High precision nuclear forces within chiral EFT

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LENPIC

Low Energy Nuclear Physics International Collaboration

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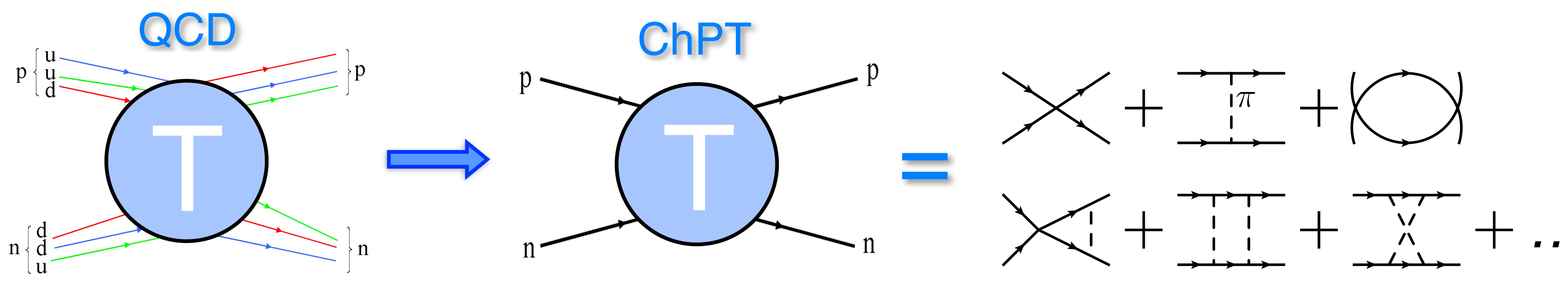
Outline

- Nuclear forces in chiral EFT
- NN up to N^4LO with (semi)local regulators
- Chiral error estimate
- 3NF observables with N^4LO NN
- PWD of the three-nucleon forces
- Summary & Outlook

ChPT pros and cons



ChPT as an effective field theory of QCD

- ✓ is the most general field theory with pions, nucleons (deltas) as dofs in line with the symmetries of QCD
- ✓ is systematically improvable
- ✓ gives a unified description of $\pi\pi$, πN, NN, (axial) vector currents etc.
- ✓ naturally explains the hierarchy $V_{2N} >> V_{3N} >> V_{4N}$
- ✓ predicts the long range behavior of nuclear forces
- ✓ allows doing precision physics with/from light nuclei

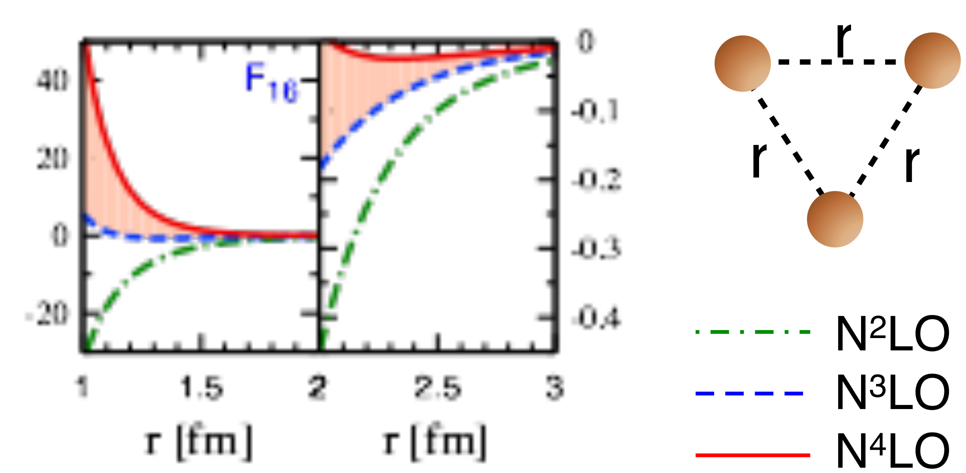
- ✖ number of free parameters (LEC) increases with increasing order in ChPT
- ✖ does not provide an explanation on the size of a particular LEC
- ✖ is only applicable in the low energy region
- ✖ convergence radius of ChPT is a priori unknown

ChPT nuclear forces

The situation before *Epelbaum, HK, Meißner arXiv:1412.0142[nucl-th], arXiv:1412.4623[nucl-th]*

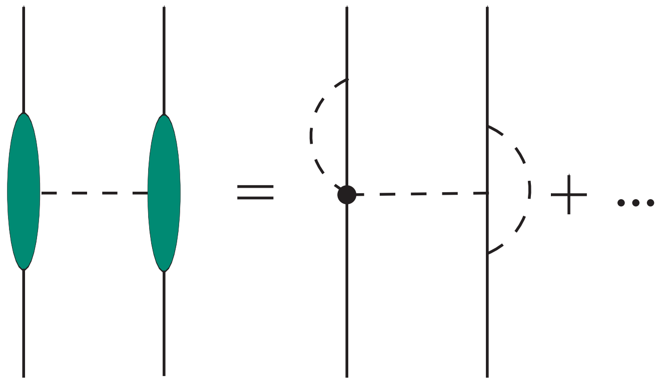
	V_{NN}	V_{3N}	V_{4N}
Worked out up to the order	N^3LO	N^3LO N^4LO in progress	N^3LO
Regularization used	SFR In order to suppress strong attraction from short-range components of N^2LO TPE diagr.	Dim. Reg. SFR implementation is not straightforward in case of e.g. ring diagr.	——

N^4LO contributions to 3NF's are not negligible due to large c_i LECs within loop diagrams

