Extrapolations in finite model spaces and EFT for deformed nuclei

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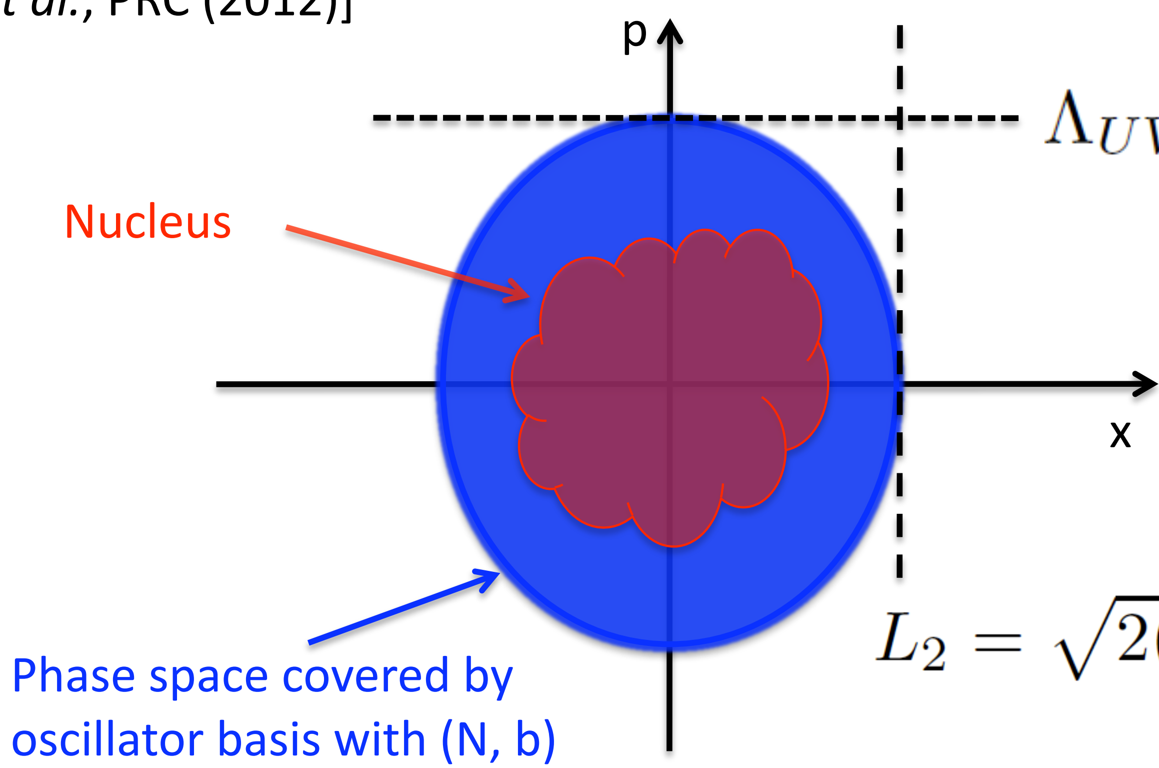
Convergence in finite oscillator spaces

Calculations are performed in finite oscillator spaces. How can one reliably extrapolate to infinity? What is the equivalent of Lüscher's formula for the harmonic oscillator basis [Lüscher, Comm. Math. Phys. 104, 177 (1986)] ?

Convergence in momentum space (UV) and in position space (IR) needed
[Stetcu *et al.*, PLB (2007); Hagen *et al.*, PRC (2010); Jurgenson *et al.*, PRC (2011); Coon *et al.*, PRC (2012)]

