Chiral Three-Body Interactions in Ab-Initio Nuclear Reactions

Joachim Langhammer

TECHNISCHE UNIVERSITÄT DARMSTADT

Outline

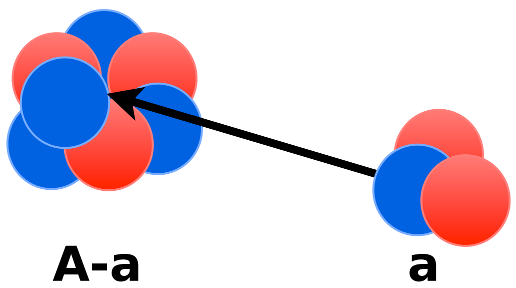
- Motivation
- Ingredients from Three-Body Technology
- No-Core Shell Model/Resonating Group Method
  - Reminder - Hamiltonian Kernel for Two-Body Interactions
  - Inclusion of Three-Body Interactions
- n+^{4}He Scattering
- Scattering with Heavier Projectiles & Targets
- Conclusions

Motivation

Realistic ab-initio description of light nuclei

Bound states
& spectroscopy

Resonances
& scattering states



(IT-)NCSM/RGM
approach

(IT-)NCSM

Ab-initio description of
nuclear clusters

RGM

Describing relative
motion of clusters

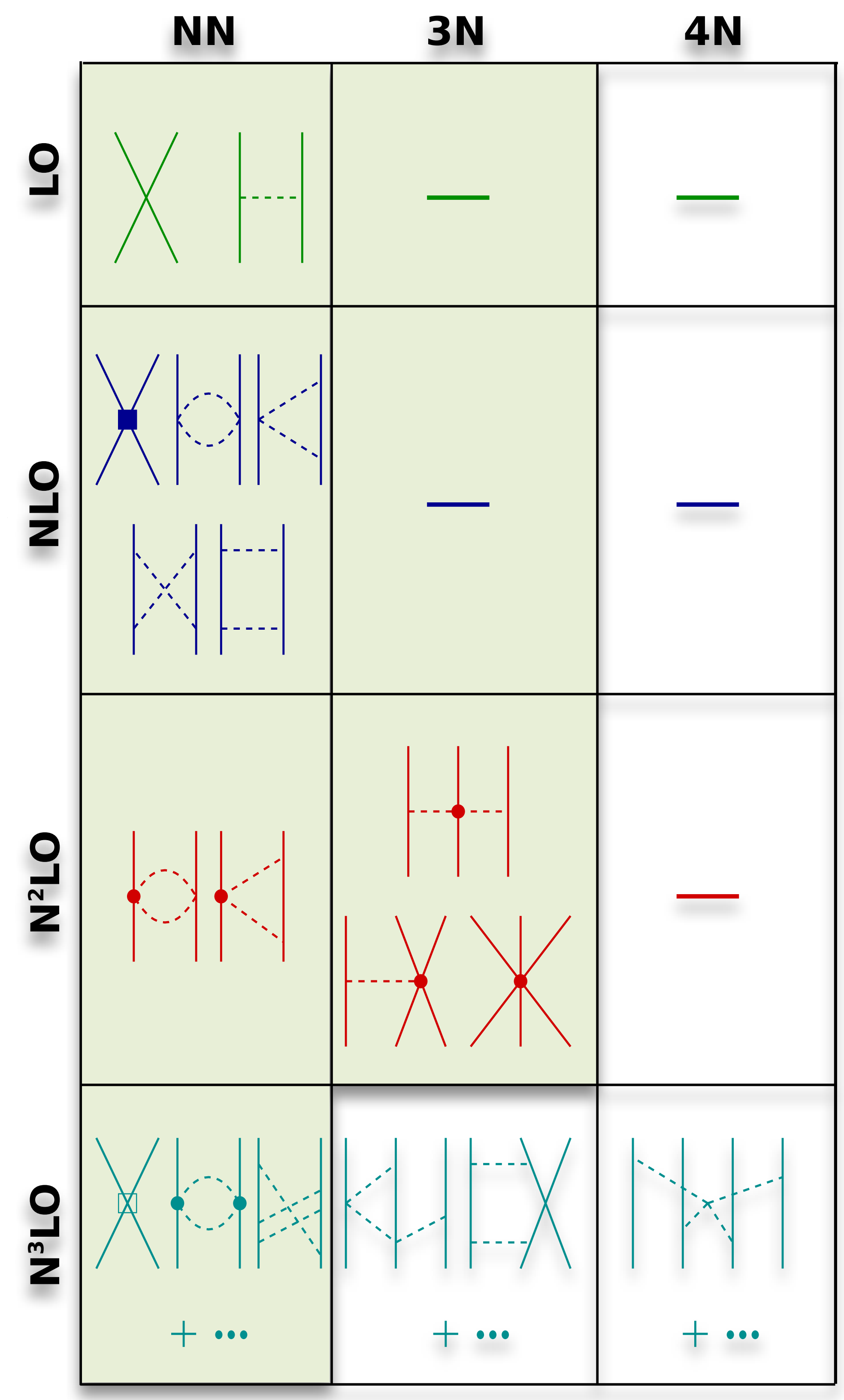
Successfully applied with NN interactions

Ingredients from Three-Body Technology

The Chiral NN+3N Hamiltonian

Weinberg, van Kolck, Machleidt, Entem, Meißner, Epelbaum, Krebs, Bernard, Skibinski, Golak...

- Hierarchy of consistent nuclear NN, 3N,... forces (and currents)
- Standard Hamiltonian
  - NN interaction @ N^{3}LO (Λ=500MeV)
    [Entem, Machleidt, Phys.Rev C **68**, 041001(R) (2003)]
  - 3N interaction @ N^{2}LO (Λ=500MeV)
    - Local form by Navrátil
    - LECs c_D, c_E fitted to β-decay halflife & binding energy of ^{3}H
      [Gazit et.al., Phys.Rev.Lett. **103**, 102502 (2009)]
- Ready for 3N forces @ N^{3}LO



The Similarity Renormalization Group

Wegner, Glazek, Wilson, Perry, Bogner, Furnstahl, Hergert, Calci, Langhammer, Roth, Jurgenson, Navrátil,...

...yields an evolved Hamiltonian with improved convergence properties in many-body calculations

- Unitary transformation of Hamiltonian $H_\alpha = U_\alpha^\dagger H U_\alpha$

Different SRG-Evolved Hamiltonians

- **NN-only**: start with NN initial Hamiltonian and keep two-body terms only
- **NN+3N-induced**: start with NN initial Hamiltonian and keep two- and three-body terms
- **NN+3N-full**: start with NN+3N initial Hamiltonian and keep two- and three-body terms

No-Core Shell Model / Resonating Group Method - Formalism -

In collaboration with

G. Hupin, S. Quaglioni, P. Navrátil & R. Roth

S. Quaglioni and P. Navrátil ----- Phys. Rev. Lett. 101, 092501 (2008)

P. Navrátil, R. Roth and S. Quaglioni ----- Phys. Rev. C 82, 034609 (2010)

S. Quaglioni, P. Navrátil, G. Hupin, J. Langhammer et al. ----- Few-Body Syst. DOI 10.1007/s00601-012-0505-0 (2012)

S. Quaglioni, P. Navrátil, R. Roth, W. Horiuchi ----- J.Phys.Conf.Ser. 402 (2012)

General Approach of NCSM/RGM

Wildermuth,Thompson, Tang, ..., Navrátil, Quaglioni, Roth, Hupin, Langhammer,...

- Represent $H\,|\psi^{J\pi T}\rangle = E\,|\psi^{J\pi T}\rangle$ using the **over-complete basis**

$$|\psi^{J\pi T}\rangle = \sum_{\nu}\int \mathrm{d}r r^2 \frac{g_{\nu}^{J\pi T}(r)}{r}\mathcal{A}_{\nu}\,|\phi_{\nu r}^{J\pi T}\rangle$$

$g_{\nu}^{J\pi T}(r)$ unknown

with the binary-cluster channel states

$$|\phi^{J\pi T}\rangle = \left\{\,|\Phi^{(A-a)}\rangle\,|\Phi^{(a)}\rangle\right\}^{J\pi T}\frac{\delta(r-r_{A-a,a})}{r r_{A-a,a}}$$

NCSM delivers $|\Phi^{(A-a)}\rangle$ and $|\Phi^{(a)}\rangle$

- Solve **generalized eigenvalue** problem

$$\sum_{\nu}\int \mathrm{d}r r^2 \left[\mathcal{H}_{\nu,\nu'}^{J\pi T}(r',r) - E\mathcal{N}_{\nu,\nu'}^{J\pi T}(r,r')\right]\frac{g_{\nu r}^{J\pi T}}{r} = 0$$

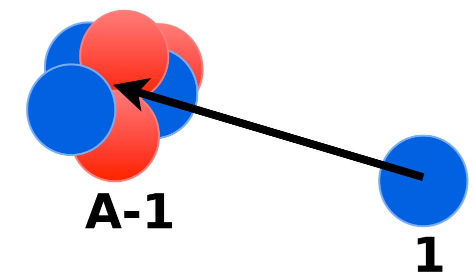
Hamiltonian kernel $\langle\phi_{\nu' r'}^{J\pi T}|\,\mathcal{A}_{\nu'}H\mathcal{A}_{\nu}\,|\phi_{\nu r}^{J\pi T}\rangle$

Norm kernel $\langle\phi_{\nu' r'}^{J\pi T}|\,\mathcal{A}_{\nu'}\mathcal{A}_{\nu}\,|\phi_{\nu r}^{J\pi T}\rangle$

The Hamiltonian Kernel: NN Diagrams

- Consider NN-interaction kernels with **single-nucleon projectiles**

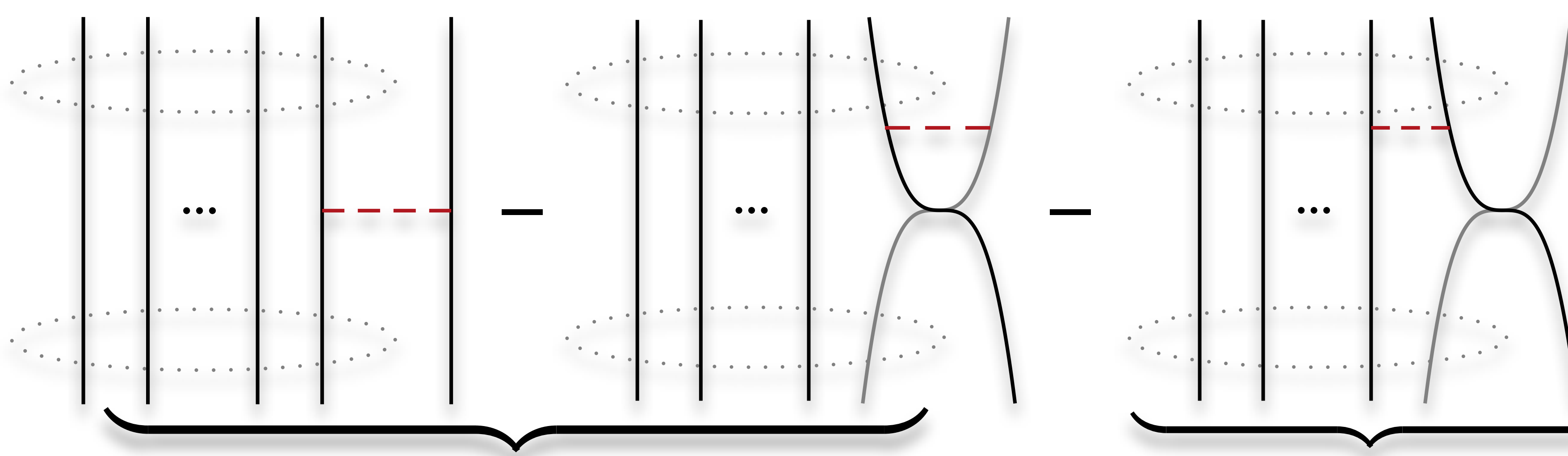
$$\langle \phi^{J\pi T}_{\nu' r'} | V_{NN} \mathcal{A}^2 | \phi^{J\pi T}_{\nu r} \rangle = \langle \phi^{J\pi T}_{\nu' r'} | V_{NN} \left[1 - \sum_{i=1}^{A-1} T_{i,A} \right] | \phi^{J\pi T}_{\nu r} \rangle$$



$$\left.\begin{aligned} &= (A-1) \langle \phi^{J\pi T}_{\nu' r'} | V_{A-1,A} | \phi^{J\pi T}_{\nu r} \rangle \\ &- (A-1) \langle \phi^{J\pi T}_{\nu' r'} | V_{A-1,A} T_{A-1,A} | \phi^{J\pi T}_{\nu r} \rangle \end{aligned}\right\} \text{“direct” kernel}$$

$$-(A-1)(A-2) \langle \phi^{J\pi T}_{\nu' r'} | V_{A-2,A} T_{A-1,A} | \phi^{J\pi T}_{\nu r} \rangle \qquad \text{“exchange” kernel}$$

=



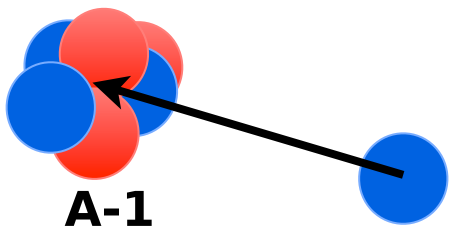
Densities managable

$\propto \langle \Phi'^{(A-1)} | a^\dagger a | \Phi^{(A-1)} \rangle$ $\qquad$ $\propto \langle \Phi'^{(A-1)} | a^\dagger a^\dagger a a | \Phi^{(A-1)} \rangle$

Towards Inclusion of Full 3N Forces

- Derive expressions for 3N-interaction kernel

$$\langle \phi^{J\pi T}_{\nu' r'} | V_{3N} \mathcal{A}^2 \, | \phi^{J\pi T}_{\nu r} \rangle = \langle \phi^{J\pi T}_{\nu' r'} | V_{3N} \left[1 - \sum_{i=1}^{A-1} T_{i,A} \right] | \phi^{J\pi T}_{\nu r} \rangle$$



$$\left.\begin{aligned} &= \frac{(A-1)(A-2)}{2} \langle \phi^{J\pi T}_{\nu' r'} | V_{A-2,A-1,A} \, | \phi^{J\pi T}_{\nu r} \rangle \\ &- \frac{(A-1)(A-2)}{2} \langle \phi^{J\pi T}_{\nu' r'} | V_{A-2,A-1,A} T_{A-2,A} \, | \phi^{J\pi T}_{\nu r} \rangle \\ &- \frac{(A-1)(A-2)}{2} \langle \phi^{J\pi T}_{\nu' r'} | V_{A-1,A-2,A} T_{A-1,A} \, | \phi^{J\pi T}_{\nu r} \rangle \end{aligned}\right\}$$

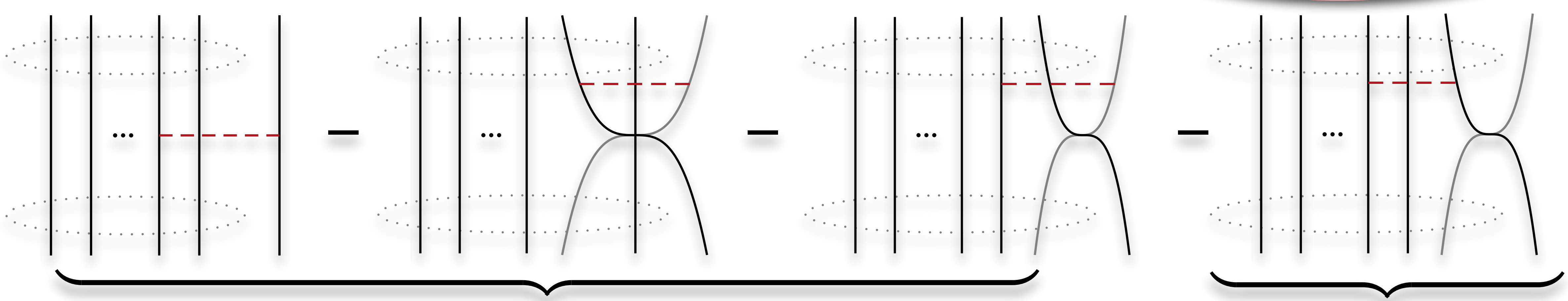
"direct" kernel

$$- \frac{(A-1)(A-2)(A-3)}{2} \langle \phi^{J\pi T}_{\nu' r'} | V_{A-3,A-2,A} T_{A-1,A} \, | \phi^{J\pi T}_{\nu r} \rangle$$

