A Direct Construction of the Nuclear Effective Interaction from Scattering Phase Shifts

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Work Done in Collaboration with Wick Haxton

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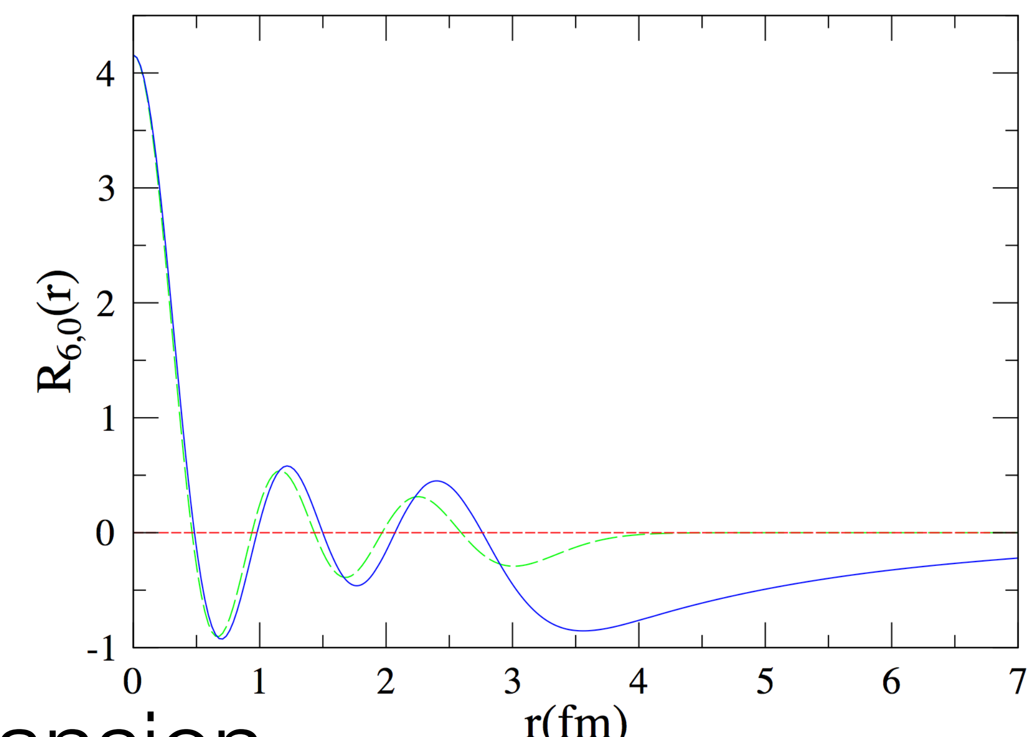
Background: Interaction From a Potential

Bloch-Horowitz: $$H_{eff} = P\left(H + H \frac{1}{E_i - QH} QH \right) P\psi_i = E_i P\psi_i$$

Haxton-Luu Form: $$H_{eff} = P \frac{E}{E - TQ}\left[T - T\frac{Q}{E}T + V + V\frac{1}{E-QH}V \right]\frac{E}{E-QT}P$$

$\frac{E}{E-QT}P$ acts only on edge state, restoring the long range waveform. The first terms in H_{eff} perform a complete sum of scattering by QT.

