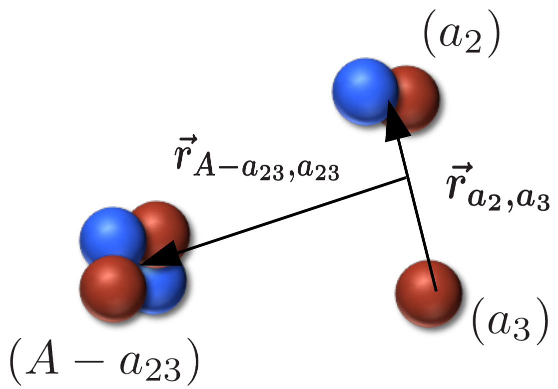
NCSM/RGM-3B



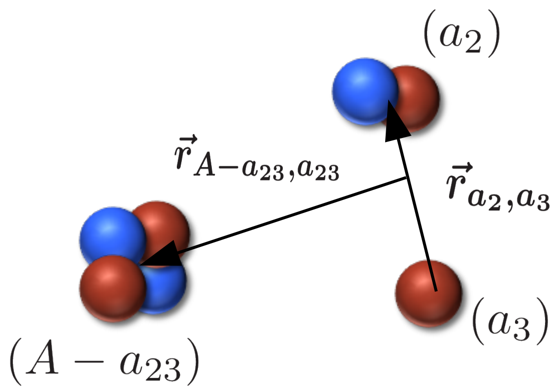
Extension to three-body cluster

C. Romero-Redondo

S. Quaglioni, P. Navrátil

In progress

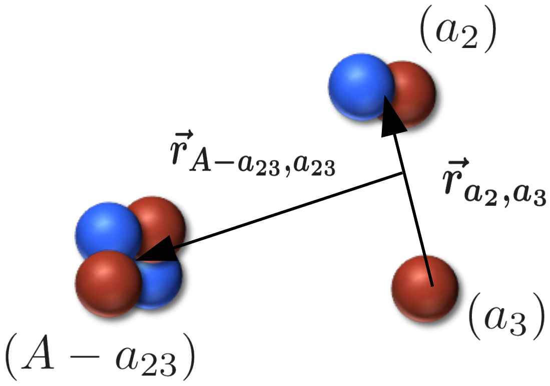
NCSM/RGM-3B



Extension to three-body cluster

Why?

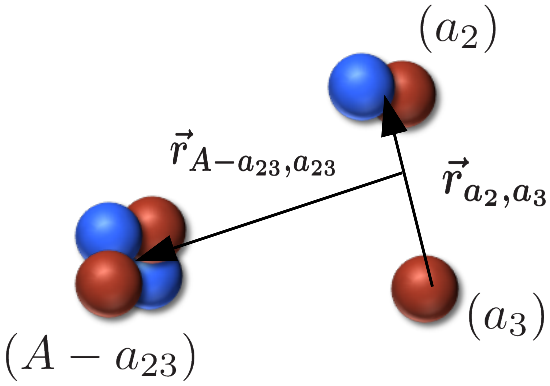
NCSM/RGM-3B



Extension to three-body cluster

Bound and resonant states:
2n Halo nuclei

NCSM/RGM-3B



Extension to three-body cluster

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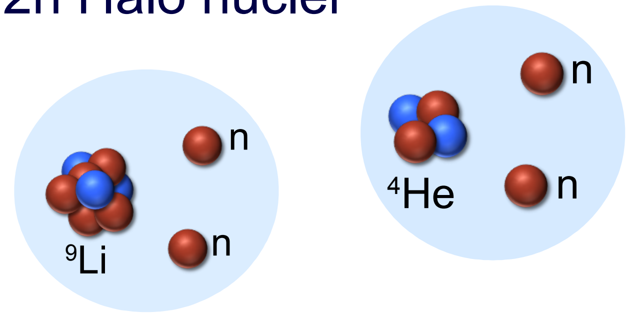

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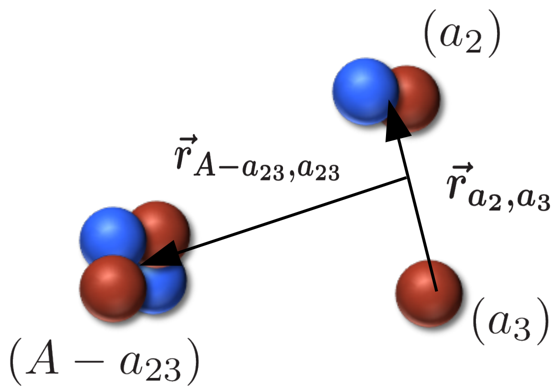
NCSM/RGM-3B



Extension to three-body cluster

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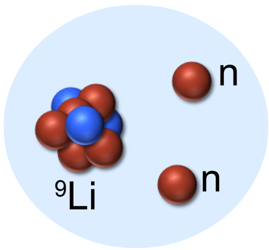
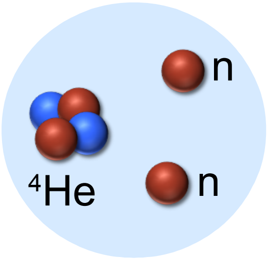
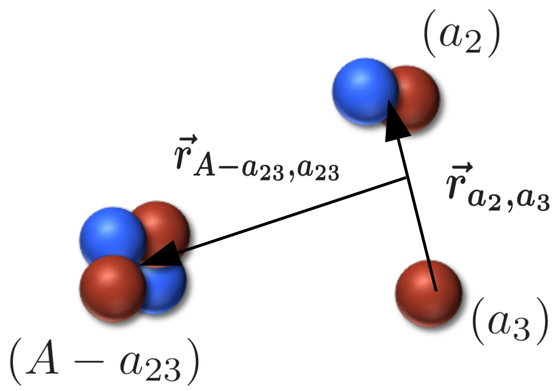




3-body continuum states:
Transfer reactions

NCSM/RGM-3B



Extension to three-body cluster

Bound and resonant states:
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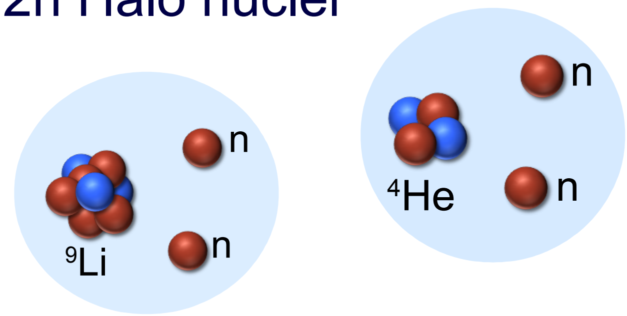

