The Fate of UV Physics with Renormalization Group Evolution

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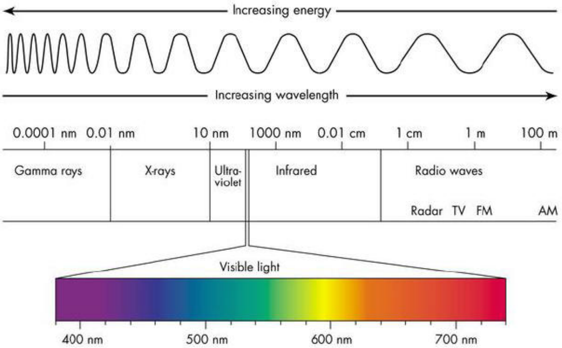


Where do we draw the line? How can we take advantage of moving the line?

Thanks: colleagues at low resolution

- ANL: L. Platter
- Darmstadt: H.-W. Hammer, K. Hebeler, R. Roth et al., A. Schwenk
- IIT (Madras): S. Ramanan
- Iowa State: P. Maris, J. Vary
- Jülich: A. Nogga
- Michigan State: S. Bogner, A. Ekstrom
- LLNL: E. Jurgenson, N. Schunck
- OSU: B. Dainton, A. Dyhdalo, H. Hergert, S. Koenig, S. More, R. Perry, S. Wesolowski
- ORNL / UofT: G. Hagen, W. Nazarewicz, T. Papenbrock, K. Wendt
- TRIUMF: S. Bacca, P. Navratil
- UNC: E. Anderson, J. Drut
- many others in NUCLEI, LENPIC, . . .

U.S. DEPARTMENT OF ENERGY

Why should we care what happens to UV physics?

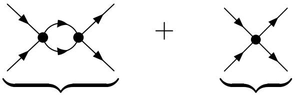
- Evolution of Hamiltonians and other operators
  - Where does UV physics go as we lower a cutoff?
  - When do many-body terms become important?
  - Flow to universal Hamiltonians: can we exploit it?
- Using the EFT cutoff (Λ) scale: Naturalness?
  - Bayesian priors for fitting LECs?
  - What is learned from regulator cutoff variation?
- Which is better: EFT at lower cutoff or SRG?
  - Is SRG decoupling the same as cutting off?
  - Does it matter how we cut off UV physics?
- UV basis extrapolation; e.g., for SRG-evolved potentials
  - Universal/dual aspects of UV vs. IR? What's different?
- Knock-out experiments: short-range correlations and all that
  - What role do the UV parts of wave functions play?
  - What factorization (separation) scale should we use?

Plan: random walk through these topics (mostly questions!)

What does changing a cutoff do in an EFT?

- (Local) field theory version in perturbation theory (diagrams)
  - Loops (sums over intermediate states) $\overset{\Delta\Lambda_c}{\Longleftrightarrow}$ LECs

$$\frac{d}{d\Lambda_c}\Big[$$

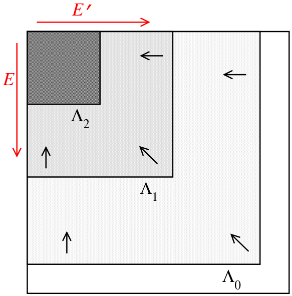


$$\Big] = 0$$

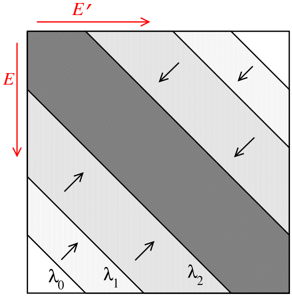
$$\int^{\Lambda_c} \frac{d^3q}{(2\pi)^3}\frac{C_0 M C_0}{k^2 - q^2 + i\epsilon} \qquad C_0(\Lambda_c) \propto \frac{\Lambda_c}{2\pi^2} + \cdots$$

  - Momentum-dependent vertices $\Longrightarrow$ Taylor expansion in k^2
- Claim: $V_{\mathrm{low}\,k}$ RG and SRG decoupling work analogously

"$V_{\mathrm{low}\,k}$"



SRG ("T" generator)



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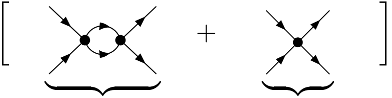

$$\frac{d}{d\Lambda_c}\left[\ \int^{\Lambda_c}\frac{d^3q}{(2\pi)^3}\frac{C_0 M C_0}{k^2-q^2+i\epsilon} + C_0(\Lambda_c)\propto\frac{\Lambda_c}{2\pi^2}+\cdots\ \right] = 0$$

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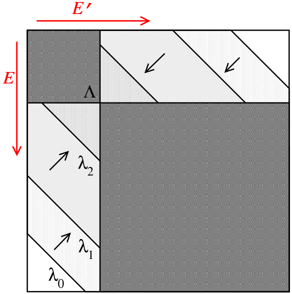

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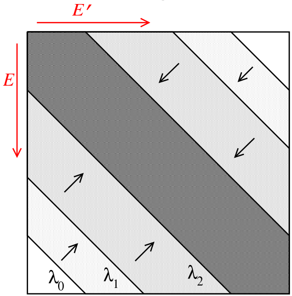
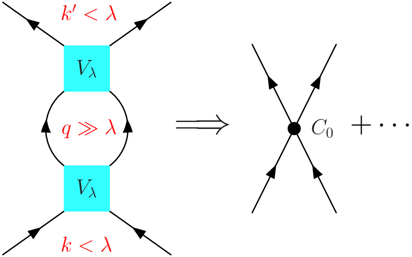
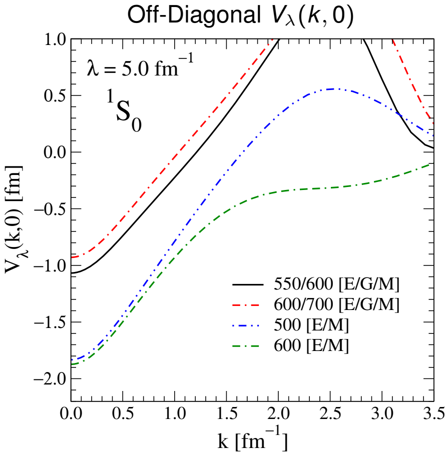


Approach to universality (fate of high-q physics!)

Run NN to lower λ via SRG $\Longrightarrow$ $\approx$Universal low-k V_{NN}





$q \gg \lambda$ (or Λ) intermediate states

$\Longrightarrow$ replace with contact terms:

$$C_0 \delta^3(\mathbf{x} - \mathbf{x}') + \cdots$$

[cf. $\mathcal{L}_{\rm eft} = \cdots + \frac{1}{2} C_0 (\psi^\dagger \psi)^2 + \cdots$]

- Similar pattern with phenomenological potentials (e.g., AV18)

Factorization: $\Delta V_\lambda(k,k') = \int U_\lambda(k,q) V_\lambda(q,q') U^\dagger_\lambda(q',k')$ for $k, k' < \lambda$, $q, q' \gg \lambda$

$$\xrightarrow{U_\lambda \to K \cdot Q} K(k)[\int Q(q) V_\lambda(q,q') Q(q')] K(k')$$ with $K(k) \approx 1$!

