

# RECENT INSIGHTS FROM THE NO-CORE SHELL MODEL: FROM LIGHT NUCLEI TO COLD ATOMS

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Main research funding by:



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# Acknowledgments

## Many thanks to my collaborators

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# Outline

- ❖ Cold atoms in deformed traps
- ❖ Ab initio NCSM towards the driplines
- ❖ NNLO (POUNDerS) in the NCSM

# From “QCD” to Nuclei

## Nuclear Structure

### Many-body Methods

- ab initio no-core shell model
  - ▶ A-body HO model space (m scheme)
  - ▶ Full-space  $N_{\max}$  energy cutoff

### Renormalization Scheme

- Renormalized for truncated model space
  - ▶ SRG flow in HO or momentum space
  - ▶ Lee-Suzuki transformation

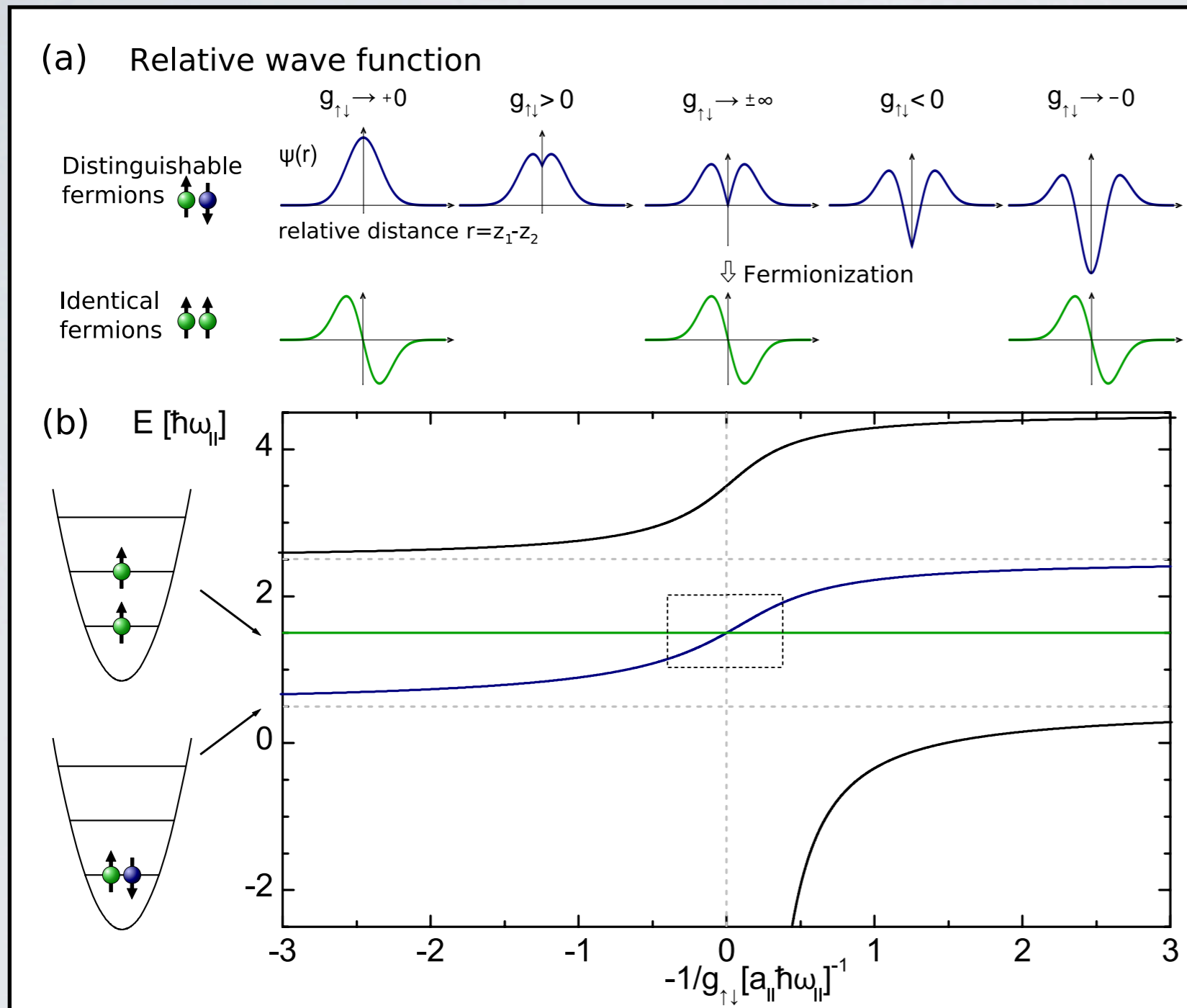
### Realistic, effective nuclear interaction

- Realistic nuclear interaction
  - ▶ fits scattering data
  - ▶ chiral EFT interaction

## Low-energy QCD

# Cold Atoms In Deformed Traps

# Busch model



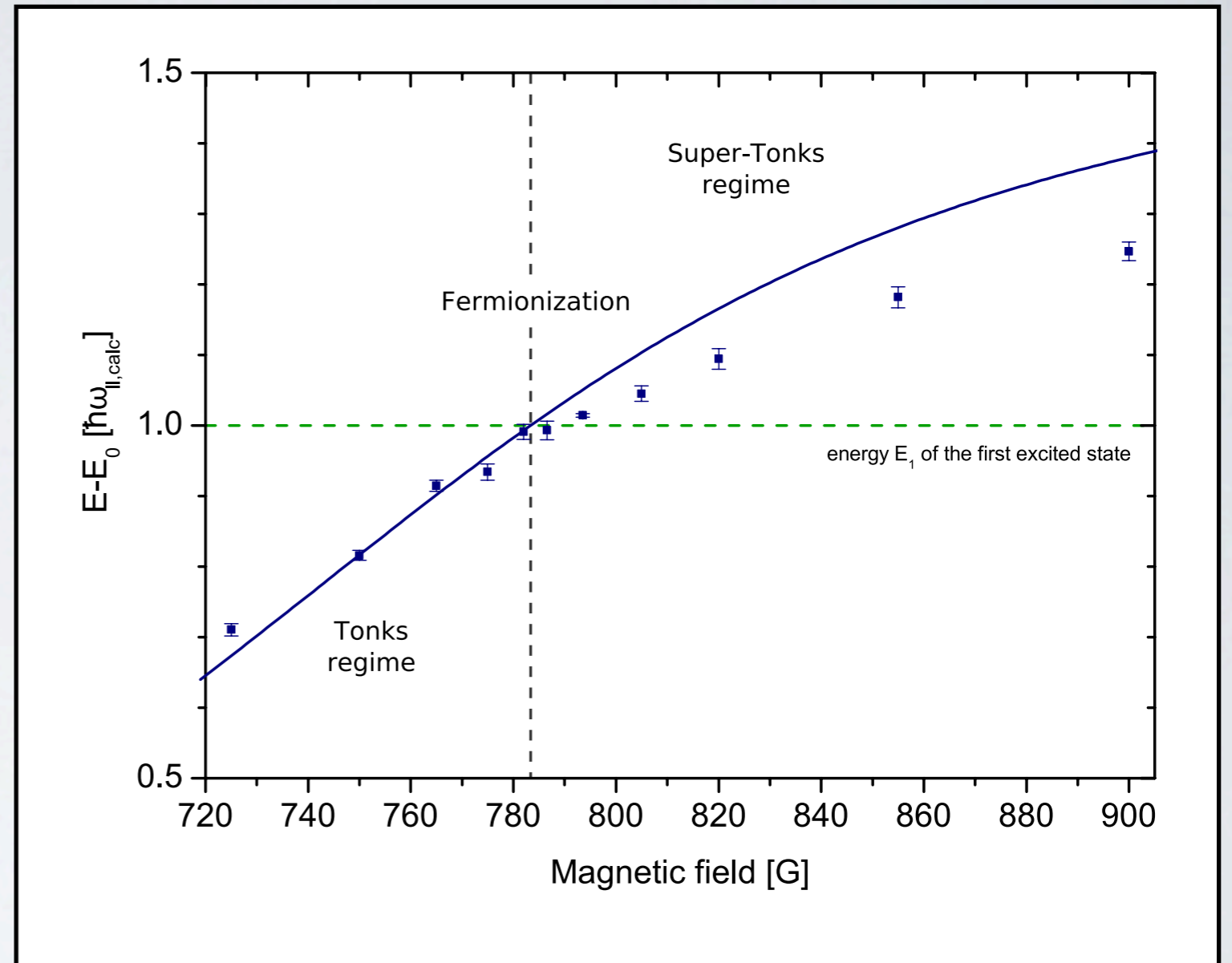
- ❖ Zero-range interaction
- ❖ Parabolic trapping potential
- ❖ Energy spectrum given by the Busch formula
 
$$\frac{\Gamma(-E/2 + 1/4)}{\Gamma(-E/2 + 3/4)} = -\frac{2}{g}$$
- ❖ Analytical expressions for wave functions

T. Busch et al., Foundations of Physics, Vol. 28, No. 4, 1998.

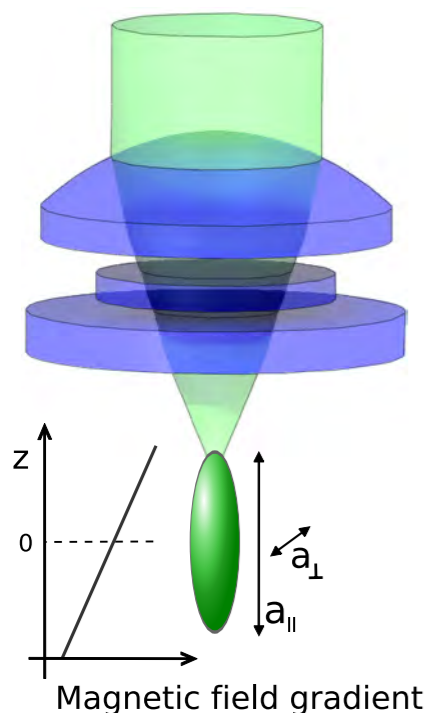
From: G. Zürn et al., Phys. Rev. Lett. 108, 075303 (2012).

# Heidelberg Experiment

- ❖ 1d system with repulsively interacting bosonic gases (Tonks-Girardeau regime)
- ❖ Two-component fermionic systems using hyperfine states of  $^6\text{Li}$
- ❖ 1:10 asymmetric opto-magnetic trap.



From: G. Zürn et al., Phys. Rev. Lett. 108, 075303 (2012).





# Model and energy spectrum

❖ Studied 1+2, 1+3, 1+4 systems

❖ Hamiltonian

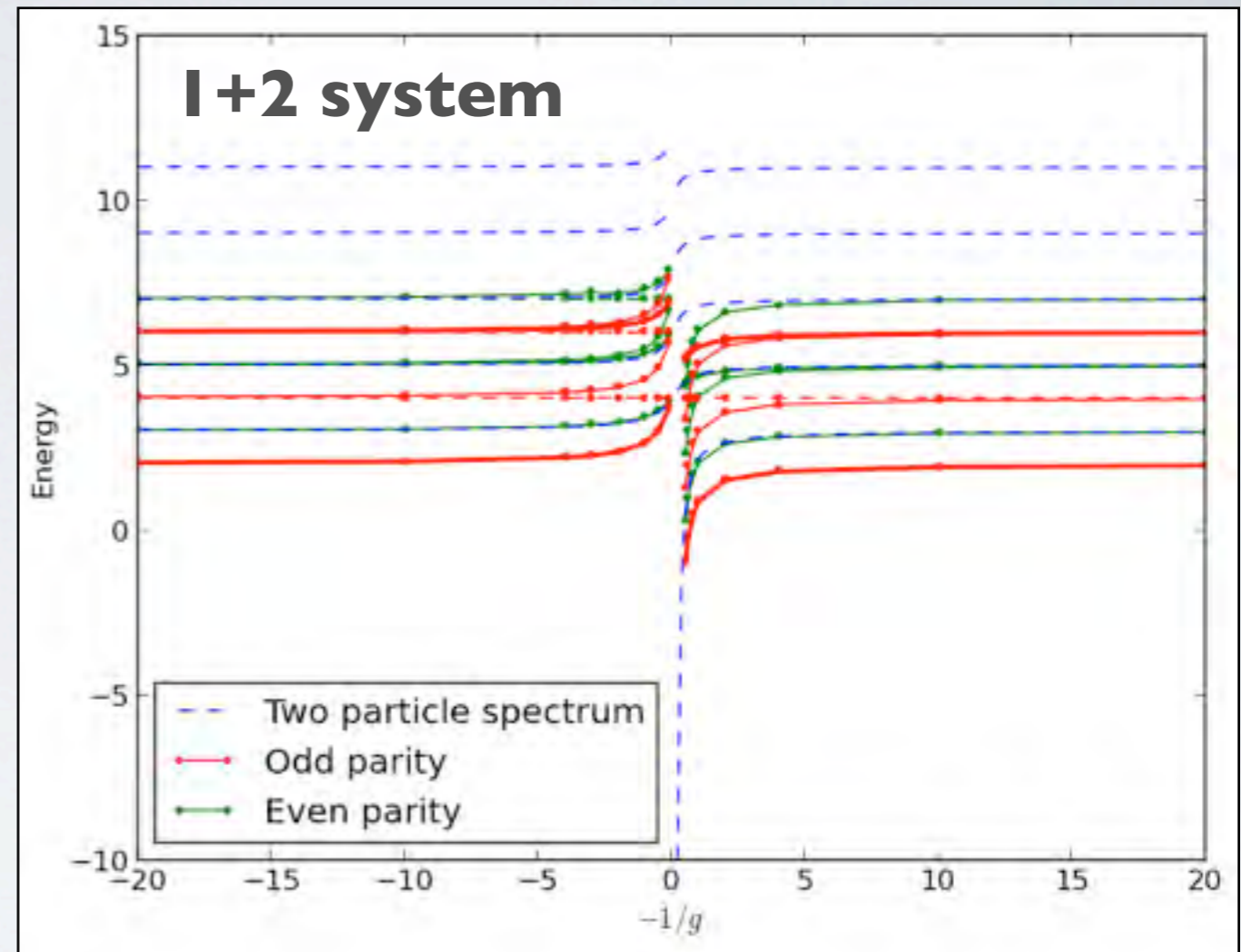
$$H = \sum_{i\sigma} \left( \frac{p^2}{2} + \frac{1}{2} x_{i\sigma}^2 \right) + g \sum_{i\sigma, j\tilde{\sigma}} \delta(x_{i\sigma} - x_{j\tilde{\sigma}})$$

with  $\sigma = \pm, \tilde{\sigma} = -\sigma$

❖ Coupling strength

$$g \propto \frac{a_{3d}}{1 - C a_{3d}/a_{\perp}}$$

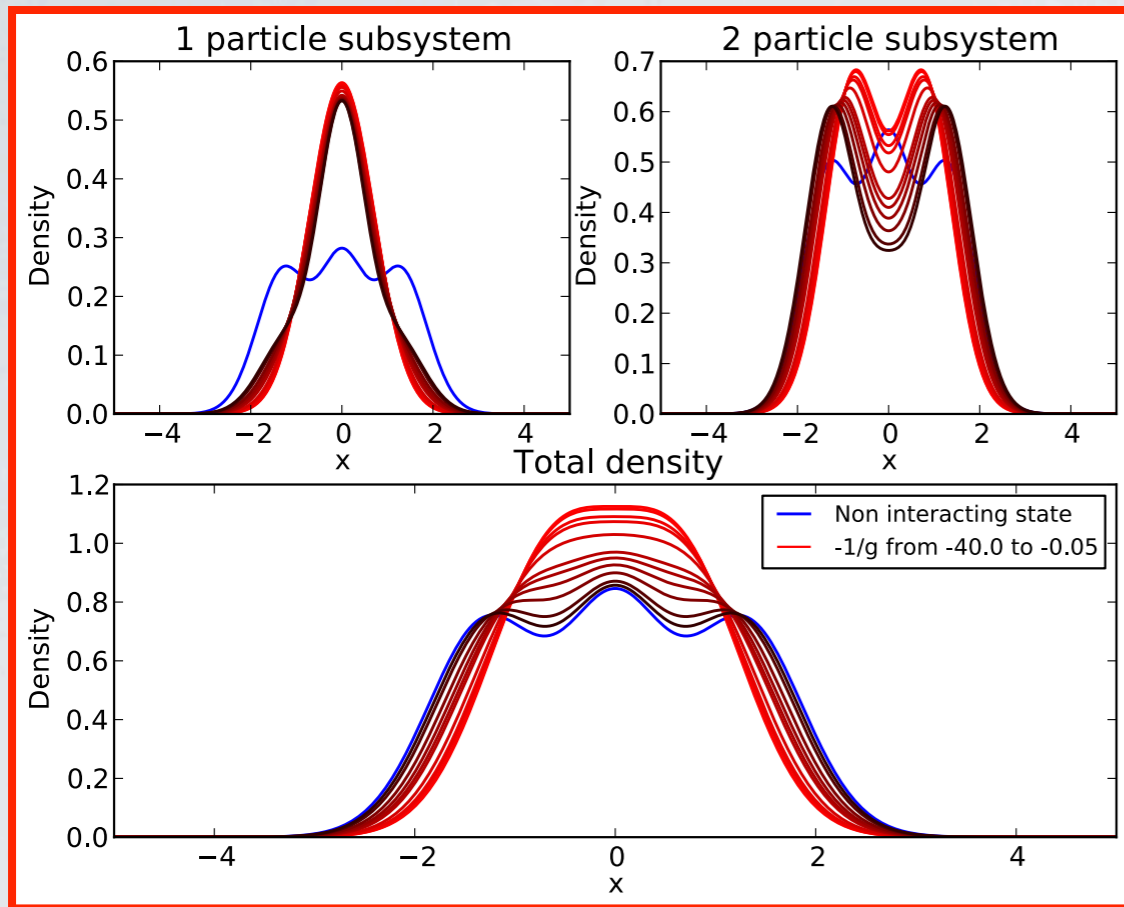
❖ LS-transformation using exact Busch model solution