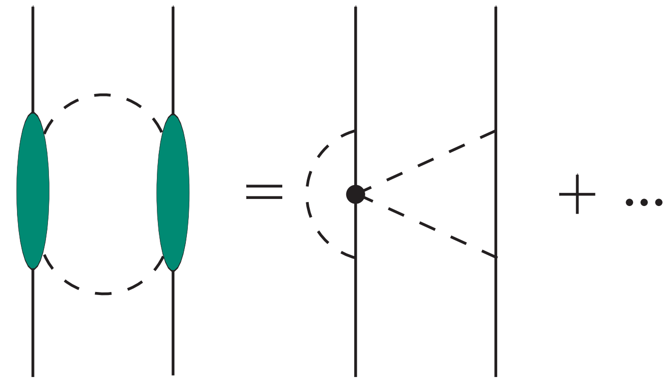


For consistency reason V_{NN} should be worked out up to N^4LO

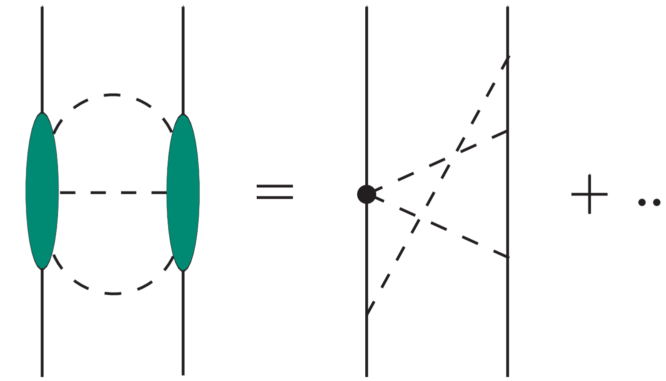
NN at N^{4}LO



Contribute only to renormalization of OPE: OPE is unchanged at N^{4}LO



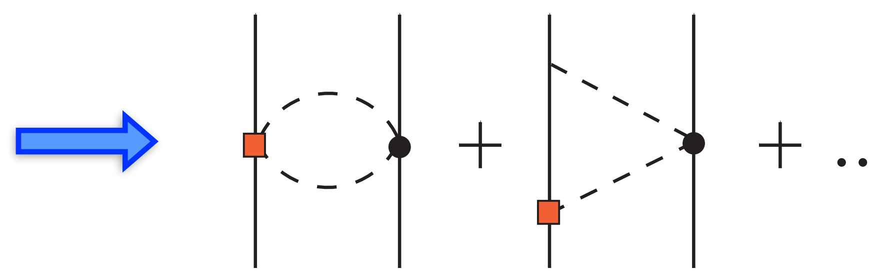
Entem, Kaiser, Machleidt, Nosyk, PRC91 (2015) 014002



Kaiser, PRC63 (2001) 044010
Due to shorter range nature can be approximated by contact interactions

Various relativistic corrections at N^{4}LO

- $1/m_N$ - corrections to N^{2}LO one-loop TPE
  Kaiser, PRC64 (2001) 057001



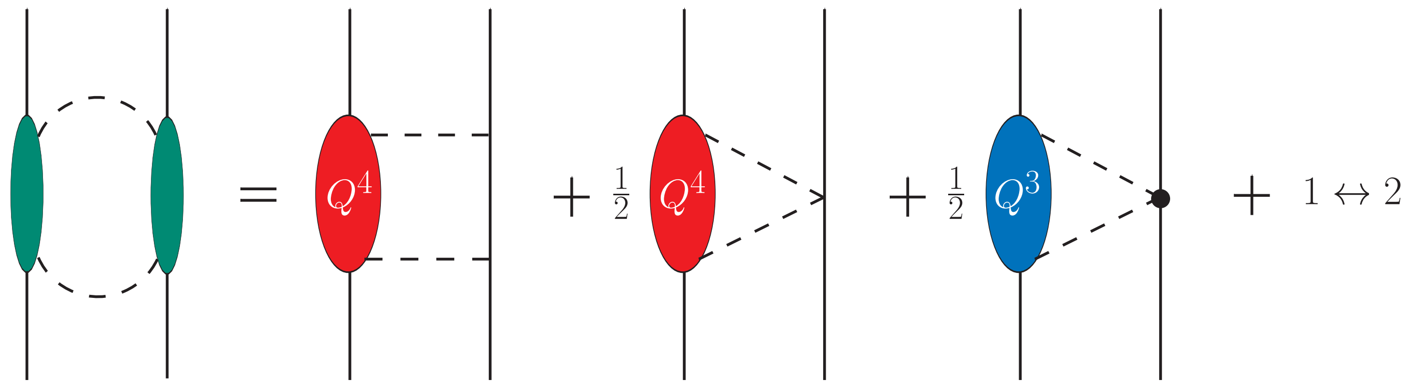
- Additional $1/m_N$ - corrections from rewriting relativistic Schrödinger eq. in the equivalent non-relativistic form
  Friar, PRC60 (1999) 034002

$$\left[2\sqrt{p^2+m_N^2}+V\right]\Psi = 2\sqrt{k^2+m_N^2}\Psi \Longrightarrow \left[\frac{p^2}{2m_N}+\tilde{V}\right]\Psi = \frac{k^2}{2m_N}\Psi \quad \text{with} \quad \tilde{V} = \left\{\frac{\sqrt{p^2+m_N^2}}{2m_N}+V\right\}+\frac{V^2}{4m_N}$$

At N^{4}LO there are no isospin-conserving contact interactions

Parity conservation forbids contact terms at Q^5

Two-loop TPE at N^{4}LO



Spectral function representation

$$V_{C,S}(q) = -\frac{2q^6}{\pi}\int_{2M_\pi}^{\infty} d\mu \frac{\mathrm{Im}\, V_{C,S}(i\mu-\epsilon)}{\mu^5(\mu^2+q^2)}$$
$$V_T(q) = \frac{2q^4}{\pi}\int_{2M_\pi}^{\infty} d\mu \frac{\mathrm{Im}\, V_T(i\mu-\epsilon)}{\mu^3(\mu^2+q^2)}$$

$$\mathcal{M}(q) = i\sum_{a,b=1}^{3}\lambda^{4-d}\int\frac{d^dl}{(2\pi)^d}T_{\pi N}(l,l-q)^{(1)}_{a,b}\frac{i}{l^2-M_\pi^2+i\epsilon}\frac{i}{(l-q)^2-M_\pi^2+i\epsilon}T_{\pi N}(-l,-l+q)^{(2)}_{a,b}$$

$$T_{\pi N}(l,l')^{(i)}_{ab} = \delta_{ab}\left(g_+^{(i)}(l_0,(l-l')^2)+i\,\vec\sigma_i\cdot\vec l'\times\vec l\,h_+^{(i)}(l_0,(l-l')^2)\right) + \sum_{c=1}^{3} i\,\epsilon_{bac}\tau_i^c\left(g_-^{(i)}(l_0,(l-l')^2)+i\,\vec\sigma_i\cdot\vec l'\times\vec l\,h_-^{(i)}(l_0,(l-l')^2)\right)$$