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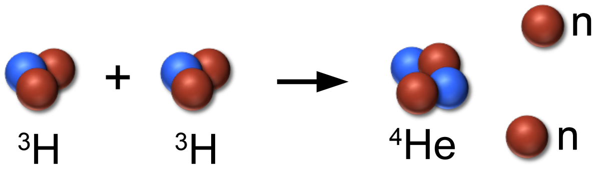
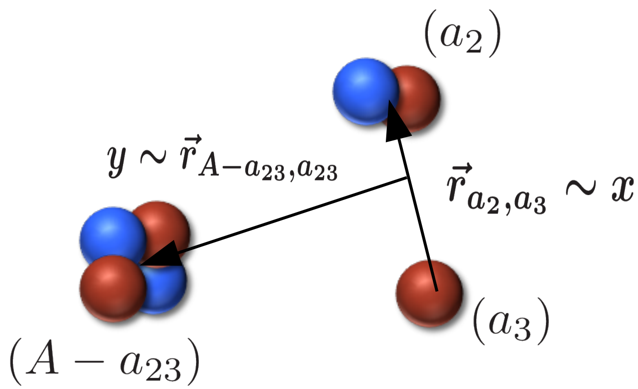


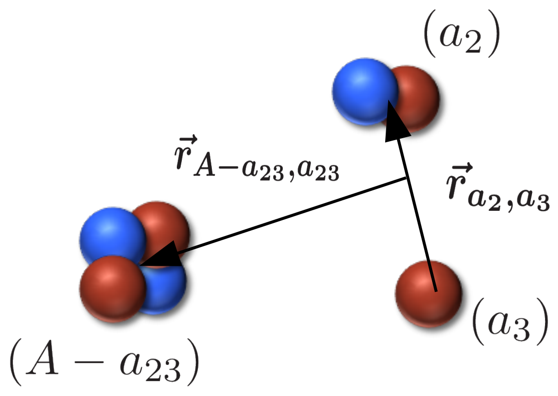
NCSM/RGM-3B



Basis

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu \underbrace{|\boldsymbol{\Phi}^{J^\pi T}_{\nu x y}\rangle}_{\text{Basis}}$$

NCSM/RGM-3B



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$$|\Phi^{J^\pi T}_{\nu r}\rangle \sim \Psi^{A-a_{23}}_{\alpha_1} \Psi^{a_2}_{\alpha_2} \Psi^{a_3}_{\alpha_3} Y_{\ell_x}(\hat{r}_{a_2,a_3}) Y_{\ell_y}(\hat{r}_{A-a_{23},a_{23}}) \delta(y - r_{A-a_{23},a_{23}}) \delta(x - r_{a_2,a_3})$$

NCSM wave functions

NCSM/RGM-3B

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu |\boldsymbol{\Phi}^{\boldsymbol{J^\pi T}}_{\boldsymbol{\nu x y}}\rangle$$

Schrödinger equation

$$(\mathcal{H} - E)\,|\Psi^{J^\pi T}\rangle = 0$$

$$\sum_\nu \int dx dy x^2 y^2 \left[\mathcal{H}_{\nu'\nu}(x,y,x',y') - E\mathcal{N}_{\nu'\nu}(x,y,x',y')\right] G_\nu^{J^\pi T}(x,y) = 0$$

Hamiltonian Kernel

$$\langle \Phi^{J^\pi T}_{\nu' r'} | \hat{A}_{\nu'} \mathcal{H} \hat{A}_\nu | \Phi^{J^\pi T}_{\nu r}\rangle$$

Norm kernel

$$\langle \Phi^{J^\pi T}_{\nu' r'} | \hat{A}^2 | \Phi^{J^\pi T}_{\nu r}\rangle$$

NCSM/RGM-3B

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu |\boldsymbol{\Phi}_{\boldsymbol{\nu x y}}^{\boldsymbol{J^\pi T}}\rangle$$

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Norm kernel

$$\langle \Phi_{\nu' r'}^{J^\pi T} | \hat{A}^2 | \Phi_{\nu r}^{J^\pi T} \rangle$$

Relative movement wavefunction

NCSM/RGM-3B

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu |\boldsymbol{\Phi}^{\boldsymbol{J^\pi T}}_{\boldsymbol{\nu x y}}\rangle$$

Schrödinger equation

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$$\sum_\nu \int dx dy x^2 y^2 \left[\mathcal{H}_{\nu'\nu}(x,y,x',y') - E\mathcal{N}_{\nu'\nu}(x,y,x',y')\right] G_\nu^{J^\pi T}(x,y) = 0$$

Orthogonalization

$$\sum_\nu \int dx dy x^2 y^2 \left[\mathbb{H}_{\nu'\nu}(x,y,x',y') - E\delta\nu'\nu \frac{\delta(x'-x)}{x'x}\frac{\delta(y'-y)}{y'y}\right] \chi_\nu^{J^\pi T}(x,y) = 0$$

NCSM/RGM-3B

$$\sum_{\nu}\int dxdyx^2y^2\left[\mathbb{H}_{\nu'\nu}(x,y,x',y') - E\delta\nu'\nu\frac{\delta(x'-x)}{x'x}\frac{\delta(y'-y)}{y'y}\right]\chi_{\nu}^{J^{\pi}T}(x,y) = 0$$

Hyperspherical coordinates: $\rho = \sqrt{x^2+y^2}, \quad \alpha = \arctan(x/y)$

NCSM/RGM-3B

$$\sum_{\nu} \int dx dy x^2 y^2 \left[\mathbb{H}_{\nu' \nu}(x, y, x', y') - E \delta \nu' \nu \frac{\delta(x' - x)}{x' x} \frac{\delta(y' - y)}{y' y} \right] \chi_{\nu}^{J^\pi T}(x, y) = 0$$

Hyperspherical coordinates: $\rho = \sqrt{x^2 + y^2}, \quad \alpha = \arctan(x/y)$

$$\chi_{\nu}^{J^\pi T}(x, y) = \sum_k C_{k\nu}(\rho) \phi_k^{\ell_x \ell_y}(\alpha)$$

$$\phi_k^{\ell_x \ell_y}(\alpha) = N_k \sin^{\ell_x}(\alpha) \cos^{l_y}(\alpha) P_{k/2}^{\ell_x + 1/2, \ell_y + 1/2}(\cos 2\alpha)$$

NCSM/RGM-3B

$$\sum_{\nu}\int dx dy x^2 y^2 \left[\mathbb{H}_{\nu'\nu}(x,y,x',y') - E\delta\nu'\nu \frac{\delta(x'-x)}{x'x}\frac{\delta(y'-y)}{y'y}\right]\chi_{\nu}^{J^\pi T}(x,y) = 0$$

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$$\chi_{\nu}^{J^\pi T}(x,y) = \sum_{k} C_{k\nu}(\rho)\phi_k^{\ell_x \ell_y}(\alpha)$$

After changing to hyperspherical coordinates and integrating in α,α':

$$\sum_{\nu k}\int d\rho \rho^5 \left[\bar{\mathcal{H}}_{\nu'\nu}^{k'k}(\rho',\rho) - E\frac{\delta(\rho-\rho')}{\rho^5}\delta_{\nu'\nu}\delta_{k'k}\right] C_{k\nu}^{J^\pi T}(\rho) = 0$$