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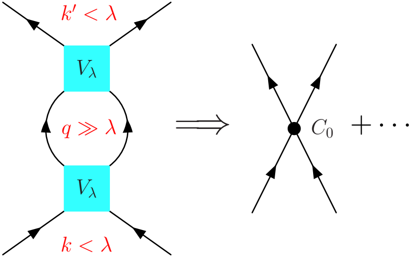

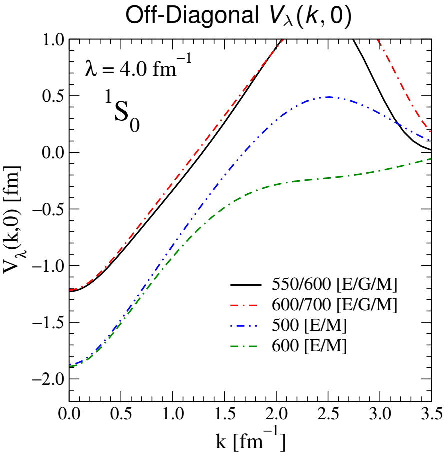

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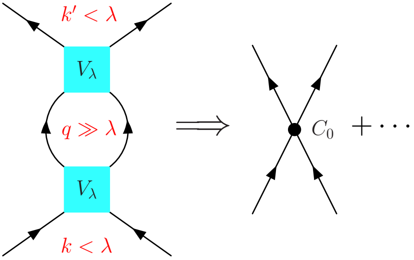

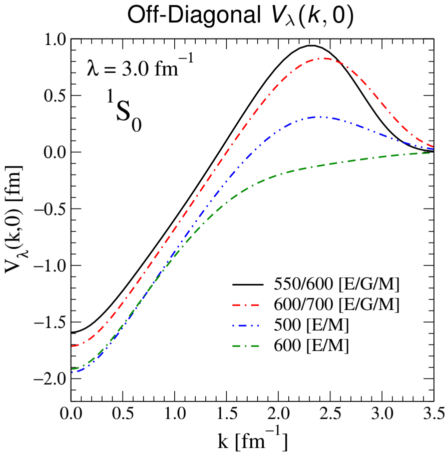

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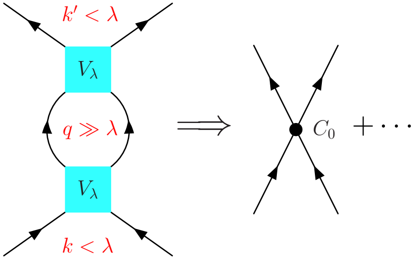

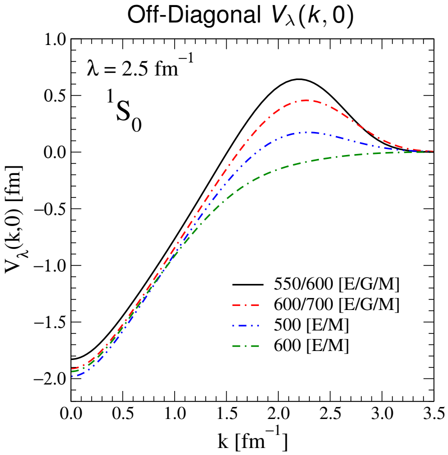

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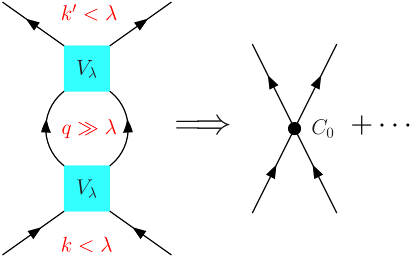

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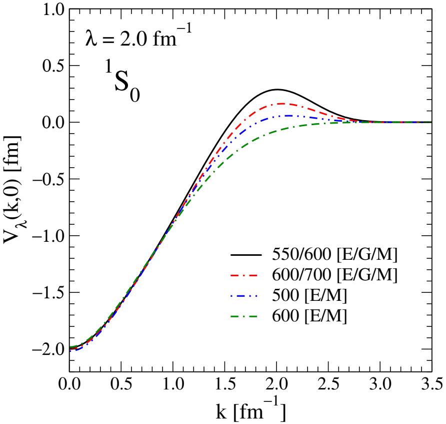
Off-Diagonal $V_\lambda(k, 0)$



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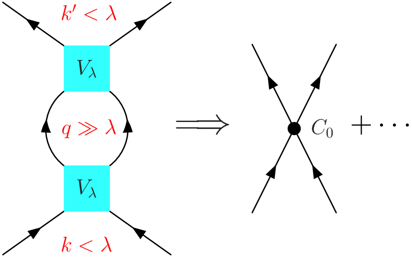

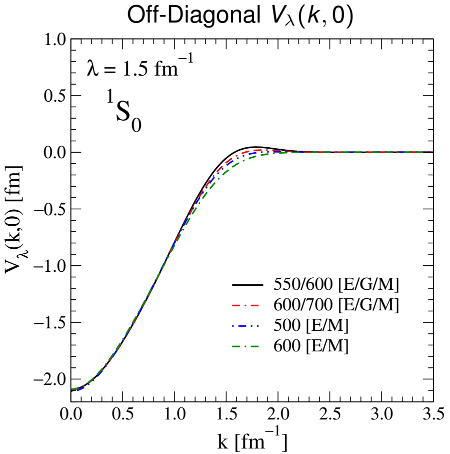

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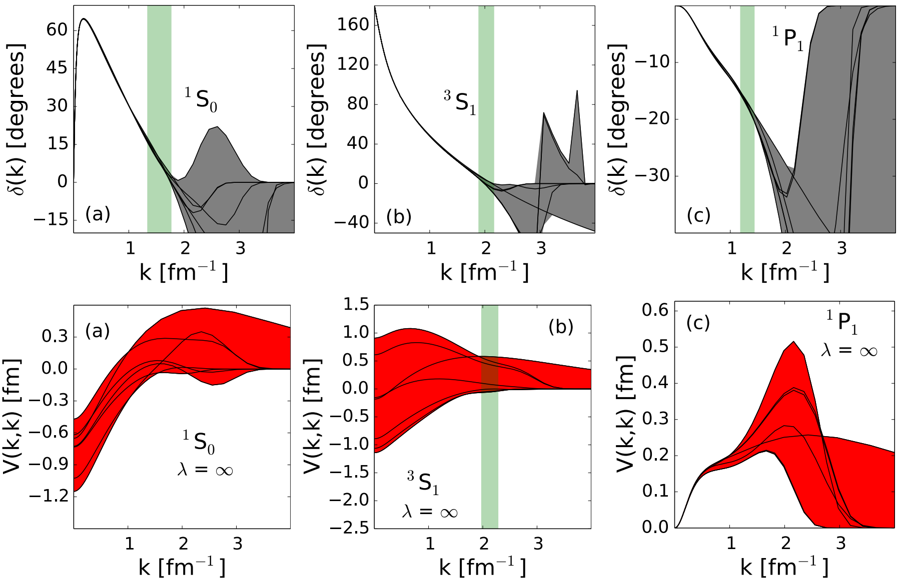
NN V_{SRG} universality from phase equivalent potentials

Diagonal elements collapse where phase equivalent [Dainton et al, 2014]