Perform a complex Lorentz transformation $l^\mu \to \Lambda^\mu_\nu l^\nu$ to bring πN - amplitudes into CMS of 2 exchanged pions

$$\begin{pmatrix}\mu\\ \vec 0\end{pmatrix} = \Lambda^{-1}\begin{pmatrix}0\\ \vec q\end{pmatrix}, \quad \text{with} \quad \vec q^{\,2} = -\mu^2. \quad \Lambda = \begin{pmatrix}0 & i\hat q\\ i\hat q & \mathbb{1}-\hat q\,\hat q^T\end{pmatrix} \quad\Longrightarrow\quad \Lambda^{-1} = \begin{pmatrix}0 & -i\hat q\\ -i\hat q & \mathbb{1}-\hat q\,\hat q^T\end{pmatrix}$$

Since $\Lambda^\mu_\nu \in \mathbb{C}$ the substitution $l^\mu \to \Lambda^\mu_\nu l^\nu$ should be taken with care: analytic continuation

$$\mathrm{Im}\, V_C(q=i\,\mu-\epsilon) = -\frac{3}{8\mu}\,\mathrm{Im}\, i\int\frac{dl_q}{2\pi}g_+^{(1)}(il_q,q^2)g_+^{(2)}(-il_q,q^2)\theta(\mu-2\omega_{l_q})|_{q^2=\mu^2+i\epsilon}$$
$$\mathrm{Im}\, V_S(q=i\,\mu-\epsilon) = -\frac{3\mu}{64}\,\mathrm{Im}\, i\int\frac{dl_q}{2\pi}h_+^{(1)}(il_q,q^2)h_+^{(2)}(-il_q,q^2)(\mu^2-4\omega_{l_q}^2)\theta(\mu-2\omega_{l_q})|_{q^2=\mu^2+i\epsilon}$$

Local regularization

Working with relatively low cut-offs $\Lambda \sim 500 \ldots 600\,\text{MeV}$ prevents appearance of NN deeply bound states

✓ Absence of deeply bound states is advantageous for few- and many-body simulations

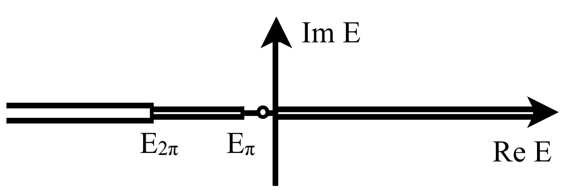
✖ Finite cut-off artefacts are manifested in residual cut-off dependence of nuclear observables

Reduce cut-off artefacts by efficient choice of regularization

Standard non-local momentum space regulator: $V(\vec{p}\,', \vec{p}\,) \to V(\vec{p}\,', \vec{p}\,) \exp\left(-\frac{p'^n + p^n}{\Lambda^n}\right)$

✓ Convenient for partial wave decomposition of nuclear force: simple multiplication

✖ Affects the discontinuity across the left-hand cuts



⇨ Distortion of analytic structure of partial-wave amplitudes near threshold. Effects proportional to inverse power of Λ.

✓ Cut-off artefacts can be partly reduced by additional introduction of SFR

Local regularization in coordinate space: $V_{\text{long-range}}(\vec{r}\,) \to V_{\text{long-range}}(\vec{r}\,) \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n$

✓ By construction long - range physics is unaffected by this regulator

✓ No additional SFR is needed

Fits of πN LECs

Epelbaum, HK, Meißner arXiv:1412.0142[nucl-th], arXiv:1412.4623[nucl-th]

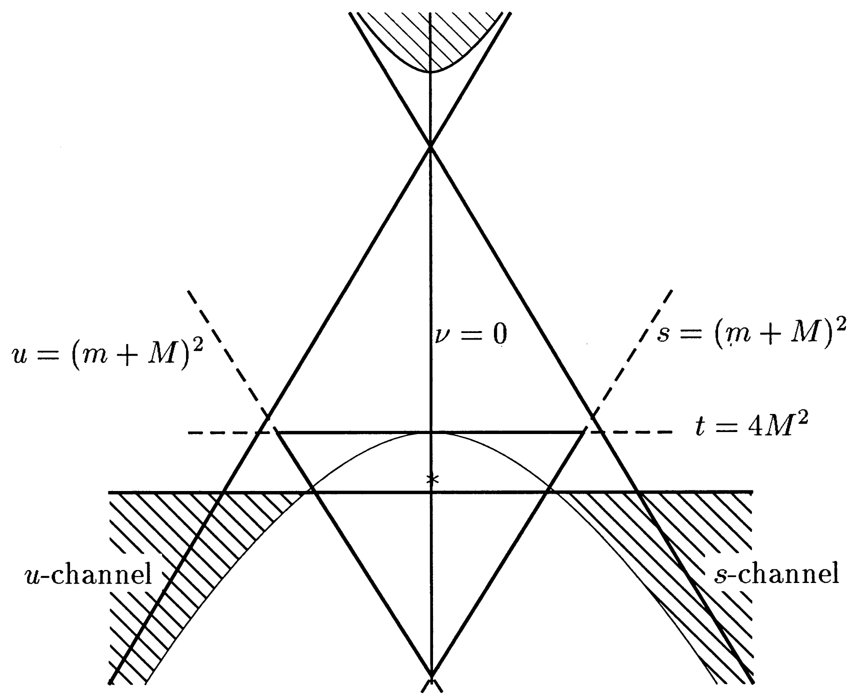
LECs which affect long range behavior and are fixed from πN - sector (c_i in GeV^{-1}, $\bar{d}_i$ in GeV^{-2}, $\bar{e}_i$ in GeV^{-3})