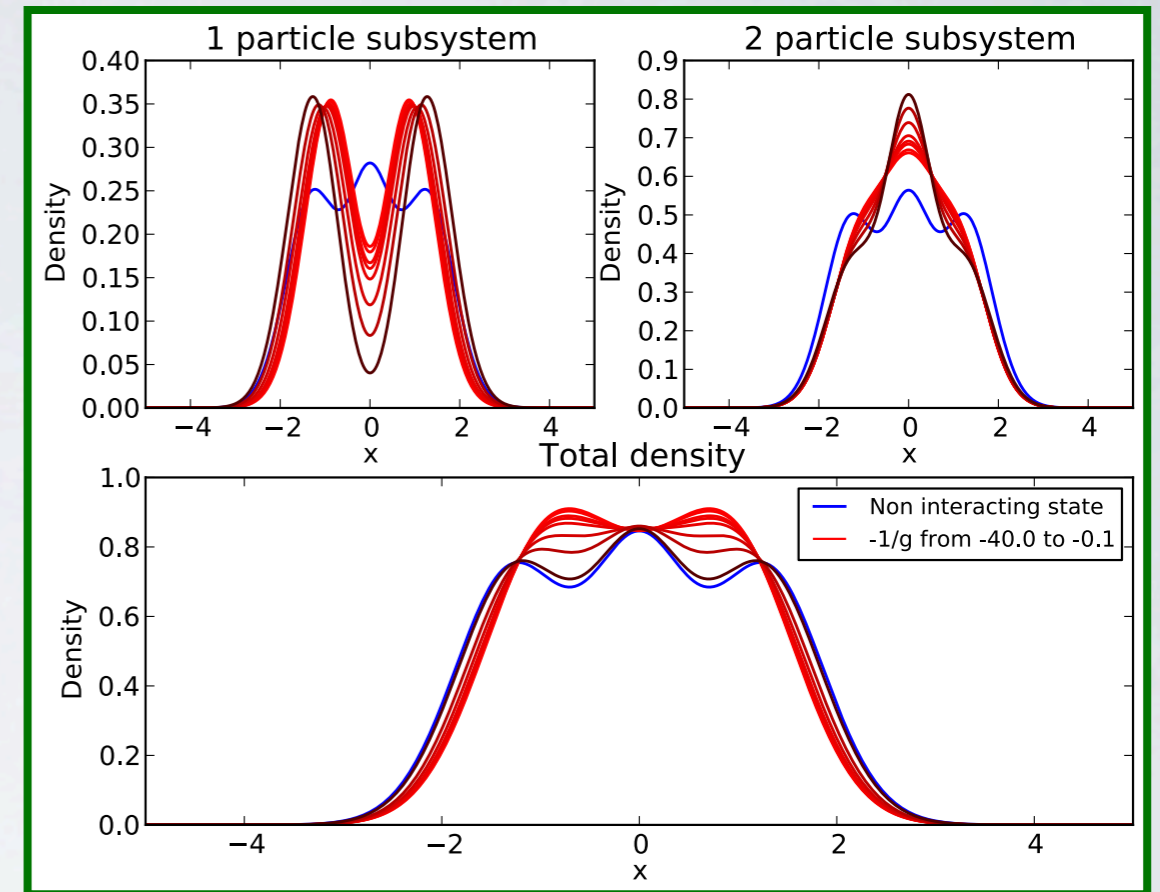
← Repulsive      Attractive →

# Densities

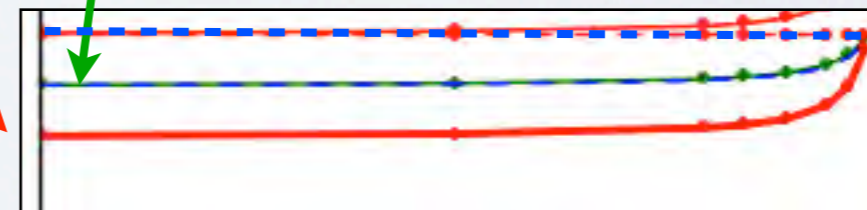
## Ground state



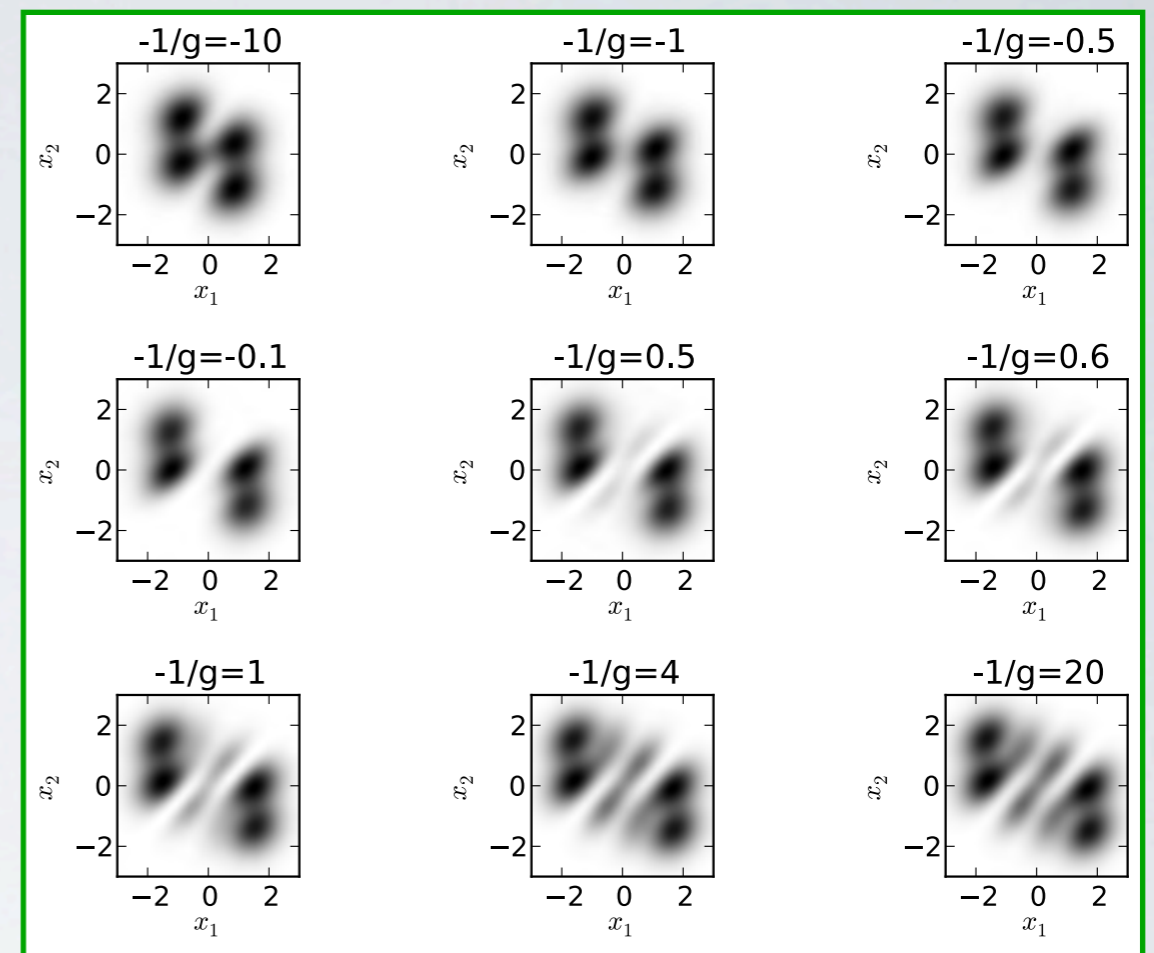
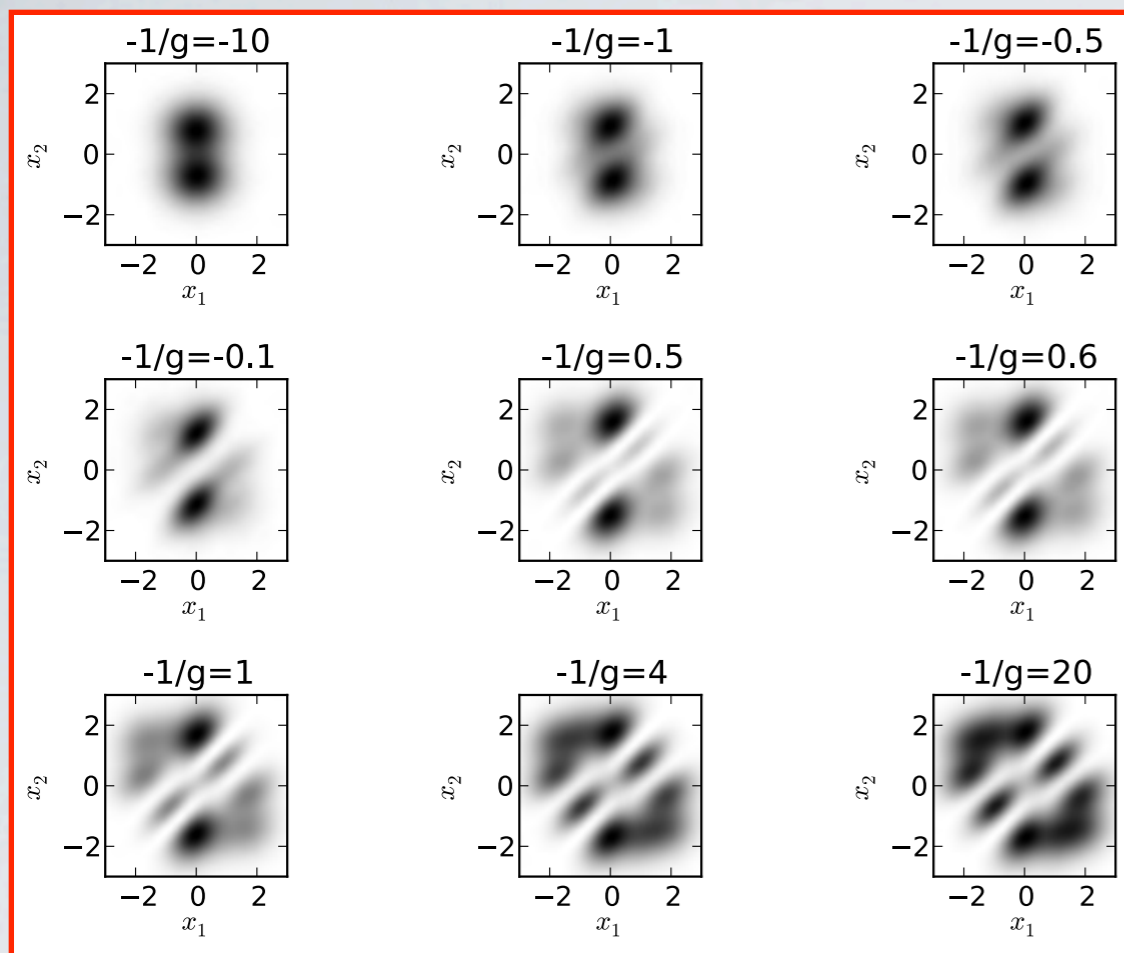
## 1st excited state



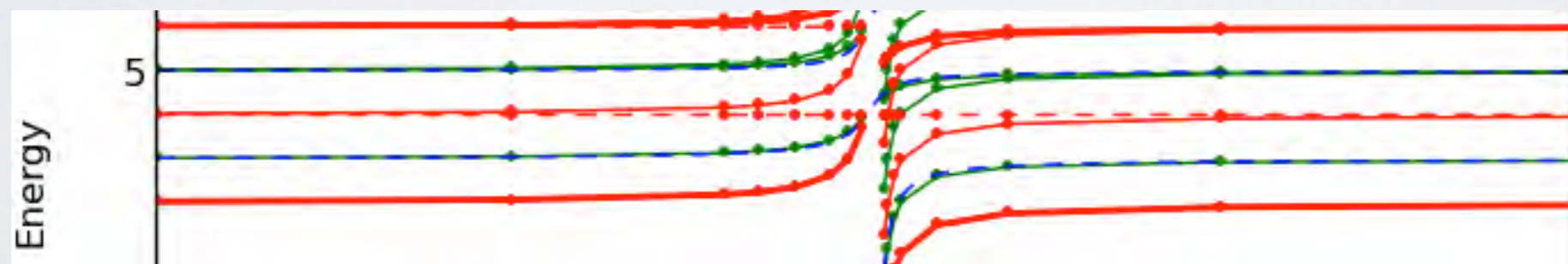
- ❖ Repulsive interaction
- ❖ From **weak** to **strong**
- ❖ **Non-interacting** state corresponds to the three-fermion ground state in HO trap
- ❖ At  $-1/g \rightarrow 0$ , these are degenerate



# Densities and correlation densities



skip



# Ab Initio Towards The Driplines



# ${}^6\text{He}$ Ground-State Properties

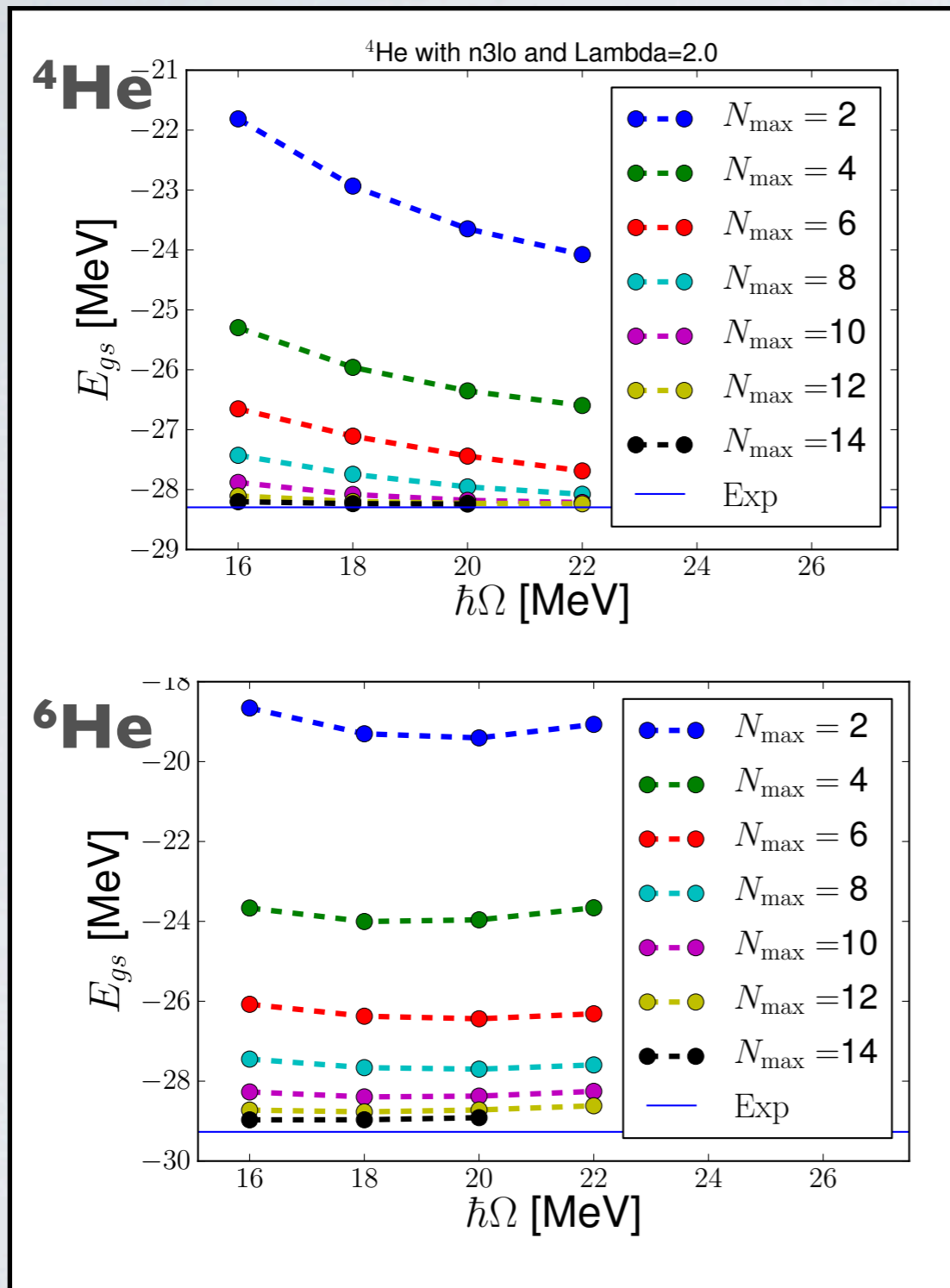
- ❖ Very accurate charge-radius measurements using laser spectroscopy.
- ❖ Very accurate mass measurement with a Penning trap mass spectrometer.
- ❖ Several ab initio calculations
- ❖ Most recently by S. Bacca et al
  - ▶ using EHH and  $V_{\text{lowk}}$  NN potential based on I-N<sup>3</sup>LO.
  - ▶ Study of  $V_{\text{lowk}}$  cutoff-dependence and observable correlations.