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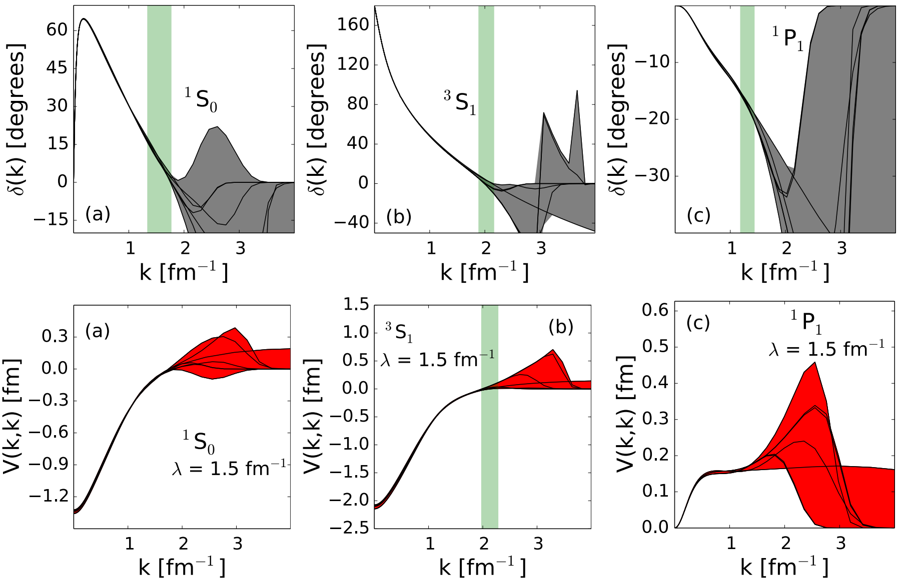
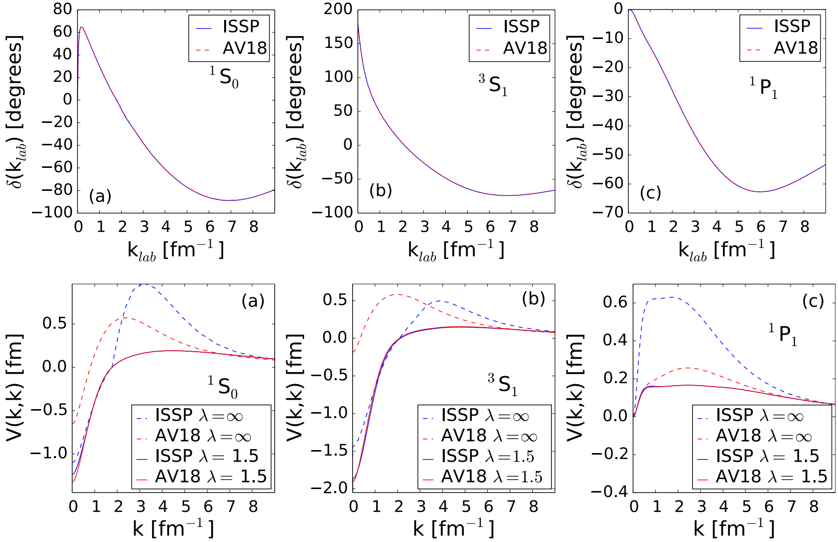


Are inverse scattering potentials sufficient? [Dainton et al]

Create a separable potential that is phase equivalent to AV18:



For the diagonal elements, yes, this is sufficient!

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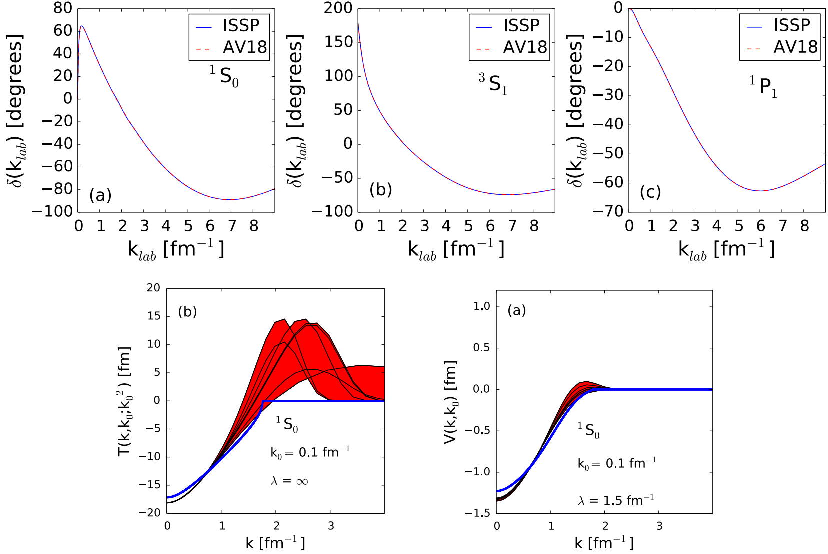

But for off-diagonal, need half-on-shell T-matrix (HOST) equivalence

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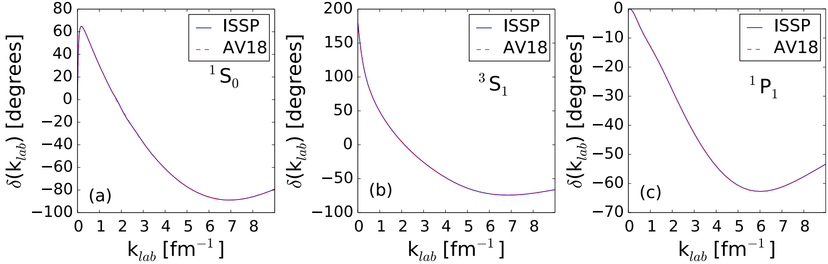
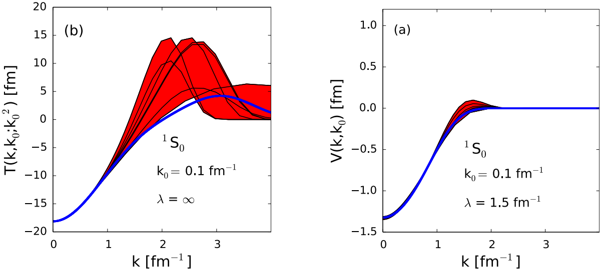
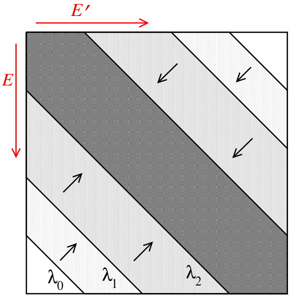


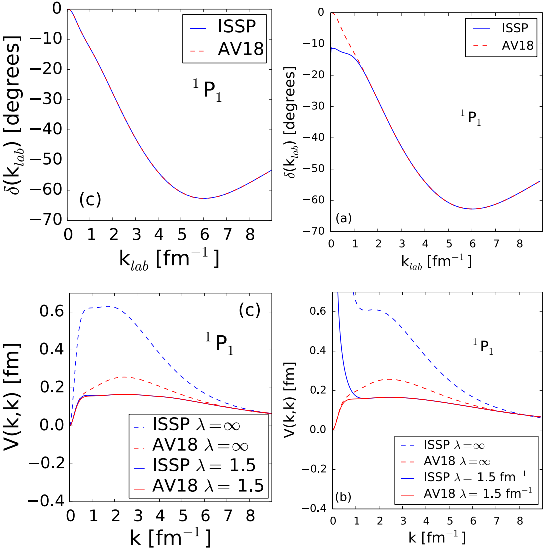


With HOST equivalence, even delta shell potential plus OPE is sufficient!

Use universality to probe decoupling

- What if not phase equivalent *everywhere*?
- Use $^1P_1$ as example (for a change :)
- Result: local decoupling!





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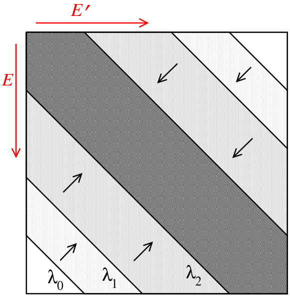
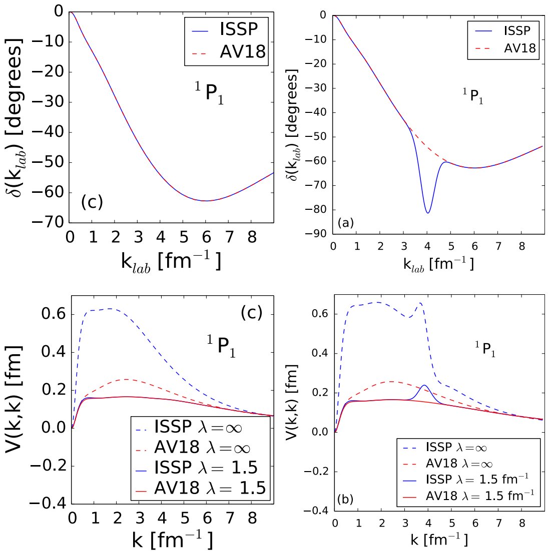
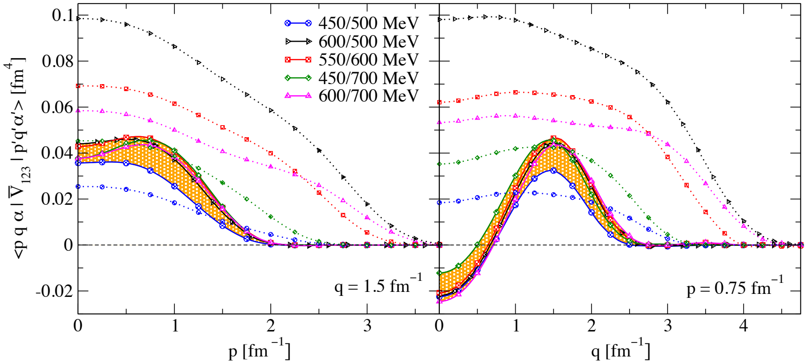




Is there 3NF universality?