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$
N^3LO	-0.81	3.28	-4.69	3.40	3.06	-3.27	0.45	-5.65	—	—
N^4LO	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-0.37

N^3LO LECs: *Büttiker & Meißner NPA668 (2000) 97*
c_2 & $\bar{d}_i$ from Fettes et al. NPA640 (1998) 199

N^4LO LECs: *Q^4 fit to πN scattering (KH PWA)*
HK, Gasparyan, Epelbaum PRC85 (2012) 054006

For the direct fit of LECs to experimental data see
Wendt, Eckström, Carlson arXiv: 1410.0646[nucl-th]



By use of dispersion relations c_1, c_3, c_4 are determined inside Mandelstam triangle where ChPT converges better.

Epelbaum, Glöckle, Meißner NPA747 (2005) 362 used c_3=-3.40 GeV^{-1}. c_3=-4.69 GeV^{-1} would lead to spurious deeply-bound states.

No deeply-bound states with c_3=-4.69 GeV^{-1} by use (semi)local regulator procedure

Fitting procedure

(Semi)local regularization used

$$V_{\rm long-range}(\vec{r}\,) \to V_{\rm long-range}(\vec{r}\,) \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6 \quad V_{\rm contact}(\vec{p}\,', \vec{p}\,) \to V_{\rm contact}(\vec{p}\,', \vec{p}\,) \exp\left(-\frac{p'^2 + p^2}{(2R^{-1})^2}\right)$$

$$R = 0.8 \ldots 1.2\,{\rm fm} \leftrightarrow \Lambda \sim 330 \ldots 500\,{\rm MeV} \qquad {\rm FT}_q \left[\exp\left(-r^2/R^2\right)\right] \propto \exp\left(-q^2 R^2/4\right) \;\rightarrow\; \Lambda \sim 2R^{-1}$$

Using Nijmegen PWA (NPWA): *Stoks et al. PRC48 (1993) 792*

- We fit all isospin-1 channels to pp phase-shifts & generate np and nn phase-shifts (with exception of $^1S_0$ partial wave)
- All isospin breaking corrections are taken in the same way as in NPWA
- 26 LECs at N³LO (and one more IB LEC at N⁴LO resulting in $C^{pp}_{^1S_0} \neq C^{np}_{^1S_0}$) are fitted for $E_{\rm lab} \leq 200\,{\rm MeV}$
- At N³LO fits in $^3S_1$ - $^3D_1$ channel become unstable (due to appearance of different solutions)
    - We require deuteron binding energy to be correctly reproduced
    - Discard solutions with unrealistic values of D - state probability: $P_D = 5\% \pm 1\%$
    - Discard solutions with too strong violation of Wigner SU(4) symmetry: $\tilde{C}_{^1S_0} \simeq \tilde{C}_{^3S_1}$

$$\chi^2_{\rm aug} = \chi^2 + \chi^2_{\rm prior} \quad \text{with} \quad \chi^2_{\rm prior} = \frac{(\tilde{C}_{^3S_1} - \tilde{C}_{^1S_0})^2}{(\Delta\tilde{C}_{^3S_1})^2} \quad \text{and} \quad \Delta\tilde{C}_{^3S_1} = \tilde{C}_{^1S_0}/4$$

Fit results

Epelbaum, HK, Meißner arXiv:1412.0142[nucl-th], arXiv:1412.4623[nucl-th]

χ^2/datum is calculated using the errors

$$\Delta_X = \max\left(\Delta_X^{\rm NPWA}, |\delta_X^{\rm NijmI} - \delta_X^{\rm NPWA}|, |\delta_X^{\rm NijmII} - \delta_X^{\rm NPWA}|, |\delta_X^{\rm Reid93} - \delta_X^{\rm NPWA}|\right)$$

statistical error of the NPWA

χ^2/datum as defined above does *not* allow for statistical interpretation

For all values of the cutoff R=0.8,…,1.2 fm we get LECs of natural size

$$V \sim \mathcal{O}\left[\frac{1}{F_\pi^2}\left(\frac{Q}{\Lambda_b}\right)^\nu\right] \longrightarrow |\tilde{C}_i| \sim \frac{4\pi}{F_\pi^2},\ |C_i| \sim \frac{4\pi}{F_\pi^2\Lambda_b^2},\ |D_i| \sim \frac{4\pi}{F_\pi^2\Lambda_b^4}$$

R=0.9 fm	E_{lab} [MeV]	LO	NLO	N^2LO	N^3LO	N^4LO
np	0-100	360	31	4.5	0.7	0.3
np	0-200	480	63	21	0.7	0.3
pp	0-100	5750	102	15	0.8	0.3
pp	0-200	9150	560	130	0.7	0.6

- Clear improvement of χ^2/datum at N^4LO
- Additional IB N^4LO LEC affects only np - results: without it we would get $\chi^2/\mathrm{datum} = 0.5$