## 6-He references

- P. Mueller et al., Phys. Rev. Lett. **99** (2007) 252501.
- M. Brodeur et al. Phys. Rev. Lett. **108** (2012) 052504
- S. Bacca. et al. Phys. Rev. C**86**, (2012) 034321

# NCSM example: Energy convergence

$N^3\text{LO}$ , SRG (NN only,  $\Lambda = 2.0 \text{ fm}^{-1}$ )



## HO basis cutoff scales

$$\Lambda_{\text{UV}} = \sqrt{2(N + 3/2)}\hbar/b$$

$$L_{\text{IR}} = L_2 = \sqrt{2(N + 3/2 + 2)}b$$

### Extrapolations from finite HO basis

- R.J. Furnstahl et al., Phys. Rev. C 86(2012)031301R
- S. Coon et al., Phys. Rev. C 86(2012)054002
- R.J. Furnstahl et al., arXiv:1302.3815 (2013)

### Previous work with $N_{\max}$ / $\hbar\Omega$ extrapolation

- C. Forssén et al., Phys. Rev. C 77(2008)024301
- P. Maris et al., Phys. Rev. C 79 (2009)014308

## Correction to the energy due to finite HO space

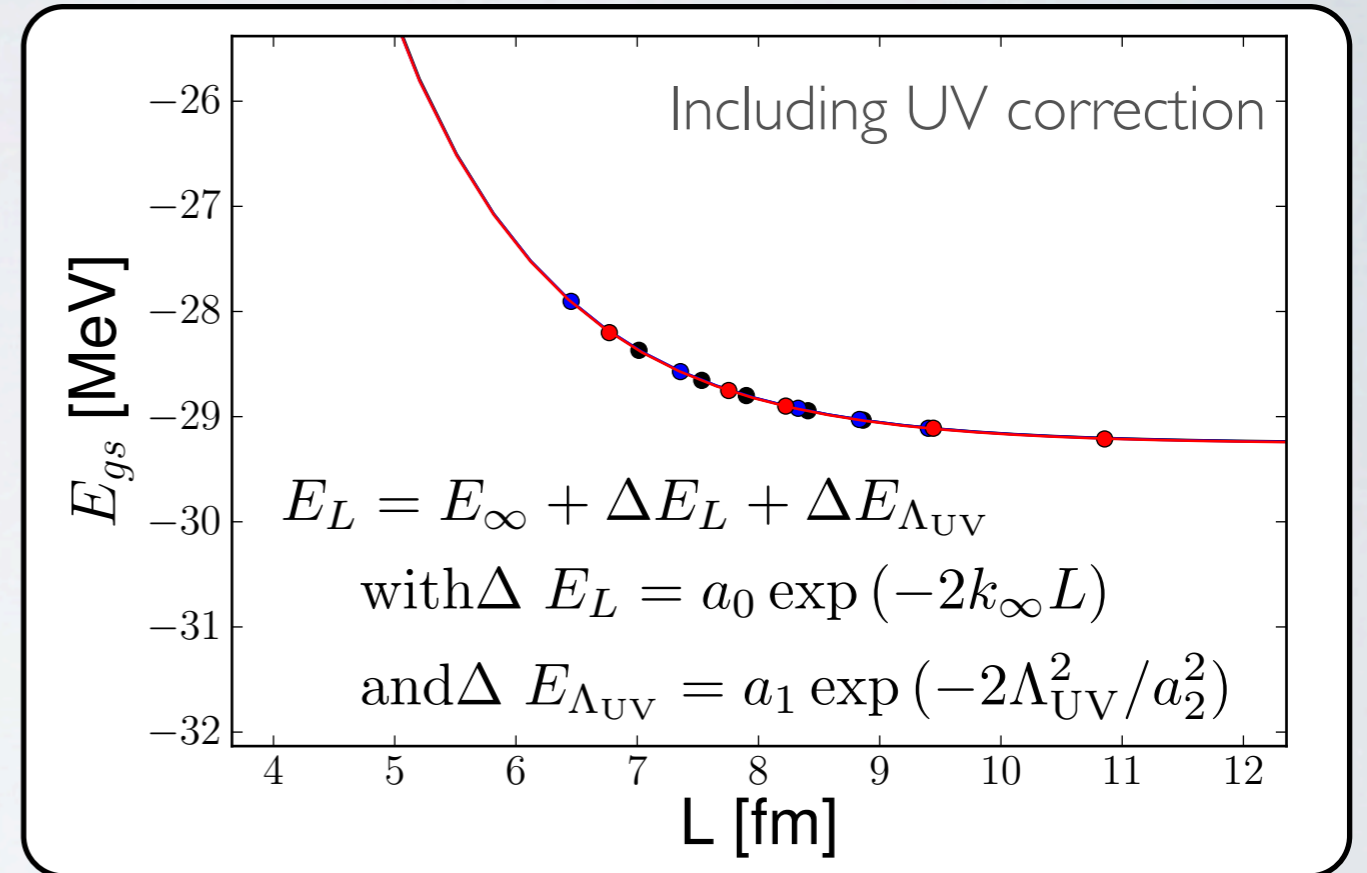
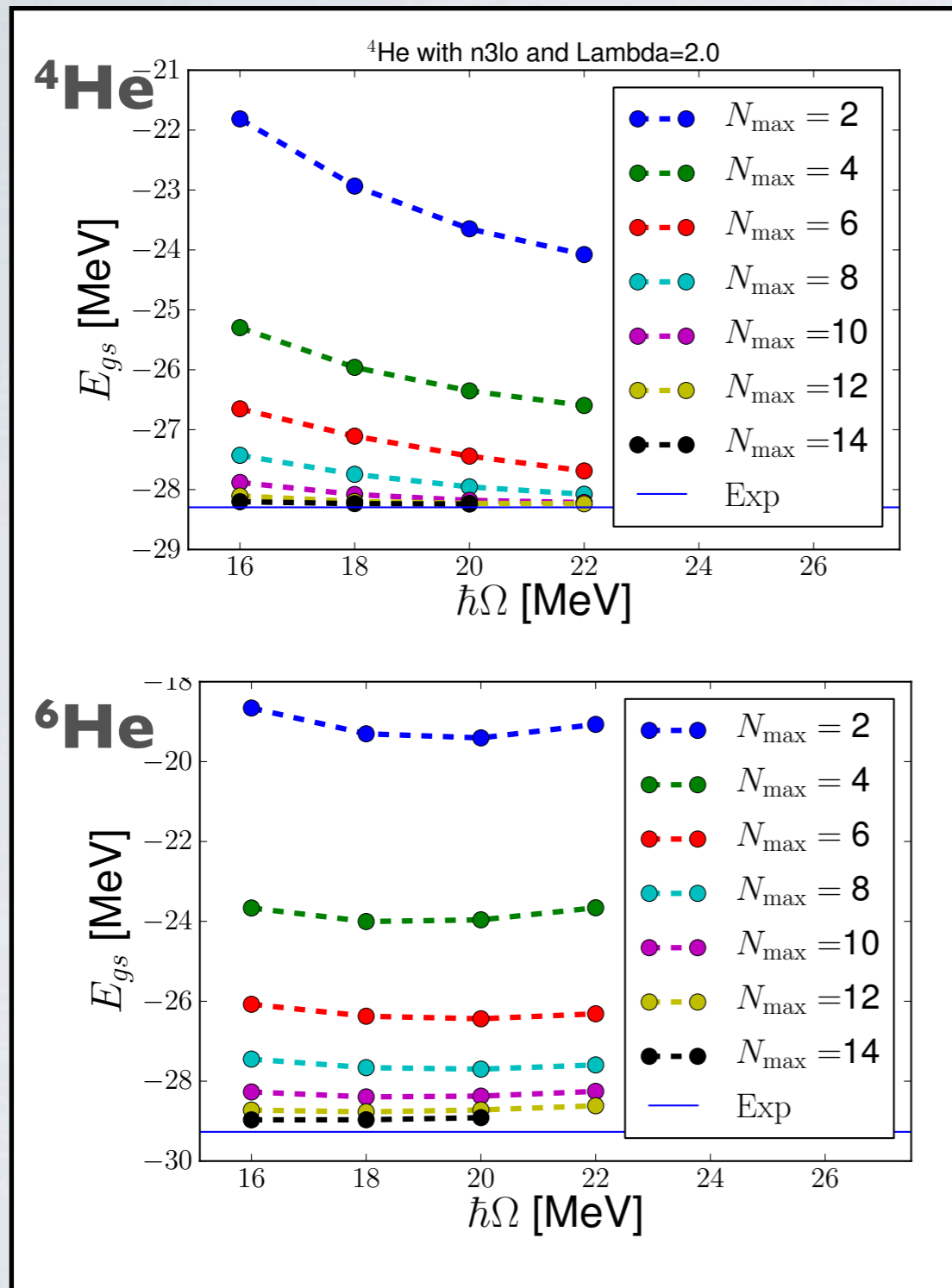
$$E_L = E_\infty + \Delta E_L$$

$$\text{with } \Delta E_L = a_0 \exp(-2k_\infty L)$$

# NCSM example: Energy convergence

## Binding energies

N<sup>3</sup>LO, SRG (NN only,  $\Lambda = 2.0 \text{ fm}^{-1}$ )

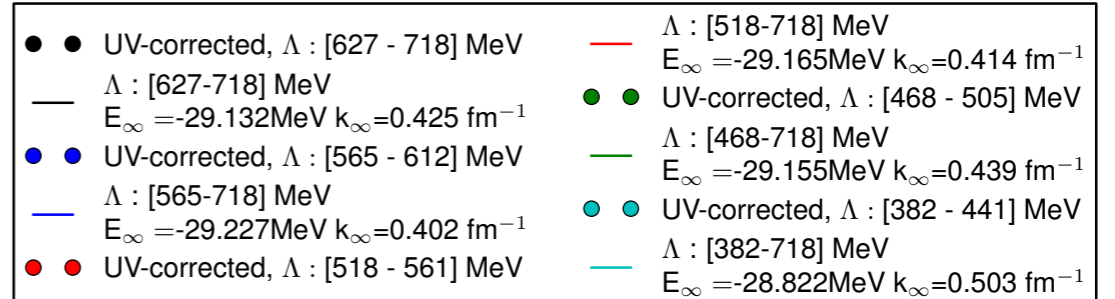
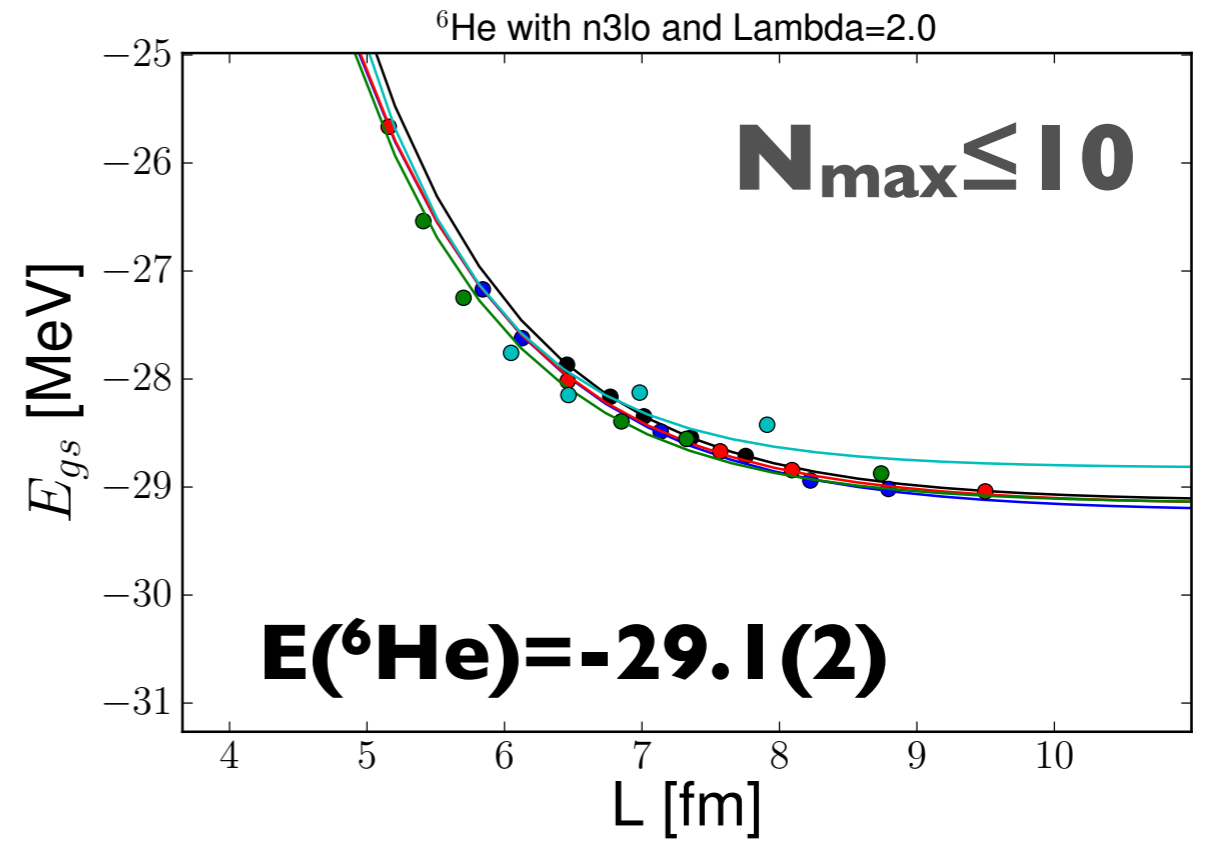
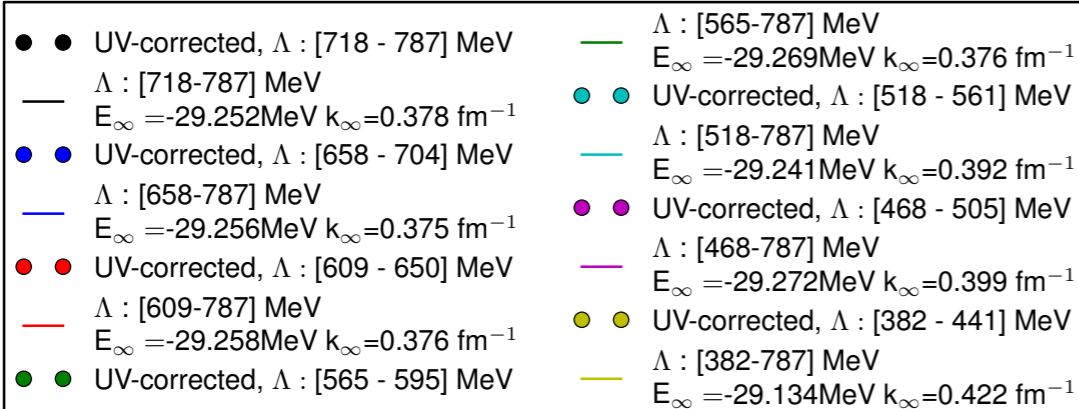
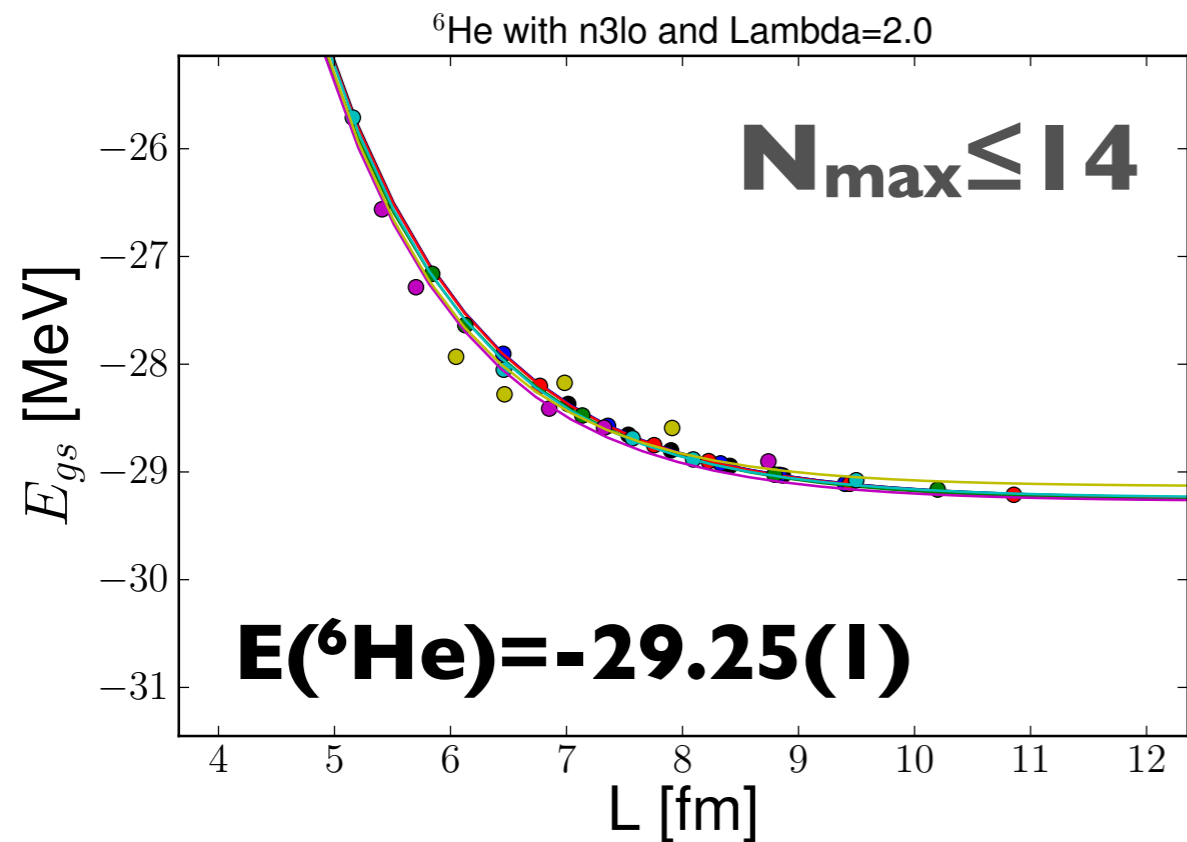