NCSM/RGM-3B

$$\sum_{\nu} \int dx dy x^2 y^2 \left[\mathbb{H}_{\nu'\nu}(x,y,x',y') - E\delta\nu'\nu \frac{\delta(x'-x)}{x'x} \frac{\delta(y'-y)}{y'y} \right] \chi_{\nu}^{J^\pi T}(x,y) = 0$$

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Coupled-channel microscopic R-matrix method on a Lagrange mesh

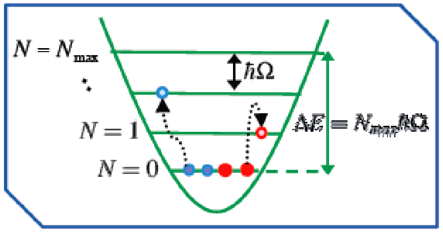
^{4}He+n+n

Input

- Accurate soft NN interaction: SRG-evolved chiral N^{3}LO potential with λ=1.5 fm^{-1}
  - Fits NN data with high accuracy
  - But: misses both chiral initial and SRG-induced NNN force
  - Fortunately: two effects mostly cancel each other

- ^{4}He ab initio wave function obtained within the NCSM

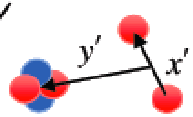
$$H^{(A-2)}\psi_{\beta_1}^{(A-2)}(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_{A-2}) = E_{\beta_1}^{(A-2)}\psi_{\beta_1}^{(A-2)}(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_{A-2})$$

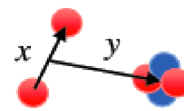


  - Large expansions in *A*-body harmonic oscillator (HO) basis
  - Preserves: 1) Pauli principle, and 2) translational invariance
  - Can include NNN interactions
  - ^{4}He binding energy close to experiment: 28.22 MeV (expt.: 28.3 MeV)

- Fully antisymmetric channel states: $\hat{A}_{\nu_3} = \sqrt{\frac{(A-2)!2!}{A!}}\left[1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{i,k}+\sum_{i<j=1}^{A-2}\hat{P}_{i,A-1}\hat{P}_{j,A}\right]\frac{1-\hat{P}_{A-1,A}}{\sqrt{2}}$

Norm Kernel

$\Big\langle$ 

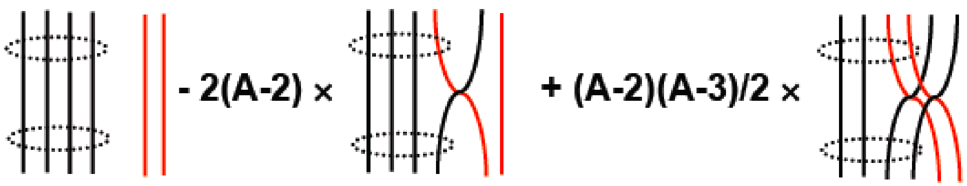
$$\left| \left(1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{i,k}+\sum_{i<j=1}^{A-2}\hat{P}_{i,A-1}\hat{P}_{j,A}\right)\left(1-\hat{P}_{A-1,A}\right) \right|$$


$\Big\rangle$

$$N_{\nu_3'\nu_3}(x',y',x,y)=\frac{1}{2}\left[1-(-1)^{\ell_x'+S_{23}'+T_{23}'}\right]\left[1-(-1)^{\ell_x+S_{23}+T_{23}}\right]\times\left\{\delta_{\nu_3'\nu_3}\frac{\delta(x'-x)}{x'x}\frac{\delta(y'-y)}{y'y}\right.$$

$$-2(A-2)\sum_{n_x'n_y'}\sum_{n_xn_y}R_{n_x'\ell_x'}(x')R_{n_y'\ell_y'}(y')\left\langle\Phi_{\nu_3'n_x'n_y'}\right|P_{A-2,A}\left|\Phi_{\nu_3n_xn_y}\right\rangle R_{n_x\ell_x}(x)R_{n_y\ell_y}(y)$$

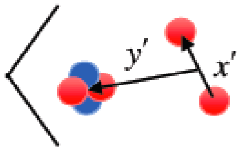
$$\left.+\frac{(A-2)(A-3)}{2}\sum_{n_x'n_y'}\sum_{n_xn_y}R_{n_x'\ell_x'}(x')R_{n_y'\ell_y'}(y')\left\langle\Phi_{\nu_3'n_x'n_y'}\right|P_{A-3,A-1}P_{A-2,A}\left|\Phi_{\nu_3n_xn_y}\right\rangle R_{n_x\ell_x}(x)R_{n_y\ell_y}(y)\right\}$$



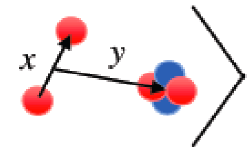
$${}_{SD}\left\langle\psi_{\mu_1}^{(A-2)}\right|a^+a\left|\psi_{\nu_1}^{(A-2)}\right\rangle_{SD}$$

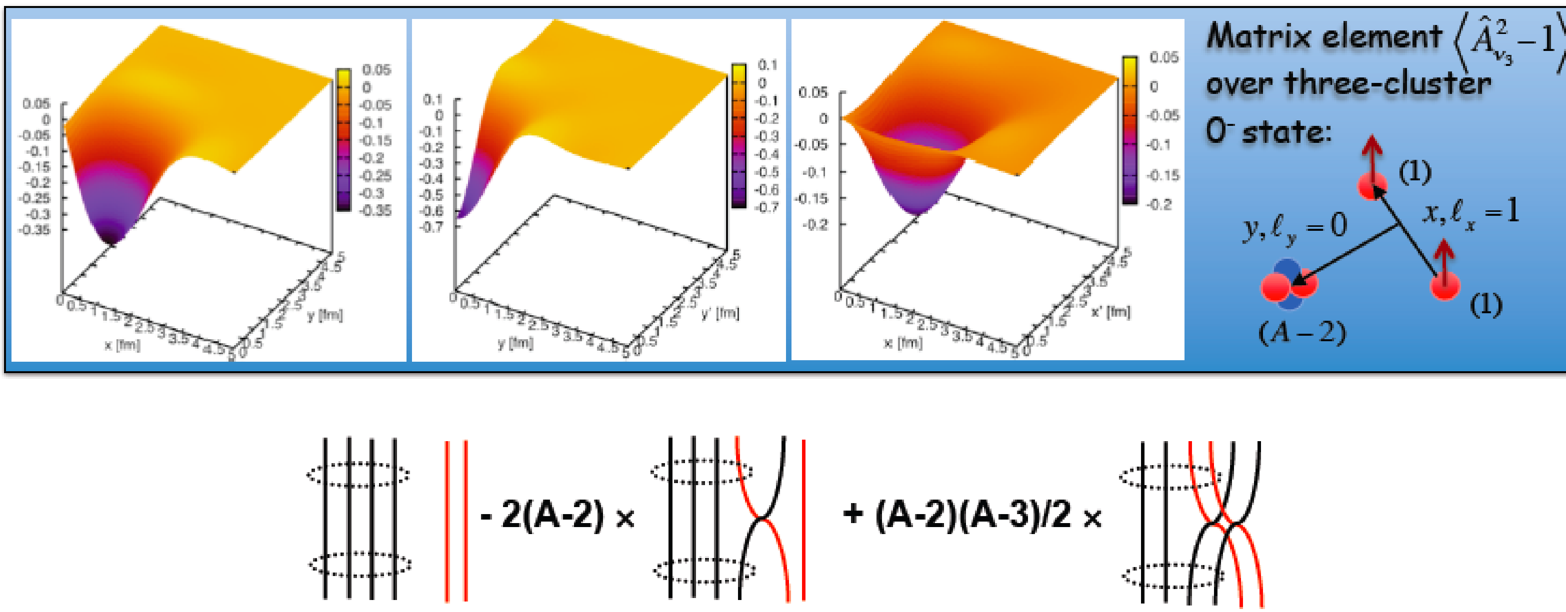
$${}_{SD}\left\langle\psi_{\mu_1}^{(A-2)}\right|a^+a^+\,a\,a\left|\psi_{\nu_1}^{(A-2)}\right\rangle_{SD}$$