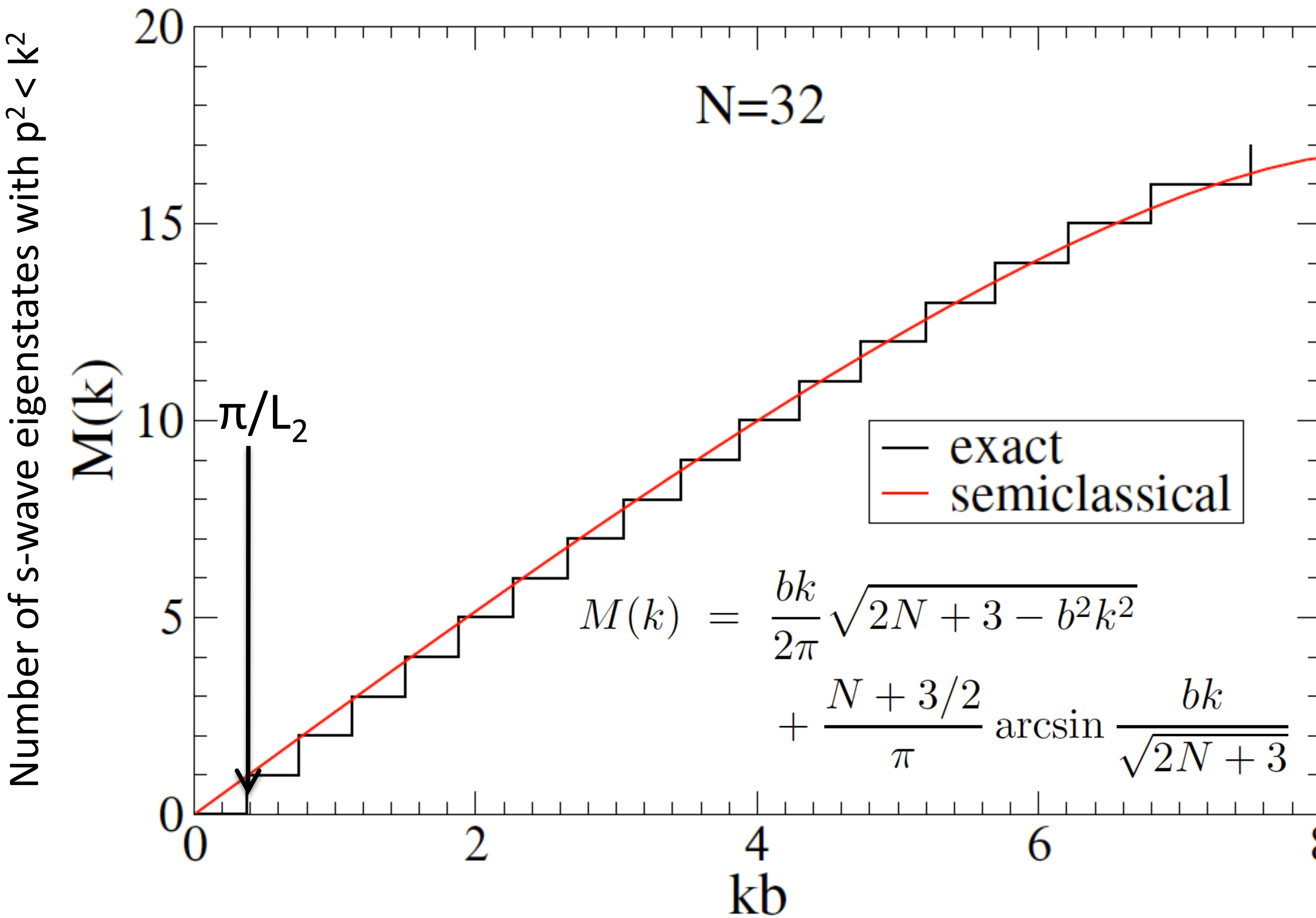


$$\Lambda_{UV} \equiv \sqrt{2(N+3/2)}\hbar/b$$

$$L_2 = \sqrt{2(N+3/2+2)}b$$

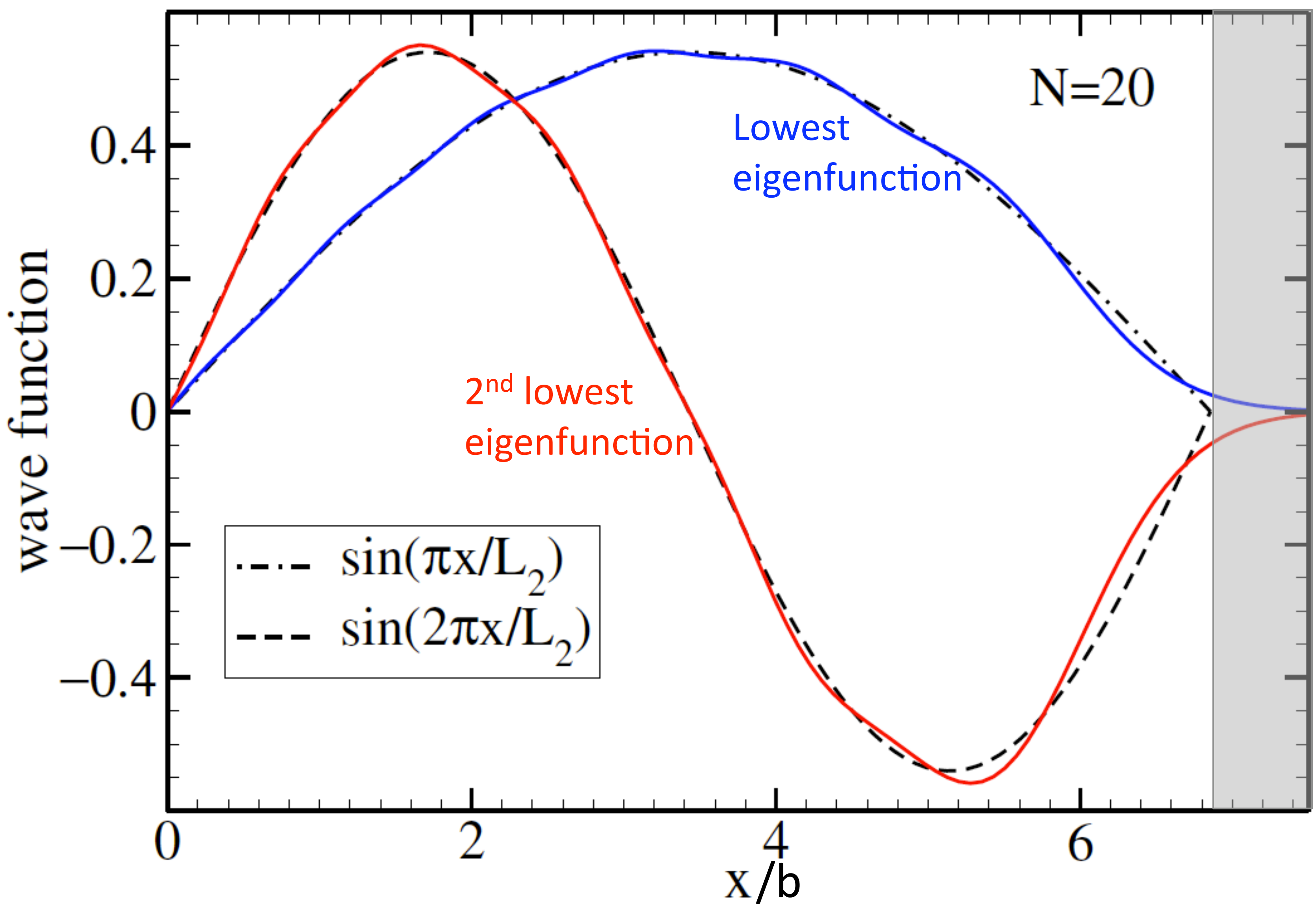
Nucleus needs to "fit" into basis:
- Nuclear radius R < L
- cutoff of interaction $\Lambda < \Lambda_{UV}$