$E(^6\text{He})$	-29.25(1) MeV
$S_{2n}$	1.01(1) MeV



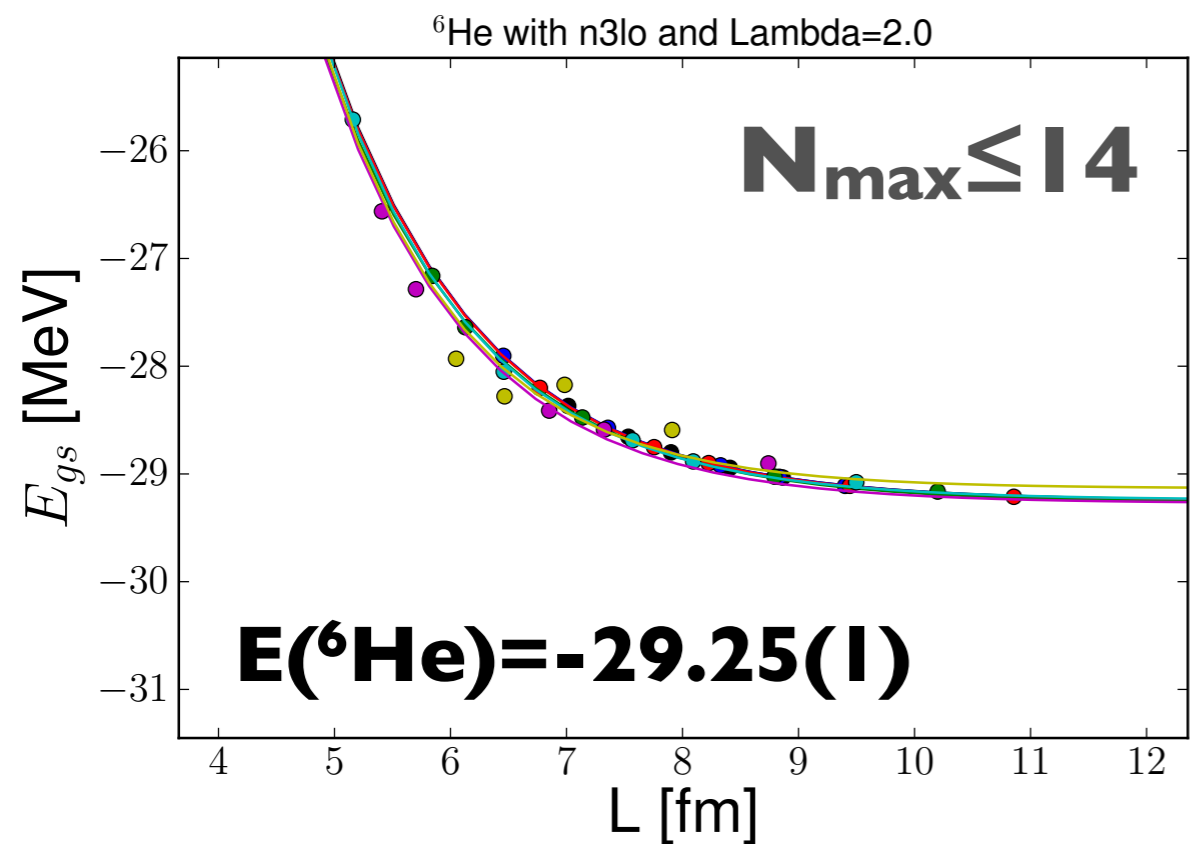
# NCSM example: Energy convergence

$N^3\text{LO}$ , SRG (NN only,  $\Lambda = 2.0 \text{ fm}^{-1}$ )

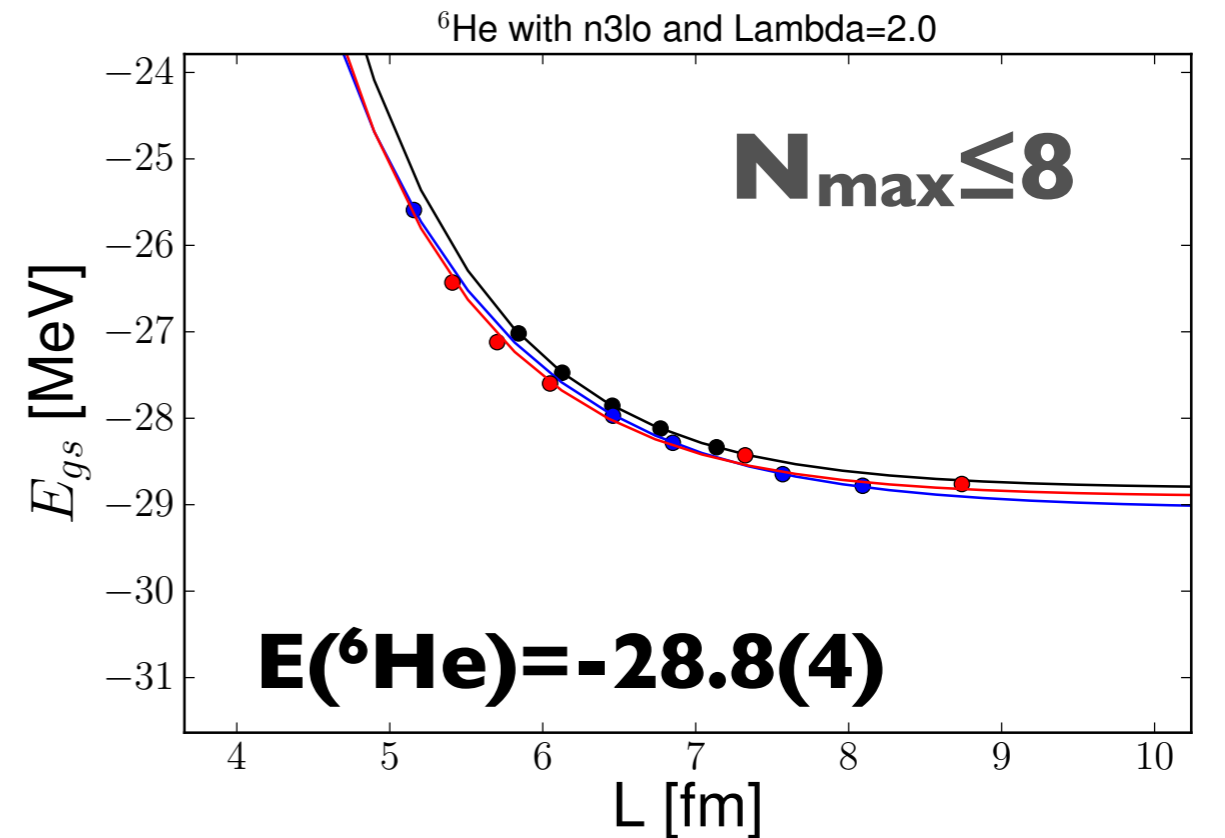


# NCSM example: Energy convergence

$N^3\text{LO}$ , SRG (NN only,  $\Lambda = 2.0 \text{ fm}^{-1}$ )



- |   |   |
|---|---|
| ● ● UV-corrected, $\Lambda$ : [718 - 787] MeV                           | — $\Lambda$ : [565-787] MeV   |
| — $E_{\infty} = -29.252\text{MeV}$ $k_{\infty} = 0.378 \text{ fm}^{-1}$ | ● ● UV-corrected, $\Lambda$ : [518 - 561] MeV                           |
| ● ● UV-corrected, $\Lambda$ : [658 - 704] MeV                           | — $\Lambda$ : [518-787] MeV   |
| — $E_{\infty} = -29.256\text{MeV}$ $k_{\infty} = 0.375 \text{ fm}^{-1}$ | — $E_{\infty} = -29.241\text{MeV}$ $k_{\infty} = 0.392 \text{ fm}^{-1}$ |
| ● ● UV-corrected, $\Lambda$ : [609 - 650] MeV                           | ● ● UV-corrected, $\Lambda$ : [468 - 505] MeV                           |
| — $\Lambda$ : [609-787] MeV   | — $\Lambda$ : [468-787] MeV   |
| — $E_{\infty} = -29.258\text{MeV}$ $k_{\infty} = 0.376 \text{ fm}^{-1}$ | — $E_{\infty} = -29.272\text{MeV}$ $k_{\infty} = 0.399 \text{ fm}^{-1}$ |
| ● ● UV-corrected, $\Lambda$ : [565 - 595] MeV                           | ● ● UV-corrected, $\Lambda$ : [382 - 441] MeV                           |
|   | — $\Lambda$ : [382-787] MeV   |
|   | — $E_{\infty} = -29.134\text{MeV}$ $k_{\infty} = 0.422 \text{ fm}^{-1}$ |



- |   |   |
|---|---|
| ● ● UV-corrected, $\Lambda$ : [565 - 658] MeV                           | — $\Lambda$ : [505-658] MeV   |
| — $E_{\infty} = -29.048\text{MeV}$ $k_{\infty} = 0.446 \text{ fm}^{-1}$ | ● ● UV-corrected, $\Lambda$ : [441 - 494] MeV                           |
| — $\Lambda$ : [565-658] MeV   | — $\Lambda$ : [441-658] MeV   |
| — $E_{\infty} = -28.811\text{MeV}$ $k_{\infty} = 0.513 \text{ fm}^{-1}$ | — $E_{\infty} = -28.908\text{MeV}$ $k_{\infty} = 0.503 \text{ fm}^{-1}$ |
| ● ● UV-corrected, $\Lambda$ : [505 - 561] MeV                           |   |