Norm Kernel



$$\left| \left(1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{i,k}+\sum_{i<j=1}^{A-2}\hat{P}_{i,A-1}\hat{P}_{j,A}\right)\left(1-\hat{P}_{A-1,A}\right) \right|$$



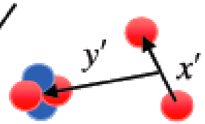


$${}_{\mathrm{SD}}\left\langle \psi^{(A-2)}_{\mu_1} \middle| a^{+}a \middle| \psi^{(A-2)}_{\nu_1} \right\rangle_{\mathrm{SD}}$$

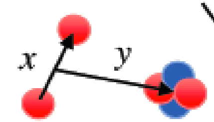
$${}_{\mathrm{SD}}\left\langle \psi^{(A-2)}_{\mu_1} \middle| a^{+}a^{+}\,a\,a \middle| \psi^{(A-2)}_{\nu_1} \right\rangle_{\mathrm{SD}}$$

C. Romero Redondo, S. Quaglioni, P. Navrátil. February 22nd, 2013

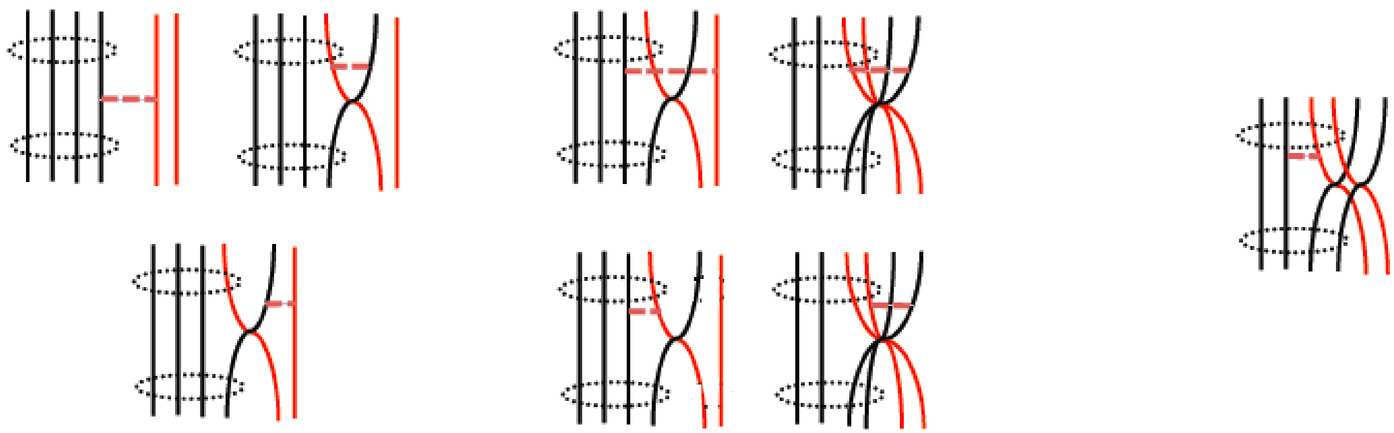
Hamiltonian Kernel



$$\left\langle \cdots \right| \left(\sum_{l=1}^{A-2}\sum_{m=A-1}^{A} V_{lm} + V_{A-1A}\right)\left(1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{i,k}+\sum_{i<j=1}^{A-2}\hat{P}_{i,A-1}\hat{P}_{j,A}\right)\left(1-\hat{P}_{A-1,A}\right)\left| \cdots \right\rangle$$



$$= V(x) N_{\nu_3'\nu_3}(x',y',x,y) +$$



$${}_{SD}\left\langle \psi_{\mu_1}^{(A-2)} \middle| a^{+} a \middle| \psi_{\nu_1}^{(A-2)} \right\rangle_{SD}$$

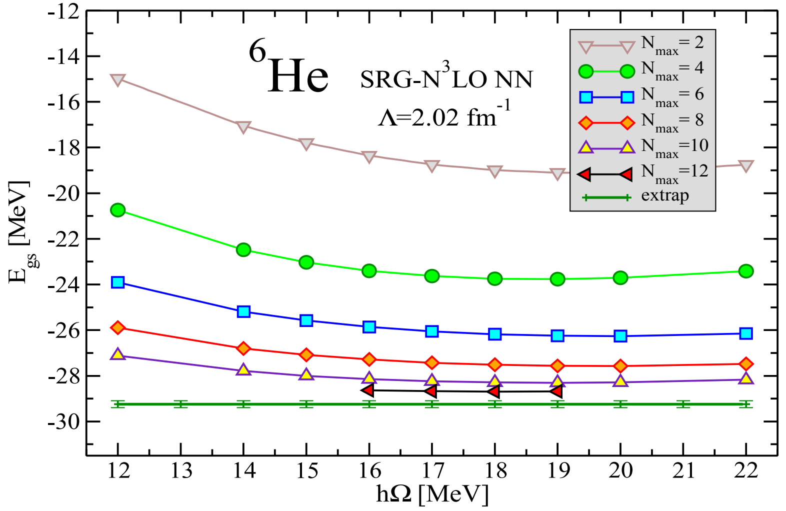
$${}_{SD}\left\langle \psi_{\mu_1}^{(A-2)} \middle| a^{+} a^{+}\, a\, a \middle| \psi_{\nu_1}^{(A-2)} \right\rangle_{SD}$$

$${}_{SD}\left\langle \psi_{\mu_1}^{(A-2)} \middle| a^{+} a^{+} a^{+}\, a\, a\, a \middle| \psi_{\nu_1}^{(A-2)} \right\rangle_{SD}$$