Spectrum of the operator p^2 in the HO basis



- At low momentum, number of states increases linearly with increasing momentum
- Spectrum looks like that of the momentum operator in a box

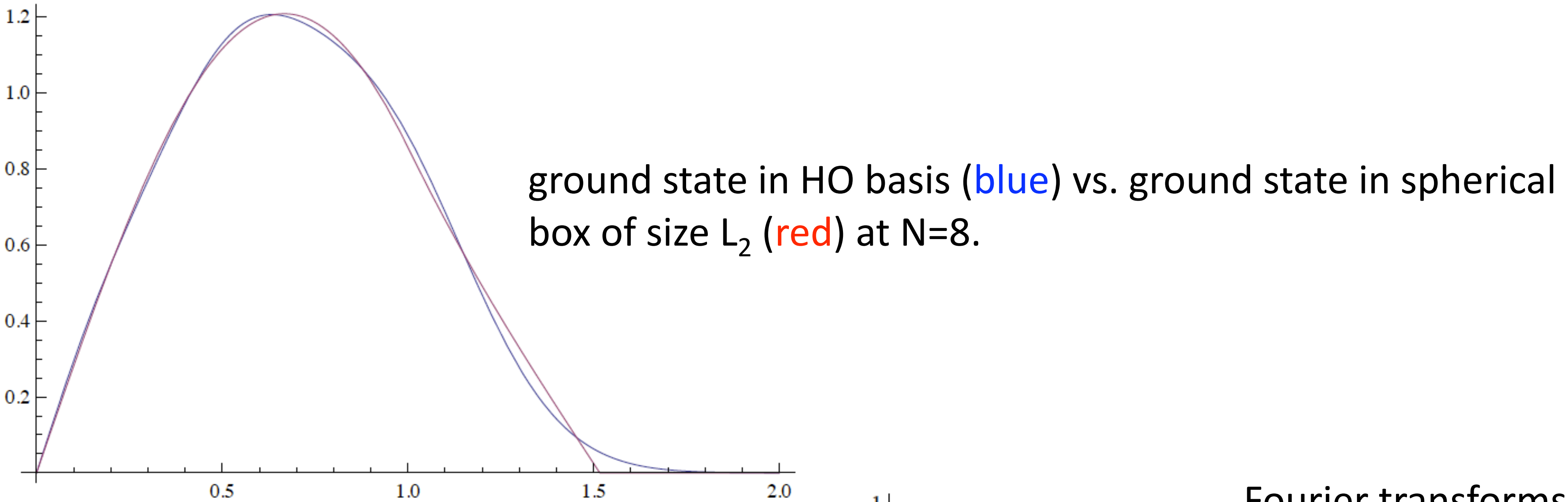
Eigenfunctions of p^2 with lowest eigenvalues in oscillator basis