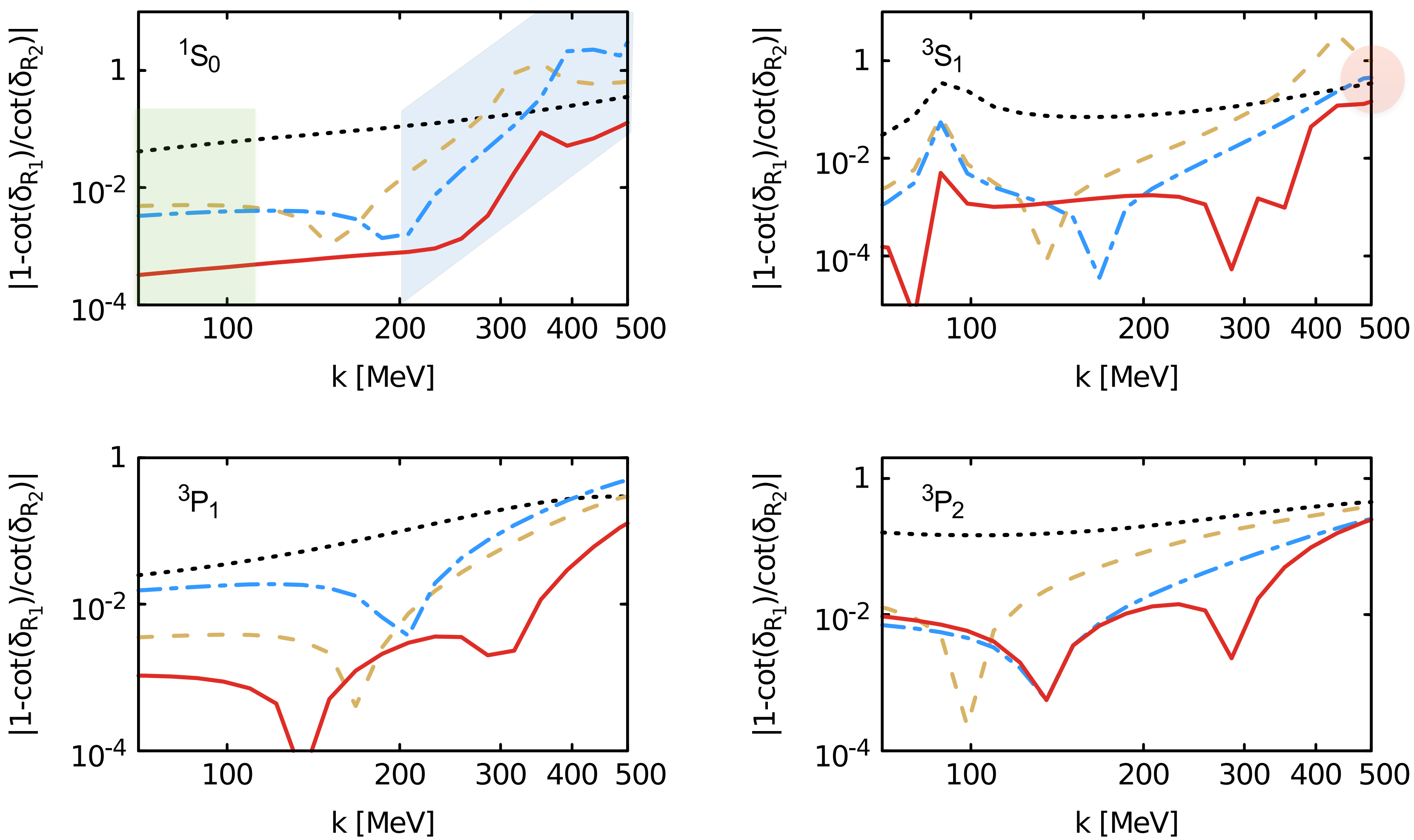
Cutoff dependence

The residual cutoff dependence ⟷ Effects of higher-order contact interactions beyond the truncation level of the potential

The residual cutoff dependence is expected

- to be reduced if LO ⟶ NLO/N^{2}LO ⟶ N^{3}LO/N^{4}LO
- not to be reduced if NLO ⟶ N^{2}LO & N^{3}LO ⟶ N^{4}LO

Residual cutoff dependence of phase shifts by looking at $[1 - \cot\delta_{R_1}(k)/\cot\delta_{R_2}(k)]$ with R_1=0.9 fm and R_2=1.0 fm



- Near horizontal cutoff dependence at $k \ll M_\pi$ due to $(k/\Lambda_b)^n \ll (M_\pi/\Lambda_b)^n$
- For $k > M_\pi$ contributions $\sim (k/\Lambda_b)^n$ start to dominate (non-vanishing slope)
- At $k \sim 500\,\text{MeV}$ N^{3}LO curve starts to cross lower order curves

⬇

No improvement at $k \sim 500\,\text{MeV}$ with increasing chiral order

⬇

$\Lambda_b \sim 400 \ldots 600\,\text{MeV}$ depends on used cutoff R

Deuteron properties

R=0.9 fm	LO	NLO	N^2LO	N^3LO	N^4LO	Exp
B_d	2.0235	2.1987	2.2311	2.2246*	2.2246*	2.224575(9)
A_S	0.8333	0.8772	0.8865	0.8845	0.8844	0.8846(9)
η	0.0212	0.0256	0.0256	0.0255	0.0255	0.0256(4)
r_d	1.990	1.968	1.966	1.972	1.972	1.97535(85)
Q	0.230	0.273	0.270	0.271	0.271	0.2859(3)
P_D	2.54	4.73	4.50	4.19	4.29	

- N^4LO & N^3LO predictions are very close to each other → good convergence (with exception of P_D which is not observable)
- Meson-exchange current and relativistic corrections are not taken into account for r_d & Q

Corresponding corrections as estimated by *Kohno JPG 9 (1983) L85 & Phillips JPG34 (2007) 365*

$$\Delta r_d^2 \simeq 0.014\,\mathrm{fm}^2 \longrightarrow \text{of the order of } 0.2\%$$

$$\Delta Q \simeq 0.008\,\mathrm{fm}^2 \longrightarrow Q + \Delta Q = 0.279\,\mathrm{fm}^2$$

N^4LO predictions for r_d and Q for different cutoffs:

r_d=1.970…1.981 fm, Q=0.270…0.281 fm^2 for cutoffs R=0.8…1.2 fm

The observed spread in Q is to be absorbed by the leading short-range NN current with natural LECs

Theoretical uncertainty

Sources of uncertainties in nuclear forces

1. Systematic uncertainty due to truncation of the chiral expansion at a given order
2. Uncertainties in the estimation of πN LECs
3. Uncertainties in the determination of contact interaction LECs
4. Uncertainties in the experimental data (NPWA in our case)