^{4}He+n+n

- Preliminary NCSM/RGM results
  - n+n+^{4}He(g.s.), N_{max} = 11, $h\Omega$ = 14 MeV
  - SRG-NN chiral with λ = 1.5 fm^{-1}
- Comparison with NCSM:
  - ~1 MeV difference in E_{gs} due to excitations of ^{4}He core, at present only included in the NCSM calculation
  - Contrary to NCSM, NCSM/RGM ^{4}He+n+n w.f. has the appropriate asymptotic behavior

^{6}He ground state, NCSM

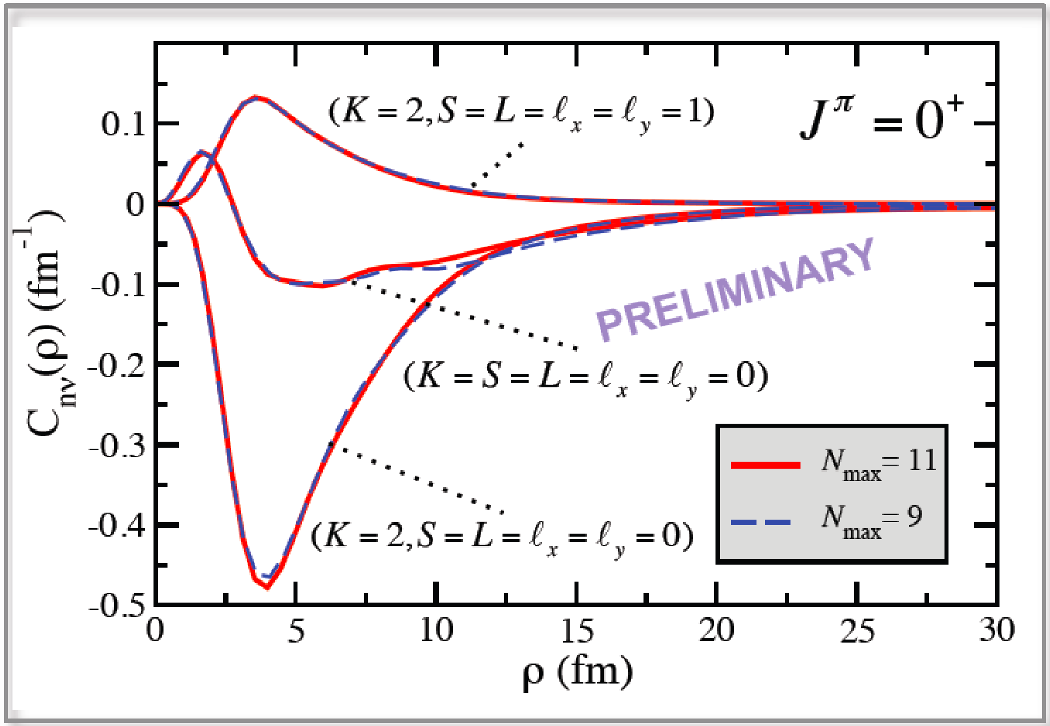


HO model space	Eg.s. (^{4}He) [MeV] (NCSM)	Eg.s. (^{6}He) [MeV] (NCSM)	Eg.s. (^{6}He) [MeV] (NCSM/RGM)
Nmax = 12	-28.22	-29.75	-28.72

PRELIMINARY

^{4}He+n+n

^{6}He ground state wave function

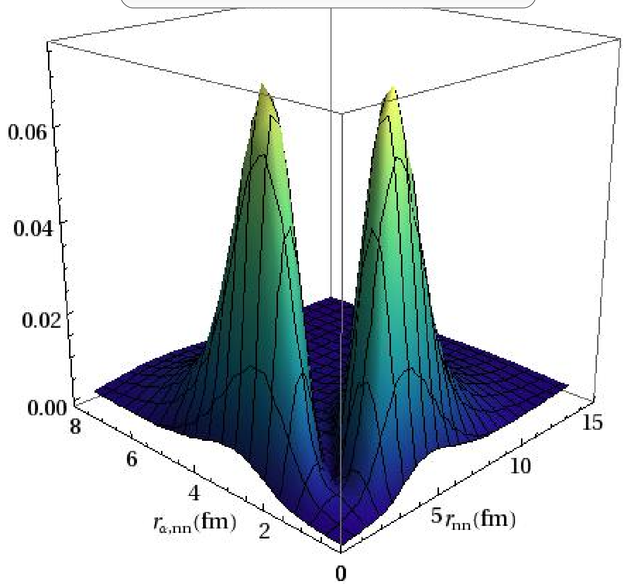


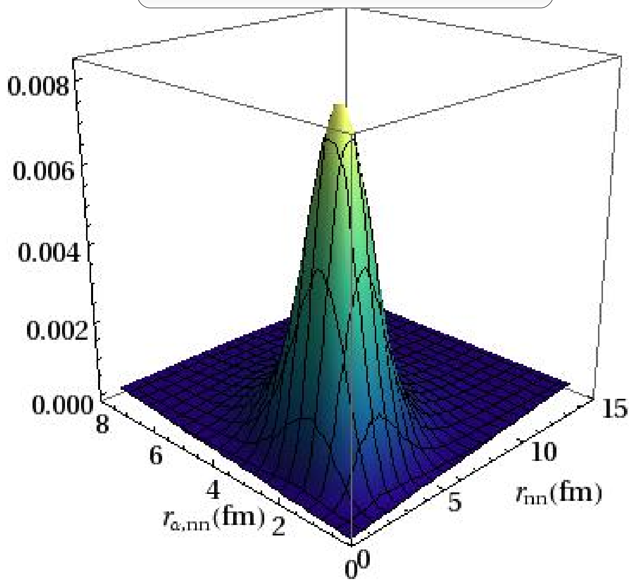
$$\chi_\nu^{J^\pi T}(x,y) = \sum_k C_{k\nu}(\rho)\phi_k^{\ell_x \ell_y}(\alpha)$$