Handling of 3-body density challenging

el

$=$



$\propto \langle \Phi'^{(A-1)} | a^\dagger a^\dagger a a \, | \Phi^{(A-1)} \rangle$ $\qquad$ $\propto \langle \Phi'^{(A-1)} | a^\dagger a^\dagger a^\dagger a a a \, | \Phi^{(A-1)} \rangle$

Handling of Three-Body Density

Hupin, Quaglioni, Navrátil

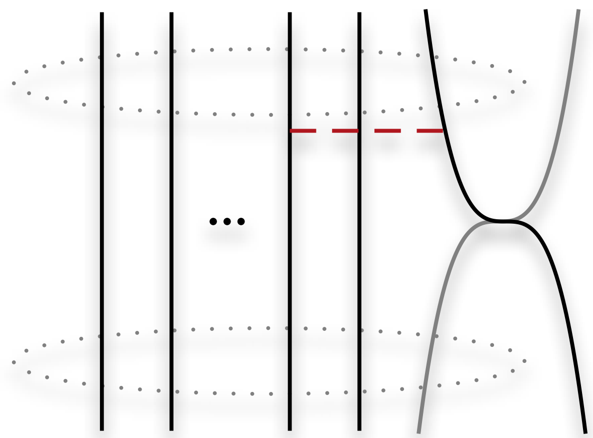
① Precomputed coupled densities

$$\sum_{\substack{j_0 j'_0 \\ K J_0}} \sum_{\substack{t_0 t'_0 \\ \tau T_0}} \sum_{\substack{n_a l_a j_a \\ n_b l_b j_b}} \sum_{\substack{n_\alpha l_\alpha j_\alpha \\ n'_a l'_a j'_a}} \sum_{\substack{n'_b l'_b j'_b \\ g' t'_g}} \frac{1}{12} \hat{\tau} \hat{K} \hat{J}_0 \hat{T}_0 \hat{g}' \hat{t}'_g (-1)^{j'_a + j'_b - j'_0 + j' + K + I_1 + J} (-1)^{3/2 - t'_0 + j' + \tau + T_1 + T}$$

$$\begin{Bmatrix} I_1 & K & I'_1 \\ j' & J & j \end{Bmatrix} \begin{Bmatrix} j' & K & j \\ g' & j'_0 & J_0 \end{Bmatrix} \begin{Bmatrix} T_1 & \tau & T'_1 \\ \frac{1}{2} & T & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \tau & \frac{1}{2} \\ t'_g & t'_0 & T_0 \end{Bmatrix}$$

$${}_a\langle ((nlj'_a, nlj'_b) j'_0 t'_0, nlj') J_0 T_0 | V_{3N} | ((nlj_\alpha, nlj_a) j_0 t_0, nlj_b) J_0 T_0 \rangle_a$$

$$\left\langle \Phi'^{(A-1)} I'_1 T'_1 \left\| \left[(a^\dagger_{nlj} (a^\dagger_{nlj'_b} a^\dagger_{nlj'_a})^{j'_0 t'_0})^{g' t'_g} ((\tilde{a}_{nlj_\alpha} \tilde{a}_{nlj_a})^{j_0 t_0} \tilde{a}_{nlj_b})^{J_0 T_0} \right]^{K\tau} \right\| \Phi^{(A-1)} I_1 T_1 \right\rangle$$



- Make use of JT-coupled 3N matrix elements

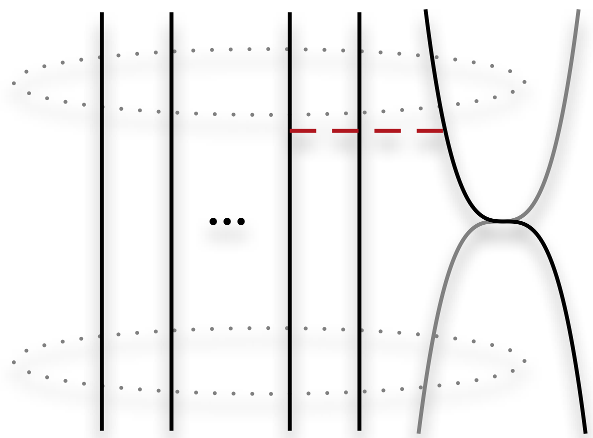
Handling of Three-Body Density

Hupin, Quaglioni, Navrátil

1 Precomputed coupled densities

$$\sum_{\substack{j_0 j_0' t_0 t_0' \\ K J_0 \tau T_0}} \sum_{\substack{n_a l_a j_a \\ n_b l_b j_b}} \sum_{\substack{n_\alpha l_\alpha j_\alpha \\ n_a' l_a' j_a'}} \sum_{\substack{n_b' l_b' j_b' \\ g' t_g'}} \frac{1}{12}\hat{\tau}\hat{K}\hat{J}_0\hat{T}_0\hat{g}'\hat{t}_g'(-1)^{j_a'+j_b'-j_0'+j'+K+I_1+J}(-1)^{3/2-t_0'+j'+\tau+T_1+T}$$

$$\begin{Bmatrix} I_1 & K & I_1' \\ j' & J & j \end{Bmatrix} \begin{Bmatrix} j' & K & j \\ g' & j_0' & J_0 \end{Bmatrix} \begin{Bmatrix} T_1 & \tau & T_1' \\ \frac{1}{2} & T & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \tau & \frac{1}{2} \\ t_g' & t_0' & T_0 \end{Bmatrix}$$



$${}_a\langle((nlj_a', nlj_b')j_0't_0', nlj')J_0T_0 | V_{3N} | ((nlj_\alpha, nlj_a)j_0t_0, nlj_b)J_0T_0\rangle_a$$

$$\left\langle \Phi'^{(A-1)} I_1' T_1' \left\| \left[\left(a^\dagger_{nlj}\left(a^\dagger_{nlj_b'} a^\dagger_{nlj_a'}\right)^{j_0' t_0'}\right)^{g' t_g'} \left(\left(\tilde{a}_{nlj_\alpha}\tilde{a}_{nlj_a}\right)^{j_0 t_0}\tilde{a}_{nlj_b}\right)^{J_0 T_0} \right]^{K\tau} \right\| \Phi^{(A-1)} I_1 T_1 \right\rangle$$

- Make use of *JT*-coupled 3N matrix elements
- Three-body density cannot be stored...use a trick

Handling of Three-Body Density

Hupin, Quaglioni, Navrátil

1 Precomputed coupled densities

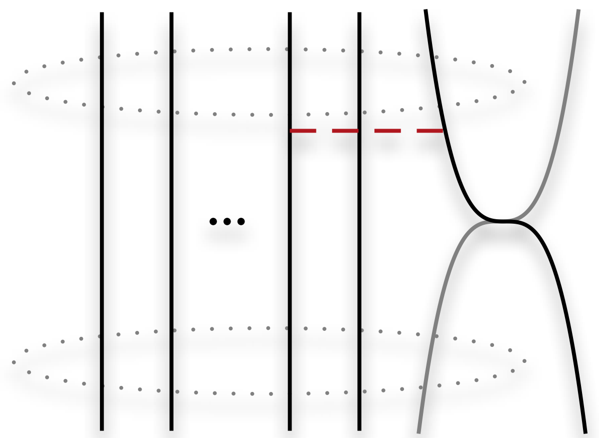
$$\sum_{\substack{j_0 j'_0 t_0 t'_0 \\ J_0 T_0}} \sum_{\substack{n_a l_a j_a \\ n_b l_b j_b}} \sum_{\substack{n_\alpha l_\alpha j_\alpha \\ n'_a l'_a j'_a}} \sum_{\substack{n'_b l'_b j'_b \\ g' t'_g}} \sum_{\phi'' I_\beta T_\beta} \tfrac{1}{12} \hat{J}_0 \hat{T}_0 \hat{g}' \hat{t}'_g (-1)^{j'_a + j'_b + J_0 + g' + I_\beta - I_1 + j} (-1)^{3/2 + T_0 + t'_g - T_1 + T_\beta}$$

$$\begin{Bmatrix} I_\beta & g' & I'_1 \\ J_0 & j'_0 & j' \\ J_1 & j & J \end{Bmatrix} \begin{Bmatrix} T_\beta & t'_g & T'_1 \\ T_0 & t'_0 & \frac{1}{2} \\ T_1 & \frac{1}{2} & T \end{Bmatrix}$$

$${}_a\langle ((nlj'_a, nlj'_b) j'_0 t'_0, nlj') J_0 T_0 | V_{3N} | ((nlj_\alpha, nlj_a) j_0 t_0, nlj_b) J_0 T_0 \rangle_a$$

$$\left\langle \Phi'^{(A-1)} I'_1 T'_1 \middle\| \left(a^\dagger_{nlj} \left(a^\dagger_{nlj'_b} a^\dagger_{nlj'_a} \right)^{j'_0 t'_0} \right)^{g' t'_g} \middle\| \Phi''^{(A-4)} I_\beta T_\beta \right\rangle$$

$$\left\langle \Phi''^{(A-4)} I_\beta T_\beta \middle\| \left((\tilde{a}_{nlj_\alpha} \tilde{a}_{nlj_a})^{j_0 t_0} \tilde{a}_{nlj_b} \right)^{J_0 T_0} \middle\| \Phi^{(A-1)} I_1 T_1 \right\rangle$$



- Make use of *JT*-coupled 3N matrix elements
- Three-body density cannot be stored…use a t
- Use reduced density matrix elements

Applicable to ^{4}He targets

Handling of Three-Body Density

Langhammer, Roth, Navrátil

② Compute uncoupled densities on-the-fly

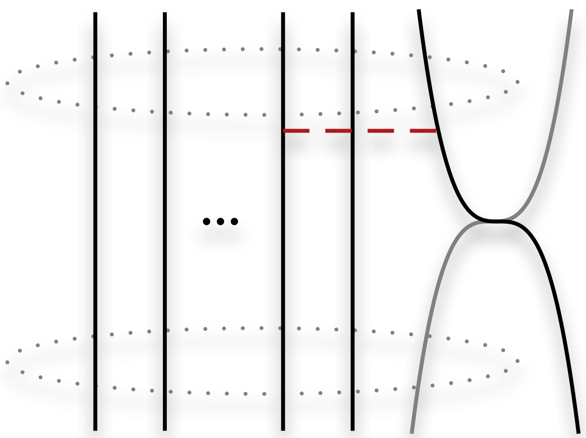
$$\sum_{jj'}\sum_{M_1 m_j}\sum_{M_{T_1} m_t}\sum_{M'_1 m'_j}\sum_{M'_{T_1} m'_t} \tfrac{1}{12}(-1)^{I_1+I'_1+2J+j+j'} \begin{Bmatrix} I_1 & \frac{1}{2} & s \\ l & J & j \end{Bmatrix} \begin{Bmatrix} I'_1 & \frac{1}{2} & s' \\ l' & J & j' \end{Bmatrix}$$

$$\left(\begin{matrix} I_1 & j \\ M_1 & m_j \end{matrix}\middle|\begin{matrix} J \\ M_J \end{matrix}\right)\left(\begin{matrix} T_1 & \frac{1}{2} \\ M_{T_1} & m_t \end{matrix}\middle|\begin{matrix} T \\ M_T \end{matrix}\right)\left(\begin{matrix} I'_1 & j' \\ M'_1 & m'_j \end{matrix}\middle|\begin{matrix} J \\ M'_J \end{matrix}\right)\left(\begin{matrix} T'_1 & \frac{1}{2} \\ M'_{T_1} & m'_t \end{matrix}\middle|\begin{matrix} T \\ M'_T \end{matrix}\right)$$

$$\sum_{\beta_{A-3}}\sum_{\beta_{A-2}}\sum_{\beta'_{A-3}}\sum_{\beta'_{A-2}}\sum_{\beta'_{A-1}}$$

$${}_a\langle \beta_{A-3}\beta_{A-2} n l j' m'_j \tfrac{1}{2} m'_t |\, V_{3N} \,|\beta'_{A-3}\beta'_{A-2}\beta'_{A-1}\rangle_a$$

$$\langle \Phi'^{(A-1)} I'_1 M'_1 T'_1 M'_{T_1} |\, a^\dagger_{nljm_j\frac{1}{2}m_t} a^\dagger_{\beta_{A-2}} a^\dagger_{\beta_{A-3}} a_{\beta'_{A-3}} a_{\beta'_{A-2}} a_{\beta'_{A-1}} \,|\Phi^{(A-1)} I_1 M_1 T_1 M_{T_1}\rangle$$

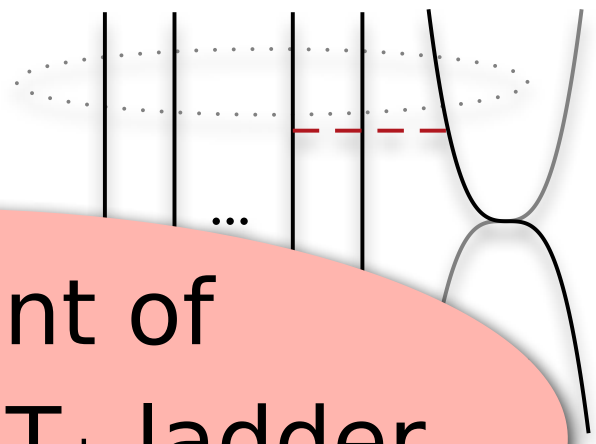


- Use m-scheme matrix elements ⇒ efficient decoupling
- Perfectly parallel

Handling of Three-Body Density

Langhammer, Roth, Navrátil

② Compute uncoupled densities on-the-fly



$$\sum_{jj'}\sum_{M_1 m_j}\sum_{M_{T_1} m_t}\sum_{M'_1 m'_j}\sum_{M'_{T_1} m'_t} \tfrac{1}{12}(-1)^{I_1+I'_1+2J+j+j'} \begin{Bmatrix} I_1 & \frac{1}{2} & s \\ l & J & j \end{Bmatrix} \begin{Bmatrix} I'_1 & \frac{1}{2} & s' \\ l' & J & j' \end{Bmatrix}$$

$$\left(\begin{matrix} I_1 & j \\ M_1 & m_j \end{matrix}\middle|\begin{matrix} J \\ M_J \end{matrix}\right)\left(\begin{matrix} T_1 & \frac{1}{2} \\ M_{T_1} & m_t \end{matrix}\middle|\begin{matrix} T \\ M_T \end{matrix}\right)\left(\begin{matrix} I'_1 & j' \\ M'_1 & m'_j \end{matrix}\middle|\begin{matrix} J \\ M'_J \end{matrix}\right)\left(\begin{matrix} T'_1 & \frac{1}{2} \\ M'_{T_1} & m' \end{matrix}\middle|\begin{matrix} T \\ \end{matrix}\right)$$

$$\sum_{\beta_{A-3}}\sum_{\beta_{A-2}}\sum_{\beta'_{A-3}}\sum_{\beta'_{A-2}}\sum_{\beta'_{A-1}}$$