In previous studies 1 was addressed by cutoff variation (not the most efficient way)

- the corresponding uncertainty depends on the taken cutoff range → some arbitrariness
- residual cutoff dependence measures the contributions due to neglected LECs
  → the same uncertainty estimation at e.g. NLO and N^2LO (LECs appear at Q^{2n})

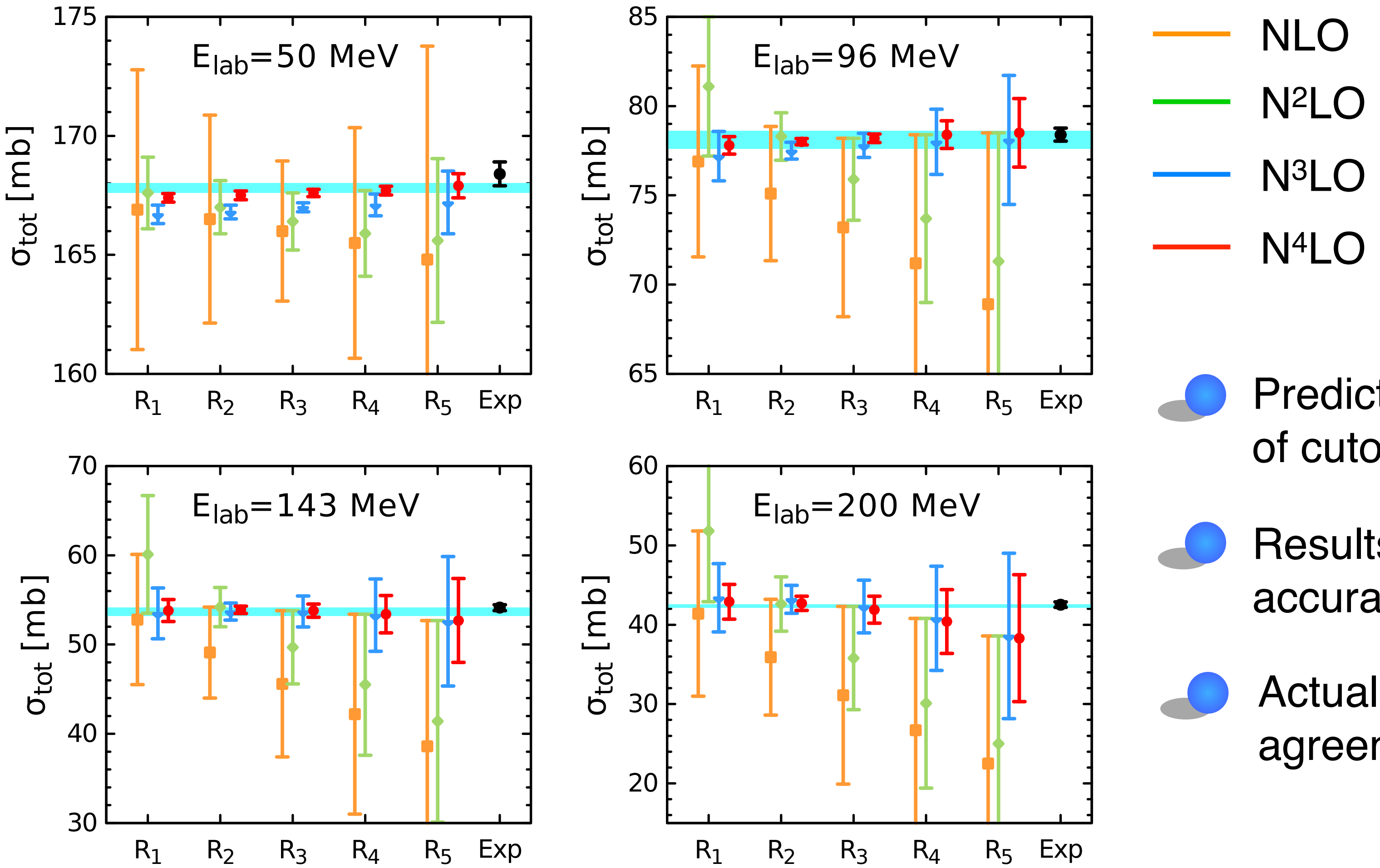
Estimate the uncertainty for a given fixed cutoff R via expected size of higher-order corrections

For a N^4LO prediction of an observable $X^{\mathrm{N^4LO}}$ we get an uncertainty

$$\begin{aligned} \Delta X^{\mathrm{N^4LO}}(p) &= \max\Big(Q \times |X^{\mathrm{N^3LO}}(p) - X^{\mathrm{N^4LO}}(p)|, Q^2 \times |X^{\mathrm{N^2LO}}(p) - X^{\mathrm{N^3LO}}(p)|, \\ &\quad Q^3 \times |X^{\mathrm{NLO}}(p) - X^{\mathrm{N^2LO}}(p)|, Q^4 \times |X^{\mathrm{LO}}(p) - X^{\mathrm{NLO}}(p)|, Q^6 \times |X^{\mathrm{LO}}(p)|\Big) \end{aligned}$$

with chiral expansion parameter $Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$