^{4}He+n+n

^{6}He ground state wave function

Probability distribution





$\ell_x = \ell_y = L = S_{nn} = 0$

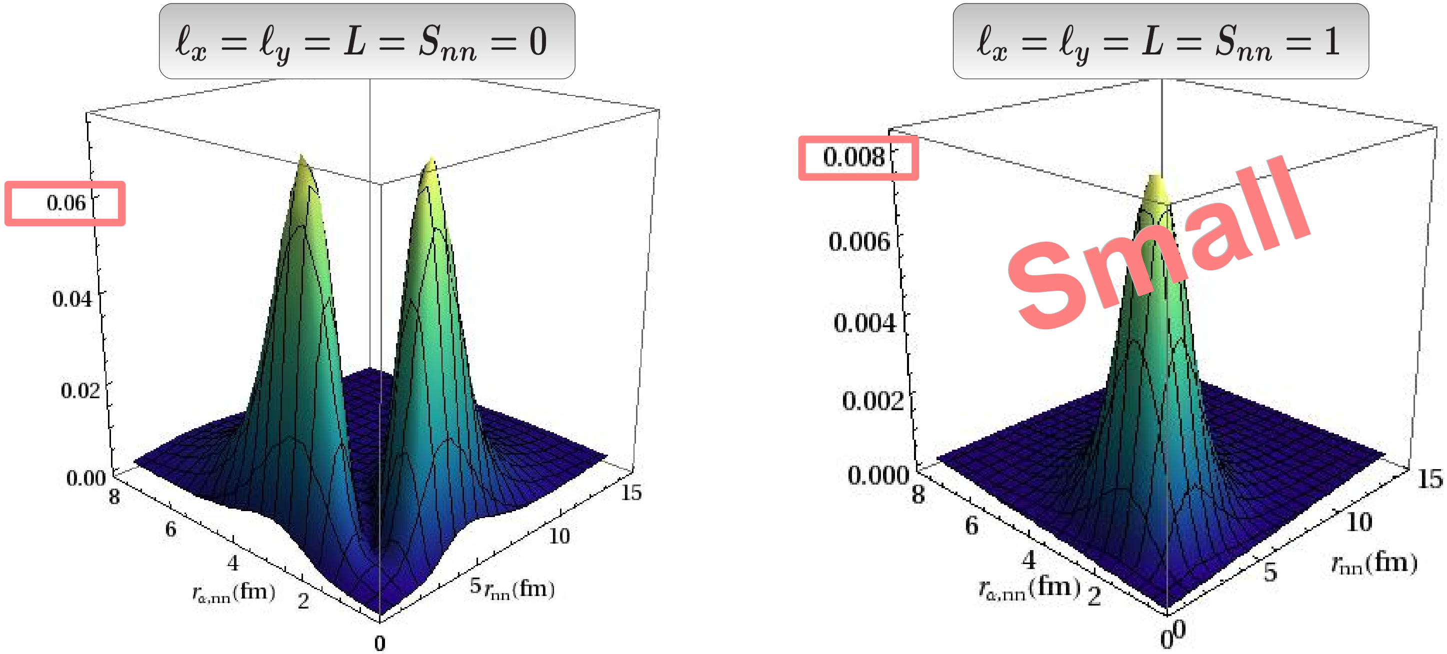
$\ell_x = \ell_y = L = S_{nn} = 1$

$$\chi_\nu^{J^\pi T}(x,y) = \sum_k C_{k\nu}(\rho)\phi_k^{\ell_x \ell_y}(\alpha)$$

^{4}He+n+n

^{6}He ground state wave function

Probability distribution

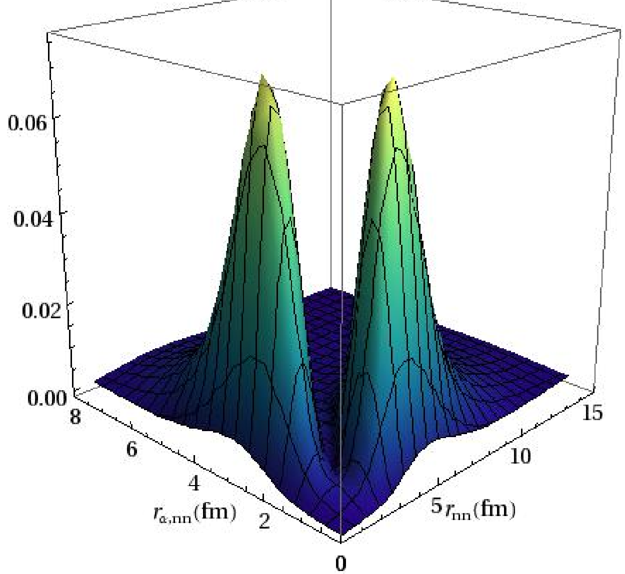


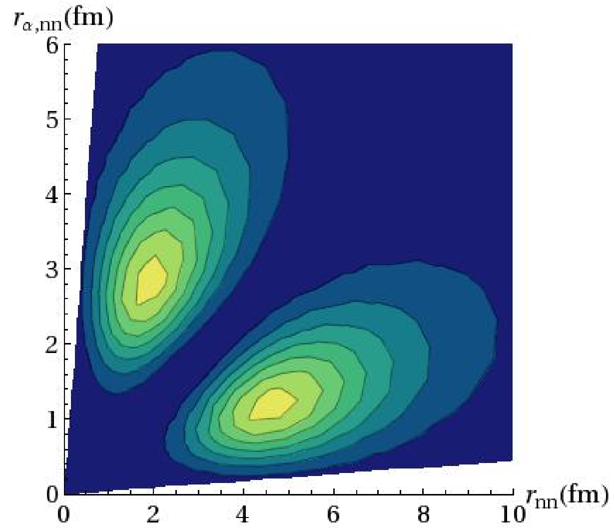
$$\chi_{\nu}^{J^{\pi}T}(x,y) = \sum_{k} C_{k\nu}(\rho)\phi_k^{\ell_x \ell_y}(\alpha)$$