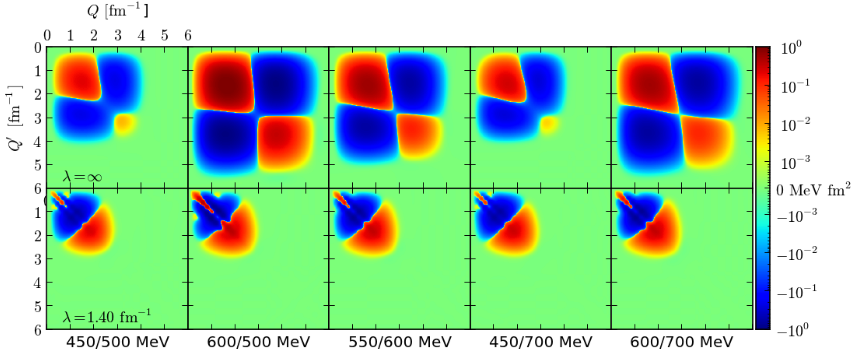
- Evolve chiral NNLO EFT potentials in momentum plane wave basis to $\lambda = 1.5\,\mathrm{fm}^{-1}$ [K. Hebeler, Phys. Rev. C85 (2012) 021002]
- In one 3-body partial wave, fix one Jacobi momentum (p, q) and plot vs. the other one:



- Collapse of curves includes non-trivial structure

Is there 3NF universality?

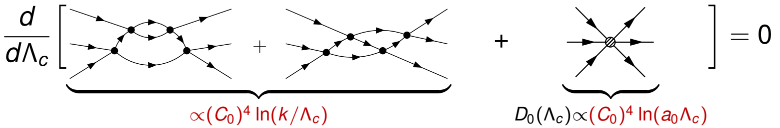
- Evolve in discretized momentum-space hyperspherical harmonics basis to $\lambda = 1.4\,\mathrm{fm}^{-1}$ [K. Wendt, Phys. Rev. C87 (2013) 061001]
- Contour plot of *integrand* for 3NF expectation value in triton



- Local projections of 3NF also show flow toward universal form
- Can we exploit universality à la Wilson? Stay tuned!

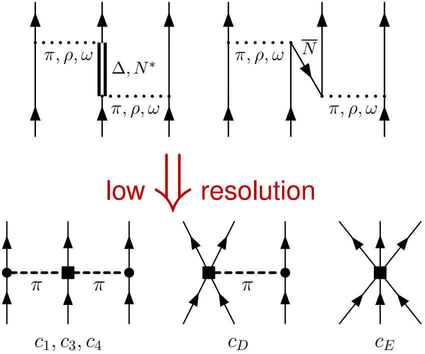
What else can we say about the flow of NN$\cdots$N potentials?

- Can arise from counterterm for new UV cutoff dependence, e.g., changes in Λ_c must be absorbed by 3-body coupling $D_0(\Lambda_c)$



RG invariance dictates 3-body coupling flow [Braaten & Nieto]

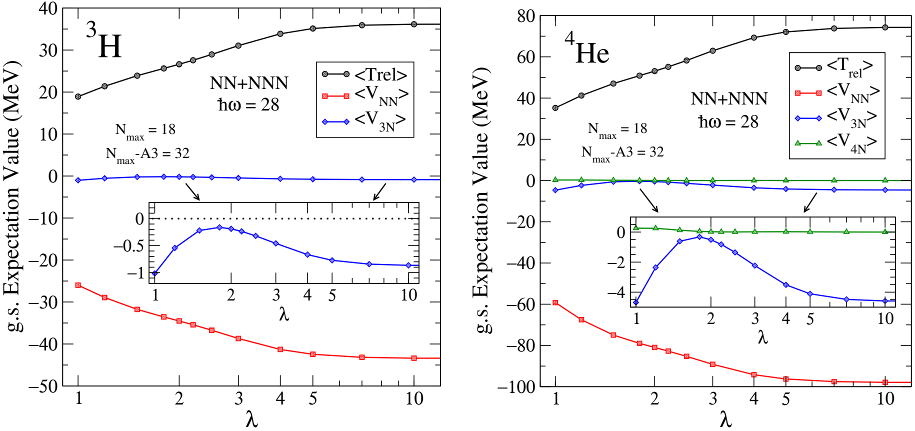
- General RG: 3NF from integrating out *or* decoupling high-*k* states



What do we know about the *growth* of NN···N potentials?

- Many interesting results have appeared, prompting questions . . .

Early results in lightest systems [Jurgenson et al. (2009)]:



How does this hierarchy evolve with *A*?

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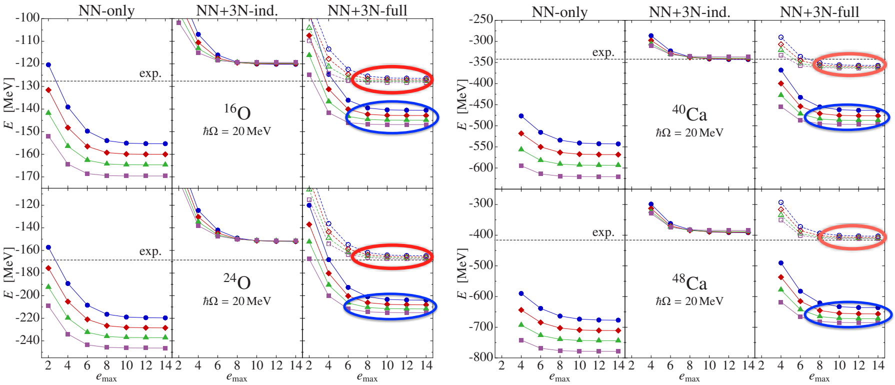
Team Roth: 4-body depends on cutoff on c_3 term.



How do we determine consistent regulators in this case?

Does local versus non-local cutoff function matter?

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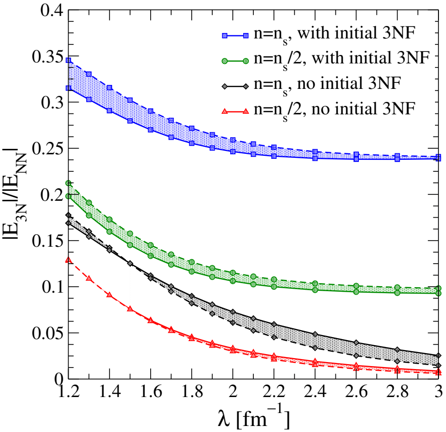
Ratio of 3NF to NN in neutron matter [Hebeler, rjf (2013)]



Density scales as you would expect (at least here :), but λ scaling?