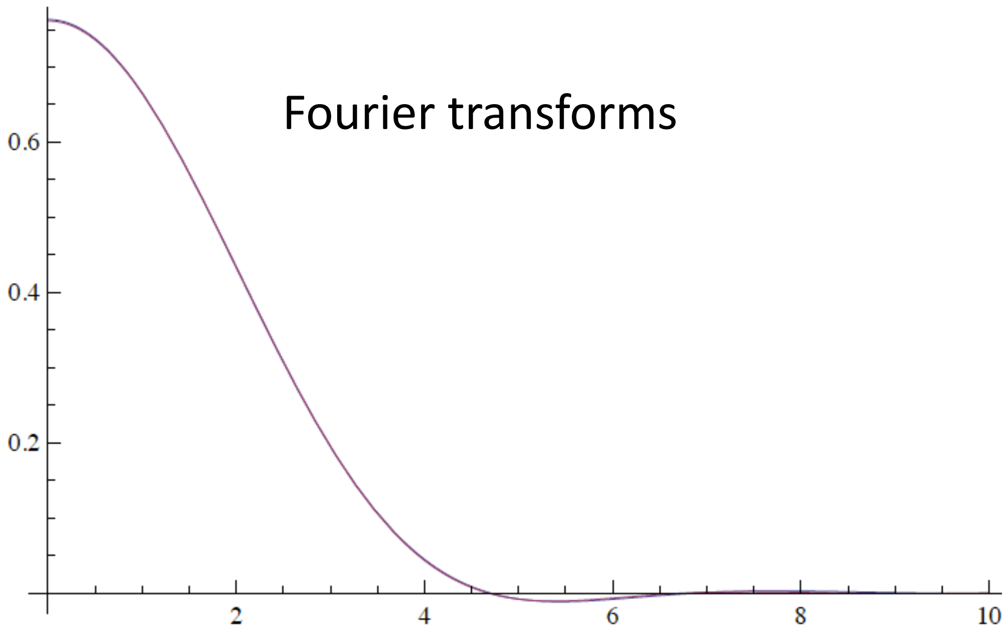


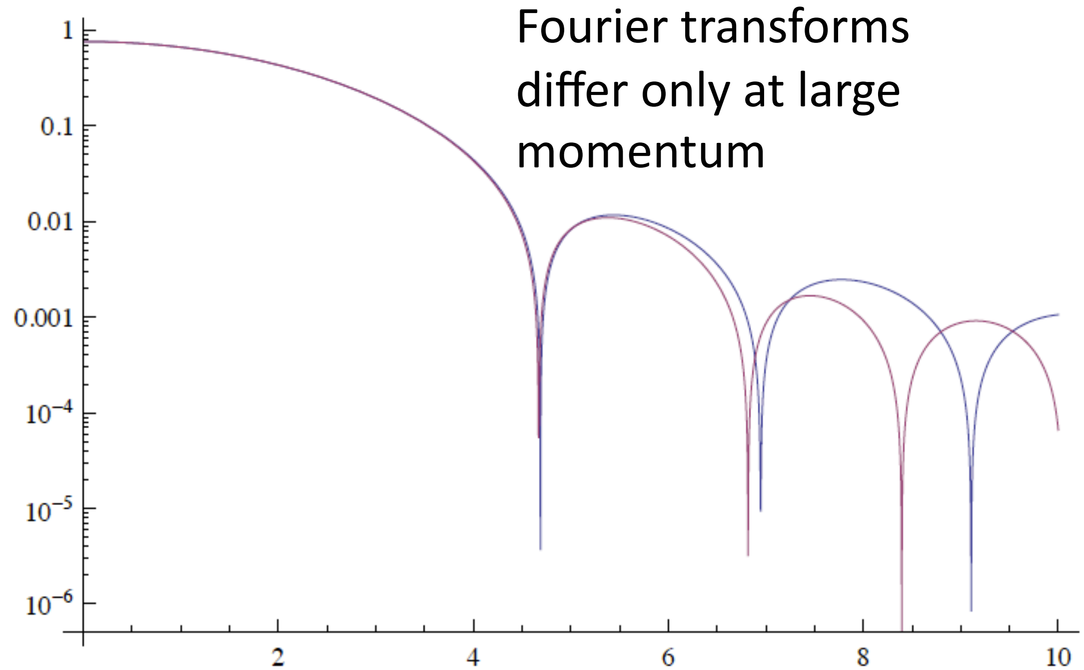
Eigenfunctions look like those from a box of size L_2.

Asymptotic (near the wall) difference is high-momentum effect.

Model of a hard wall at L_2 is in the spirit of EFT







The difference between the HO basis and a box of size L_2 can not be resolved at low momentum

Model-independent approach via *S*-matrix

The wall at *L* is in the asymptotic region → *S*-matrix $u_L(r) \stackrel{r\gg R}{\longrightarrow} (e^{-k_L r} - e^{-2k_L L} e^{k_L r})$

$$e^{-2k_L L} = [s_0(ik_L)]^{-1}$$

Parameterize *S*-matrix

$$s_0(k) = \frac{k\ \cot\delta_0(k) + ik}{k\ \cot\delta_0(k) - ik}$$

in terms of expansion around bound-state pole

$$k\ \cot\delta_0(k) = -k_\infty + \frac{1}{2}\rho_d(k^2 + k_\infty^2) + w_2(k^2 + k_\infty^2)^2 + \cdots$$

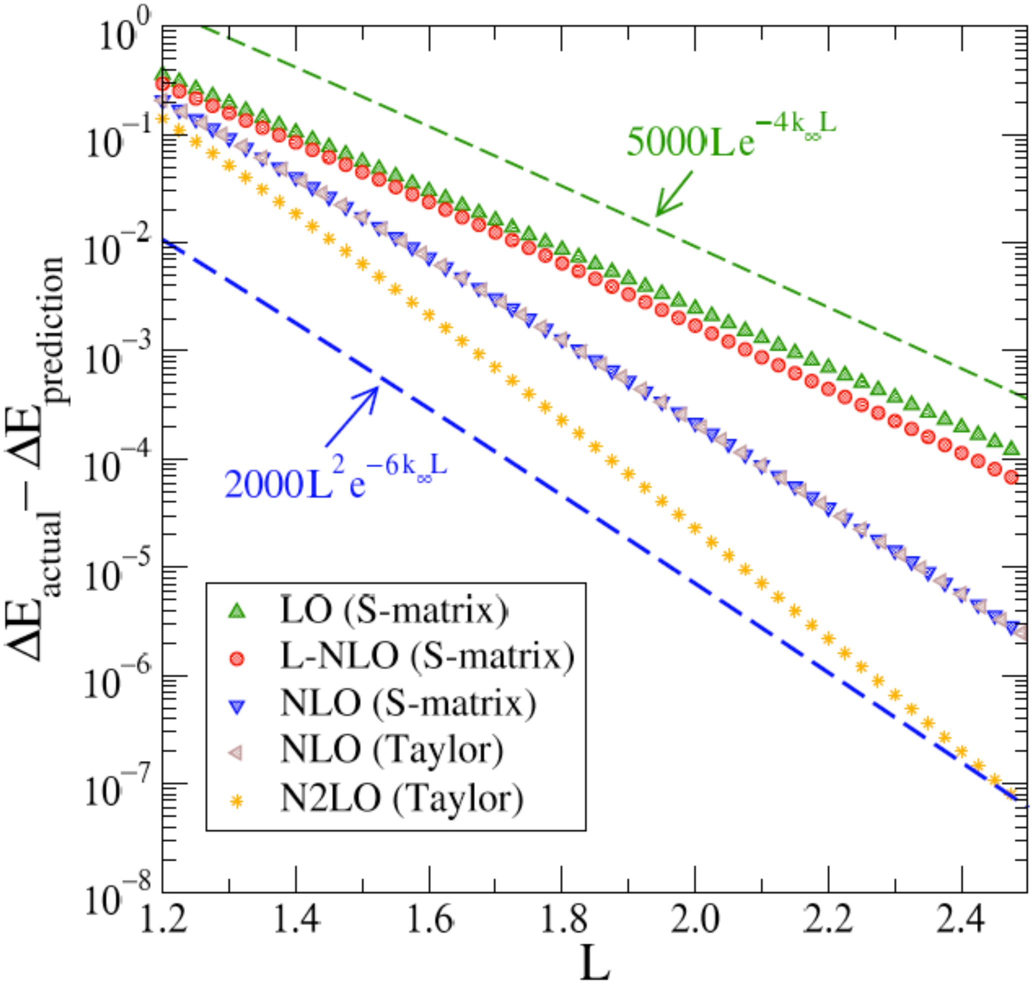
$$\rho_d = \frac{1}{k_\infty} - \frac{2}{\gamma_\infty^2}$$