Uncertainty due to chiral expansion



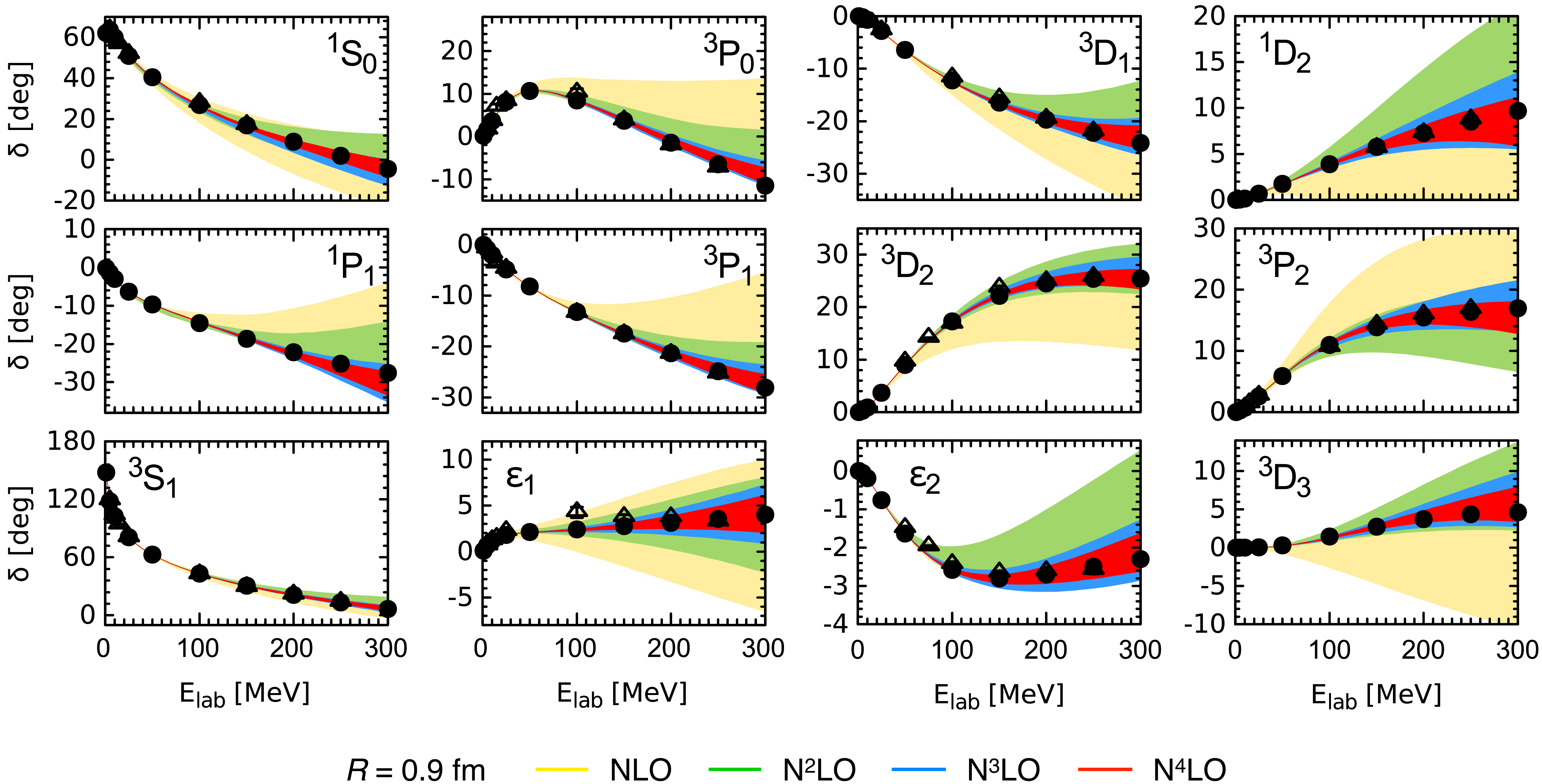
np total cross section at several cutoffs $R_i = (0.7 + i \times 0.1)\,\mathrm{fm}$

- Predictions based on different values of cutoff *R* are consistent with each other
- Results with larger value of *R* are less accurate due to larger cutoff artefacts
- Actual size of N^4LO corrections is in good agreement with estimated uncertainty at N^3LO

- Somewhat large N^4LO corrections at the lowest energy is a consequence of the adopted fitting protocol
- The most accurate results (judging on the size of error bars) are for the cutoffs R_2=0.9 fm and R_3=1.0 fm
- At lowest energy the uncertainty due to cutoff variation of NLO results is underestimated. This pattern changes at higher energies

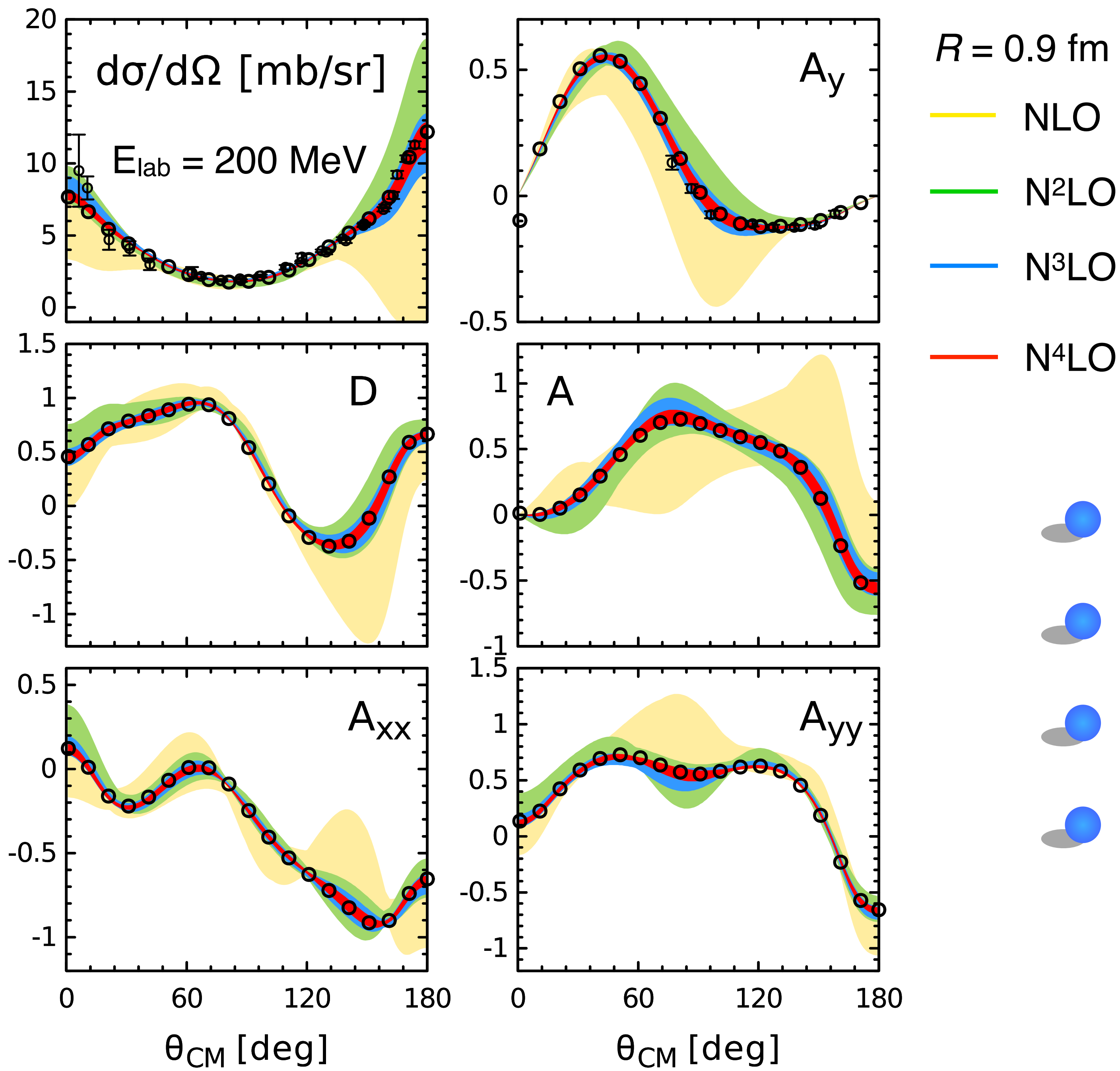
The suggested chiral error estimation is more reliable than the cutoff variation procedure