# Ab initio $\langle {}^6\text{He} \mid {}^4\text{He}+n+n \rangle$ overlap

$\langle {}^6\text{He} (0^+) \mid {}^4\text{He} (0^+) + n + n \rangle$

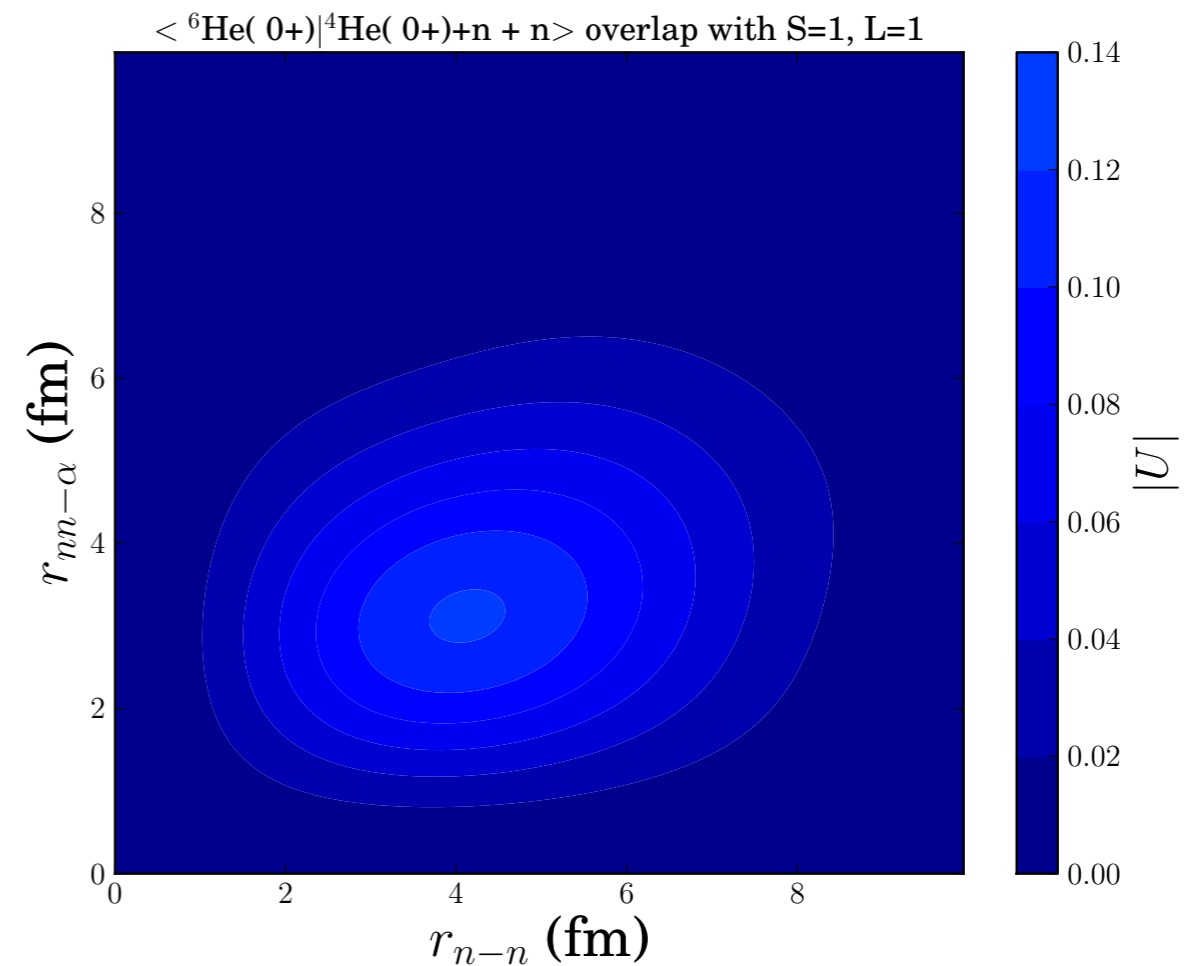
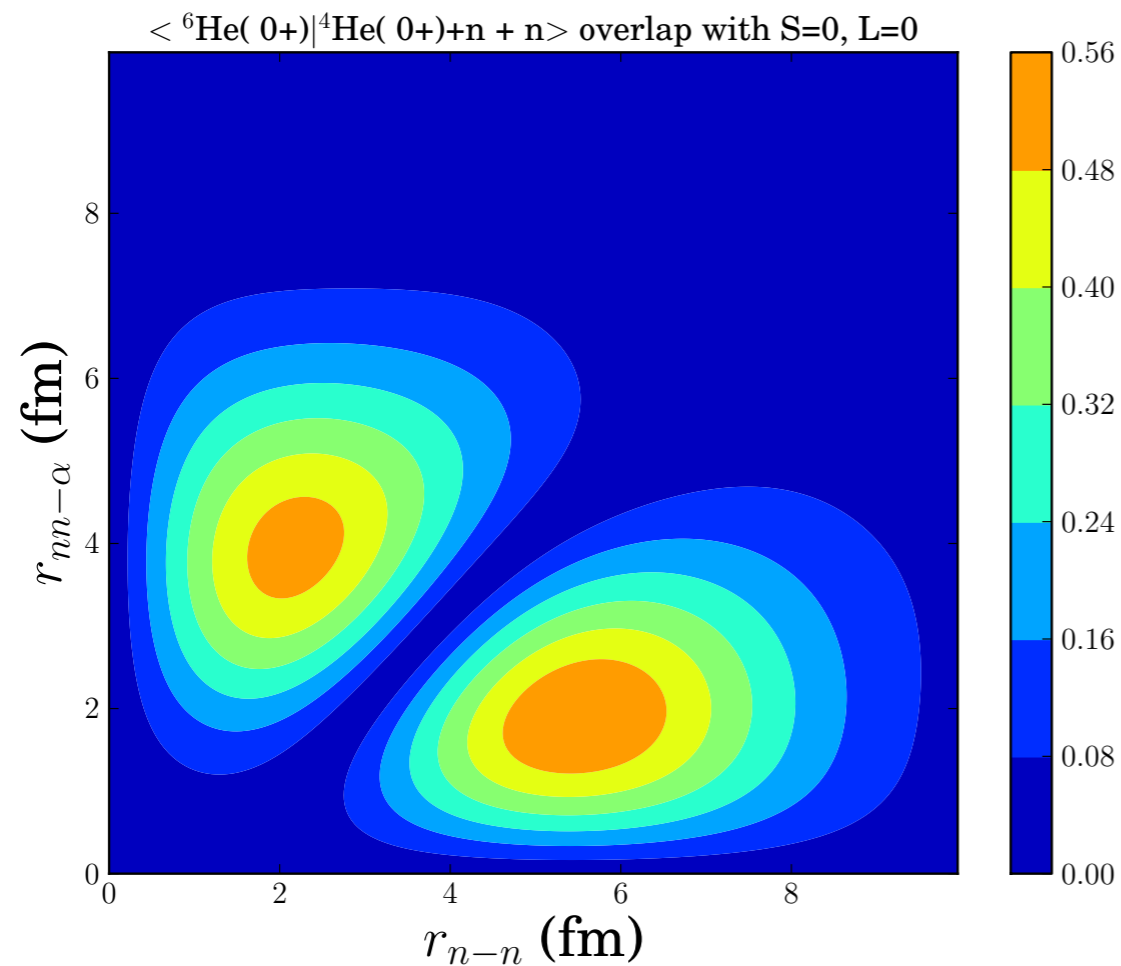
$L=S=0$

$N^3\text{LO, SRG}$

NN only,  $\Lambda = 2.0 \text{ fm}^{-1}$ ,

$N_{\text{max}}=14, \text{HO}=20 \text{ MeV}$

$L=S=1$



$$u_{(A-2)I_1T_1;LS}^{AJT}(x,y) = \sum_{ij} R_i(x)R_j(y)_{\text{SD}} \langle (A)JT \mid \mid a_i^\dagger a_j^\dagger \mid \mid (A-2)I_1T_1 \rangle_{\text{SD}}$$

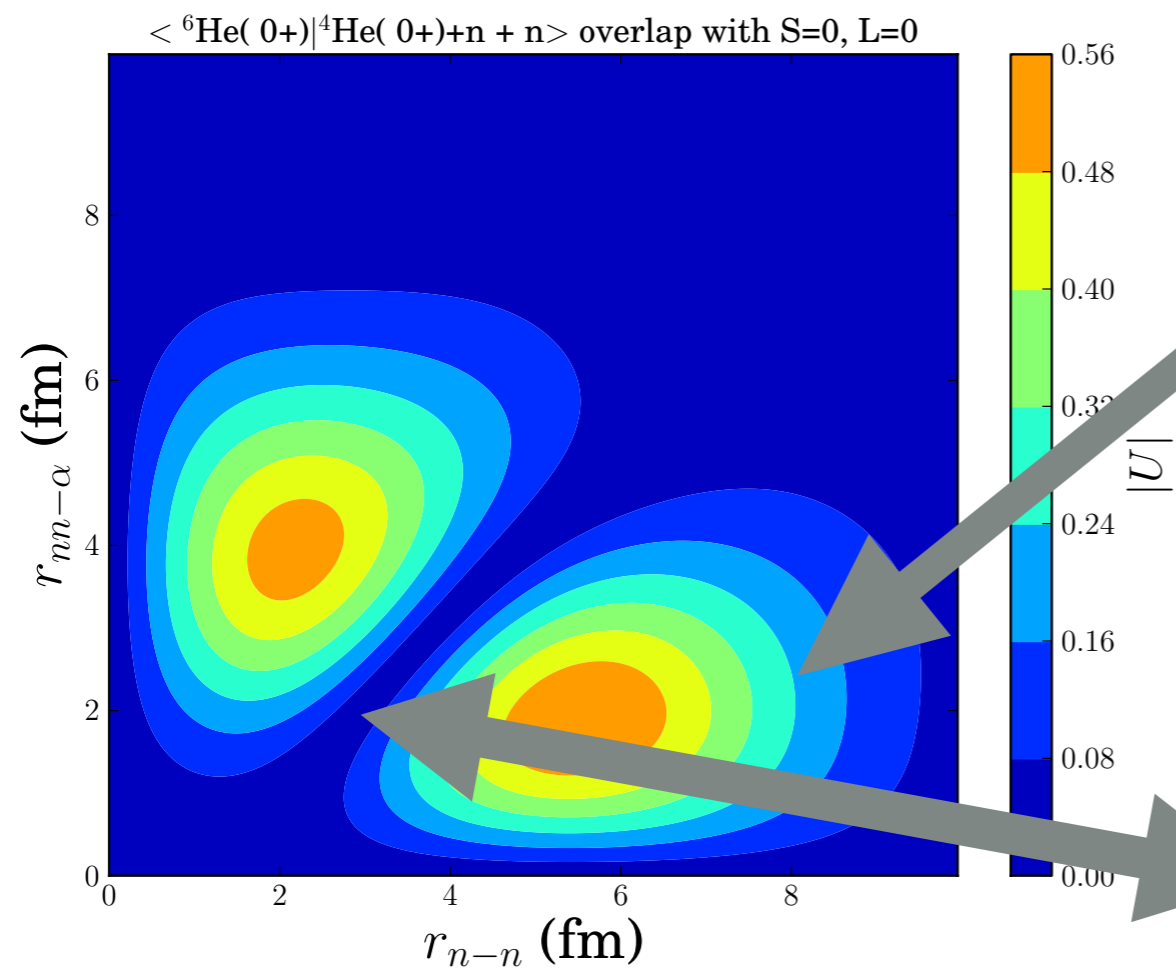
# Ab initio $\langle {}^6\text{He} \mid {}^4\text{He}+n+n \rangle$ overlap

$N^3\text{LO}$ , SRG

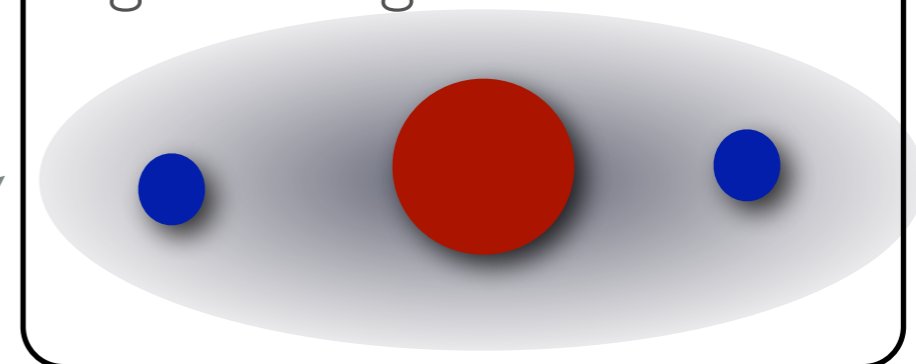
NN only,  $\Lambda = 2.0 \text{ fm}^{-1}$ ,

$N_{\text{max}}=14$ , HO=20 MeV

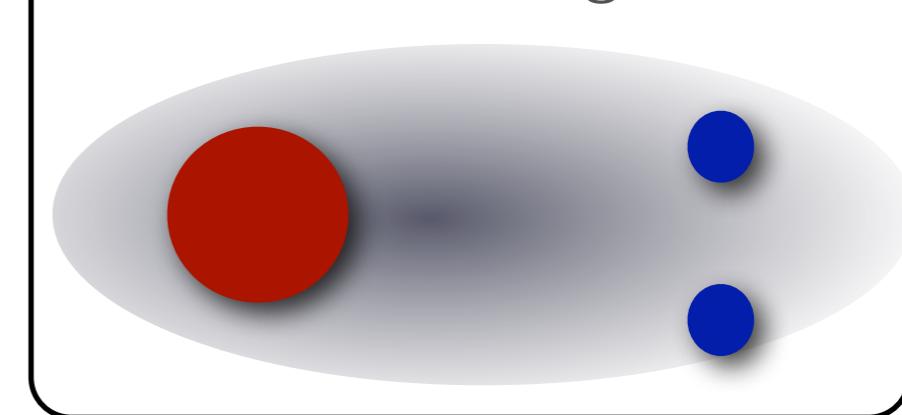
$$\langle {}^6\text{He} (0^+) \mid {}^4\text{He} (0^+) + n + n \rangle$$



Cigar configuration



Di-neutron configuration



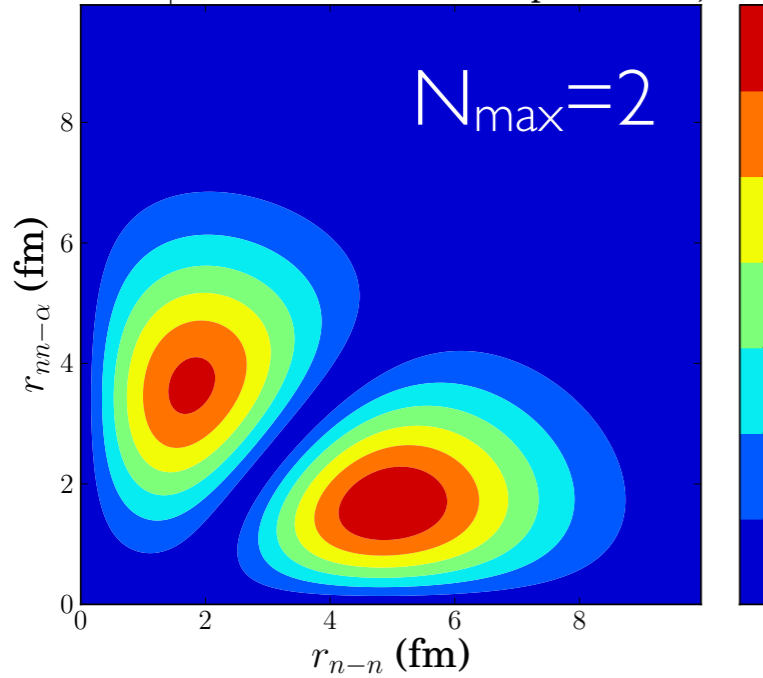
# $\langle {}^6\text{He} \mid {}^4\text{He}+n+n \rangle$ overlap: $N_{\text{max}}$ dependence

$$\langle {}^6\text{He} (0^+) \mid {}^4\text{He} (0^+) + n + n \rangle$$

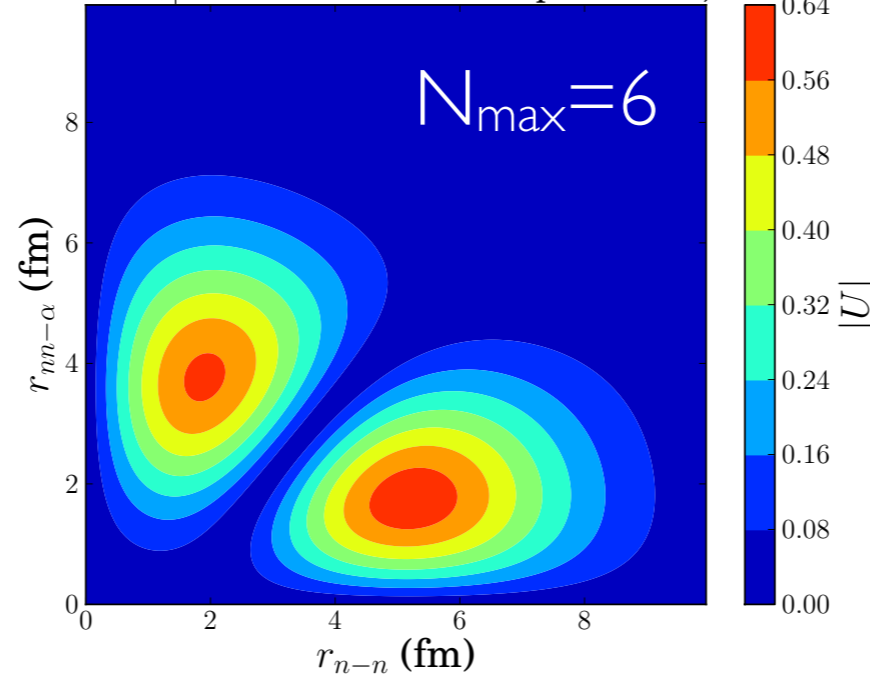
$N^3\text{LO}$ , SRG

NN only,  $\Lambda = 2.0 \text{ fm}^{-1}$ ,  
 $N_{\text{max}} = 14$ , HO = 20 MeV