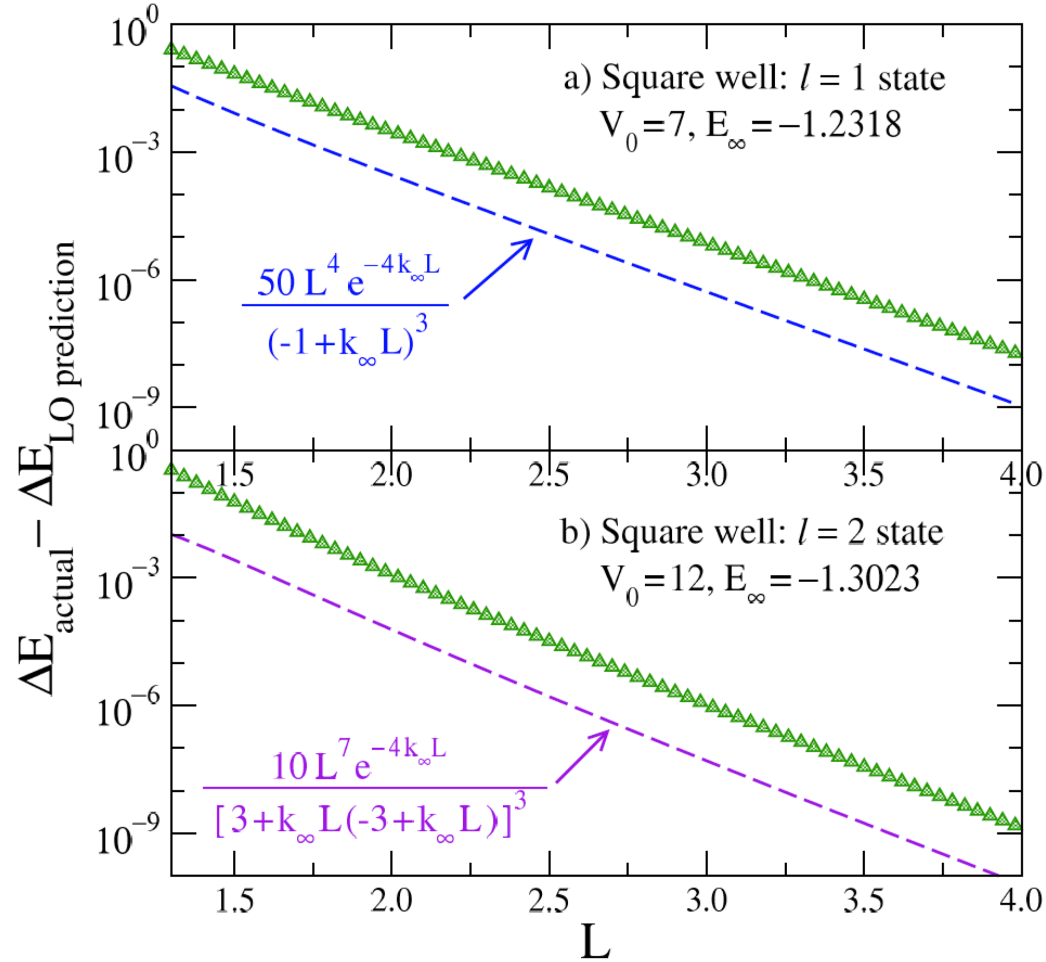
Only observables enter the *S*-matrix (and the IR correction)

$$[\Delta E_L]_{\rm NLO} = k_\infty \gamma_\infty^2 e^{-2k_\infty L} + 2k_\infty L \gamma_\infty^4 e^{-4k_\infty L} + k_\infty \gamma_\infty^2 \left(1 - \frac{\gamma_\infty^2}{k_\infty} - \frac{\gamma_\infty^4}{4k_\infty^2} + 2k_\infty w_2 \gamma_\infty^4\right) e^{-4k_\infty L}$$

Furnstahl, More, TP, arXiv:1312.6876

Error plots for square well (angular momenta l=0,1,2)