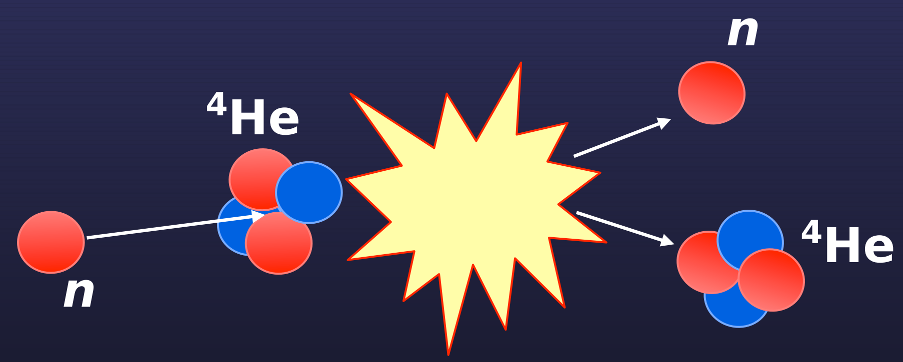
$${}_a\langle \beta_{A-3}\beta_{A-2} n l j' m'_j \tfrac{1}{2} m'_t | V_{3N} | \beta'_{A-3}\beta'_{A-2}\beta'_{A-1}\rangle_a$$

$$\langle \Phi'^{(A-1)} I'_1 M'_1 T'_1 M'_{T_1} | a^\dagger_{nljm_j\frac{1}{2}m_t} a^\dagger_{\beta_{A-2}} a^\dagger_{\beta_{A-3}} a_{\beta'_{A-3}} a_{\beta'_{A-2}} a_{\beta'_{A-1}} | \Phi^{(A-1)} I_1 M_1 T_1 M_{T_1}\rangle$$