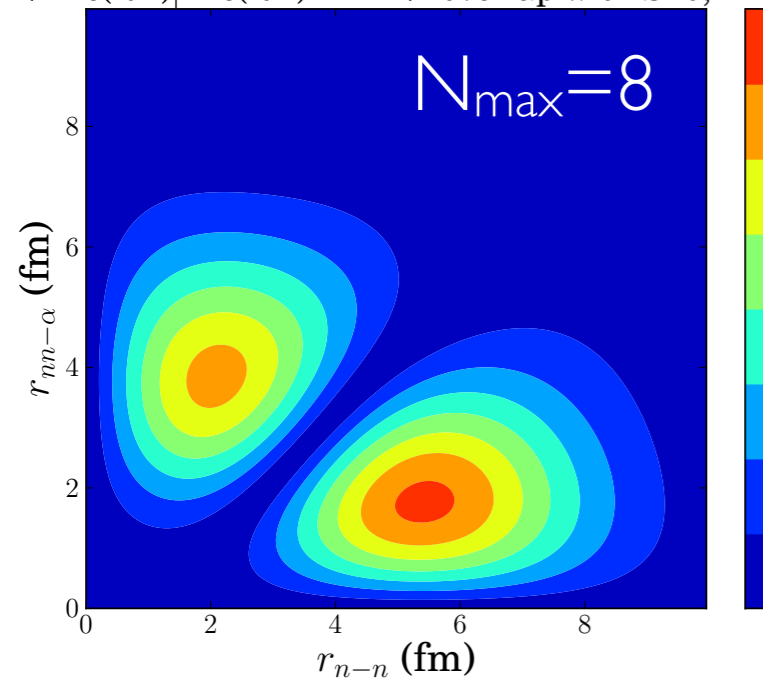
$\langle {}^6\text{He} (0^+) \mid {}^4\text{He} (0^+) + n + n \rangle$  overlap with  $S=0, L=0$



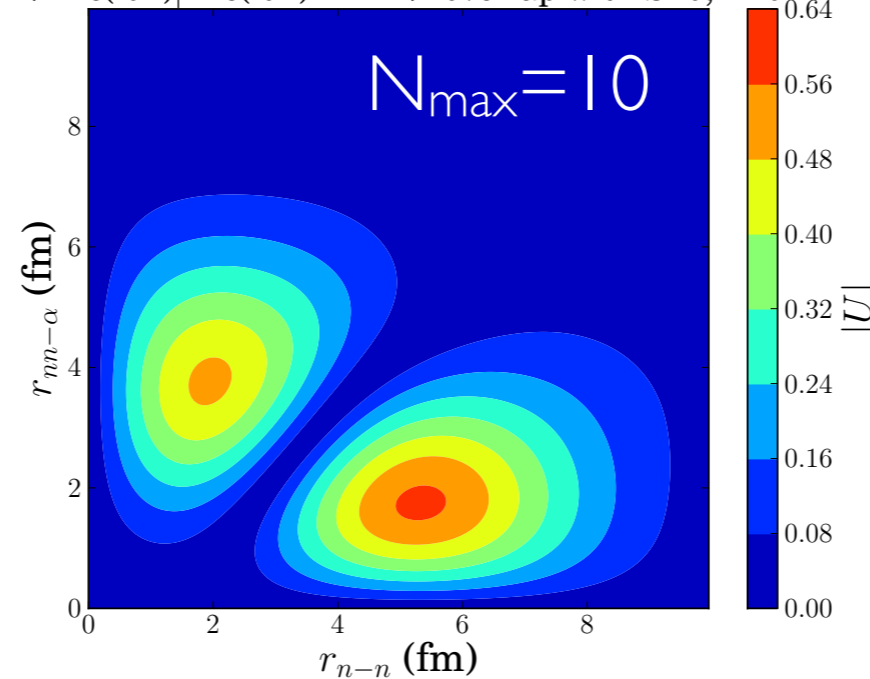
$\langle {}^6\text{He} (0^+) \mid {}^4\text{He} (0^+) + n + n \rangle$  overlap with  $S=0, L=0$



$\langle {}^6\text{He} (0^+) \mid {}^4\text{He} (0^+) + n + n \rangle$  overlap with  $S=0, L=0$



$\langle {}^6\text{He} (0^+) \mid {}^4\text{He} (0^+) + n + n \rangle$  overlap with  $S=0, L=0$



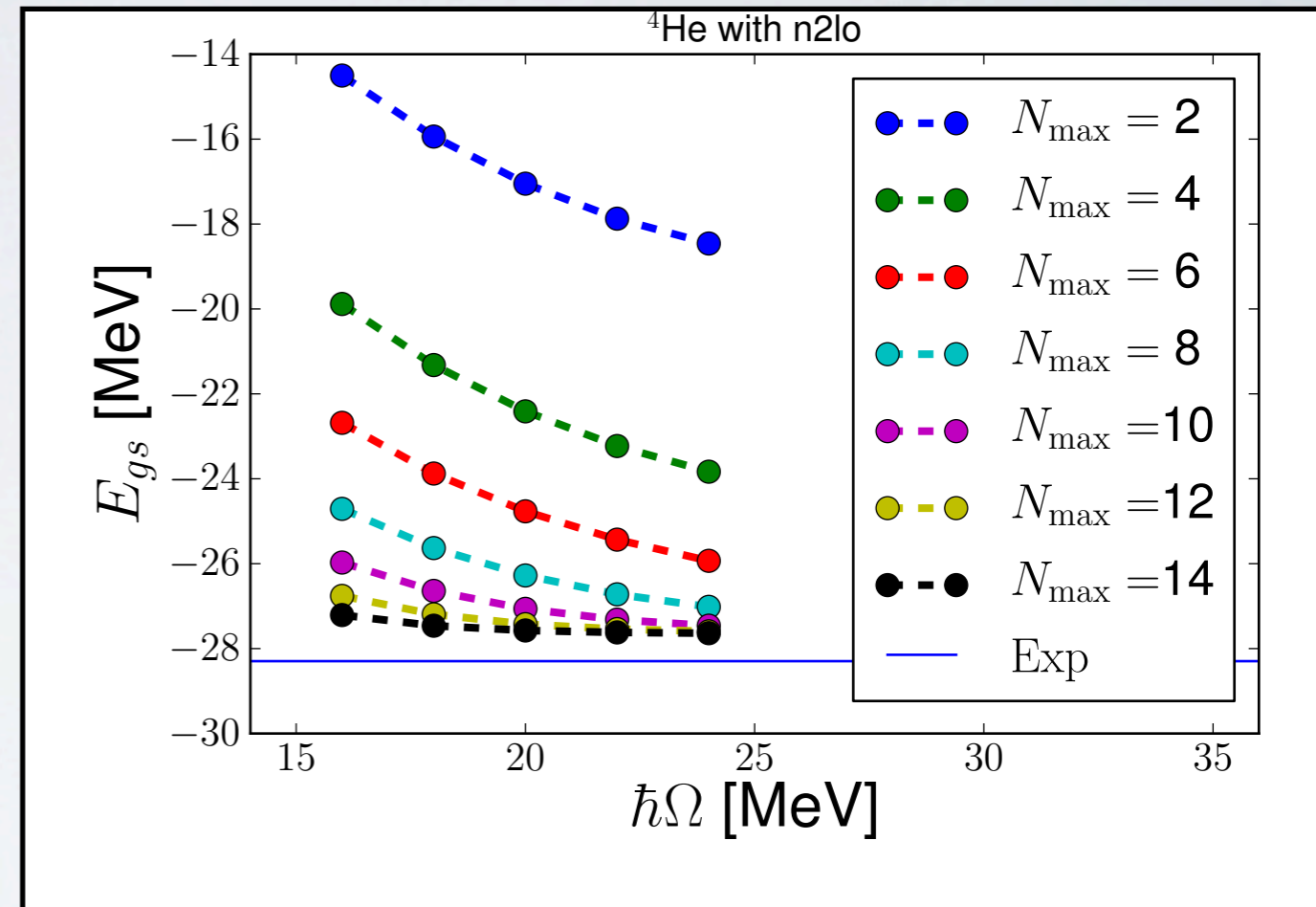
**Conclusion:**  
 This is a Pauli  
 focusing effect

# NNLO (POUNDerS) in the NCSM

# NNLO (POUNDerS) in NCSM

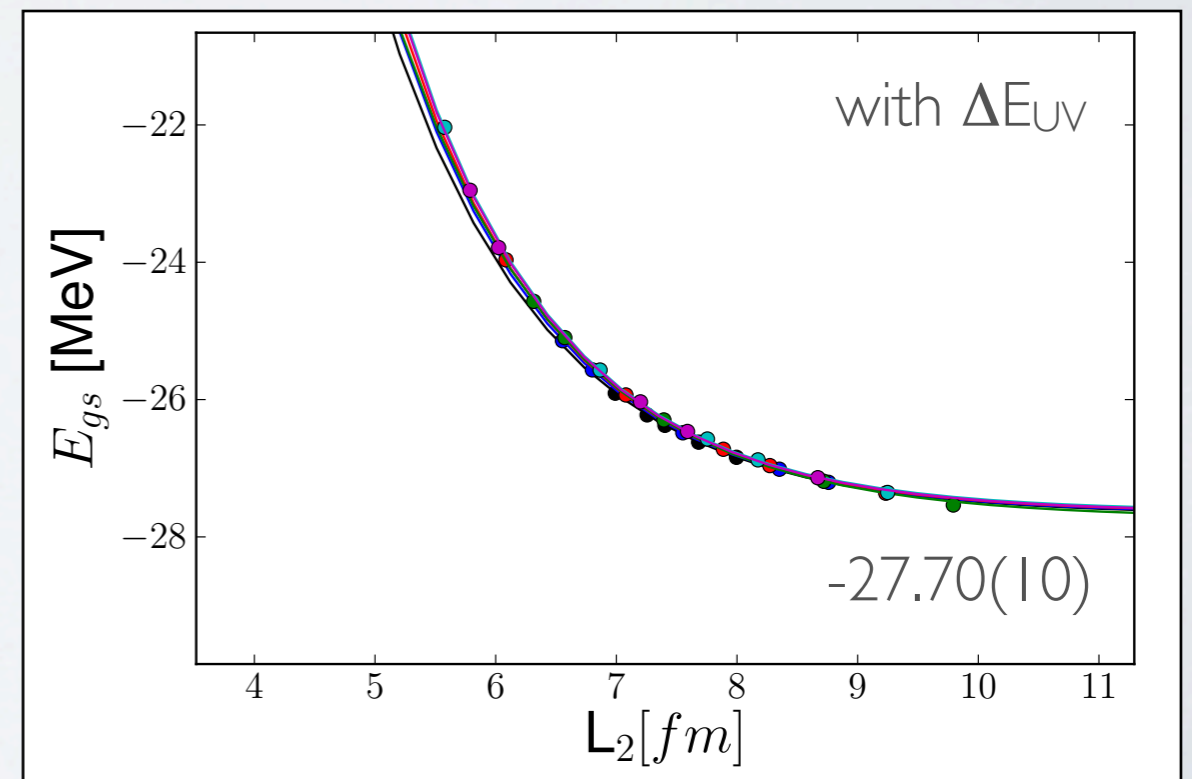
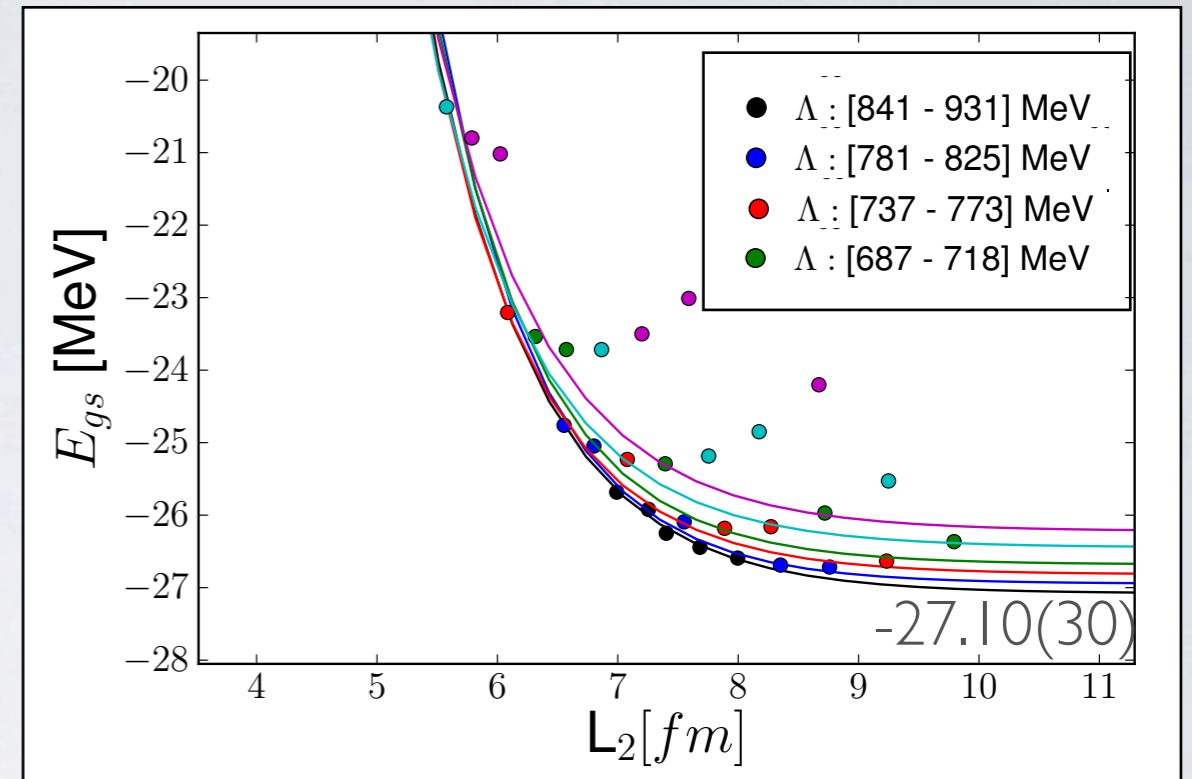
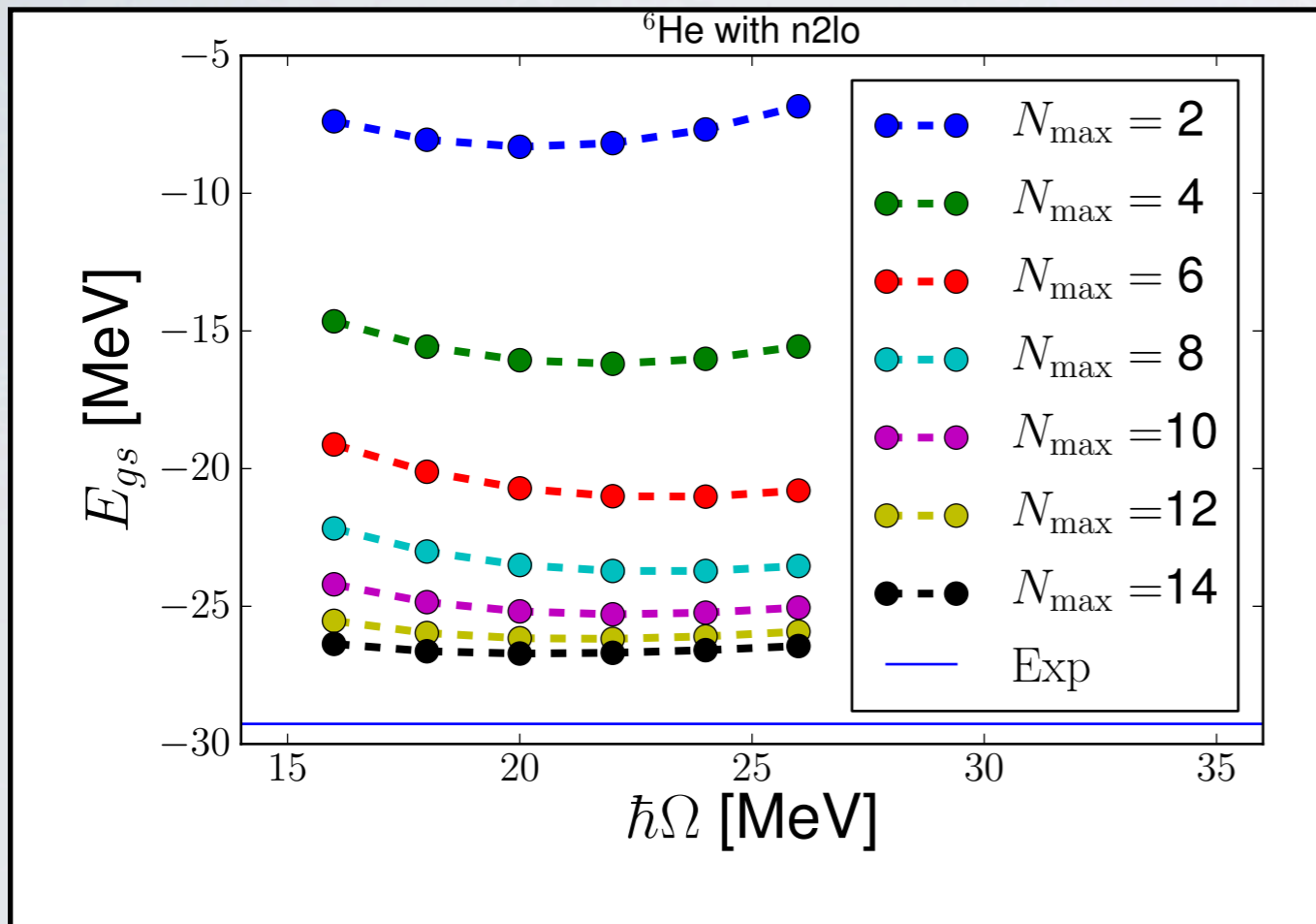
- ❖ NNLO (POUNDerS) **optimized** to NN phase shifts (see A. Ekström's talk)
- ❖ It is **soft**: we will show bare interaction results in NCSM up to  $A \sim 10$ .
- ❖ Study effects on the **structure** of light nuclei
- ❖ Technical **developments** of shell model code  $\Rightarrow$   
 $d \sim 1.7 \times 10^9$  - one Lanczos iteration in 35 min on **one** node.