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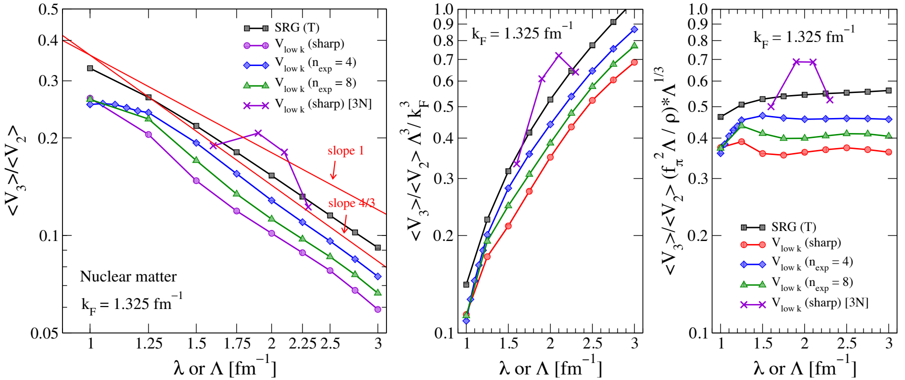
Nuclear matter scaling: use NN results at saturation $\Longrightarrow \langle V_3\rangle/\langle V_2\rangle$



Simple dimensional scaling (e.g., $(k_F/\Lambda)^3$ or $(k_F/\lambda)^3$) doesn't work

but a different scaling ... $\dfrac{\langle V_3\rangle}{\langle V_2\rangle}\dfrac{f_\pi^2\Lambda}{\rho}\Lambda^{1/3} \sim \mathcal{O}(1)$ [where did $\rho/f_\pi^2\Lambda$ come from?]

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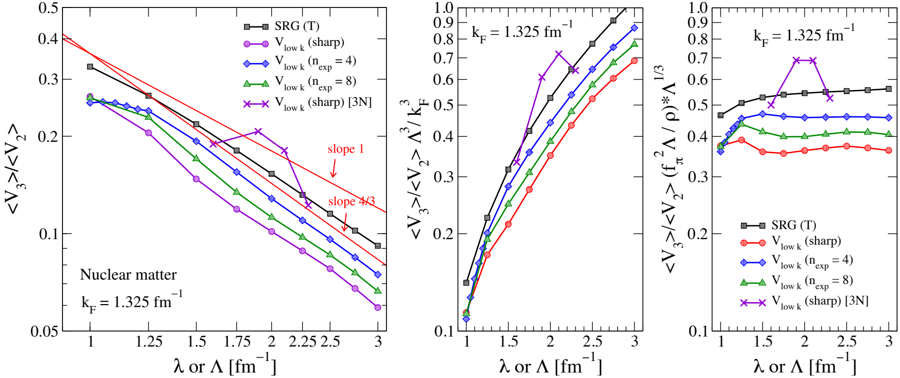

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Current answer: not enough yet! But tools in place to make progress!

How big *should* different contributions be?

- Enable chiral EFT power counting $\Longrightarrow$ NDA and naturalness

$$\mathcal{L}_{\chi\,\mathrm{eft}} = c_{lmn} \left(\frac{N^\dagger(\cdots)N}{f_\pi^2 \Lambda_\chi} \right)^l \left(\frac{\pi}{f_\pi} \right)^m \left(\frac{\partial^\mu, m_\pi}{\Lambda_\chi} \right)^n f_\pi^2 \Lambda_\chi^2 \qquad f_\pi \sim 100\,\mathrm{MeV}$$

- Georgi (1993): f_π for strongly interacting fields; rest is Λ_χ
- Cohen et al. (1997). Uncanonical scaled EFT action at Λ:

$$S_\Lambda = \frac{1}{g^2} \int d^4x\, \widehat{\mathcal{L}}_\Lambda \left(\frac{\pi'}{\Lambda}, \frac{N'}{\Lambda^{3/2}}, \frac{\partial}{\Lambda} \right) \qquad \text{"natural" if loops} \le \text{trees}$$

  - NDA: that bound is saturated: $g \sim 4\pi$ with $\Lambda \sim \Lambda_\chi$
  - Rescale to canonical kinetic normalization $\Longrightarrow$ NDA
- Claim: should *match* choosing $\Lambda \sim \Lambda_\chi$ scale $\Longrightarrow$ NDA estimates
  - Λ_χ is not itself an adjustable cutoff but a physics scale
  - e.g., from non-Goldstone-boson exchange such m_ρ
  - Need calculations for quantitative Λ_χ
- Other refs: Dugan and Golden (1993), Friar (1997)

How big *should* different contributions be?

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$$\mathcal{L}_{\chi\,\mathrm{eft}} = c_{lmn} \left(\frac{N^\dagger(\cdots)N}{f_\pi^2\Lambda_\chi}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{\partial^\mu, m_\pi}{\Lambda_\chi}\right)^n f_\pi^2\Lambda_\chi^2 \qquad f_\pi \sim 100\,\text{MeV}$$

- E.g., check NLO, NNLO constants from $\mathcal{L}_{NN}$ [Epelbaum et al.]
  Take $\Lambda_\chi \Longrightarrow$ cutoff Λ: 500...600 MeV):

$f_\pi^2\, C_S$	$-1.079\ldots-0.953$	$f_\pi^2\, C_T$	$0.002\ldots0.040$
$f_\pi^2\Lambda_\chi^2\, C_1$	$3.143\ldots2.665$	$4\, f_\pi^2\Lambda_\chi^2\, C_2$	$2.029\ldots2.251$
$f_\pi^2\Lambda_\chi^2\, C_3$	$0.403\ldots0.281$	$4\, f_\pi^2\Lambda_\chi^2\, C_4$	$-0.364\ldots-0.428$
$2\, f_\pi^2\Lambda_\chi^2\, C_5$	$2.846\ldots3.410$	$f_\pi^2\Lambda_\chi^2\, C_6$	$-0.728\ldots-0.668$
$4\, f_\pi^2\Lambda_\chi^2\, C_7$	$-1.929\ldots-1.681$		

- $1/3 \lesssim c_{lmn} \lesssim 3 \Longrightarrow$ natural! $\Longrightarrow$ truncation error estimates
- If unnaturally large, signal of missing long-distance physics (e.g., Δ in c_i's) or over-fitting
- $f_\pi^2 C_T$ *unnaturally small* $\Longrightarrow$ $SU(4)$ spin-isospin symmetry

How big *should* different contributions be?

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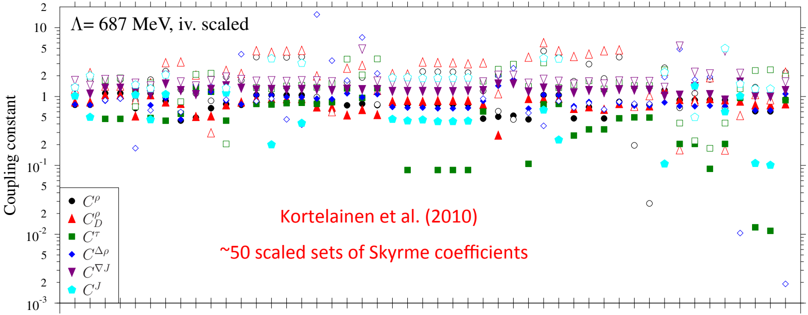
$$\mathcal{L}_{\chi\,\mathrm{eft}} = c_{lmn} \left(\frac{N^\dagger(\cdots)N}{f_\pi^2 \Lambda_\chi} \right)^l \left(\frac{\pi}{f_\pi} \right)^m \left(\frac{\partial^\mu, m_\pi}{\Lambda_\chi} \right)^n f_\pi^2 \Lambda_\chi^2 \qquad f_\pi \sim 100\,\mathrm{MeV}$$

Applications to coefficients in relativistic and Skyrme density functionals



- Identify unnaturally large *and* small Skyrme coefficients
- Guide fitting attempts with generalized EDF's?

How big *should* different contributions be?

- Enable chiral EFT power counting $\Longrightarrow$ NDA and naturalness

$$\mathcal{L}_{\chi\,\mathrm{eft}} = c_{lmn}\left(\frac{N^\dagger(\cdots)N}{f_\pi^2\Lambda_\chi}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{\partial^\mu, m_\pi}{\Lambda_\chi}\right)^n f_\pi^2\Lambda_\chi^2 \qquad f_\pi \sim 100\,\mathrm{MeV}$$

- Old chiral NDA analysis for EDFs: [Friar et al., rjf et al.]