Treatment of M_J, M_T via J±, T± ladder operators

- Use m-scheme matrix elements ⇒ effici
- Perfectly parallel

Access to heavier targets



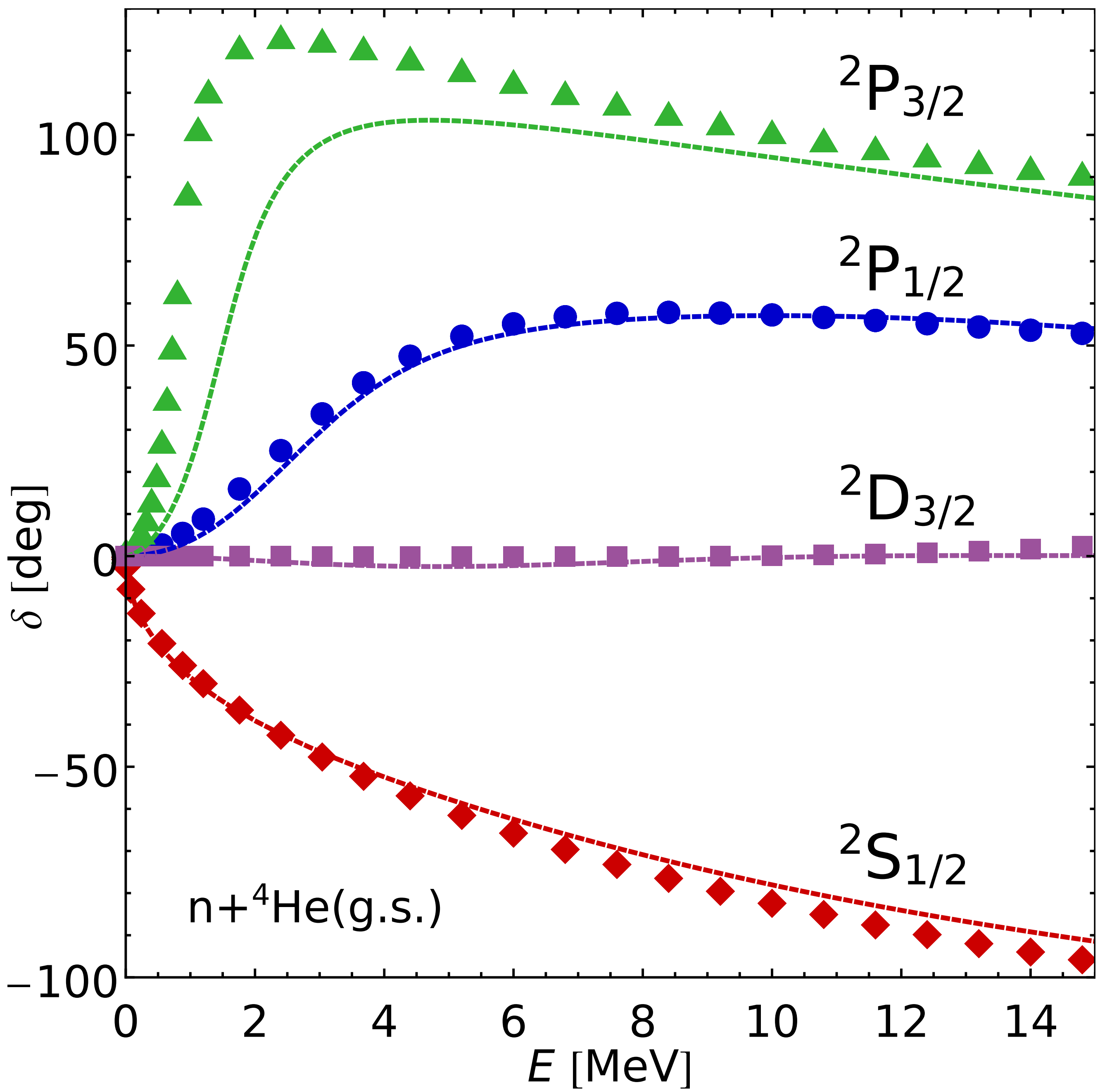
n+^{4}He Scattering

In collaboration with

G. Hupin, S. Quaglioni, P. Navrátil & R. Roth

G. Hupin, J. Langhammer et al. ----- in prep.

3N Force Effects on Phase Shifts



◆▲●■ Experiment

······· NN-only

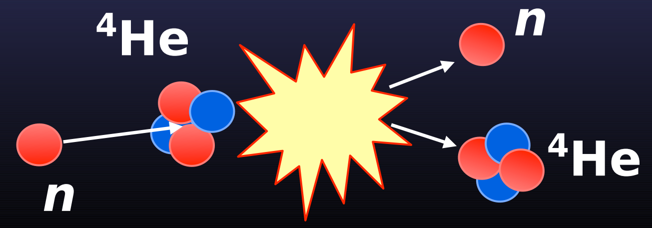
$N_{max} = 12$

$E_{3max} = 14$

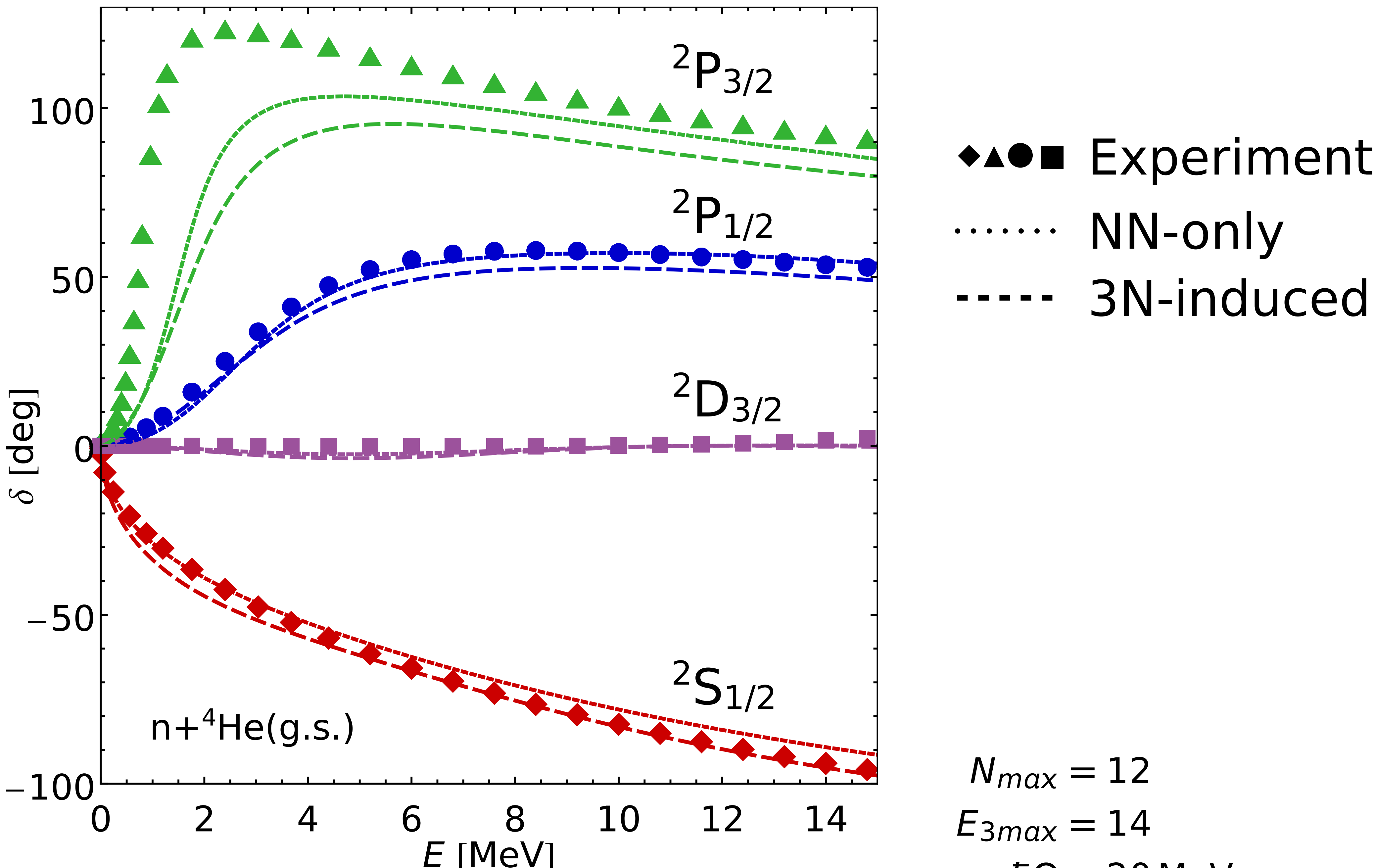
$\hbar\Omega = 20\,\mathrm{MeV}$

$\alpha = 0.0625\,\mathrm{fm}^4$

$\lambda = 2.0\,\mathrm{fm}^{-1}$



3N Force Effects on Phase Shifts



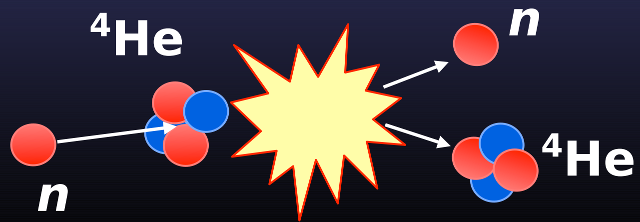
$N_{max} = 12$

$E_{3max} = 14$

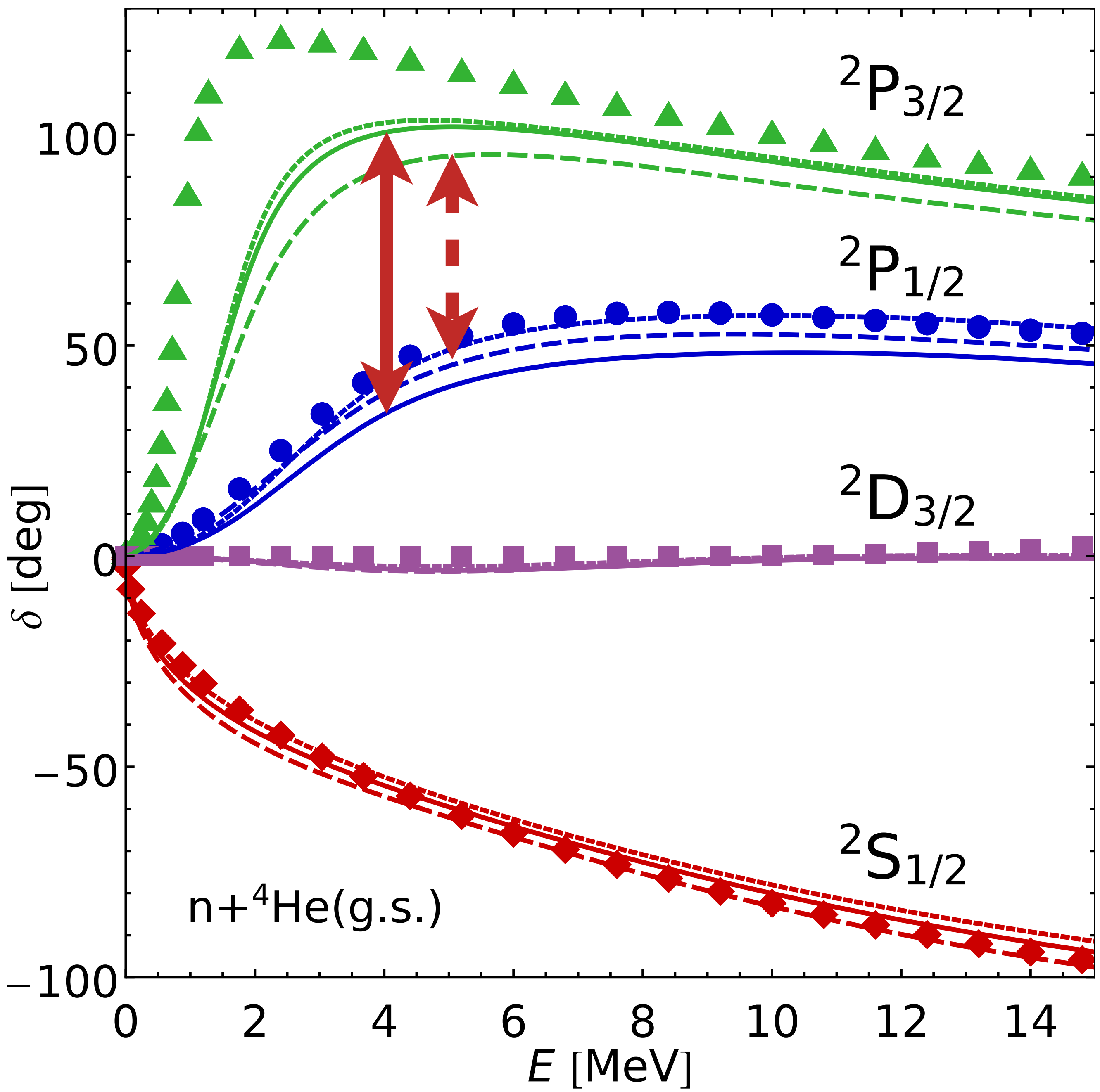
$\hbar\Omega = 20\,\mathrm{MeV}$

$\alpha = 0.0625\,\mathrm{fm}^4$

$\lambda = 2.0\,\mathrm{fm}^{-1}$



3N Force Effects on Phase Shifts



◆▲●■ Experiment

⋯⋯ NN-only

- - - - 3N-induced

—— 3N-full

3N increases spin-orbit splitting

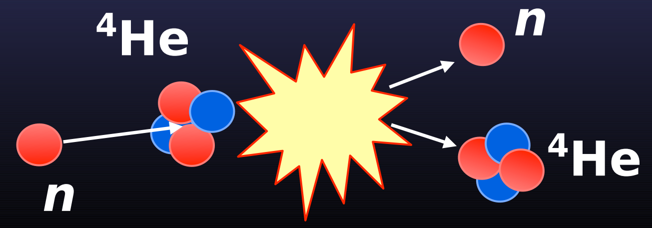
$N_{max} = 12$

$E_{3max} = 14$

$\hbar\Omega = 20\,\mathrm{MeV}$

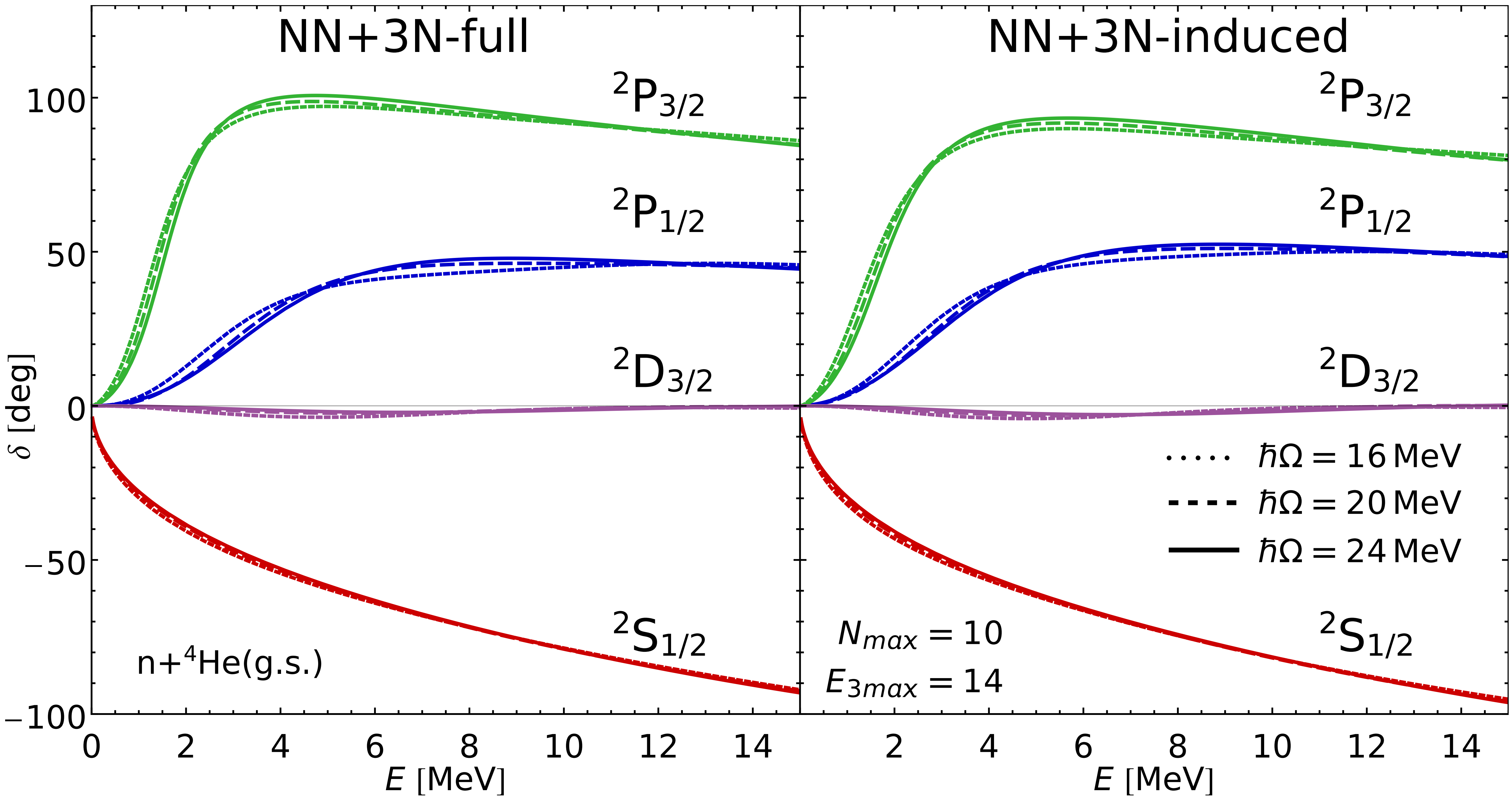
$\alpha = 0.0625\,\mathrm{fm}^4$

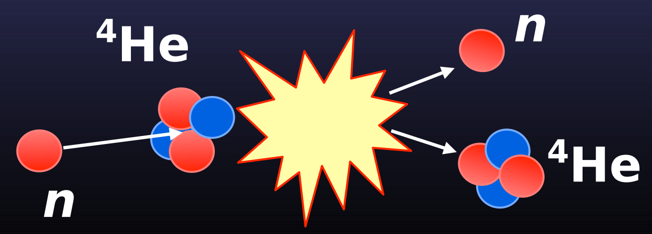
$\lambda = 2.0\,\mathrm{fm}^{-1}$



3N Force Effects on Phase Shifts

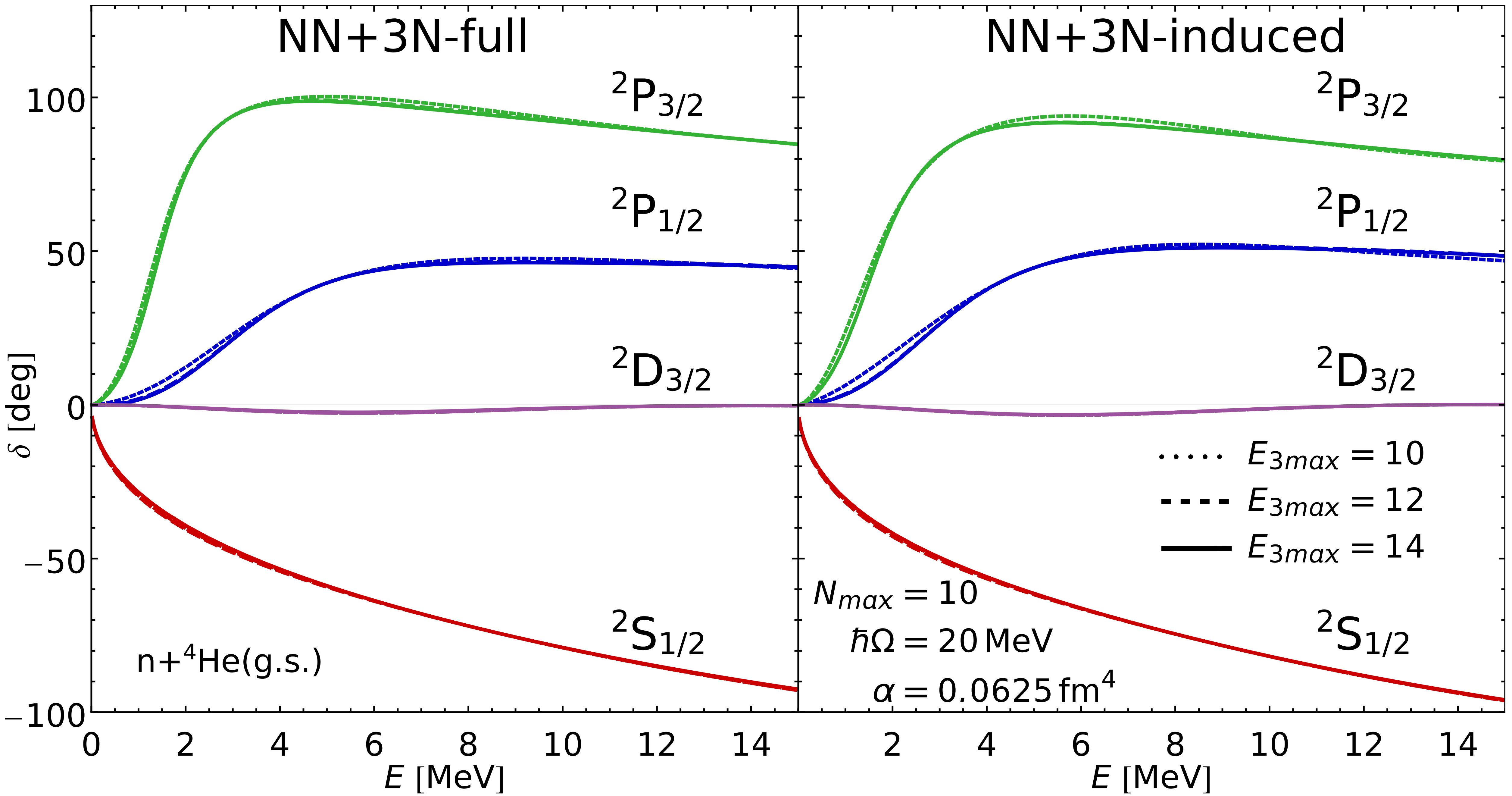