$^4\text{He}$  with NNLO (POUNDerS) -bare



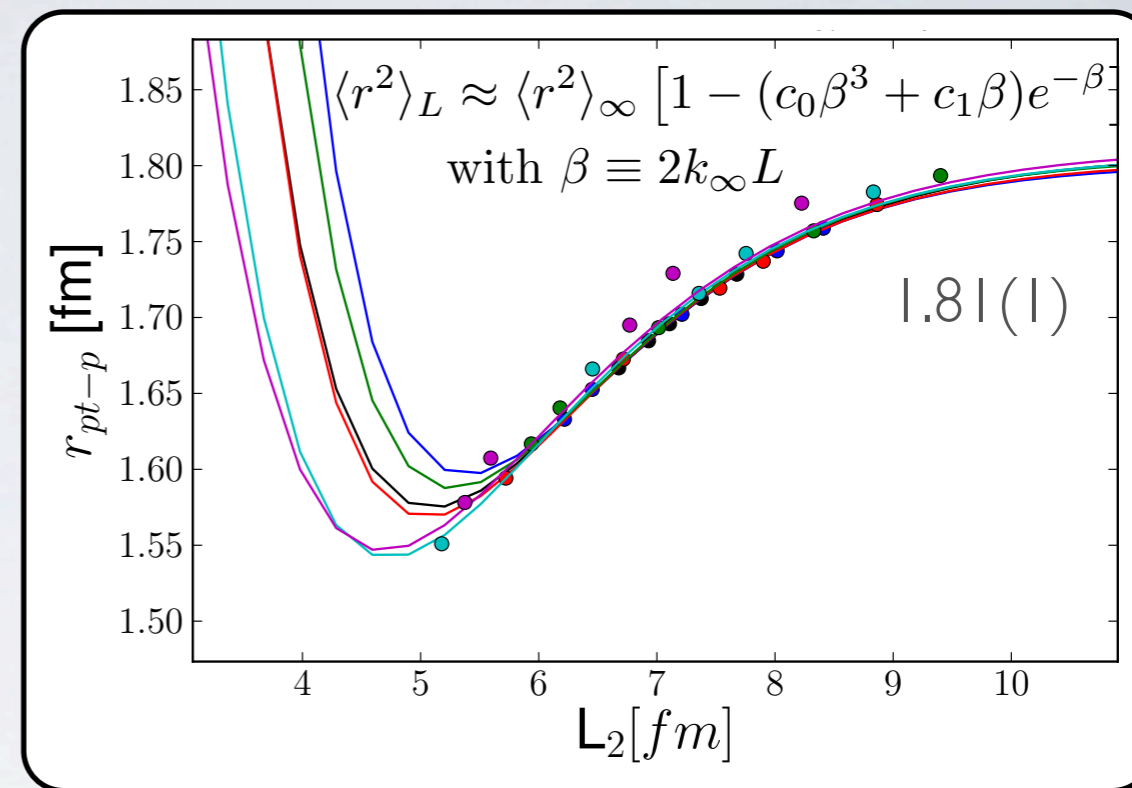
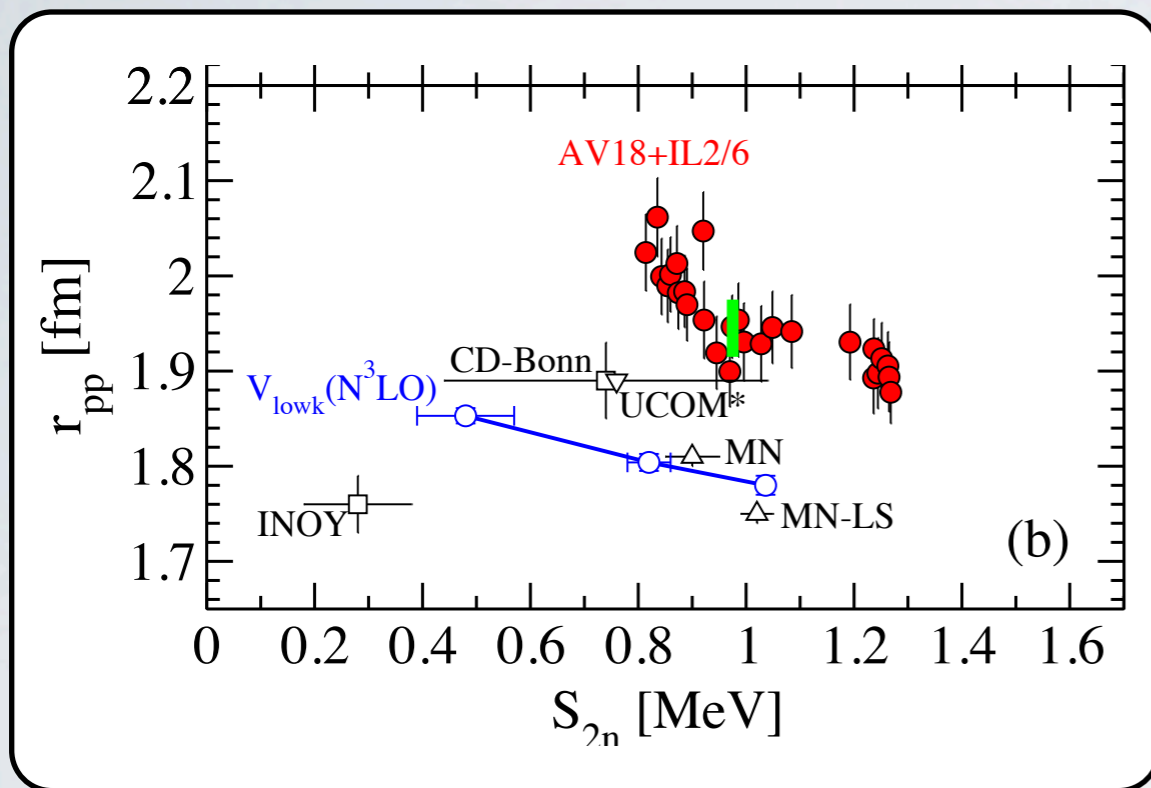
# A=6 energy

${}^6\text{He}$  with NNLO (POUNDerS) -bare





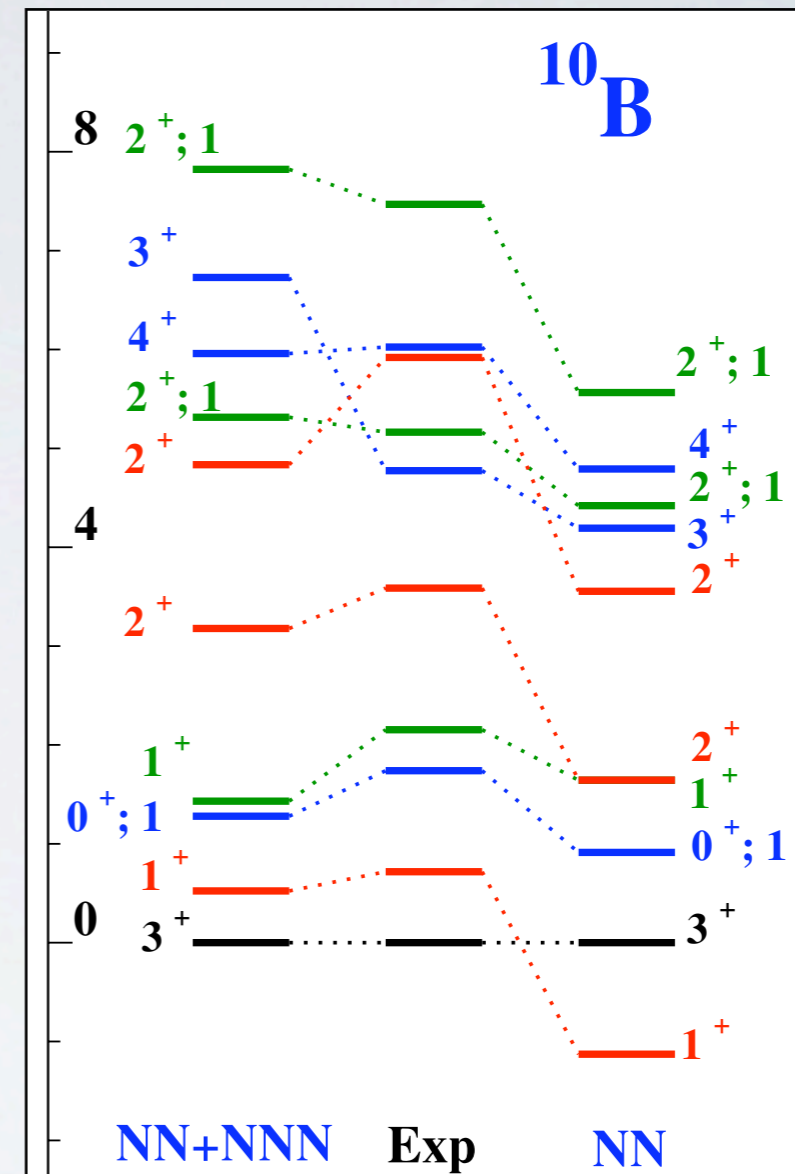
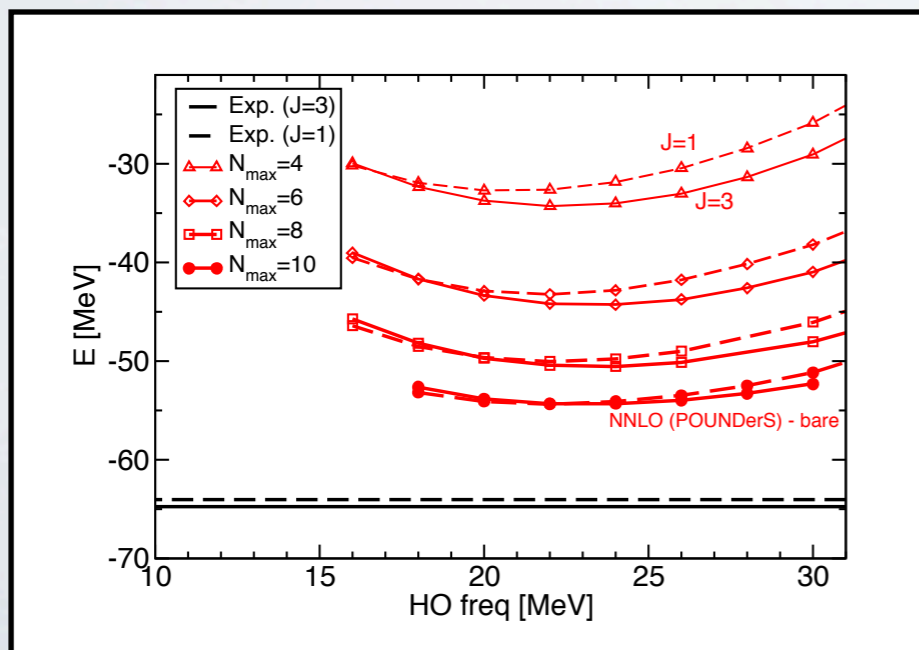
# A=6 radii and $S_{2n}$



	$r_{pt-p}(^6\text{He})$ [fm]	$S_{2n}(^6\text{He})$ [fm]
Exp.	1.938(23)	0.97
NCSM (SRG $\Lambda = 2.0 \text{ fm}^{-1}$ ): N3LO (EM)	1.83(1)	1.01(4)
HH (S. Bacca) (LS+ $V_{\text{lowk}}$ , $2.0 \text{ fm}^{-1}$ ): N3LO (EM)	1.804(9)	$\sim 0.8$
NCSM (bare): NNLO (POUNDerS)	1.81(1)	0.1(1)

# $^{10}\text{B}$ ground state

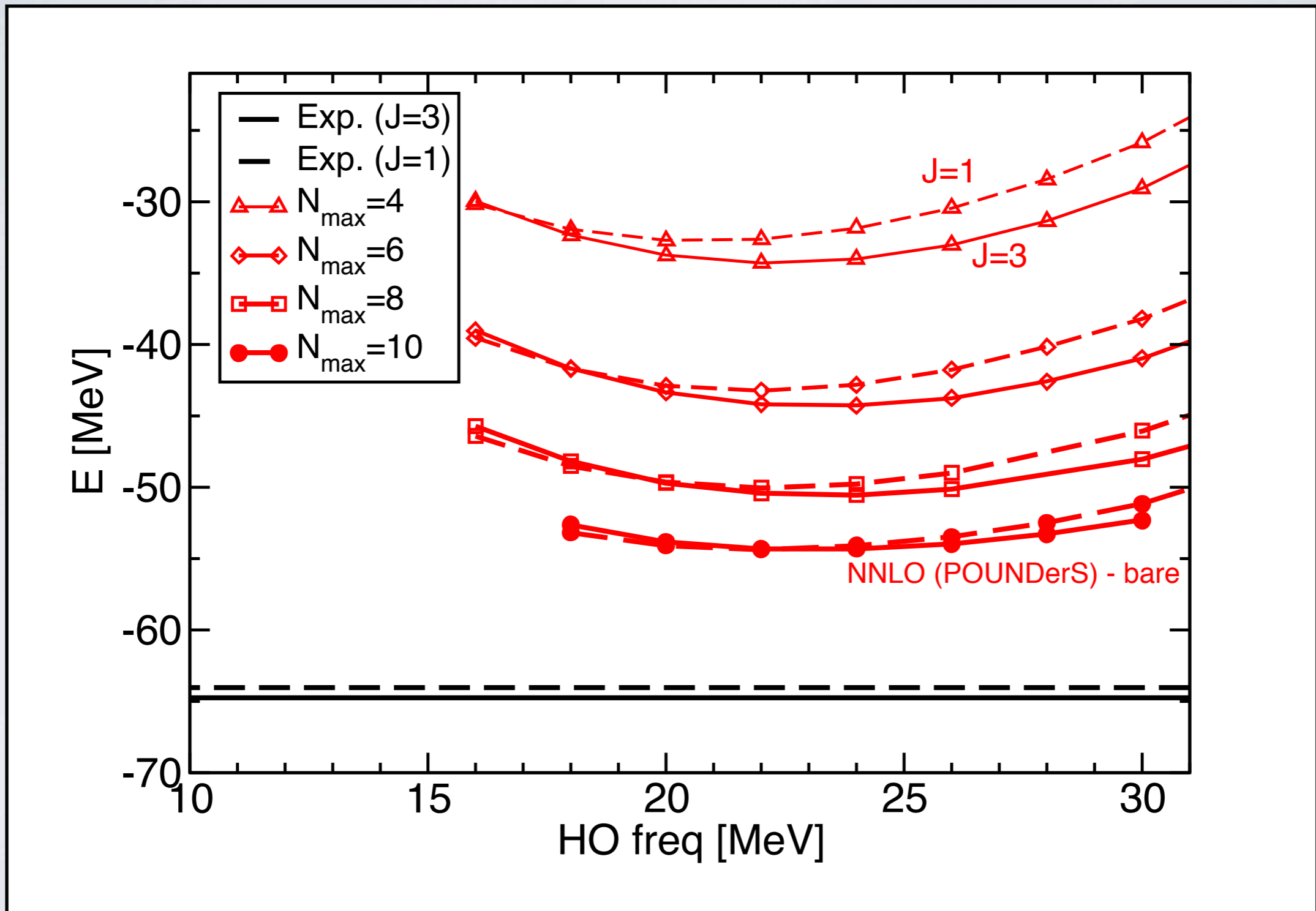
- ❖ Realistic NN interactions predict the incorrect ground-state spin of  $^{10}\text{B}$
- ❖ It has been shown that NNN terms in the Hamiltonian are needed to remedy this situation.
- ❖ With the NNLO (POUNDerS) NN interaction we find a very small gap between the  $3^+, 1^+$  levels.