^{6}He ground state wave function

Probability distribution

$$\ell_x = \ell_y = L = S_{nn} = 0$$





^{6}He ground state wave function

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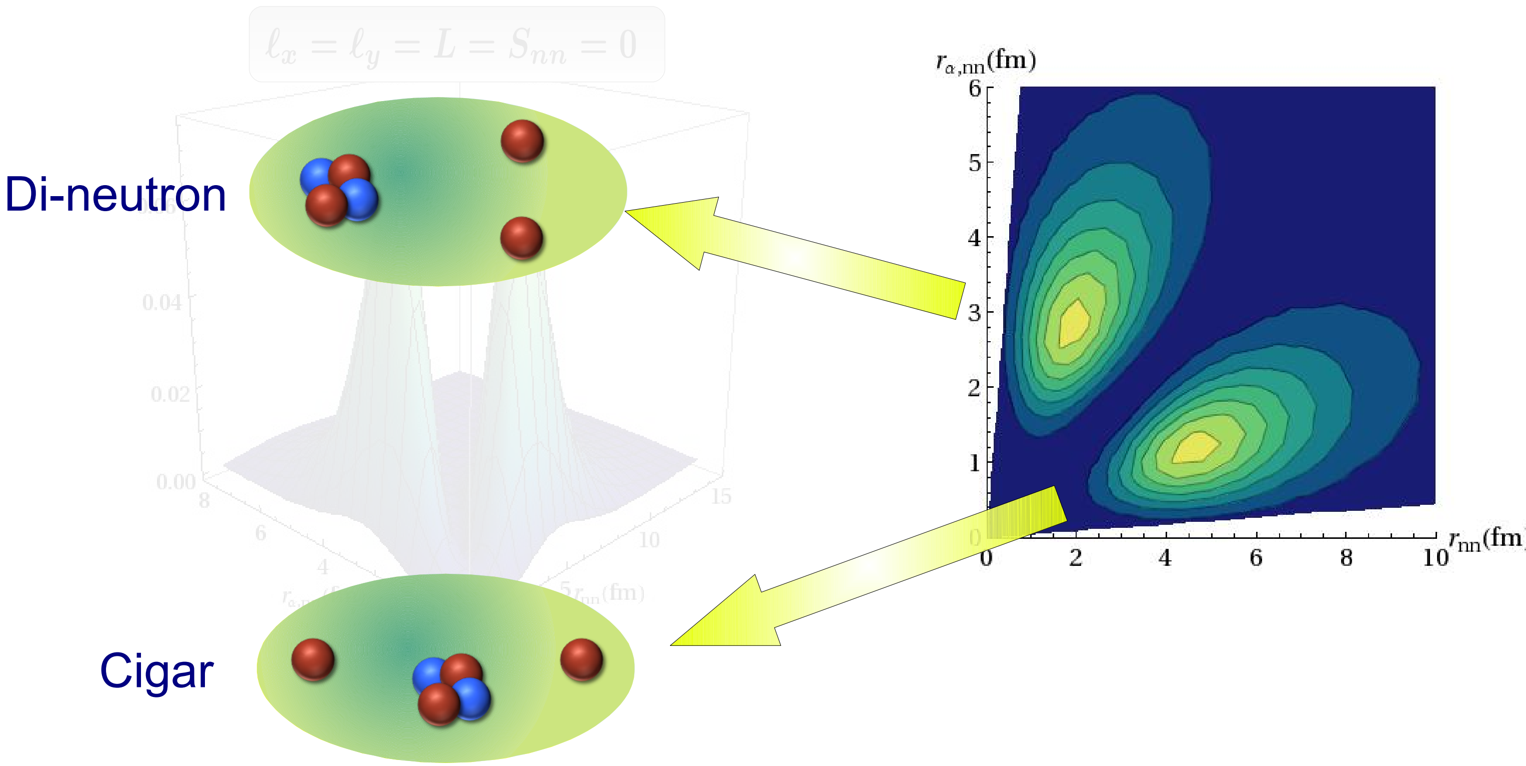
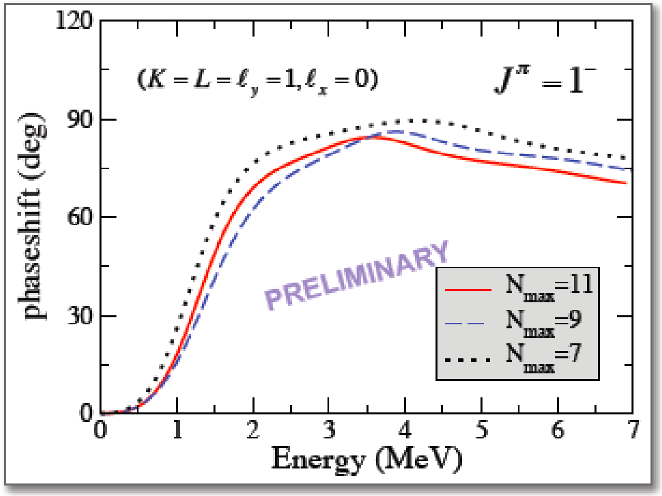
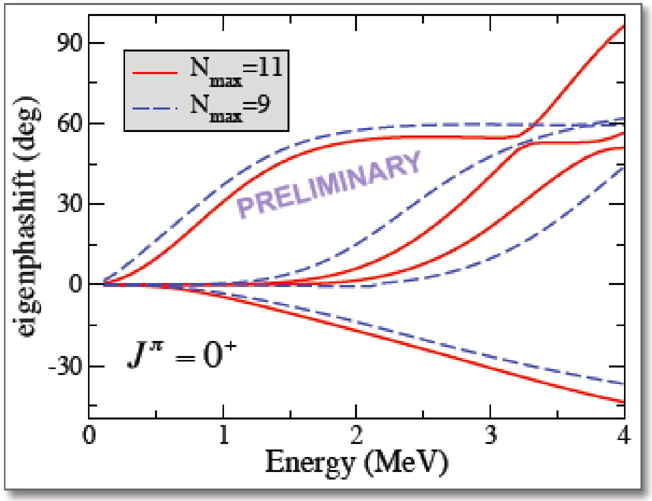
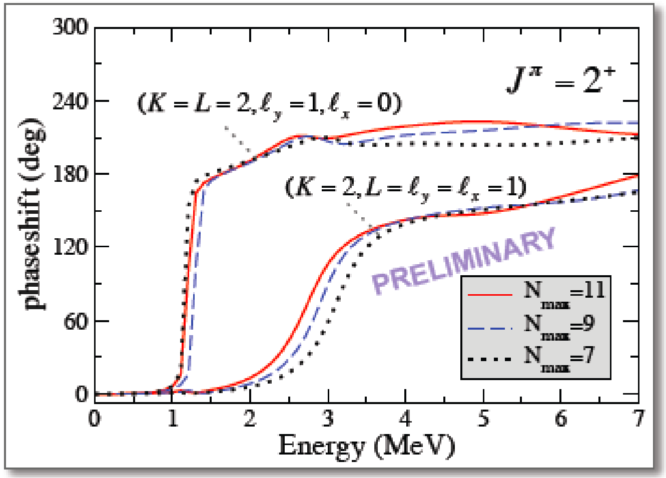


^{4}He+n+n

- Preliminary NCSM/RGM results
  - n+n+^{4}He(g.s.), N_{max} = 7,9,11; $h\Omega$ = 14 MeV
  - SRG-NN chiral with λ = 1.5 fm^{-1}
  - Matching at ρ_o = 30 fm (no propagation)
- Convergence of 1$^-$ and 2$^-$ is reasonable