Study dependence
on HO frequency

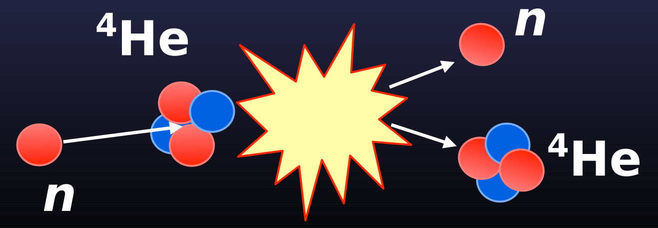




3N Force Effects on Phase Shifts

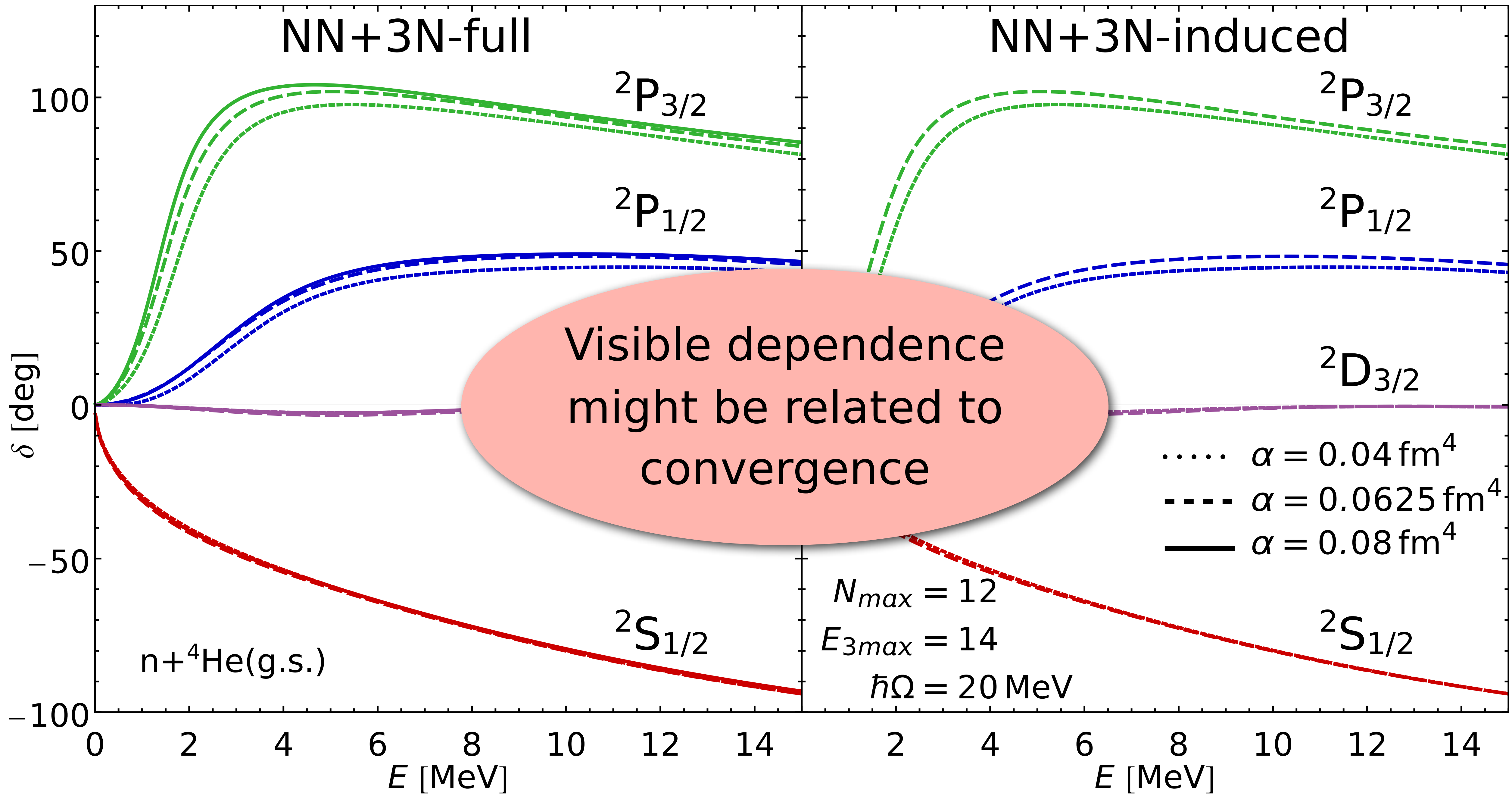
Study dependence on E_{3max} parameter





3N Force Effects on Phase Shifts

Study dependence on SRG parameter

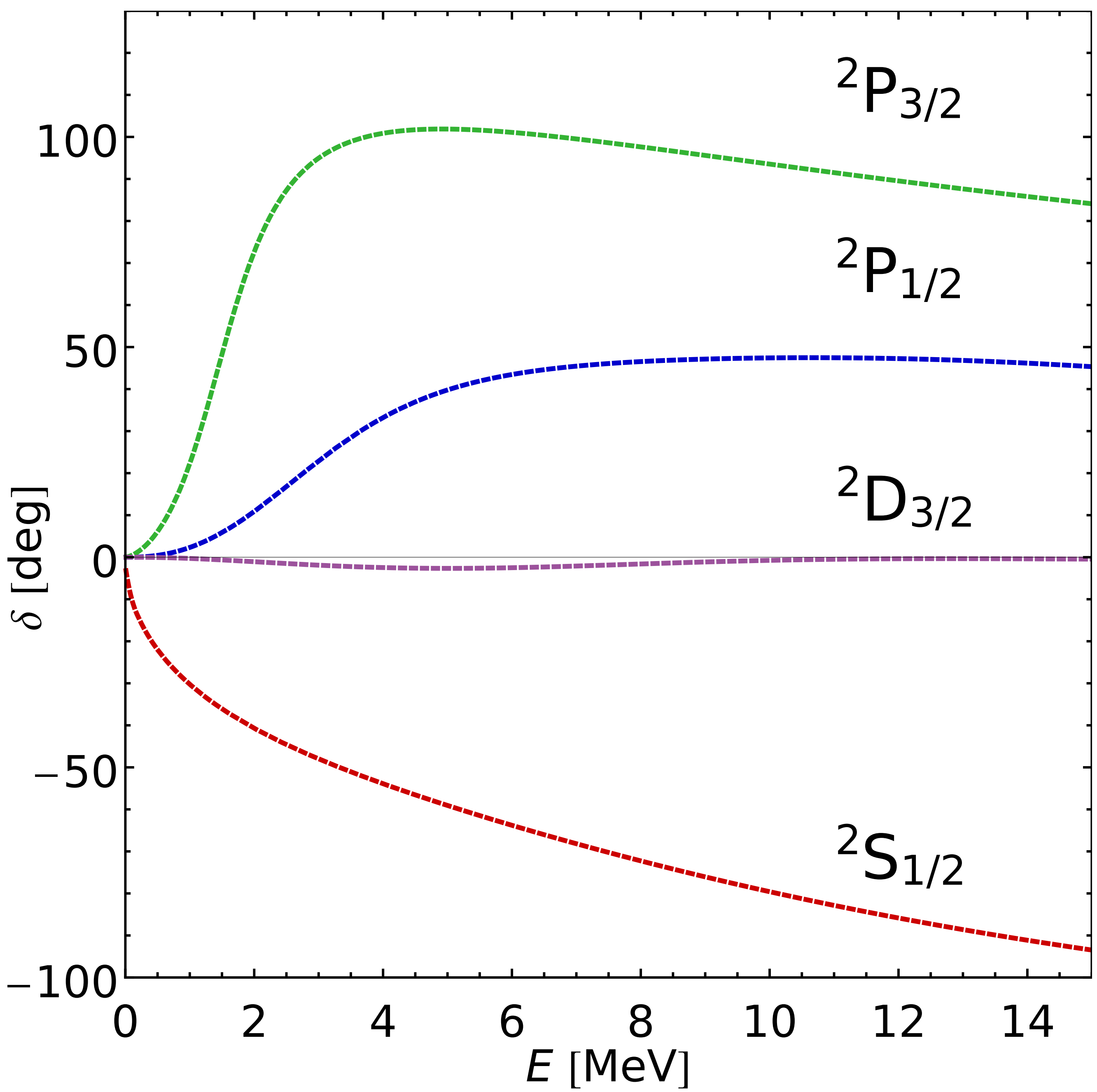


Visible dependence might be related to convergence

3N Force Effects on Phase Shifts

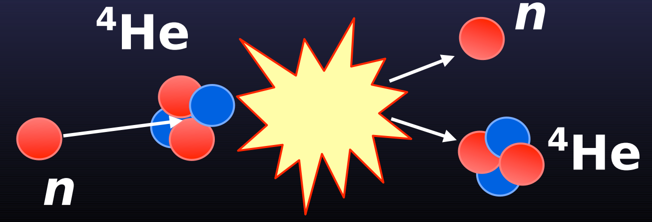
Inclusion of more excited states

NN+3N-full



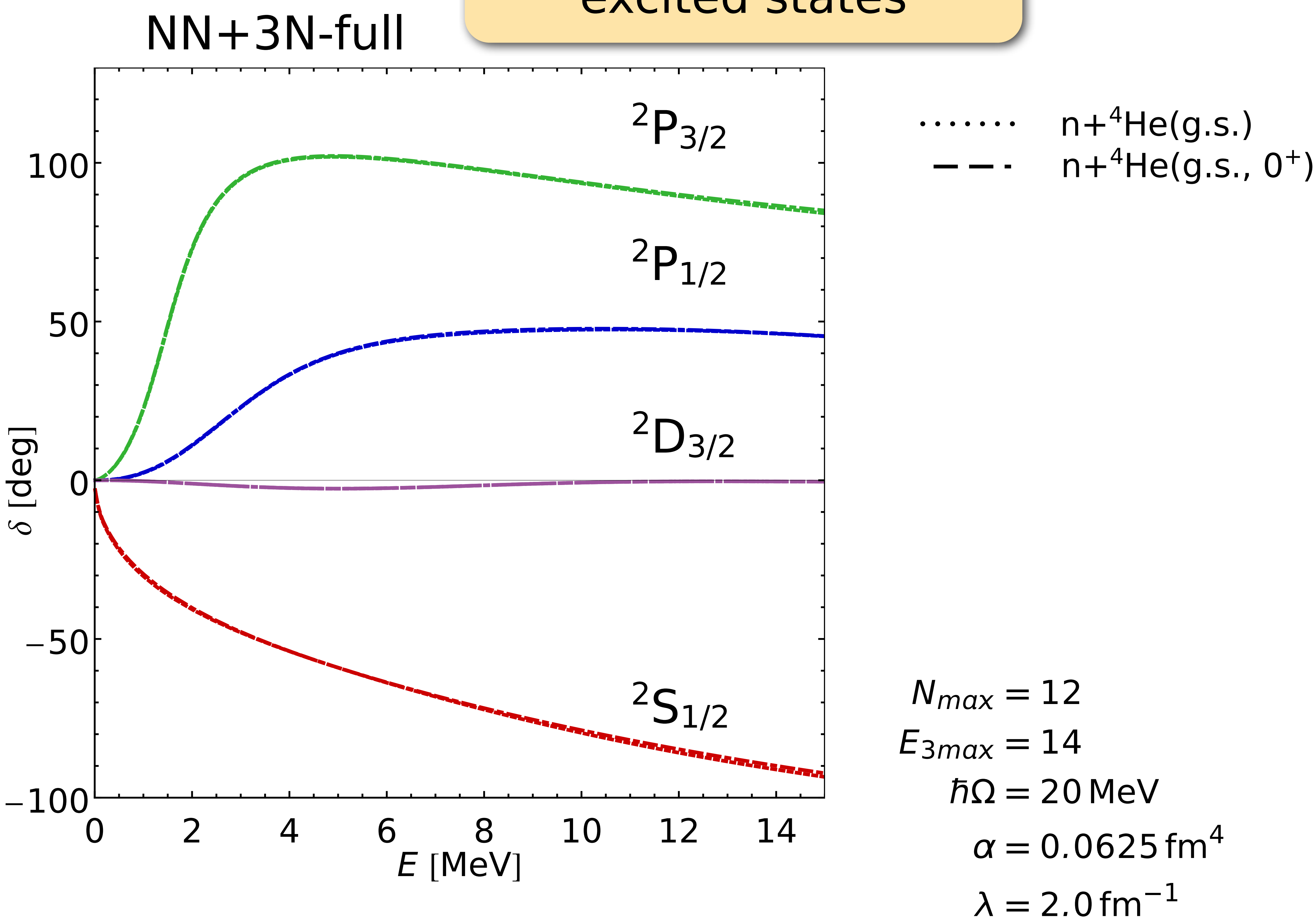
······· n+^{4}He(g.s.)

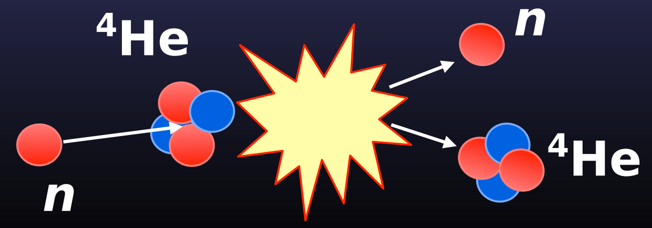
$N_{max} = 12$
$E_{3max} = 14$
$\hbar\Omega = 20\,\text{MeV}$
$\alpha = 0.0625\,\text{fm}^4$
$\lambda = 2.0\,\text{fm}^{-1}$



3N Force Effects on Phase Shifts

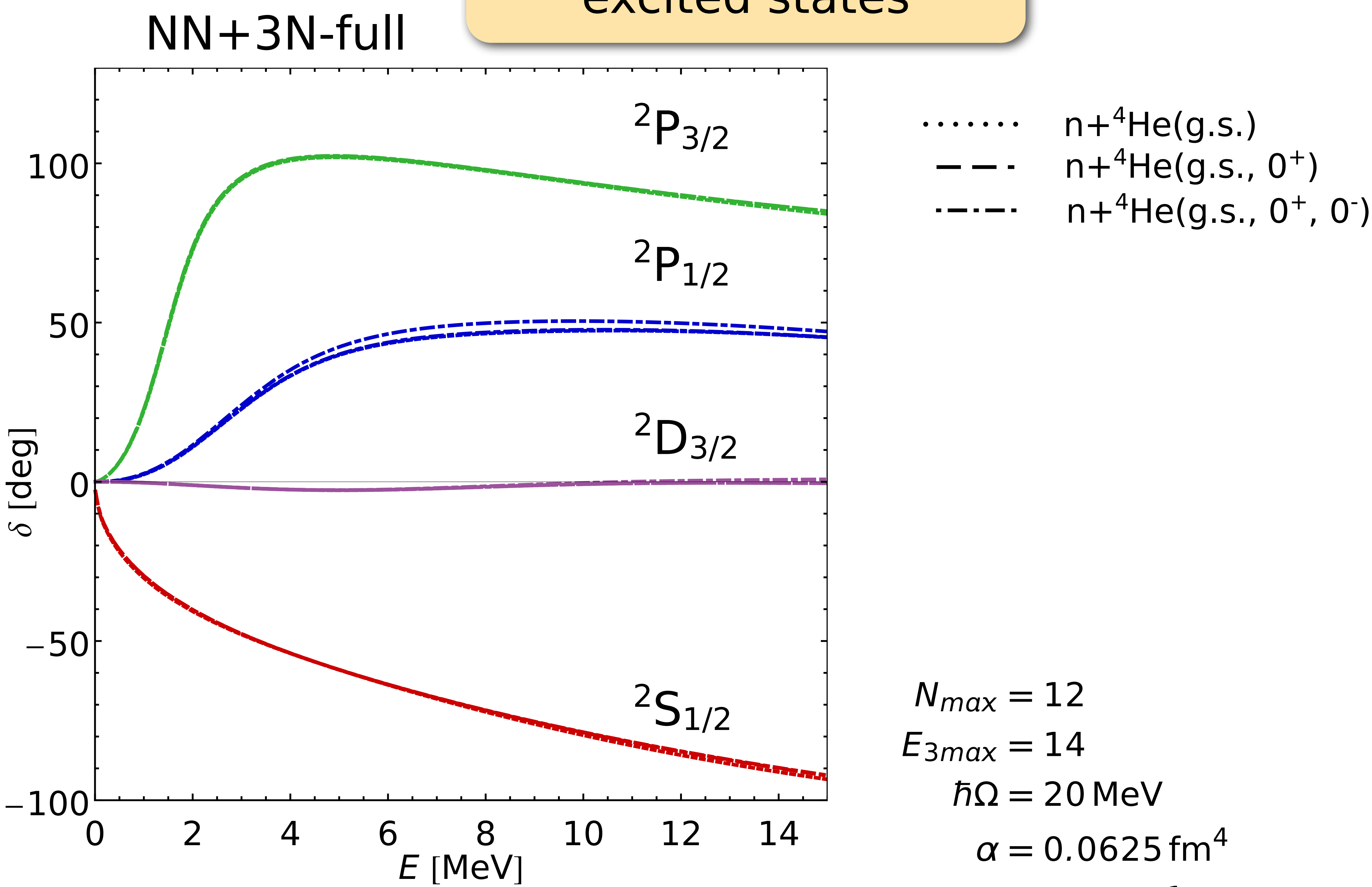
Inclusion of more excited states



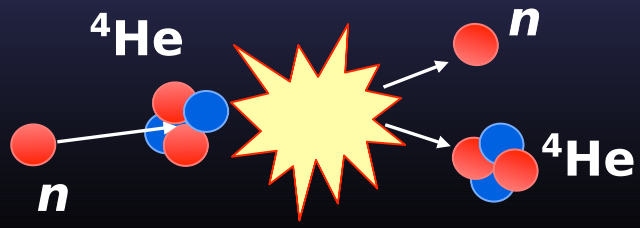


3N Force Effects on Phase Shifts

Inclusion of more excited states



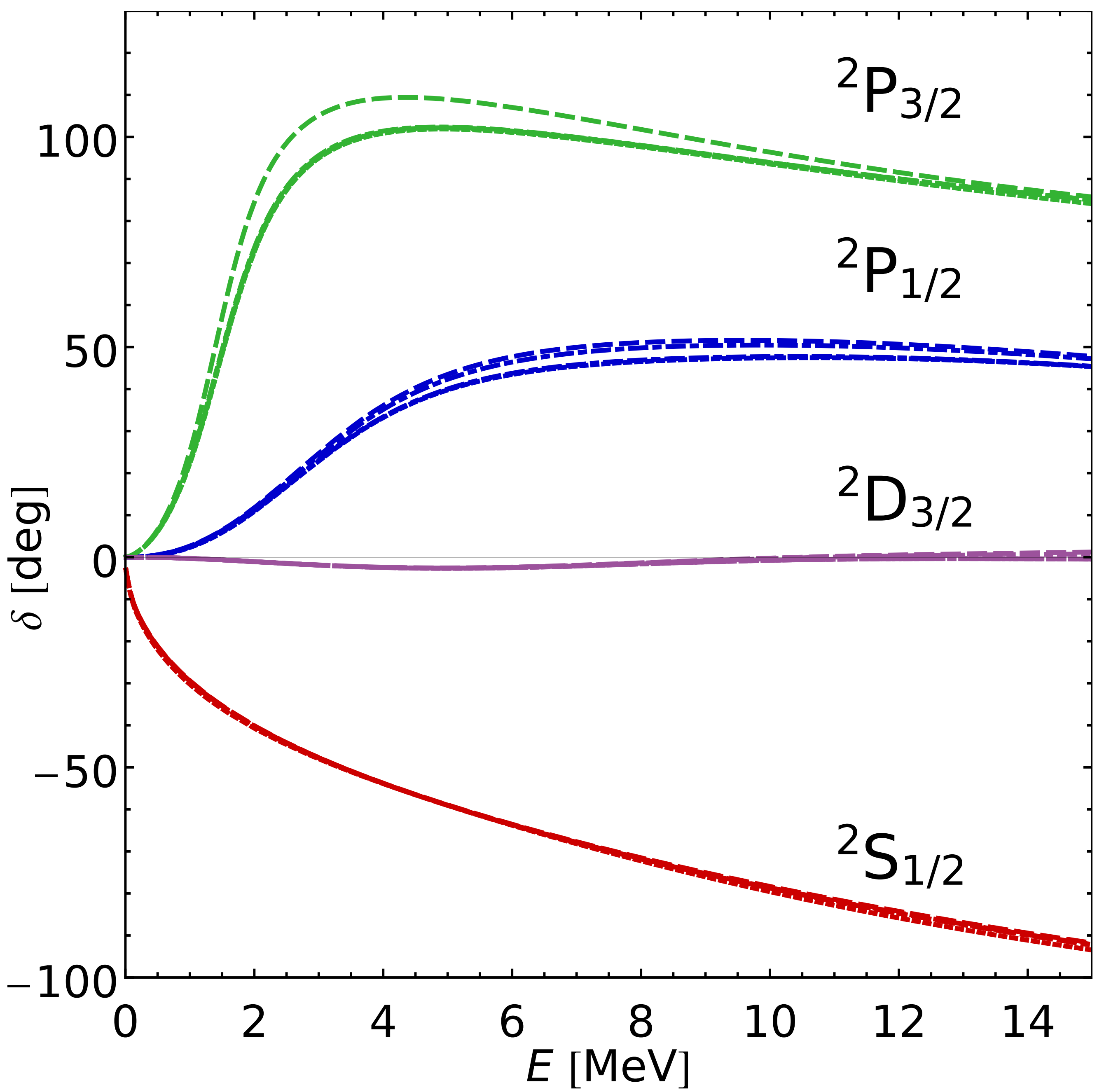
$N_{max} = 12$
$E_{3max} = 14$
$\hbar\Omega = 20\,\mathrm{MeV}$
$\alpha = 0.0625\,\mathrm{fm}^4$
$\lambda = 2.0\,\mathrm{fm}^{-1}$



3N Force Effects on Phase Shifts

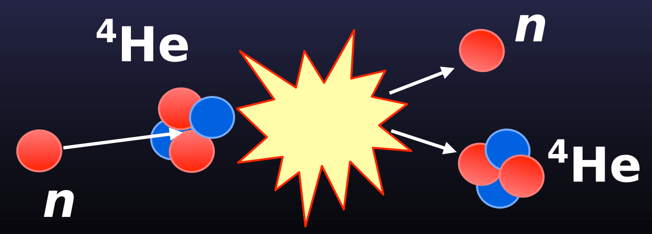
Inclusion of more excited states

NN+3N-full



n+^{4}He(g.s.)
n+^{4}He(g.s., 0^+)
n+^{4}He(g.s., 0^+, 0^-)
n+^{4}He(g.s., 0^+, 0^-, 2^-)