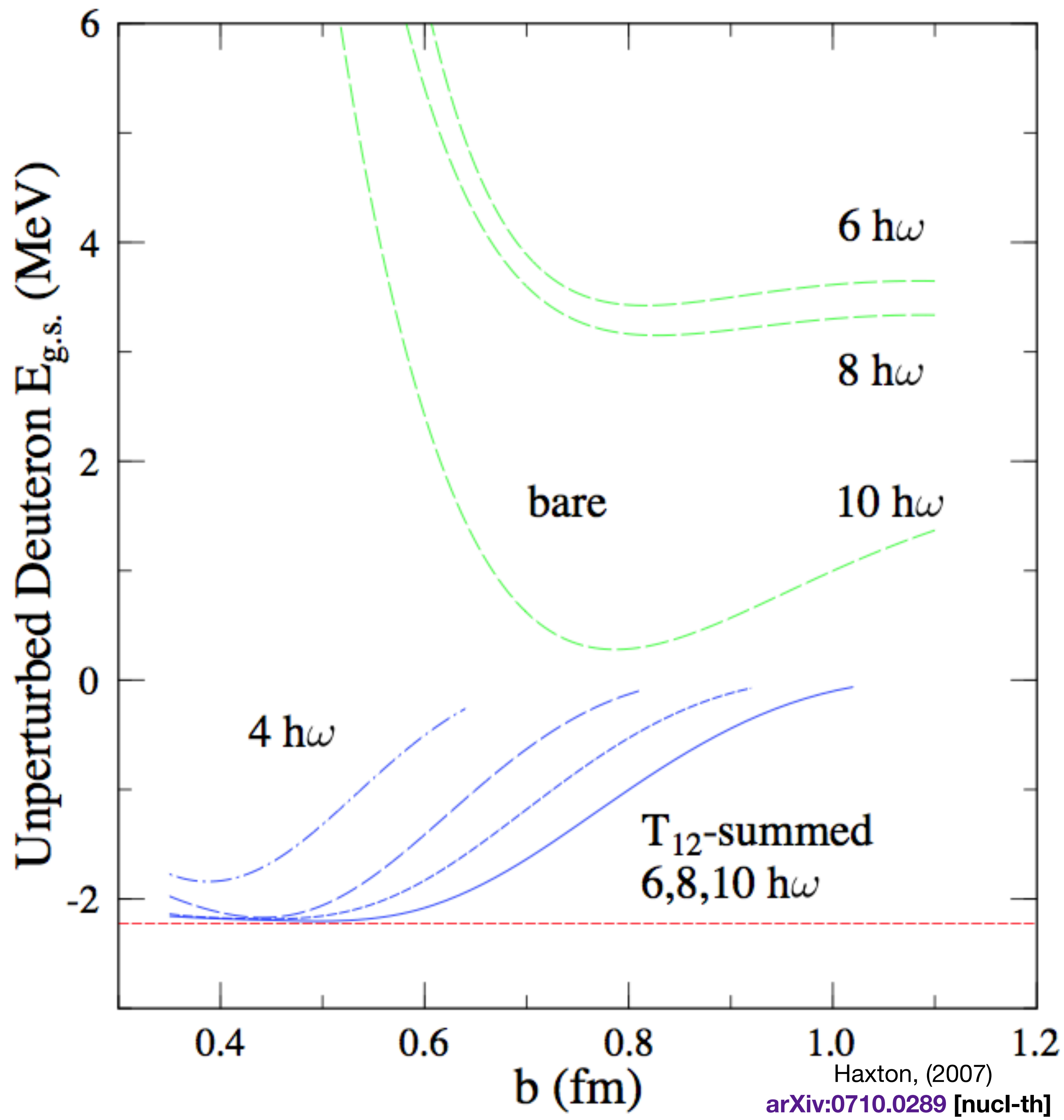


$P\frac{E}{E-TQ}V\frac{1}{E-QH}V\frac{E}{E-QT}P$ is short range and is replaced by a contact gradient expansion which can be easily fit by taking matrix elements of the known potential.

Deuteron Binding Energy Convergence

With QT summed to all orders, shrinking the HO length scale b enables the capture of the important part of V with no ET terms in a very small basis.

Shows the power of summing the IR contribution.