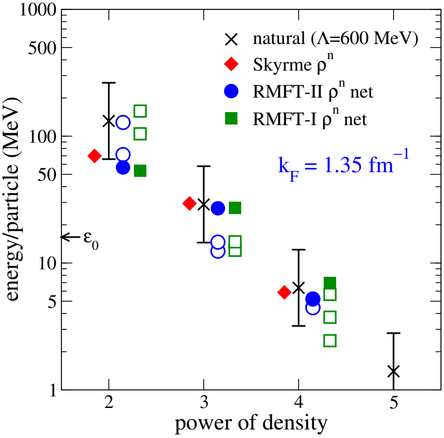
$$c\left[\frac{N^\dagger N}{f_\pi^2\Lambda}\right]^l \left[\frac{\nabla}{\Lambda}\right]^n f_\pi^2\Lambda^2$$

$$\Longrightarrow \quad \begin{array}{l} \rho \longleftrightarrow N^\dagger N \\ \tau \longleftrightarrow \nabla N^\dagger \cdot \nabla N \\ \mathbf{J} \longleftrightarrow N^\dagger \nabla N \end{array}$$

- Density expansion?

$$1000 \geq \Lambda \geq 500 \Longrightarrow \frac{1}{7} \leq \frac{\rho_0}{f_\pi^2\Lambda} \leq \frac{1}{4}$$

- Also gradient expansion
- Applied to RMF, Skyrme EDFs



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- Enable chiral EFT power counting $\Longrightarrow$ NDA and naturalness

$$\mathcal{L}_{\chi\,\rm eft} = c_{lmn} \left(\frac{N^\dagger(\cdots)N}{f_\pi^2\Lambda_\chi}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{\partial^\mu, m_\pi}{\Lambda_\chi}\right)^n f_\pi^2\Lambda_\chi^2 \qquad f_\pi \sim 100\,\text{MeV}$$

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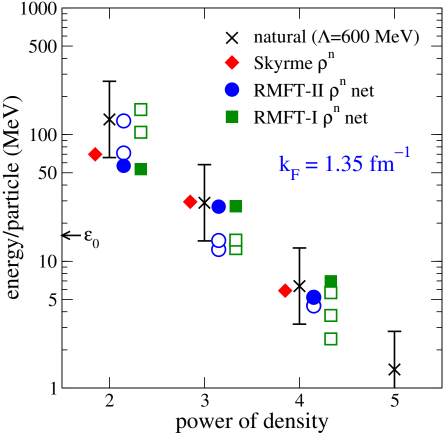
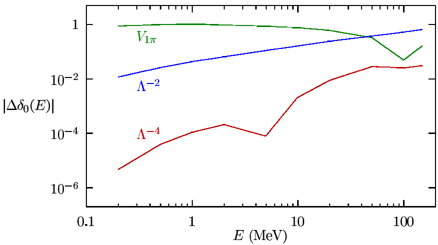
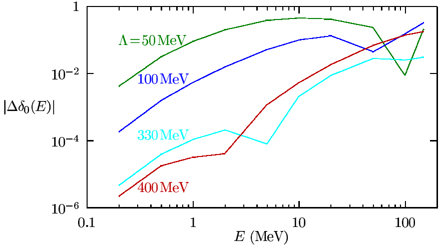


What is the breakdown scale? Not clear for χEFT! How do we analyze?

Error (“Lepage”) plots revisited [Lepage (1997); Steele, rjf (1999)]

- What is the evidence that the EFT is working as it should and we’re not just fitting (or over-fitting) elephants with many parameters?





- Slope of error curve with energy *should* increase with EFT order
- Breakdown scale (Λ_χ) where error curves intersect or where error stops improving (stabilized prediction)
  - Can we apply to observables other than phase shifts?
  - Investigations with toy models in progress [S. Wesolowski]
- What about error bands from regulator cutoff Λ variations?

How should we fit the LECs? Constrained curve fitting

- A new era for fitting and testing chiral Hamiltonians [see A. Ekstrom]
  - Deficiencies revealed; more advanced interactions coming
- Practical/theory motivations for Bayesian priors [Lepage (2001)]:
  - Constraints consistent with Lepage plots (can be tricky)
  - Would like to be independent of where we stop fitting (E, order)
  - Want the theory error at each order incorporated appropriately
  - Do not want constants to play off each other
- Bayesian fits in 30 seconds. Suppose we have parameters $\mathbf{a} = \{a_0, a_1, \cdots, a_M\}$, a data set $\mathbf{d} = \{d_1, d_2, \cdots, d_N\}$, and a theory f.
  - Goal: what $\mathbf{a}$ to use (with error) given a data set $\mathbf{d} \Longrightarrow pr(\mathbf{a}|\mathbf{d}, f)$
  - Known: given $\mathbf{a}$, what is the chance we get $\mathbf{d} \Longrightarrow pr(\mathbf{d}|\mathbf{a}, f)$
- Joint probability $pr(\mathbf{d}, \mathbf{a})$ can be decomposed into conditional probabilities two ways (and so are equal):

$$pr(\mathbf{a}|\mathbf{d}, f)pr(\mathbf{d}|f) = pr(\mathbf{d}|\mathbf{a}, f)pr(\mathbf{a}|f) \qquad \text{e.g., } pr(\mathbf{d}|\mathbf{a}, f) \propto \prod_{k=1}^{N} e^{-\chi^2/2}$$

Now just put $pr(\mathbf{d}|f)$ on the other side. The “priors” are $pr(\mathbf{a}|f)$.

"Prior" work by Schindler/Phillips: naturalness as a prior

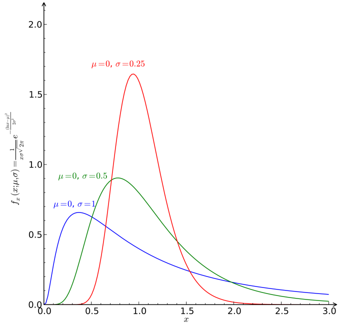
- "Bayesian Methods for Parameter Estimation in Effective Field Theories"
- Test application to chiral perturbation theory
- M coefficients naturalness values in normal distribution

$$pr(\mathbf{a}|M,R) = \left(\prod_{i=0}^{M} \frac{1}{\sqrt{2\pi}R}\right) e^{-\frac{1}{2}\sum_{i=0}^{M} a_i^2/R^2} \implies R \text{ is width}$$

- In progress: revisit by S. Wesolowski, D. Phillips, rjf for NN$\cdots$N

- Is normal distribution for natural $\mathbf{a} = \{a_i\}$ appropriate given we expect $1/n < a_i < n$?
- Maybe *log normal* distribution instead for $|a_i|$

$$f(x;\mu,\sigma) = \frac{1}{x}\frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, \quad x > 0\,.$$



How does this prior relate to weighting by the order of expansion?

“Prior” work by Schindler/Phillips: naturalness as a prior

- Schindler/Phillips toy problem: find M lowest-order coefficients in expansion of

$$g(x) = \left(\frac{1}{2} + \tan(\frac{\pi}{2}x)\right)^2 = \sum_{i=0}^{\infty} a_i x^i$$

$$\approx 0.25 + 1.57x + 2.47x^2 + 1.29x^3 + \cdots$$

  by ordinary “χ^2” fitting and using Bayesian priors on the “naturalness” of coefficients.