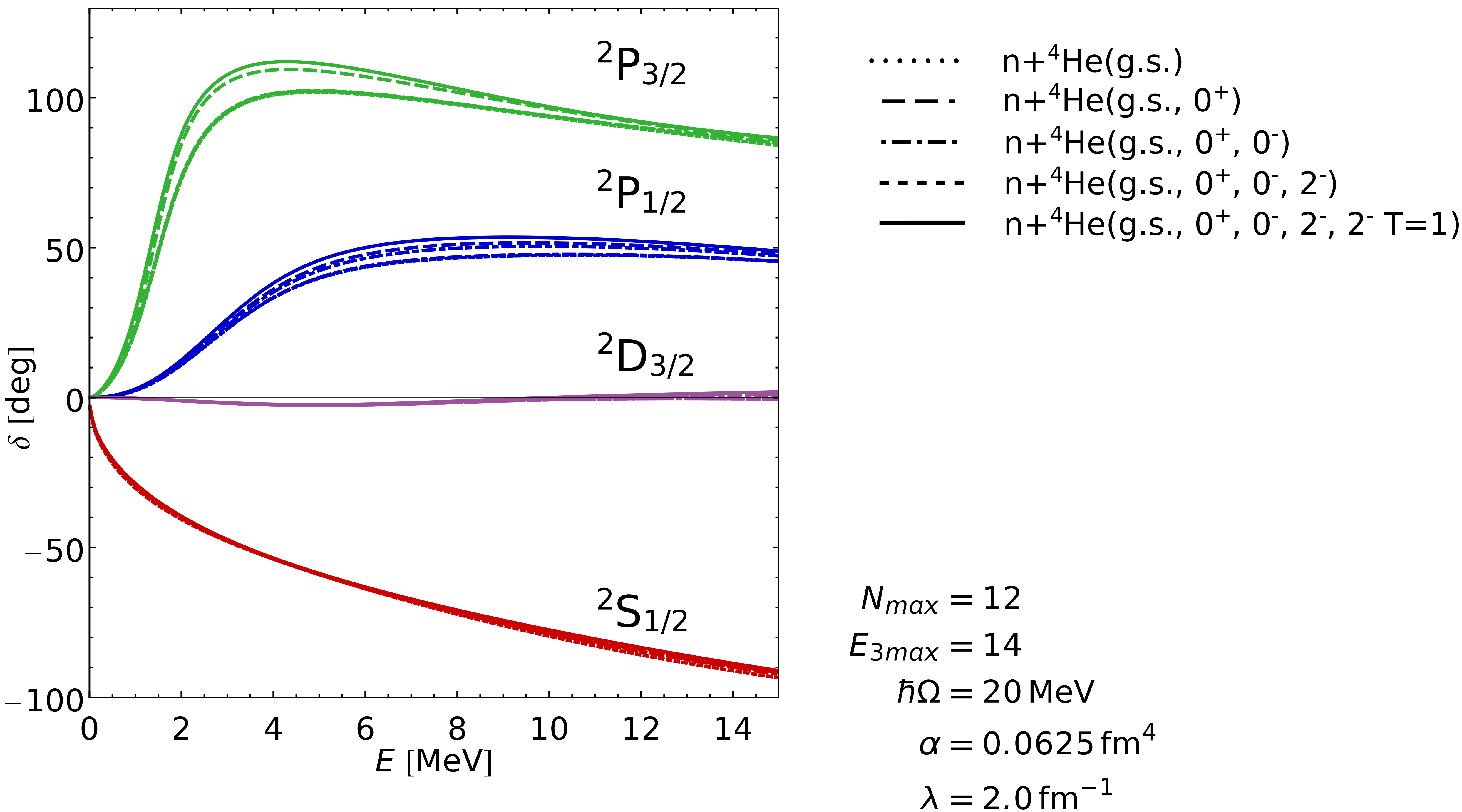
$N_{max} = 12$
$E_{3max} = 14$
$\hbar\Omega = 20\,\mathrm{MeV}$
$\alpha = 0.0625\,\mathrm{fm}^4$
$\lambda = 2.0\,\mathrm{fm}^{-1}$

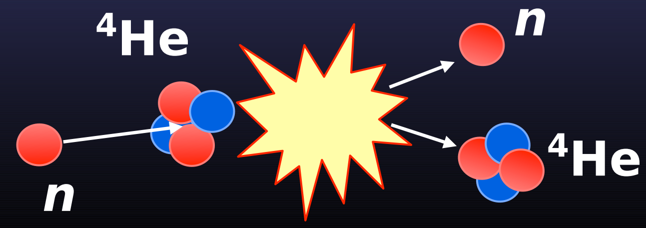


3N Force Effects on Phase Shifts

Inclusion of more excited states

NN+3N-full

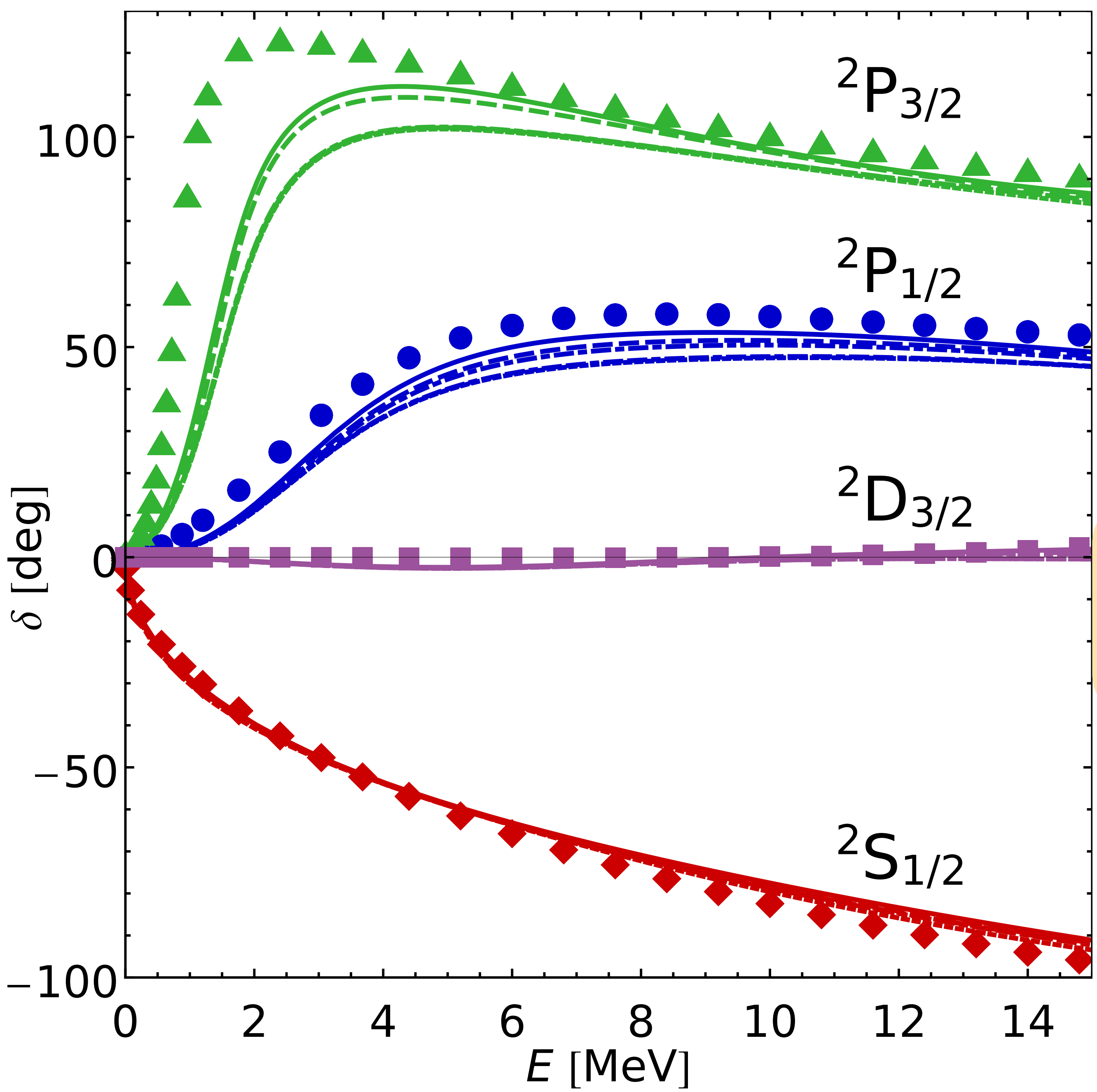




3N Force Effects on Phase Shifts

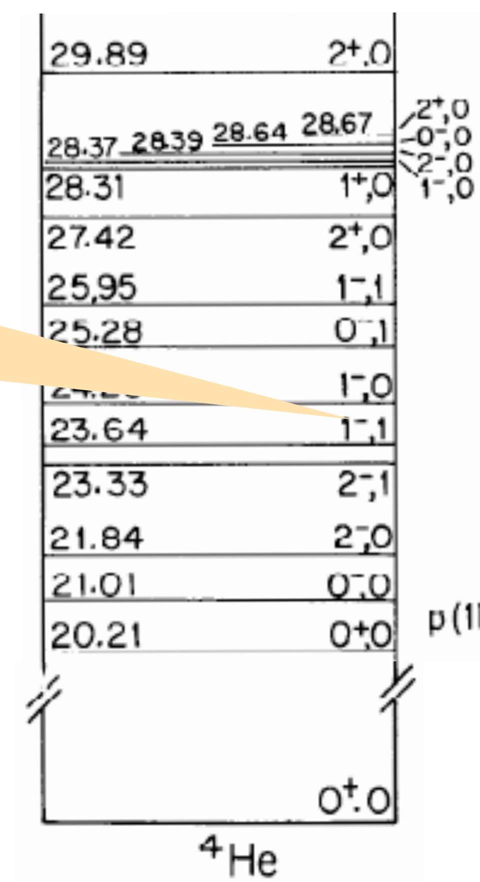
Inclusion of more excited states

NN+3N-full



n+^{4}He(g.s.)
n+^{4}He(g.s., 0^+)
n+^{4}He(g.s., 0^+, 0^-)
n+^{4}He(g.s., 0^+, 0^-, 2^-)
n+^{4}He(g.s., 0^+, 0^-, 2^-, 2^- T=1)
Experiment

Further inclusion of 1-,1 & 1-,0 states necessary



$N_{max} = 12$

$E_{3max} = 14$

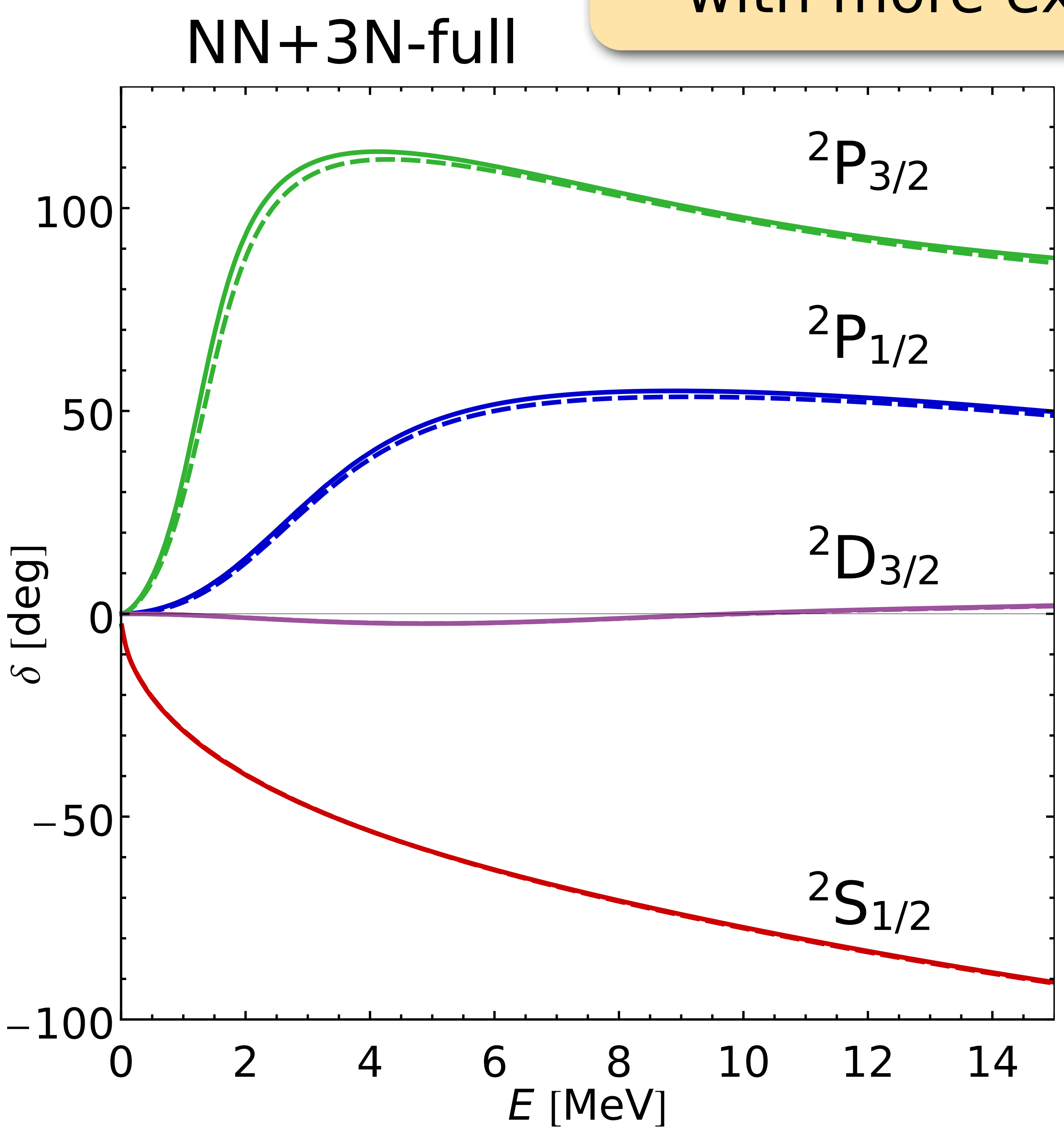
$\hbar\Omega = 20\,\mathrm{MeV}$

$\alpha = 0.0625\,\mathrm{fm}^4$

$\lambda = 2.0\,\mathrm{fm}^{-1}$

3N Force Effects on Phase Shifts

SRG parameter dependence with more excited states

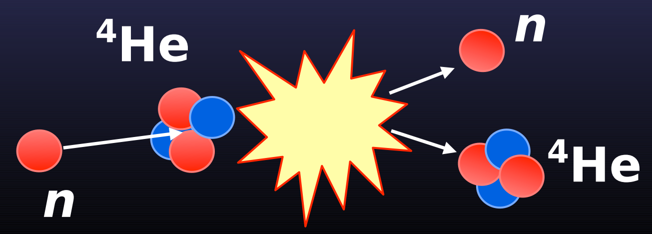


n+^{4}He(g.s., 0^+,0^-,2^-,2^- T=1)

- - - - $\alpha = 0.0625\,\mathrm{fm}^4$
—— $\alpha = 0.08\,\mathrm{fm}^4$

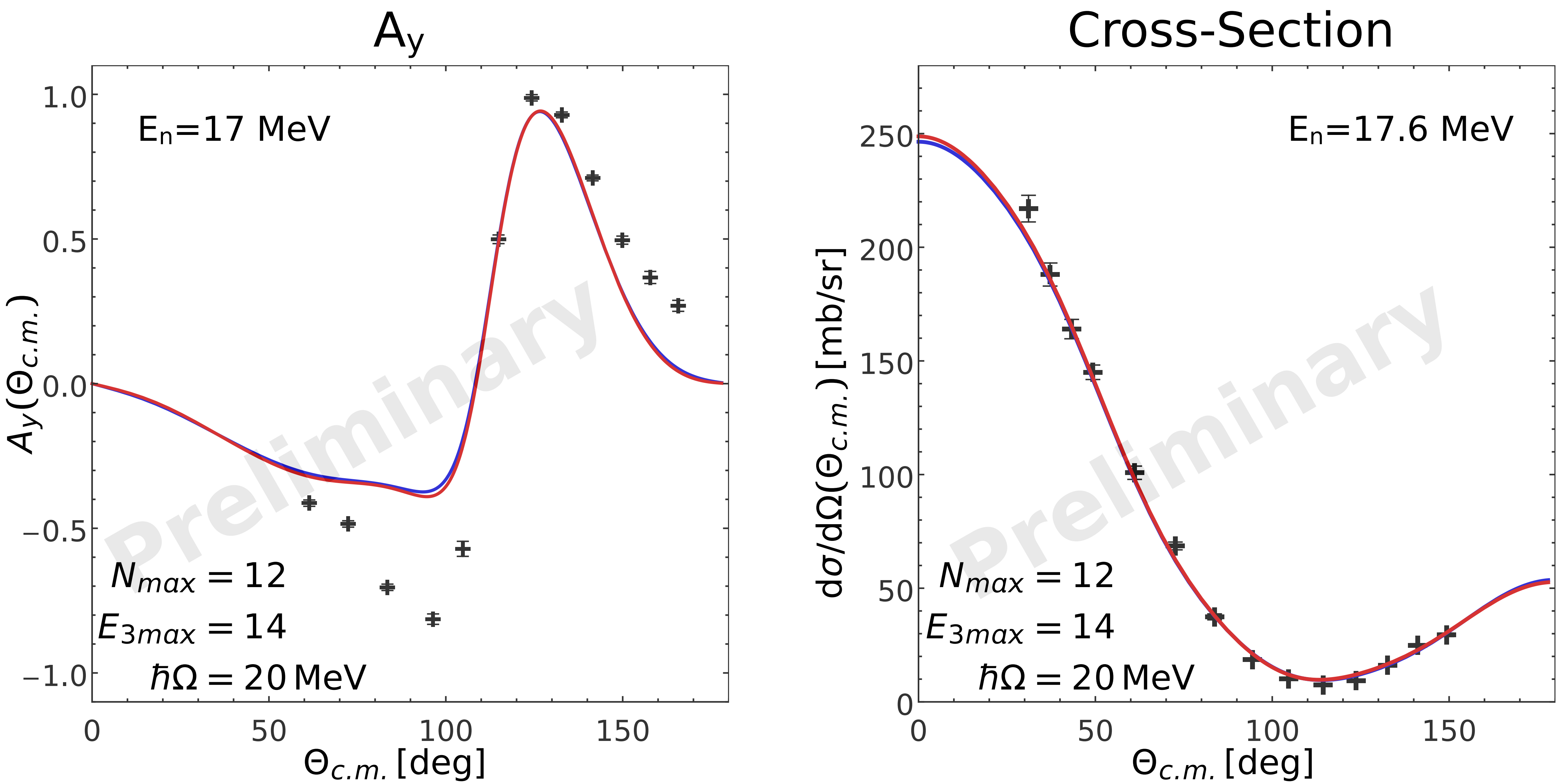
Minimal dependence on SRG parameter

$N_{max} = 12$
$E_{3max} = 14$
$\hbar\Omega = 20\,\mathrm{MeV}$
$\alpha = 0.0625\,\mathrm{fm}^4$
$\lambda = 2.0\,\mathrm{fm}^{-1}$



Cross Section & Analyzing Power

n+^{4}He(g.s., 0^+,0^-,2^-,2^- T=1)



$\alpha = 0.0625\,\mathrm{fm}^4$, $\lambda = 2.0\,\mathrm{fm}^{-1}$ (blue)

$\alpha = 0.08\,\mathrm{fm}^4$, $\lambda = 1.88\,\mathrm{fm}^{-1}$ (red)

+ Exp.