From: P. Navrátil et al., Phys. Rev. Lett. 99(2007)042501.

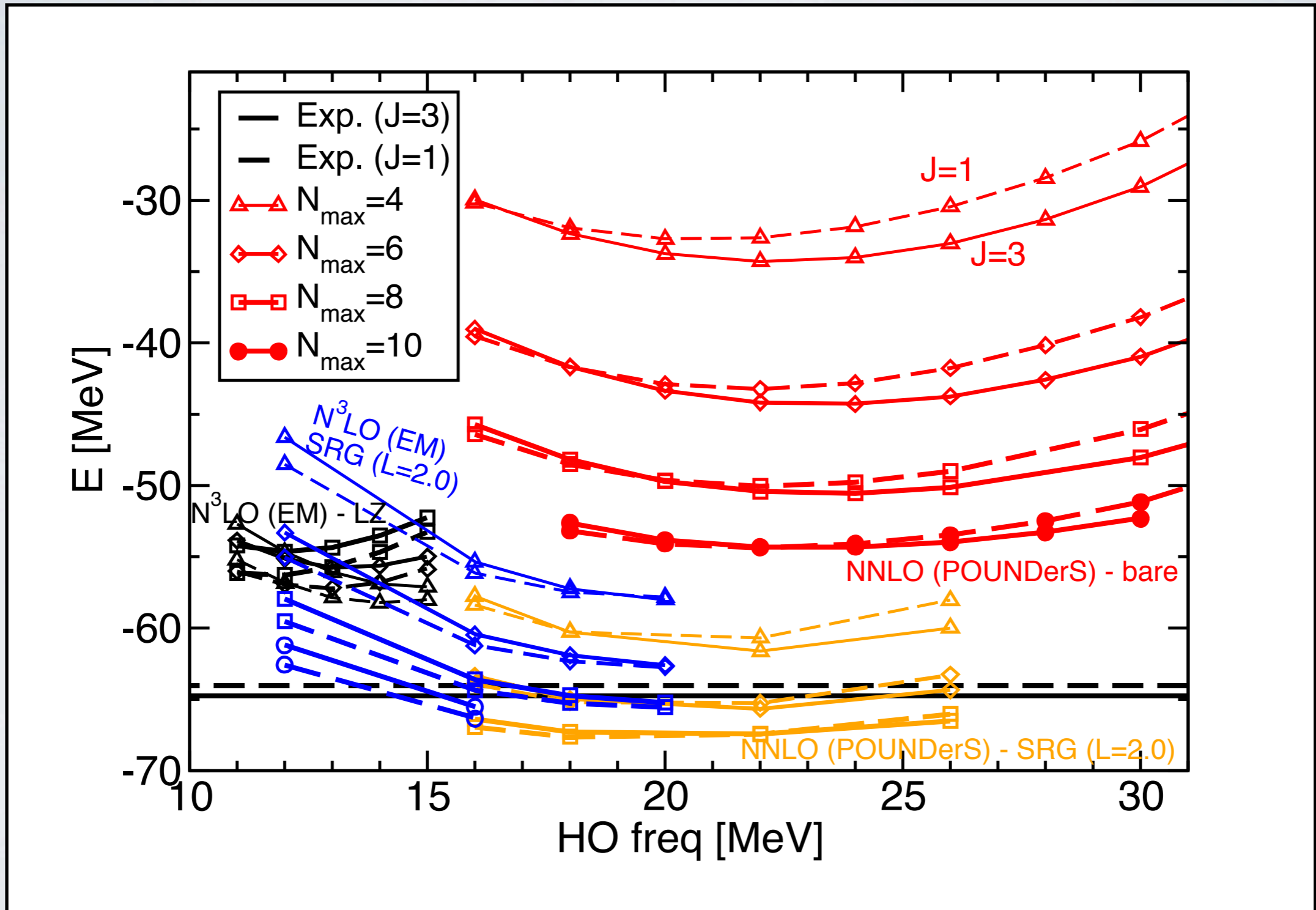
# $^{10}\text{B}$ ground state

$^{10}\text{B}$  with NNLO (POUNDerS) -bare



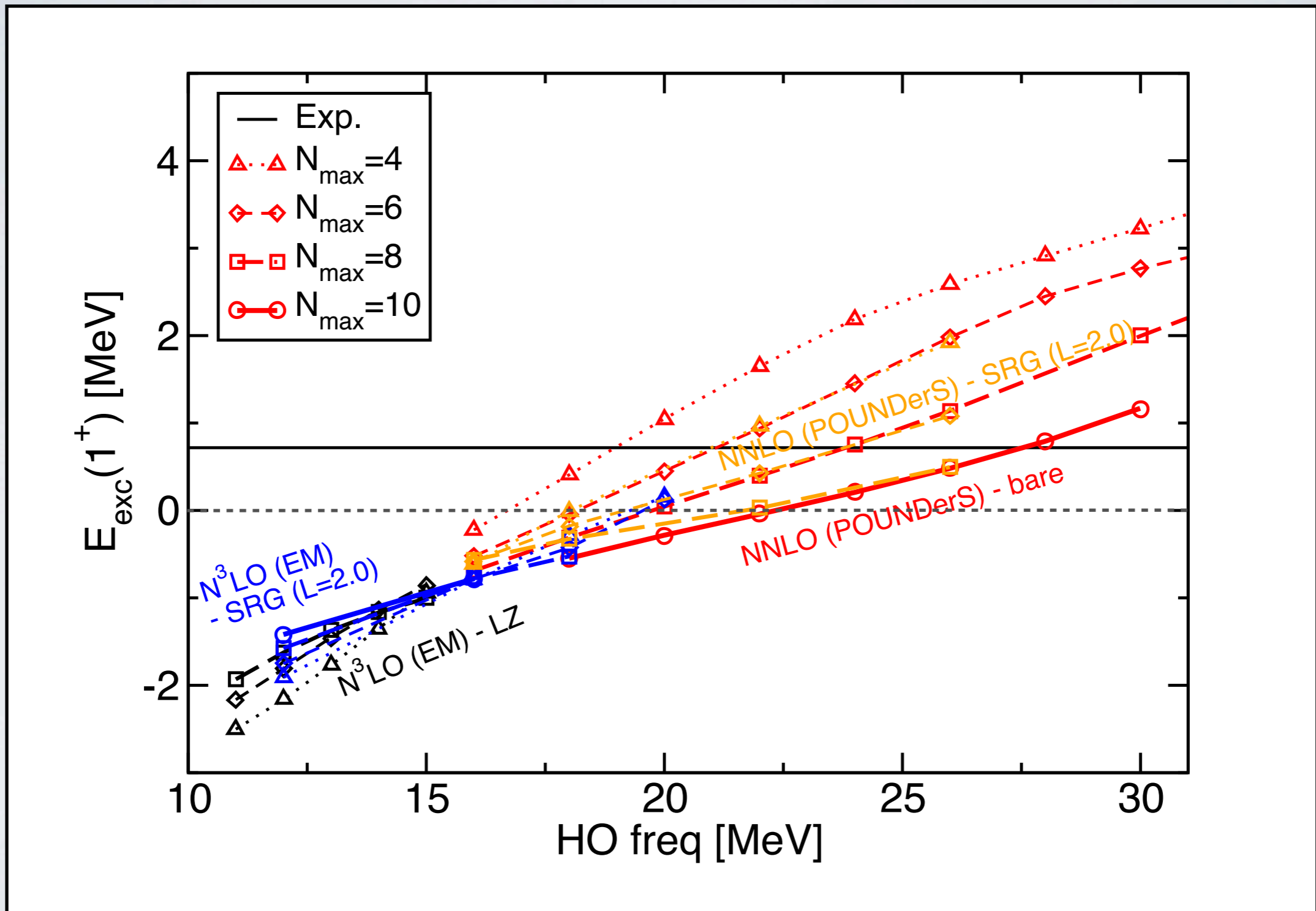
# $^{10}\text{B}$ ground state

$^{10}\text{B}$  with NNLO (POUNDerS) -bare



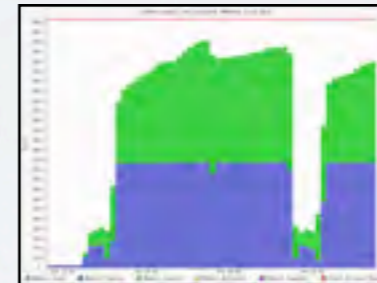
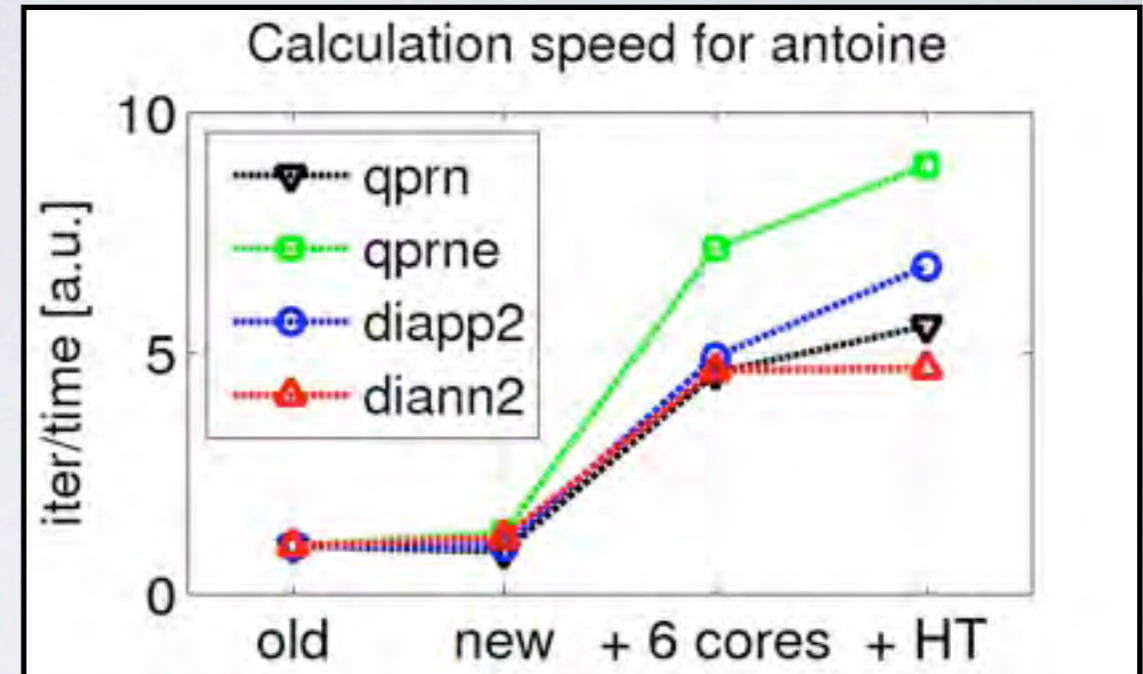
# $^{10}\text{B}$ ground state assignment

$^{10}\text{B}$  with NNLO (POUNDerS) -bare



# Technical developments

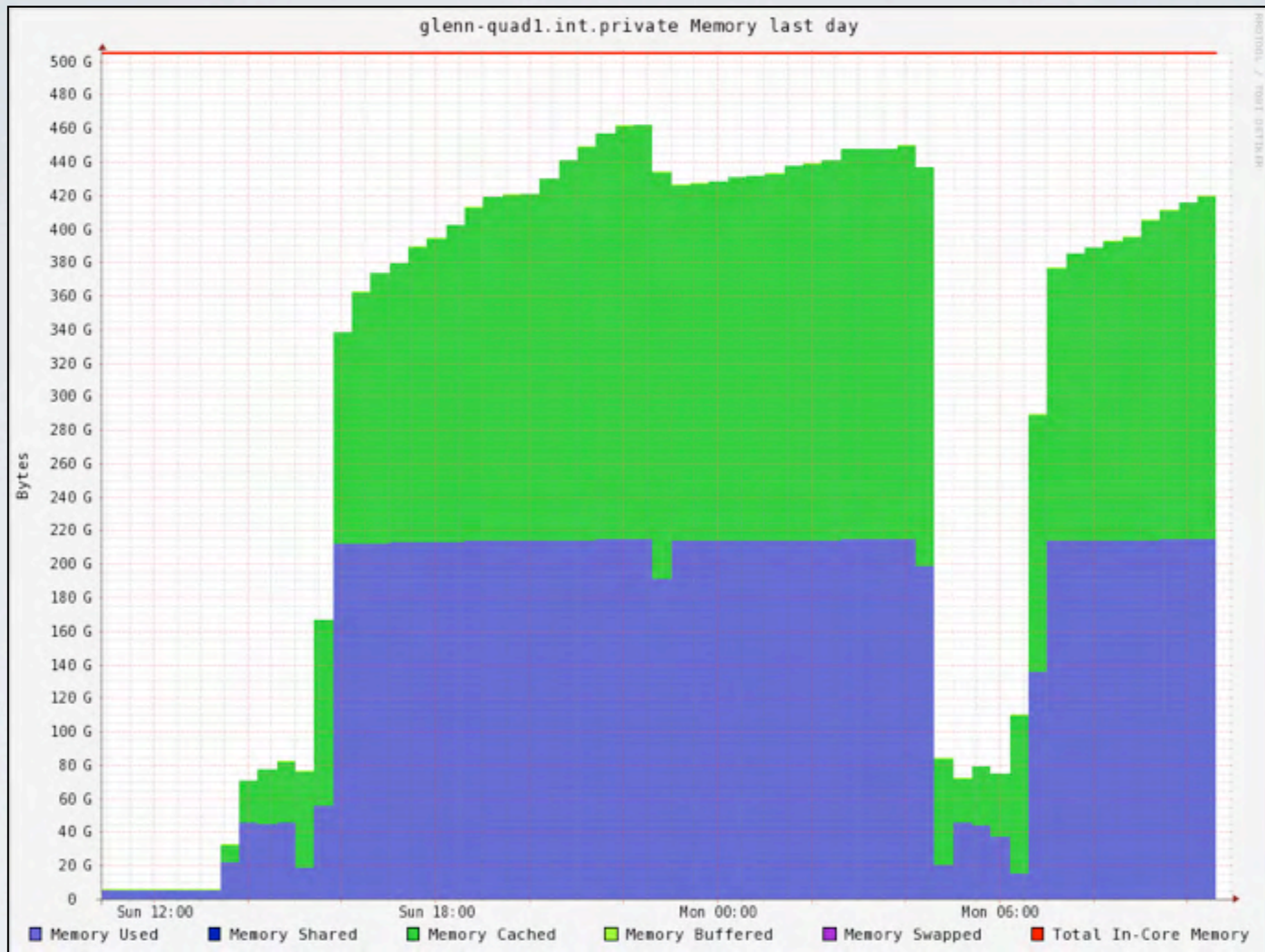
- ❖ Antoine employs the (smaller) p/n **subspaces**:  
MB state  $i = q_p + r_n$
- ❖ MB matrix generated **on-the-fly**.  
MatVec operations the largest bottleneck.
- ❖ New version: **PANTOINE**  
 $M * x = y$ , split into subsets  
 $(M_1 + M_2 + \dots) * x = y$
- ❖ Shared memory / **multithread**  
Close to theoretical max of Miter / s on multicore.
- ❖ Timing: qprn used in average 31.8 of 32 cores on Glenn no. 5



Glenn no. 5

Opteron 6220 (4 sockets, 32 cores, 512 GB)

# Technical developments



$$N_{\max} = 10 \text{ for } 10^9 \text{ B} \Rightarrow d \sim 1.7 \times 10^9$$

Ground state obtained in seven hours on **one** node.

# Conclusion and Outlook

- ❖ Effective interactions for cold atoms in deformed traps
- ❖ Introduction of UV and IR scales; optimization of run sequence
  - ▶ BUT still need several large  $N_{\max}$  computations
  - ▶ Technical development - pAntoine
- ❖ Microscopic description of clustering
  - ▶ Calculate core swelling:  $r_{pp}$
  - ▶ Study projection on HH basis
- ❖ First results from NNLO (POUNDerS) in the NCSM
  - ▶ Soft - bare interaction used
  - ▶  $^{10}\text{B}(1^+, 3^+)$  states almost degenerate