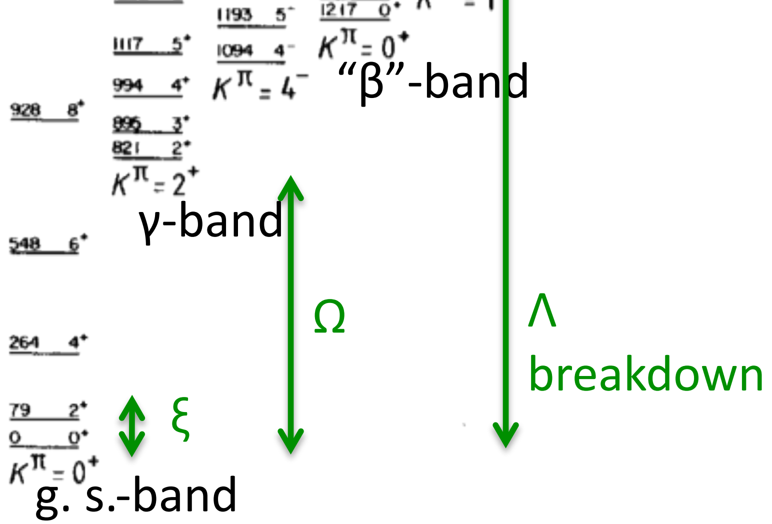


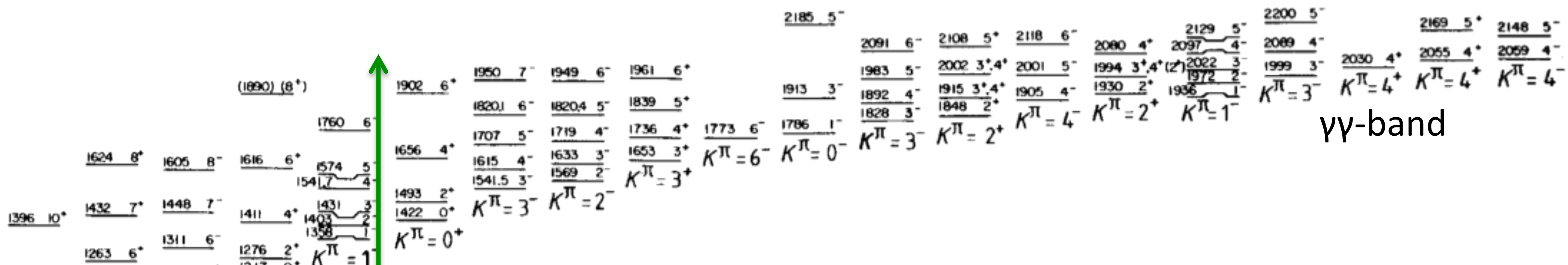
$$[\Delta E]_{\mathrm{LO}} = -k_\infty \left(\gamma_\infty^{(l)}\right)^2 \frac{h_l^{(1)}(ik_L L)}{h_l^{(1)}(-ik_L L)} \quad \text{with} \quad \frac{h_l^{(1)}(ix)}{h_l^{(1)}(-ix)} \approx -e^{-2x} \quad \text{for } x \gg 1$$

Furnstahl, More, TP, arXiv:1312.6876

Electromagnetic transitions in deformed nuclei

"Complete" spectrum of ^{168}Er [Davidson *et al.*, J. Phys. G 7, 455 (1981)]





Key features of geometric collective model

- ✓ Rotational bands on top of vibrational band heads
- ✓ Strong in-band E2 transitions
- ✓ Weaker inter-band E2 transitions
- ✓ Spectra are reproduced rather well

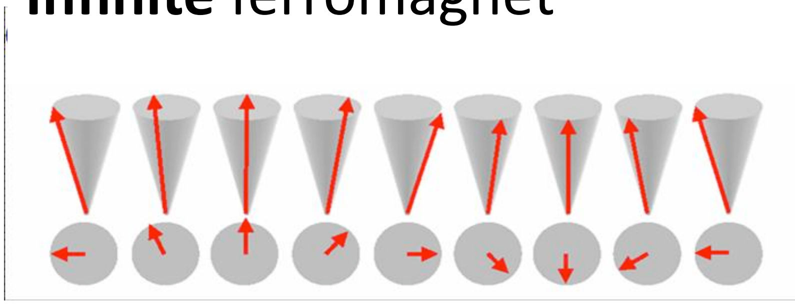
✗Inter-band transitions are factors 2-10 too strong [Garrett, J. Phys. G 27 (2001) R1; Rowe & Wood "Fundamentals of Nuclear Models" (2010)]

Separation of scale: ξ << Ω<<Λ

Consistent coupling of EM fields addresses this problem

Effective theory: SSB from SO(3)→SO(2)

Infinite ferromagnet



Rotation of individual (local) spins

- Operator

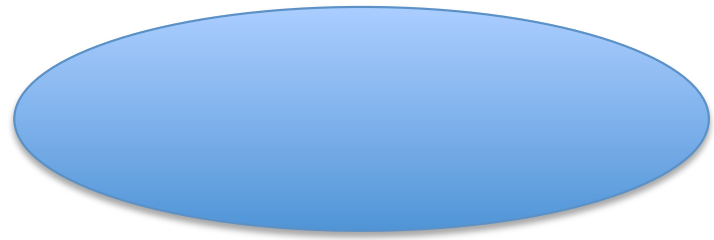
  $\exp(-i\ \psi_x(x,y,z,t)\ J_x - i\ \psi_y(x,y,z,t)\ J_y)$

- nonlinear realization of SO(3)
- Nambu-Goldstone modes: spin waves

- A total rotation (purely time-dependent mode with no spatial dependence) is excluded because it connects inequivalent Hilbert spaces

[Leutwyler 1994; Román & Soto 1999; Hofmann 1999; Bär, Imboden, Wiese 2004; Kämpfer, Moser, Wiese 2005]

Finite deformed nucleus



Rotation of individual nucleons

- Operator **g u** with

  $g = \exp(-i\ \alpha_x(t)\ J_x - i\ \alpha_y(t)\ J_y)$

  $u = \exp(-i\ \psi_x(x,y,z,t)\ J_x - i\ \psi_y(x,y,z,t)\ J_y)$

- nonlinear realization of SO(3)
- Nambu-Goldstone modes: quantized vibrations
- A total rotation (purely time-dependent mode with no spatial dependence) is allowed! It connects equivalent Hilbert spaces, and restores rotational symmetry
- In a large system with A nucleons

  $E_{rot} \sim A^{-5/3} << E_{vib} \sim A^{-2/3}$

[TP 2011; TP & Weidenmüller 2014]