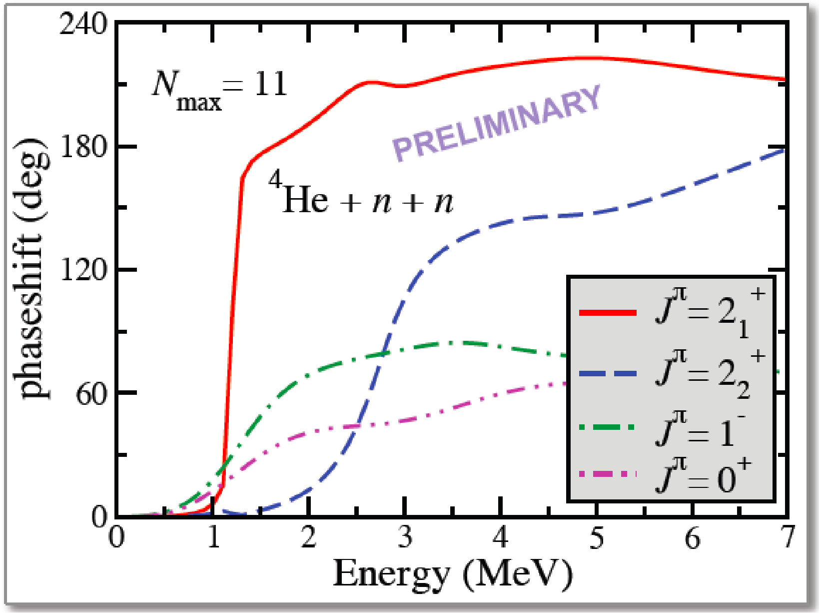


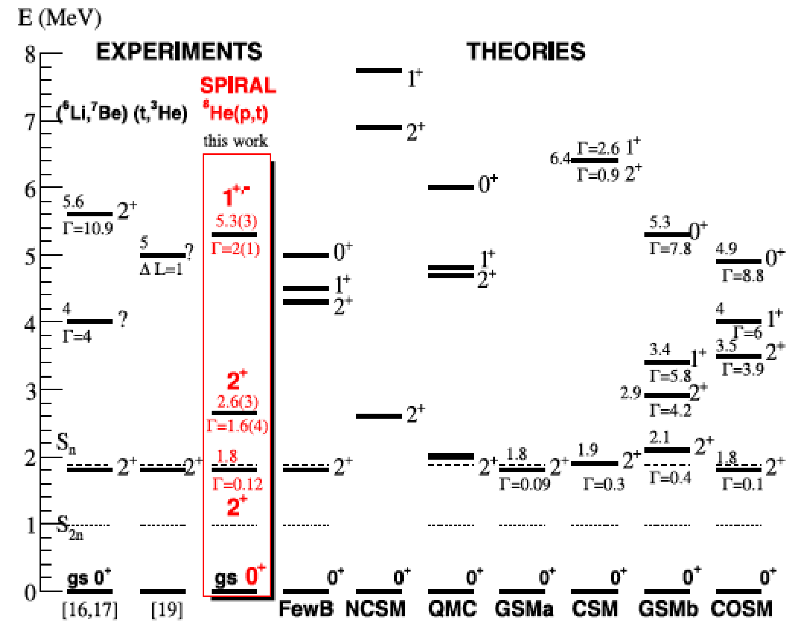




^{4}He+n+n

Preliminary results



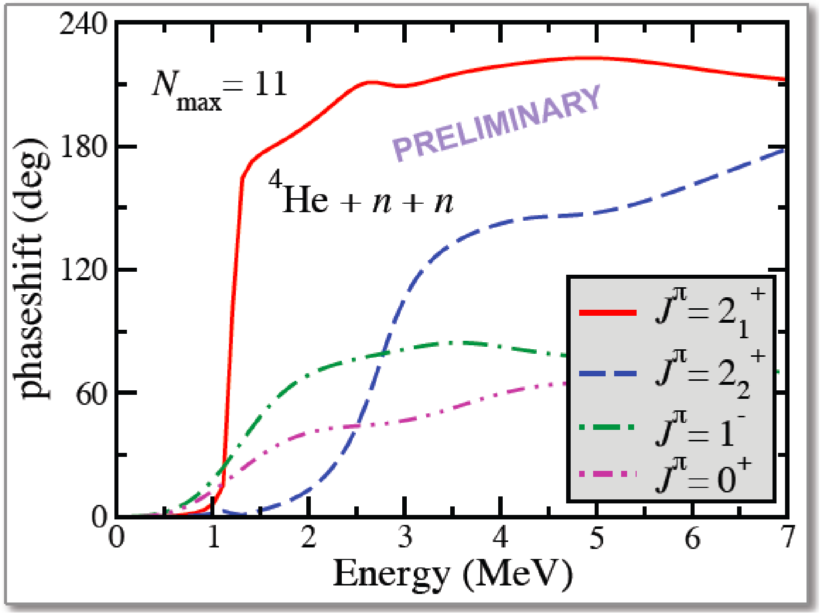


- Very narrow resonance in 2^+_1 at 1.1 MeV (Experimental 0.824MeV)
- Second resonance in 2^+_2 at 2.6 MeV Γ~ 800KeV (New exp. at Ganil 1.67 MeV, Γ=1.6MeV)
- Broad structures 1^- at ~1.2MeV, Γ=1.8MeV

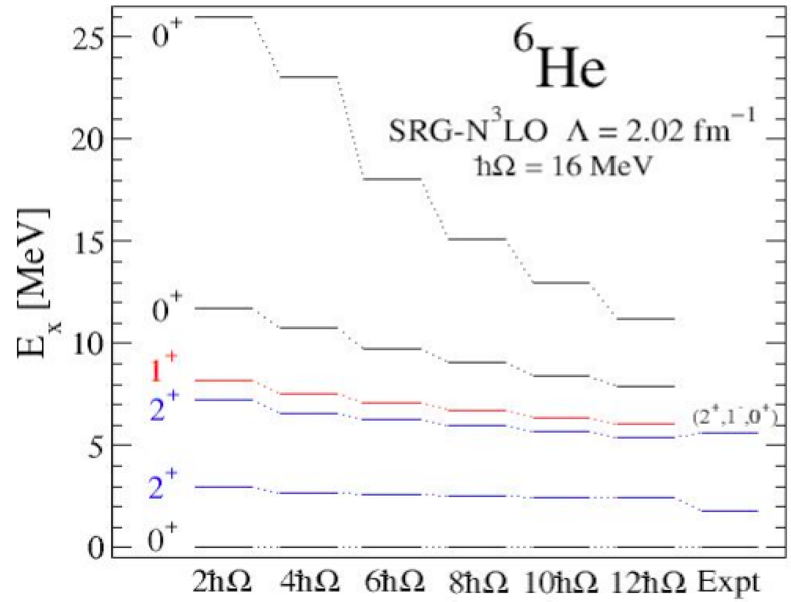
  and in 0^+ at ~1MeV, Γ=2.6MeV

$$\Gamma = \frac{2}{\frac{d\delta(E)}{dE}}\Bigg|_{E=E_R}$$

^{4}He+n+n



NCSM



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Summary and Outlook

Approach is very versatile for studying different types of nuclear systems

Bound and resonant states in structure problems

Continuum states for reaction problems

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Bound and resonant states in structure problems

Continuum states for reaction problems

Preliminary results are very promising

Ground state of ^{6}He

Continuum $^{4}He+n+n$

Work to do

Study more deeply the stability of the results

Introduce ^{4}He excitations

Is matching at 30fm enough?

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Introduce ^{4}He excitations

Is matching at 30fm enough?

Transfer reactions, i.e, $^{3}H(^{3}H,2n)^{4}He$

Derive and calculate couplings between two and three body clusters

TRIUMF: Alberta | British Columbia | Calgary
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Montréal | Northern British Columbia | Queen's
Regina | Saint Mary's | Simon Fraser | Toronto
Victoria | Winnipeg | York

Thank you!
Merci