- Coefficients are of order unity: $1/4 < a_i < 4$
- Limited measurements and experimental noise
- Goal: determine a_0 and a_1

Usual χ^2 fit

M	χ^2	a_0	a_1	a_2
1	2.49	0.22±0.02	2.47±0.11	
2	0.85	0.29±0.02	1.04±0.40	4.91±1.31
3	0.85	0.26±0.04	2.00±1.12	-2.55±8.27
4	0.60	0.18±0.07	5.74±2.81	-50.4±34.0
5	0.57	0.28±0.14	0.24±7.08	46.9±120.0

With natural prior

M	a_0	a_1	a_2
1	0.23±0.14	2.42±0.11	
2	0.27±0.03	1.50±0.35	3.21±1.21
3	0.27±0.03	1.54±0.33	2.80±1.19
4	0.27±0.03	1.54±0.35	2.76±1.18
5	0.28±0.05	1.57±0.21	2.79±1.11

$\Longrightarrow$ marginalize over M and log normal parameters

Controlled fitting protocol needed for consistent “running” of EFT

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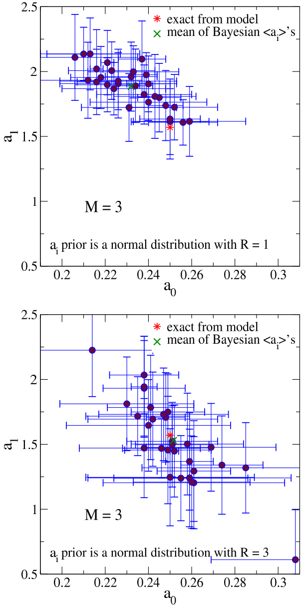

Controlled fitting protocol needed for consistent "running" of EFT

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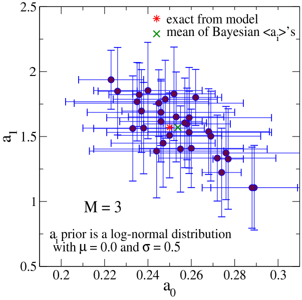
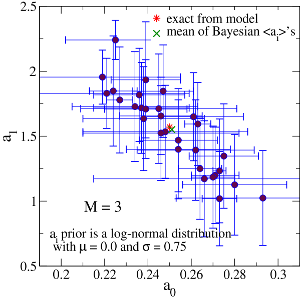




Controlled fitting protocol needed for consistent "running" of EFT

Is there a motivation for lower EFT cutoffs?

- Recent examples of calculations with soft EFT interactions
  - Nuclear matter calculations with soft *smooth cutoff* EFT potential [Corraggio et al., arXiv:1402.0965]
  - Lattice chiral EFT: coarse lattices $\Longrightarrow$ low Λ cutoff
    $\Longrightarrow$ but many successes [see D. Lee]
- How is an EFT at two different scales related to an RG running via SRG or Vlowk?
  - First, distinguish breakdown Λ_χ from regulator Λ
  - For matching, choose $\Lambda \sim \Lambda_\chi$ for Weinberg counting
- Integrating out momenta in a local EFT (à la Georgi)
  - Integrate out momenta $\Longrightarrow$ non-local action
  - Derivative expansion and drop higher terms $\Longrightarrow$ back to local
  - Requires sufficient scale separation or error grows from dropped terms
  - cf. SRG $\Longrightarrow$ error is unchanged with softening
  - But what is happening if we instead refit the EFT?
- Which is better *in practice*? We need more comparisons!

Does it matter *how* we cutoff UV physics?

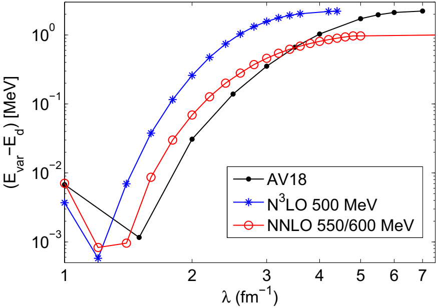
- Perhaps not in principle, but certainly in practice!
- What form does the T-generator SRG cutoff take?
  - Decoupling (*roughly*) imposes off-diagonal form for $V_\lambda(k,q)$

$$V_\lambda(k,q) \xrightarrow{q \gg k} V_\lambda(0,q) \sim V_\infty(0,q)\, e^{-(q^4/\lambda^4)}$$

- Test with a simple variational ansatz (from k-space S-eqn)

$$u(k) = \frac{1}{(k^2+\gamma^2)(k^2+\mu^2)} e^{-(k^4/\lambda^4)} \qquad w(k) = \frac{ak^2}{(k^2+\gamma^2)(k^2+\nu^2)^2} e^{-(k^4/\lambda^4)}$$

- error in deuteron energy for different initial potentials
- small λ works pretty well
- $V_{\text{low }k}$ works even better!



What if we "lower" cutoff by a truncated oscillator basis?

[Work by S. Bogner, S. Koenig, S. More, T. Papenbrock, rjf . . .]

- S. Coon: Finite oscillator basis imposes both IR and UV cutoffs
- Nature of UV vs. IR cutoff in light of dual nature of HO
  - Low-momentum (IR) spectrum is the same as hard-wall at

    $$L_\Delta = \sqrt{2(N_{\max} + 3/2 + \Delta)}\, b_{\rm osc} \qquad \text{with } b_{\rm osc} \equiv \sqrt{\hbar/(\mu\Omega)}$$

    with $\Delta = 2$ [see T. Papenbrock]
  - Duality $\Longrightarrow$ short distance (UV) same as hard wall in momentum with $b_{\rm osc} \to \hbar/b_{\rm osc}$ in L_2 $\Longrightarrow$ we expect

    $$\Lambda_\Delta = \sqrt{2(N_{\max} + 3/2 + \Delta)}\, \hbar/b_{\rm osc} \qquad \text{with } \Delta = 2$$

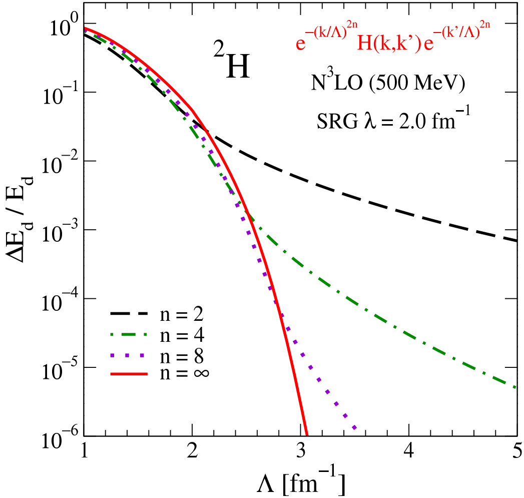
- Analytic result for separable potential with hard cutoff Λ:

$$V_\lambda(k,k') = g f_\lambda(k) f_\lambda(k') \ \text{with}\ f_\lambda(k) = e^{-(k/\lambda)^n} \ \Longrightarrow\ \Delta E \overset{\Lambda \gg \lambda}{\longrightarrow} C \int_\Lambda^\infty dk\, f_\lambda^2(k)$$

- Expect asymptotic form of energy correction for SRG or smooth $V_{\text{low}\,k}$ to (roughly) follow this form (with additional Λ dependence)

Examples for deuteron and RG-evolved potentials

[Thanks to K. Wendt for generating deuteron energies in IR-converged spaces]



$\Longrightarrow \Delta E_d / E_d$ for different cutoff forms; hard wall is $n = \infty$

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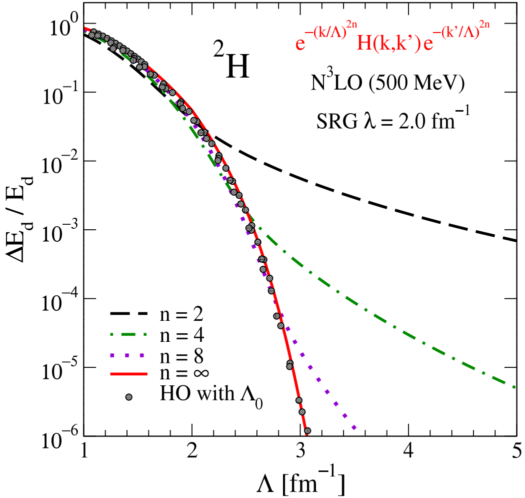

$\Longrightarrow \Delta E_d/E_d$ for Λ_0; looks like $n = \infty$ but noticable scatter