Effective theory for deformed nuclei: rotations

Nambu-Goldstone modes parameterize coset SO(3)/SO(2) ~ S^2 (the 2-sphere)
Pure rotations equivalent to particle on a sphere

$$\hat{H}_{\rm LO} = -\frac{1}{2C_0}\Delta_\Omega = \frac{1}{2C_0}\hat{L}^2$$

$$-i\nabla_\Omega = -\vec{e}_r \times \hat{L}$$

Electro-magnetic coupling is standard (minimal coupling)

$$\hat{H}_{\rm EM} = \frac{1}{2C_0}\left(-i\nabla_\Omega - q\vec{A}\right)^2 = \frac{1}{2C_0}\left(\hat{L} - q\vec{e}_r \times \vec{A}\right)^2$$

$$\vec{A} = A_r\vec{e}_r + A_\theta\vec{e}_\theta + A_\phi\vec{e}_\phi$$

Formally identical to Haldane's 1983 treatment of quantum Hall effect in spherical geometry

Effective theory for deformed nuclei: vibrations

Vibrations are quadrupole modes (3 additional degrees of freedom because 2 are realized as rotations)

$$\tilde{\psi}_0 = v + \psi_0 \qquad \tilde{\psi}_{\pm 2} = \psi_2 e^{\pm i 2\gamma}$$

$$H_{NLO} = \frac{p_0^2}{2} + \frac{\omega_0^2}{2}\psi_0^2 + \frac{p_2^2}{4} + \frac{1}{4\psi_2^2}\left(\frac{p_\gamma}{2}\right)^2 + \frac{\omega_2^2}{4}\psi_2^2$$

vibrations

$$+\frac{1}{2C_0}\left(J^2 - p_\gamma^2\right)$$

coupled to rotations

$$J_{+1} = L_{+1} - \sqrt{\frac{1}{2}} e^{i\phi} \frac{p_\gamma}{\sin\theta}$$

$$J_0 = L_0$$

$$J_{-1} = L_{-1} + \sqrt{\frac{1}{2}} e^{-i\phi} \frac{p_\gamma}{\sin\theta}$$

total angular momentum

Electro-magnetic coupling is again minimal coupling (seems not to be used before)

$$\hat{J} \to \hat{J} - q\vec{e}_r \times \vec{A}$$

Results

1. At leading order in EM couplings: (strong) in-band transitions
2. At next-to-leading order in EM couplings: weaker inter-band transitions

$$\Delta H_{N^2LO} = -\frac{1}{2C_0}\frac{C_1}{C_0}\psi_0 \mathbf{p}^2_{\Omega\gamma} - \frac{1}{2C_0}\frac{C_2}{C_0}\psi_2 \mathbf{p}^T_{\Omega\gamma}\begin{pmatrix}\cos 2\gamma & \sin 2\gamma \\ \sin 2\gamma & -\cos 2\gamma\end{pmatrix}\mathbf{p}_{\Omega\gamma}$$

Difference to phenomenological models

• Consistent description of EM currents and Hamiltonian

• Higher-order kinetic terms (models focus on variation of the potential)

• Power counting in ξ/Ω and Ω/Λ and k_{EM}/Mass

• The deviations between the Bohr model and data for inter-band transitions (β→ g.s.) is interpreted as a need to include pairing. EFT suggests that this is not necessarily so.

• EFT view: Geometric models exclude physics (because they model surface oscillations)

Results for ^{168}Er

Effective theory:

- Gauging of Lagrangian yields EM currents consistent with Hamiltonian
- Power counting also for EM couplings
- Richer structure than geometric model; more parameters in a systematic expansion

a Baglin, Nucl. Data Sheets 111, 1807 (2010)
b Lehmann et al., Phys. Rev. C 57, 569 (1998)
c Value employed to adjust low-energy constant

[Toño Coello and TP, in preparation]

B(E2, i→f) in e^2b^2

$i \to f$	$B(E2)^{a}_{\rm exp}$	$B(E2)_{\rm ET}$	Adiabatic Bohr model
$0^+_{g.s.} \to 2^+_{g.s.}$	5.72 (20)	5.92[c]	5.92
$2^+_{g.s.} \to 4^+_{g.s.}$	3.07 (19)	3.04 (30)	
$4^+_{g.s.} \to 6^+_{g.s.}$	3.30 (20)	2.69 (26)	
$6^+_{g.s.} \to 8^+_{g.s.}$	2.57 (15)	2.55 (25)	
$3^+_{\gamma} \to 2^+_{\gamma}$	1.65 (37)	2.11 (21)	
$2^+_{\gamma} \to 4^+_{\gamma}$	1.66 (10)	1.26 (12)	
$4^+_{\gamma} \to 3^+_{\gamma}$	2.78 (71)	1.57 (15)	
$0^+_{g.s.} \to 2^+_{\gamma}$	0.129 (4)	0.143 (14)	0.568
$2^+_{\gamma} \to 2^+_{g.s.}$	0.0442 (38)	0.0410[c]	
$2^+_{\gamma} \to 4^+_{g.s.}$	0.00242 (22)	0.00200 (20)	
$3^+_{\gamma} \to 2^+_{g.s.}$	0.042 (11)	0.051 (5)	
$3^+_{\gamma} \to 4^+_{g.s.}$	0.028 (12)	0.020 (2)	
$4^+_{\gamma} \to 2^+_{g.s.}$	0.0114 (7)	0.0171 (17)	
$4^+_{\gamma} \to 4^+_{g.s.}$	0.0576 (32)	0.0503 (50)	
$4^+_{\gamma} \to 6^+_{g.s.}$	0.0049 (26)	0.0043 (4)	
$6^+_{\gamma} \to 4^+_{g.s.}$	0.00481 (27)	0.0140 (14)	
$6^+_{\gamma} \to 6^+_{g.s.}$	0.0421 (45)	0.0521 (52)	
$6^+_{\gamma} \to 8^+_{g.s.}$	0.0110 (80)	0.0055 (5)	
$2^+_{\beta} \to 0^+_{g.s.}$	0.0022[b]	0.0022[c]	0.0391
$2^+_{\beta} \to 2^+_{g.s.}$	0.0027[b]	0.0031 (3)	
$2^+_{\beta} \to 4^+_{g.s.}$	0.0121[b]	0.0056 (5)	