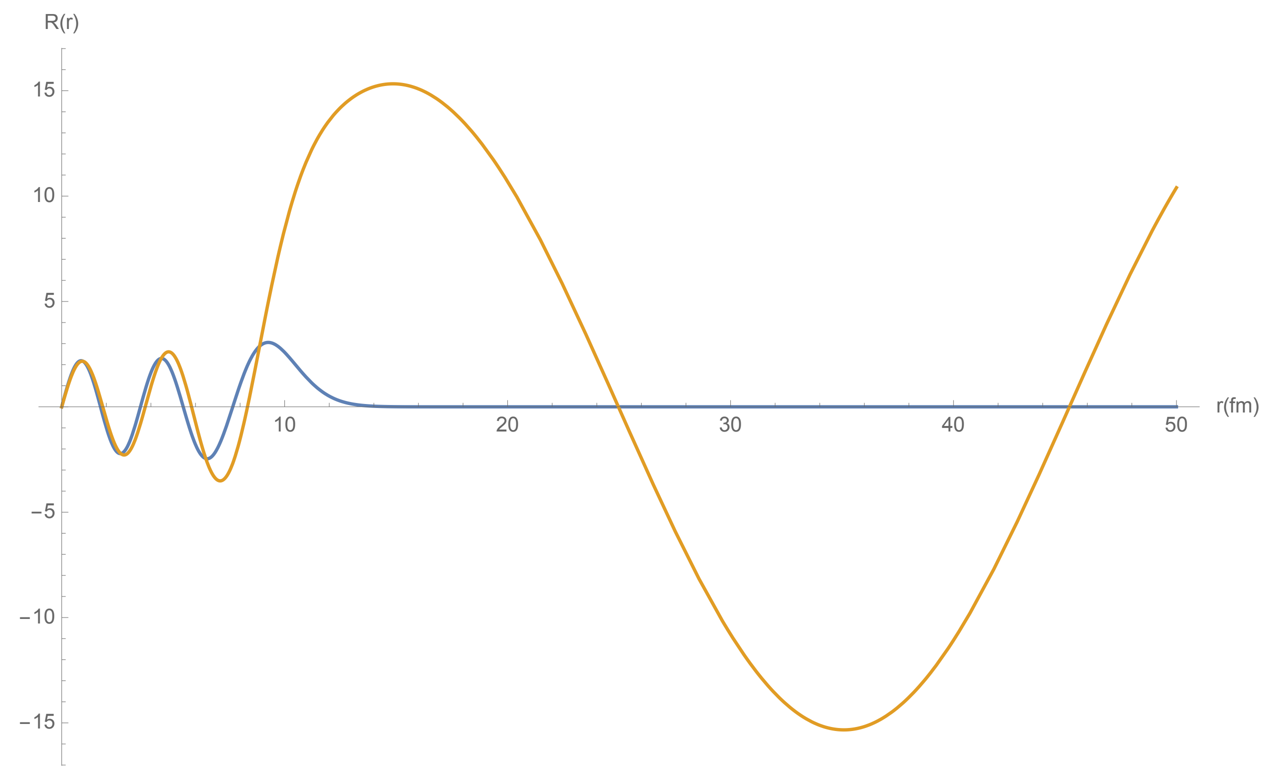
Haxton, (2007)
arXiv:0710.0289 [nucl-th]

Fitting to Continuum Scattering Phase Shifts

Same H_{eff}

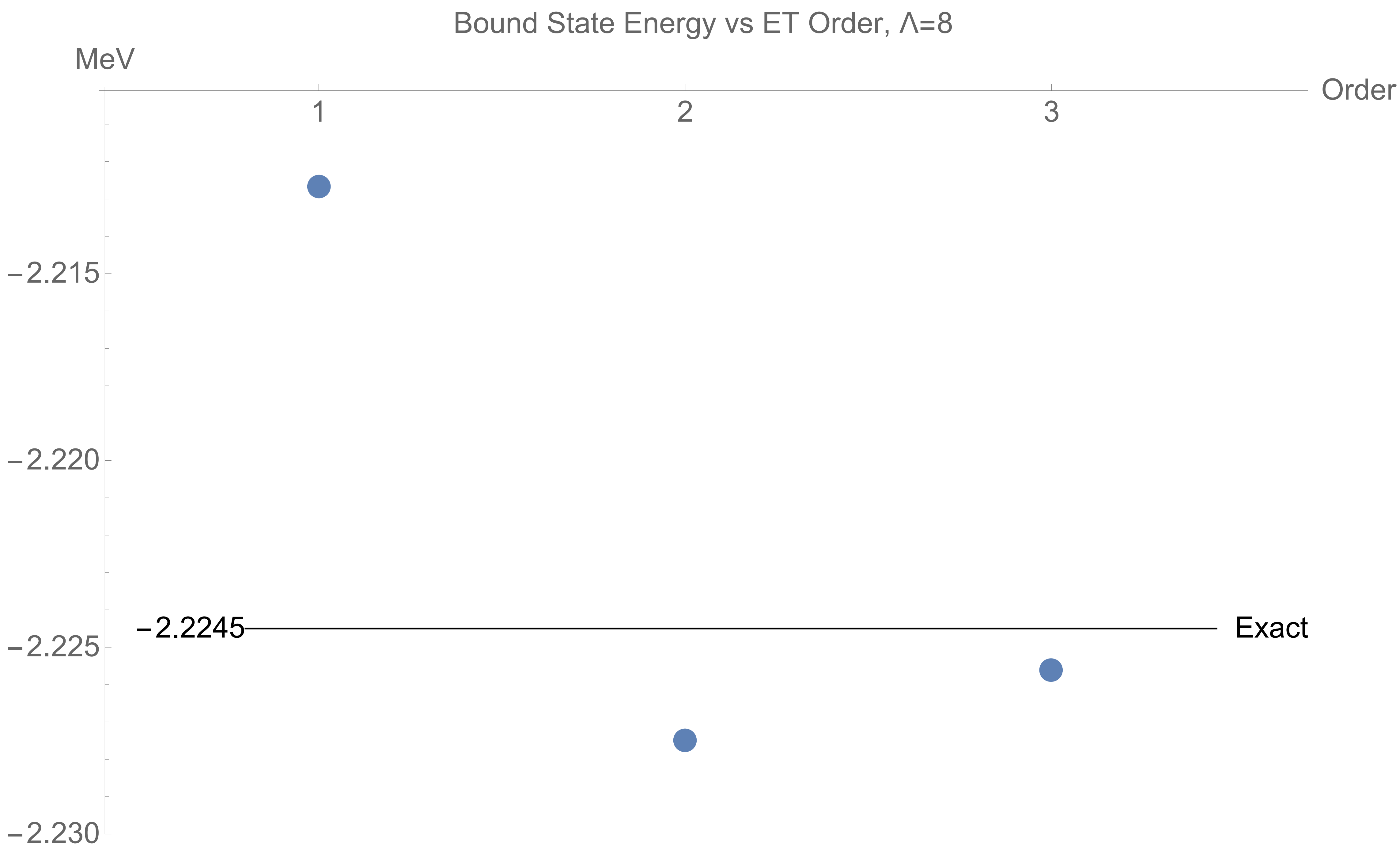
$$H_{eff} = P \frac{E}{E - TQ} \left[T - T \frac{Q}{E} T + V + V \frac{1}{E - QH} V \right] \frac{E}{E - QT} P$$