Examples for deuteron and RG-evolved potentials

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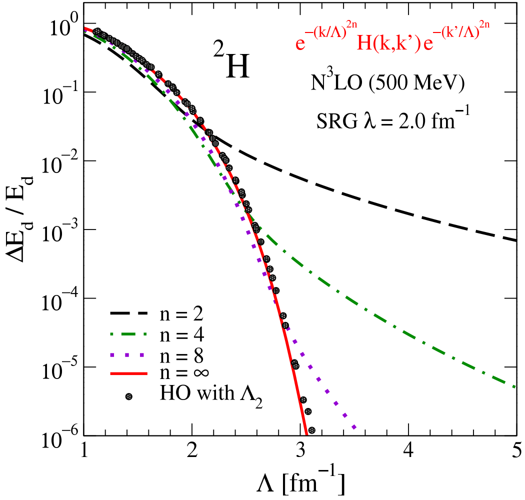

$\Longrightarrow \Delta E_d/E_d$ for Λ_2; looks like $n = \infty$ and *no* scatter

Examples for deuteron and RG-evolved potentials

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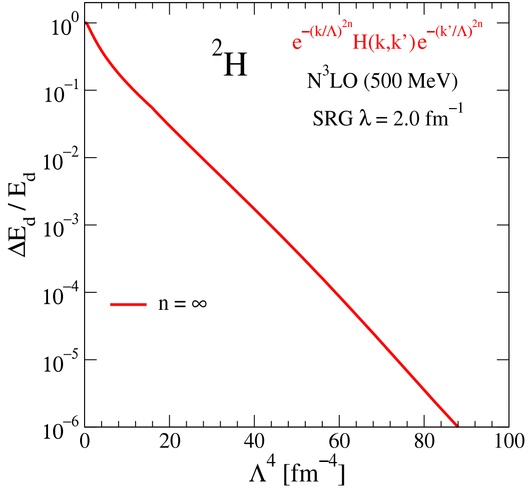

For $\Lambda > \lambda$, $\Delta E_d / E_d \propto g(\Lambda) e^{-2(\Lambda/\lambda)^4}$ (??)

Examples for deuteron and RG-evolved potentials

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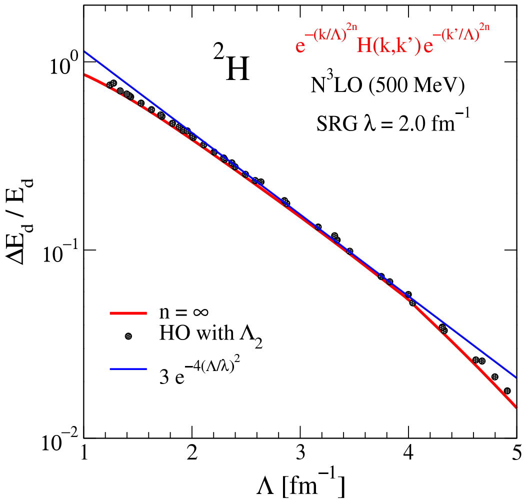
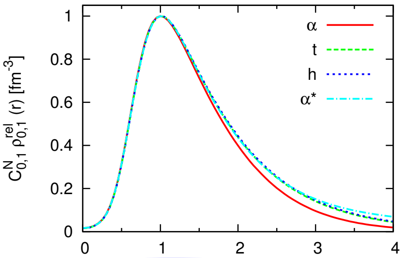
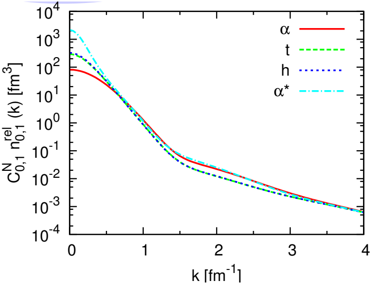


For $\Lambda \lesssim \lambda$, $\Delta E_d / E_d \propto e^{-4(\Lambda/\lambda)^2}$ (roughly), as used empirically

SRG/Vlowk wave functions versus “measured” SRCs

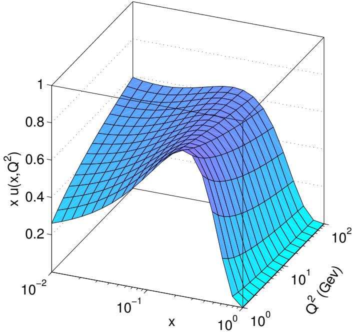
- Universal aspects of UV and IR truncations?
  - IR dictated by asymptotic many-body wave function
    $\Longrightarrow$ break-up channels $\Longrightarrow$ depends only on observables
    $\Longrightarrow$ independent of RG running (and intial potential)
  - UV depends on potential; e.g., changes with RG running because UV potential and wave function do
  - But expect similar (scaled) ΔE for $A > 2$
- Similar to discussions of short-range correlation physics
  - Frankfurt/Strikman arguments on asymptotic k-space wf
  - E.g., T. Neff et al. 2-body $S = 0$, $T = 1$ densities:

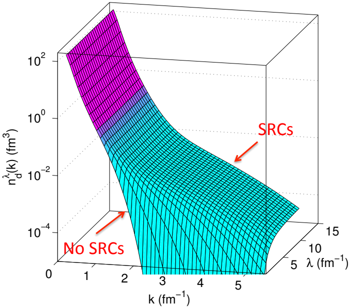




Is any of this UV physics "measurable"? [see rjf, 1309.5771]

- Relevant to knock-out experiments of various types
- Issues of scale and scheme dependence (RG invariants?)
  - We have (implicitly or explicitly) established a separation or factorization scale when we calculate observables
  - If sufficient separation of scales, then impulse approximation can be good, and no ambiguities.
  - Generally scale dependent, e.g. parton vs. momentum distributions:

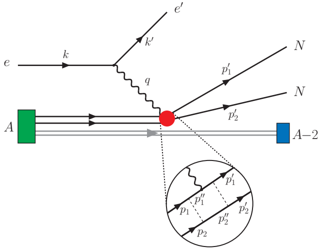


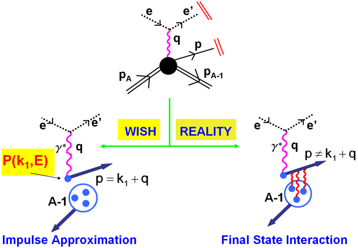


Which scale to use for experiment? Clear for QCD (gauge theory) but EFT?

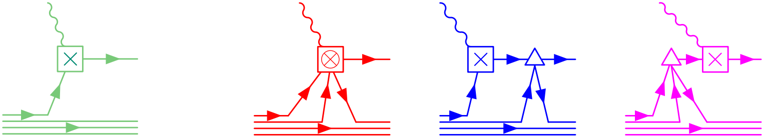
Start with simplest problem: deuteron electrodisintegration

- In progress by S. More, K. Hebeler, rjf





- Build on Yang and Phillips EFT calculations, but beyond the EFT ("high-resolution probes of low-resolution nuclei")
- Old field redefinition arguments of Hammer, rjf; also with $U_\lambda(k, q)$



- Understand mixing of structure, FSI, and currents (can't isolate!)
- Can we make money on factorization?

Additional comments (prejudices) on UV physics

- The fate of UV physics cuts across and unites many topics
- Calculational methods with microscopic forces are maturing
  - Deficiencies of current Hamiltonians clearly revealed
  - Opportunities: revisit old EFT technology while inventing new
  - Structure component ahead of reactions but RG can shift between; treating one in isolation can be dangerous
- Knock-out experiments need to be understood better
  - EFT and RG provide tools to do this
  - Different factorization scale for expt. analysis and calculation?
- Don't be too narrow with "*ab initio*" for microscopic NN$\cdots$N forces
  - Use sounds provincial in light of QCD
  - Low-energy paradigm: tower of effective theories (or turtles)
- Where should we think about the next rung on the EFT tower?
  - pionless EFT for halo nuclei
  - low-lying excitations in deformed nuclei [see T. Papenbrock]
  - DFT? [e.g., J. Dobaczewski et al.; revisit Landau-Migdal?]