Rotational constants of two-phonon γγ bands

Effective theory predicts rotational constants depend linearly on number of phonons

$$A_{\text{theo}}[I(I+1)-K^2]$$

$$A_{\text{theo}} = A_{\text{g.s.}} - a_\beta n_0 - a_\gamma(2n_2 + |K|/2).$$

	^{168}Er			^{166}Er			^{232}Th		
E	0	821	2056	0	786	2028	0	785	1414
K	0	2	4	0	2	4	0	2	4
A	13.17	12.33	11.37	13.43	12.25	10.56	8.23	7.38	7.27
A_{theo}	13.17	12.33	11.49	13.43	12.25	11.07	8.23	7.38	6.53

prediction

Zhang, Papenbrock / Phys. Rev. C 87, 034323 (2013)

Summary

- Extrapolations to infinite model spaces
  - Finite oscillator space practically induces hard wall at $r=L$
  - Identify lowest momentum as π/L
  - Model-independent IR extrapolation formulae for energies and radii

- Effective theory for deformed nuclei
  - consistent description of EM currents and Hamiltonian
  - correctly describes strength of inter-band E2 transitions
  - small decrease of rotational constants for $\gamma\gamma$ bands predicted

Eigenvalues κ² of p² in oscillator basis for angular momenta l=0,1,2

x_l= 1st zero of j_l

Furnstahl, More, TP, arXiv:1312.6876

l	n	κ	x_l/L_2	l	n	κ	x_l/L_2	l	n	κ	x_l/L_2
0	0	1.2247	1.1874	1	0	1.5811	1.4978	2	0	1.8708	1.7378
0	1	0.9586	0.9472	1	1	1.2764	1.2463	2	1	1.5423	1.4881
0	2	0.8163	0.8112	1	2	1.1047	1.0898	2	2	1.3509	1.3222
0	3	0.7236	0.7207	1	3	0.9892	0.9805	2	3	1.2191	1.2018
0	4	0.6568	0.6551	1	4	0.9042	0.8987	2	4	1.1207	1.1092
0	5	0.6058	0.6046	1	5	0.8382	0.8344	2	5	1.0432	1.0352
0	6	0.5651	0.5642	1	6	0.7850	0.7822	2	6	0.9801	0.9742
0	7	0.5316	0.5310	1	7	0.7408	0.7387	2	7	0.9274	0.9229
0	8	0.5035	0.5031	1	8	0.7033	0.7018	2	8	0.8824	0.8789
0	9	0.4795	0.4791	1	9	0.6711	0.6698	2	9	0.8435	0.8407
0	10	0.4585	0.4582	1	10	0.6429	0.6419	2	10	0.8093	0.8070

Results for ^{166}Er

B(E2, i→f) in e^2b^2

$i \rightarrow f$	$B(E2)^{a}_{exp}$	$B(E2)_{ET}$
$2^+_{g.s.} \rightarrow 0^+_{g.s.}$	1.04 (10)	1.14[b]
$4^+_{g.s.} \rightarrow 2^+_{g.s.}$	1.65 (10)	1.63 (16)
$6^+_{g.s.} \rightarrow 4^+_{g.s.}$	1.70 (14)	1.79 (17)
$8^+_{g.s.} \rightarrow 6^+_{g.s.}$	1.98 (14)	1.88 (19)
$4^+_{\gamma} \rightarrow 2^+_{\gamma}$	0.75 (7)	0.68 (7)
$5^+_{\gamma} \rightarrow 3^+_{\gamma}$	1.59 (92)	1.09 (11)
$6^+_{\gamma} \rightarrow 4^+_{\gamma}$	1.51 (15)	1.34 (13)
$2^+_{\gamma} \rightarrow 0^+_{g.s.}$	0.028 (3)	0.034 (3)
$2^+_{\gamma} \rightarrow 2^+_{g.s.}$	0.054 (5)	0.049[b]
$2^+_{\gamma} \rightarrow 4^+_{g.s.}$	0.0052 (17)	0.0024 (2)
$3^+_{\gamma} \rightarrow 2^+_{g.s.}$	0.017 (5)	0.061 (6)
$3^+_{\gamma} \rightarrow 4^+_{g.s.}$	0.026 (5)	0.024 (2)
$4^+_{\gamma} \rightarrow 2^+_{g.s.}$	0.011 (1)	0.020 (2)
$4^+_{\gamma} \rightarrow 4^+_{g.s.}$	0.059 (6)	0.060 (6)
$4^+_{\gamma} \rightarrow 6^+_{g.s.}$	0.0120 (30)	0.0052 (5)
$6^+_{\gamma} \rightarrow 4^+_{g.s.}$	0.0045 (4)	0.0168 (17)
$6^+_{\gamma} \rightarrow 6^+_{g.s.}$	0.053 (5)	0.062 (6)
$6^+_{\gamma} \rightarrow 8^+_{g.s.}$	0.0084 (157)	0.0066 (7)

a Fahlander et al, Nucl. Phys. A 537, 183 (1992).
b Value employed to adjust low-energy constant

[Toño Coello and TP, in preparation]