But … $\frac{E}{E-QT}P$ is no longer unique. The boundary condition at infinity is a periodic boundary condition corresponding to the phase shift.



The contact gradient expansion coefficients are used to satisfy the self consistent energy constraint at a set of continuum energies. Then we predict …

Practice Problem: S Channel Only, Hard Core + Well



$$Solve\ for\ E\ self\ consistently: H_{eff}(E)\psi_P = E\psi_P$$

Coupled Channel Deuteron Binding Energy

Phase shifts generated from Argonne v18 potential.

Eight parameters fit through NNLO to produce self consistent energies.

	alo3S1	anlo3S1	anloSD	N^2	Binding Energy
Reference	-1.7589	-0.02221	-0.1271	yes	-2.22
Reference-	-1.7589	-0.02221	-0.1271	no	-2.08
Fit	-1.7803	-0.02167	-0.1268	no	-2.27

@Λ=8

The short range part of the interaction comes from the phase shifts.   The IR contribution comes from the transform of the edge state, yielding a sum of the scattering by QT.

Summary

- Key Ideas for generalization to continuum
  - Match phase shifts at each energy by constraining Greens' function
  - Fit contact gradient coefficients to produce self consistent energies across a range of energies

- The result is an accurate small basis interaction
  - Due to separation of UV and IR and complete sum of IR contributions
  - Only need long range part of V to be accurate. The rest of the potential contribution is captured from phase shifts

See me at the poster session for questions!

Thank You!

NN ET Operators

$$^3S_1 \leftrightarrow {}^3S_1: \quad a_{LO}^{3S1}\delta(\vec{r}) + a_{NLO}^{3S1}\left(\overleftarrow{\nabla}^2\delta(\vec{r}) + \delta(\vec{r})\vec{\nabla}^2\right) + \cdots$$

$$^3S_1 \leftrightarrow {}^3D_1: \quad a_{NLO}^{SD}\left(\overleftarrow{D}^0\delta(\vec{r}) + \delta(\vec{r})\vec{D}^0\right) + \cdots$$

$$^3D_1 \leftrightarrow {}^3D_1: \quad a_{NNLO}^{3D1}\left(\overleftarrow{D}^2 \cdot \delta(\vec{r})\vec{D}^2\right) + \cdots$$

$$\vec{D}^0 = \left[\left(\sigma(1) \otimes \sigma(2)\right)^{(2)} \otimes \left(\vec{\nabla} \otimes \vec{\nabla}\right)^{(2)}\right]_0^0 \sim S_{12}$$

$$\vec{D}_M^2 = \left[\vec{\nabla} \otimes \vec{\nabla}\right]_M^2$$