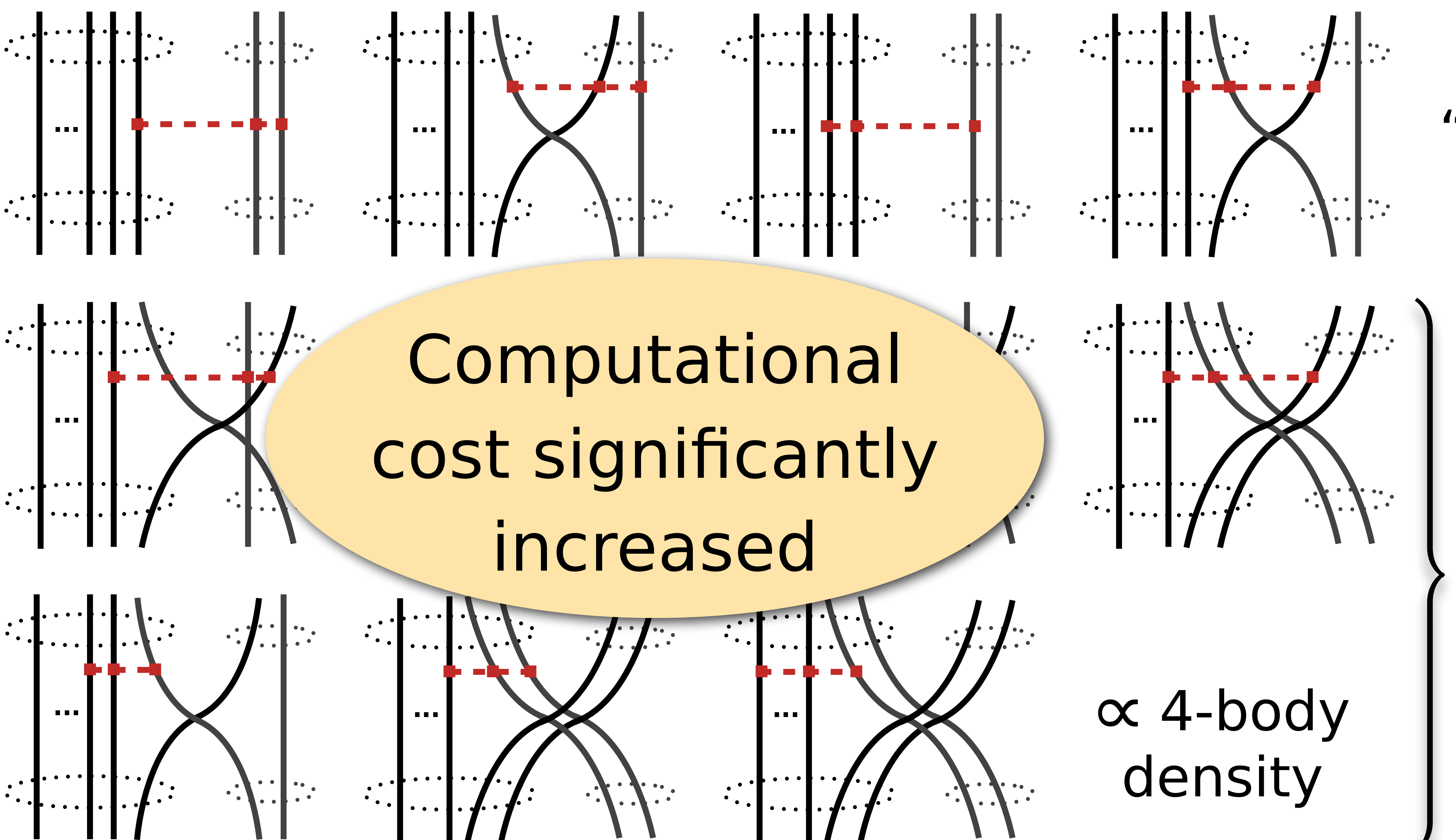
[Krupp et al., Phys.Rev.C **30**, 1810]
[M. Drosg, Los Alamos Scientific Laboratory Report, LA-7269-MS]

Scattering with Heavier Projectiles & Targets

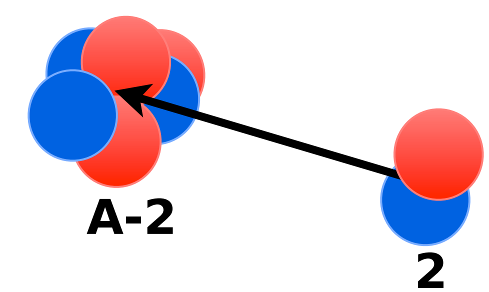
d+^{4}He Scattering with 3N Forces

- Consider 3N-interaction kernels with **two-nucleon projectiles**

$$\langle \phi^{J\pi T}_{\nu' r'} | V_{3N} \mathcal{A}^2 | \phi^{J\pi T}_{\nu r} \rangle = \langle \phi^{J\pi T}_{\nu' r'} | V_{3N} \left[1 - \sum_{i=1}^{A-2} \sum_{k=A-1}^{A} T_{i,k} \sum_{i<j=1}^{A-1} T_{i,A-1} T_{j,A} \right] | \phi^{J\pi T}_{\nu r} \rangle$$

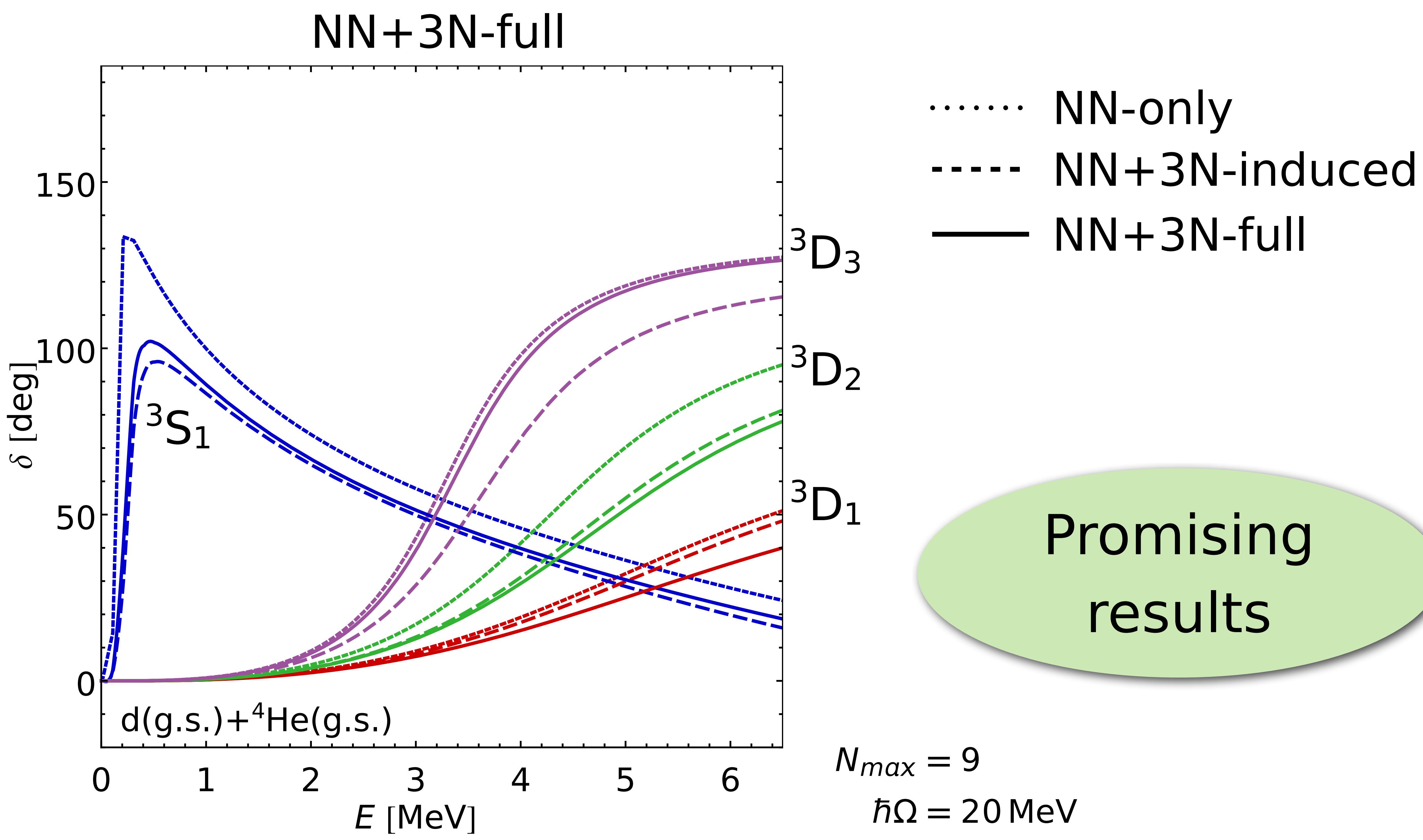


"direct" kernels



"exchange" kernels

d+^{4}He Scattering Phase Shifts



Promising results

$N_{max} = 9$

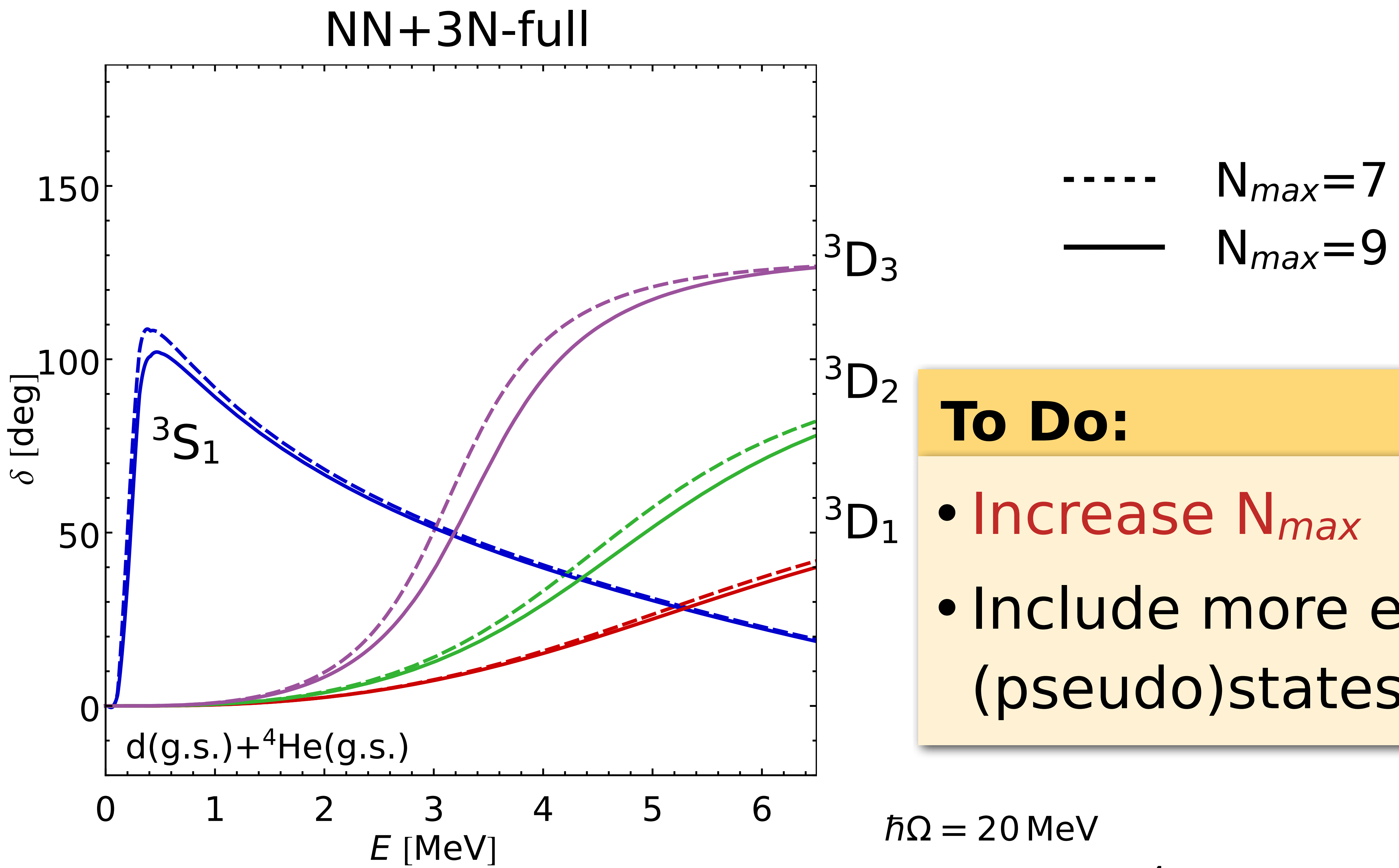
$\hbar\Omega = 20\,\text{MeV}$

$\alpha = 0.0625\,\text{fm}^4$

$\lambda = 2.0\,\text{fm}^{-1}$

$E_{3max} = 14$

d+^{4}He Scattering Phase Shifts



To Do:

- Increase N_{max}
- Include more excited (pseudo)states

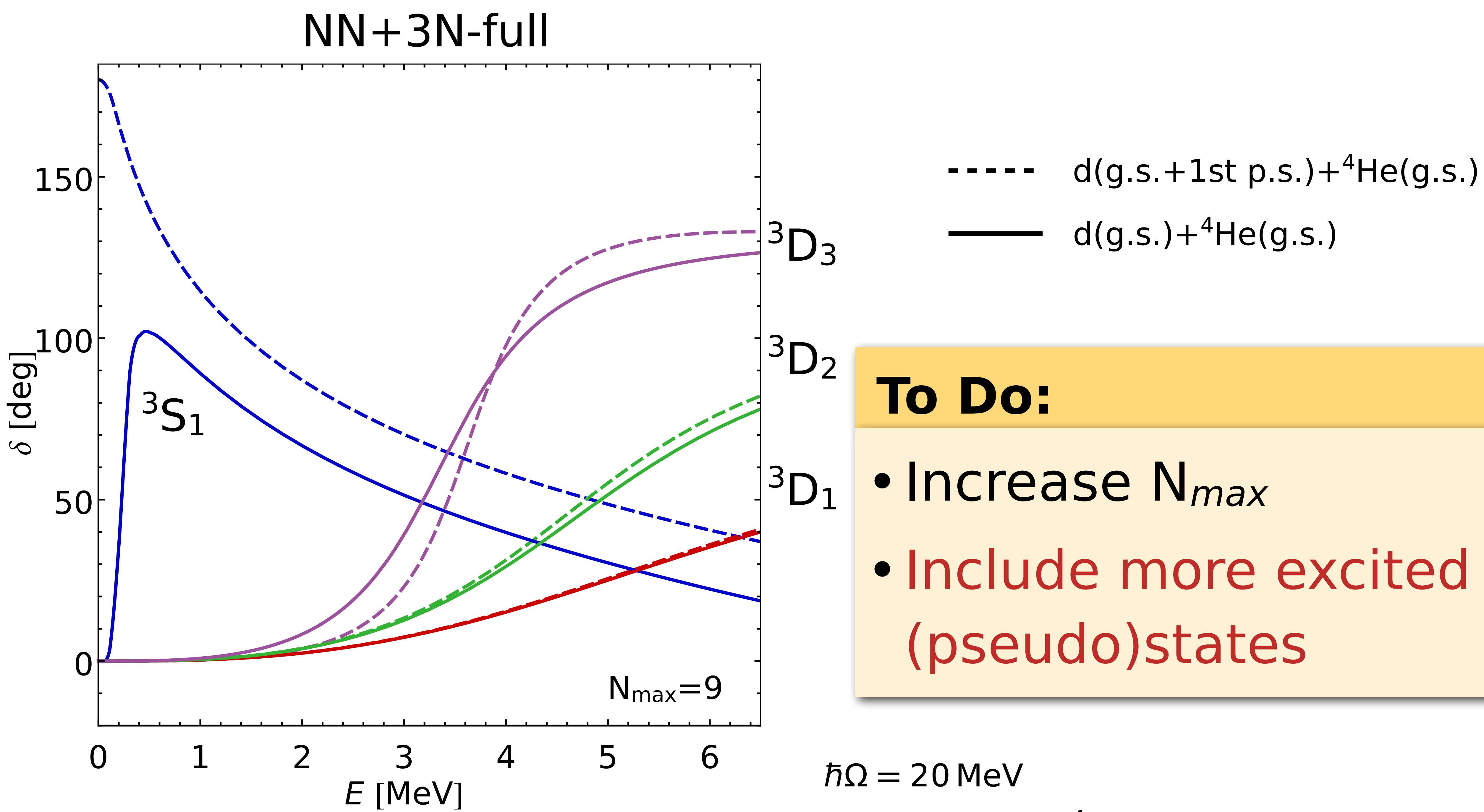
$\hbar\Omega = 20\,\mathrm{MeV}$

$\alpha = 0.0625\,\mathrm{fm}^4$

$\lambda = 2.0\,\mathrm{fm}^{-1}$

$E_{3max} = 14$

d+^{4}He Scattering Phase Shifts



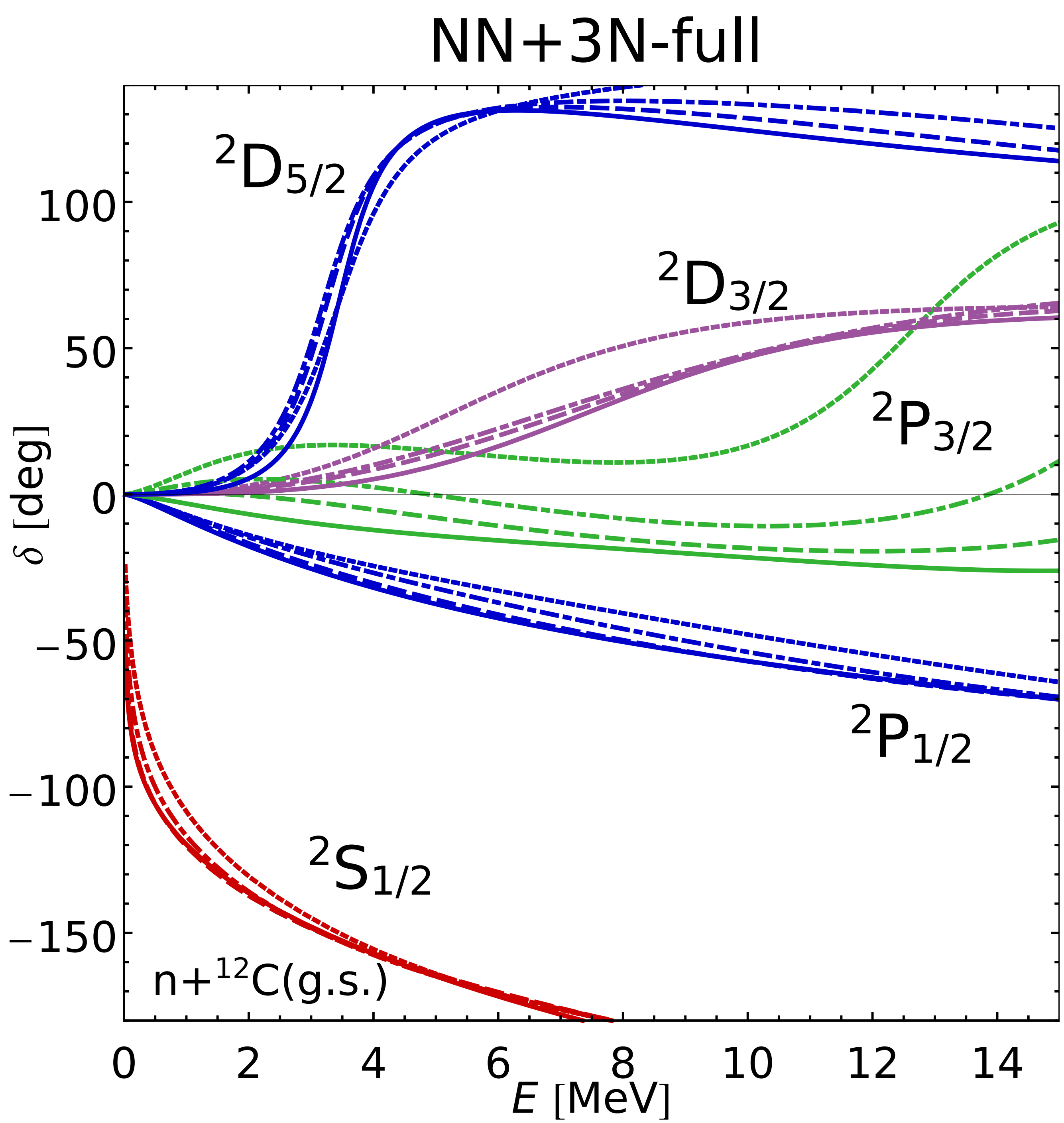
To Do:

- Increase N_{max}
- Include more excited (pseudo)states

$$\hbar\Omega = 20\,\text{MeV}$$
$$\alpha = 0.0625\,\text{fm}^4$$
$$\lambda = 2.0\,\text{fm}^{-1}$$
$$E_{3max} = 14$$

n+^{12}C Scattering

- Accessible due to new computational scheme



Study N_{max} dependence

Convergence looks promising

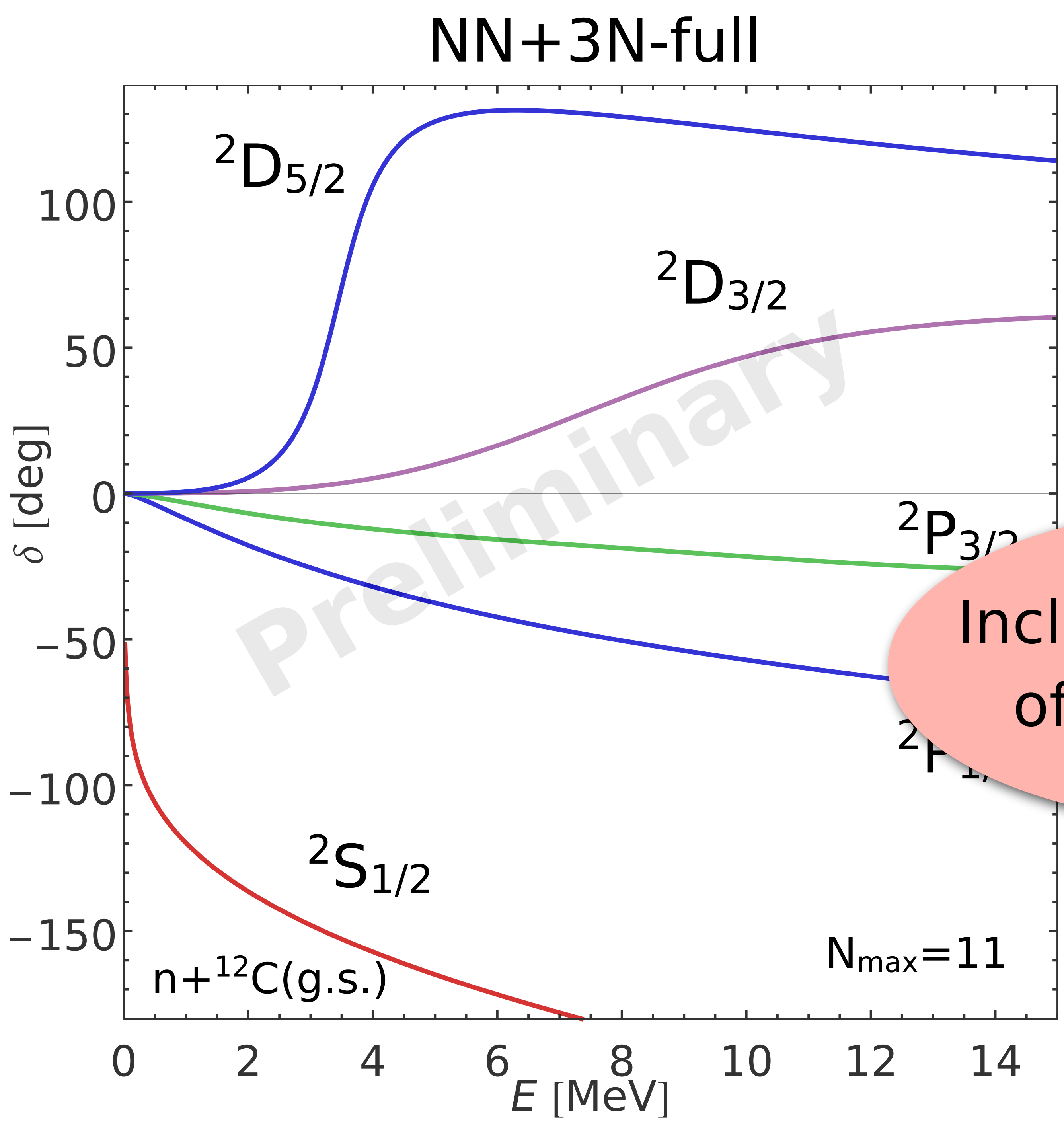
$\hbar\Omega = 20\,\text{MeV}$

$\alpha = 0.0625\,\text{fm}^4$

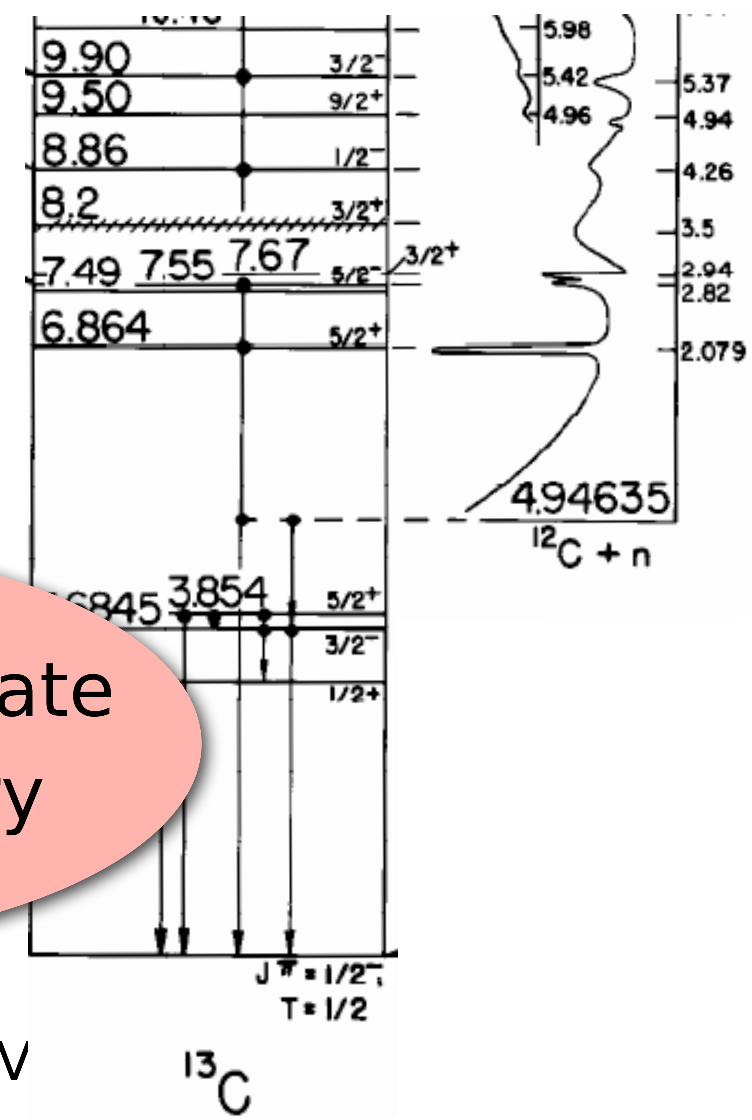
$\lambda = 2.0\,\text{fm}^{-1}$

$E_{3max} = 14$

n+^{12}C Scattering



Inclusion of 2^+ state of ^{12}C necessary



$\hbar\Omega = 20\,\mathrm{MeV}$

$\alpha = 0.0625\,\mathrm{fm}^4$

$\lambda = 2.0\,\mathrm{fm}^{-1}$

$E_{3max} = 14$

Conclusions

Conclusions

NCSM/RGM
delivers ab-initio description of low-energy nuclear reactions

- **Strict test** of predictive power of **chiral Hamiltonians**
- Inclusion of 3N forces challenging but practically completed
  - n+^{4}He scattering phase shifts show expected **enhanced spin-orbit splitting**
  - Consideration of more excited states of ^{4}He necessary
  - New computational scheme $\Longrightarrow$ **heavier targets accessible**
- First results also for d+ ^{4}He and heavier targets, e.g. n+^{12}C
- Stay tuned...

Epilogue

- **thanks to my group & collaborators**
  - S. Binder, A. Calci, E. Gebrerufael, P. Isserstedt, H. Krutsch, S. Reinhard, R. Roth, S. Schulz, C. Stumpf, A. Tichai, R. Trippel, R. Wirth

- P. Navrátil
  TRIUMF, Vancouver, Canada
- G. Hupin, S. Quaglioni
  LLNL, Livermore, USA
- J. Vary, P. Maris
  Iowa State University, USA
- H. Hergert, K. Hebeler
  The Ohio State University, USA

- C. Forssén
  Chalmers University, Sweden
- H. Feldmeier, T. Neff
  GSI Helmholtzzentrum
- P. Pa … inou

Thanks for your attention!

Computing Time

JÜLICH FORSCHUNGSZENTRUM

NERSC

CSC Center for Scientific Computing Frankfurt

SFB 634

Deutsche Forschungsgemeinschaft
DFG

HIC for FAIR
Helmholtz International Center

LOEWE
Exzellente Forschung für Hessens Zukunft

HELMHOLTZ | GEMEINSCHAFT

Bundesministerium für Bildung und Forschung