Phase shifts and mixing angles



- Good convergence of chiral expansion
- Error bands are consistent with each other → strong support of chiral uncertainty estimation
- Excellent agreement with NPWA data

Further NN observables

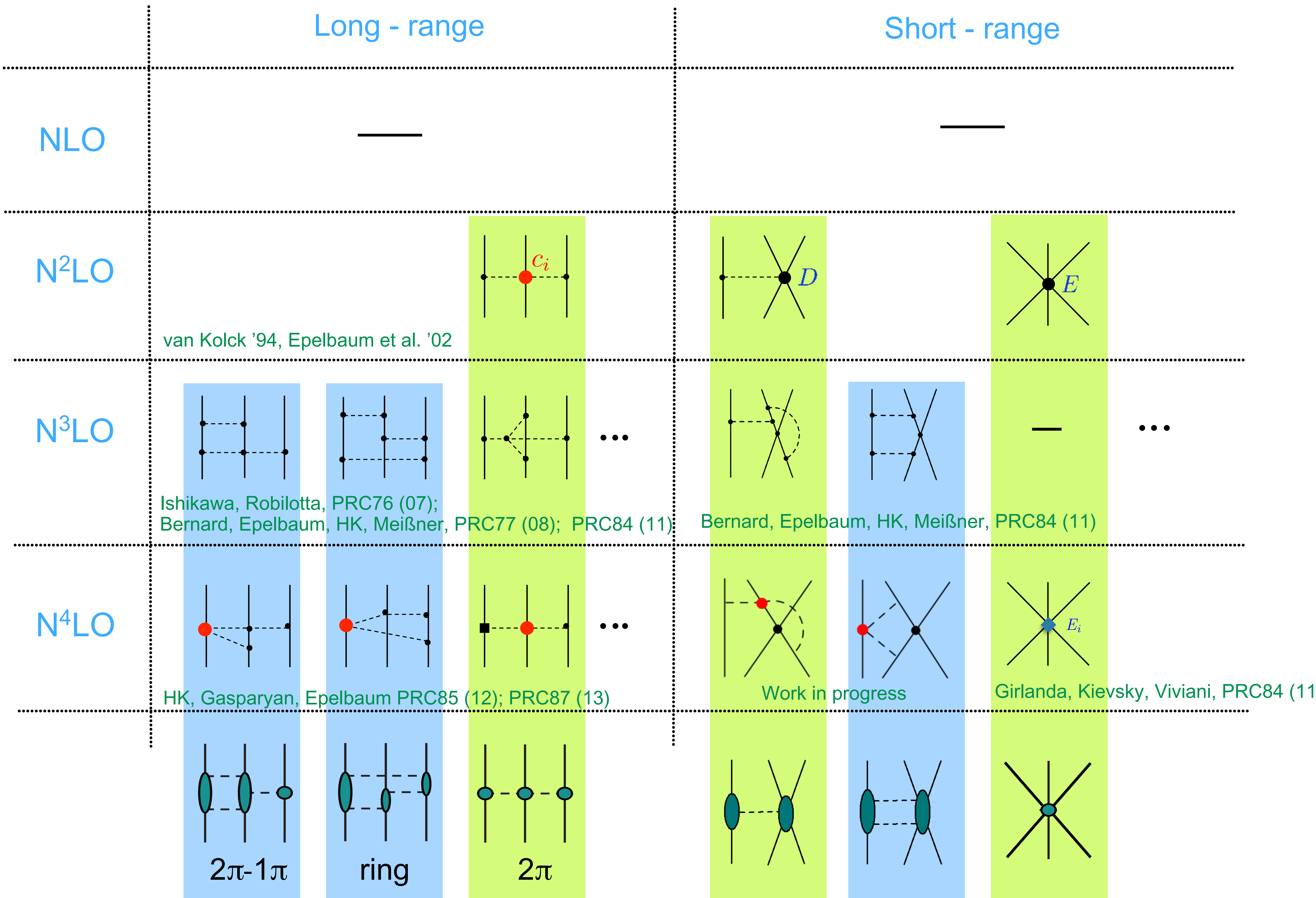


Predictions for the np:

- Differential cross section $d\sigma/d\Omega$
- Vector analyzing power A_y
- Polarization transfer coefficients D and A
- Spin correlation parameters A_{xx} and A_{yy}

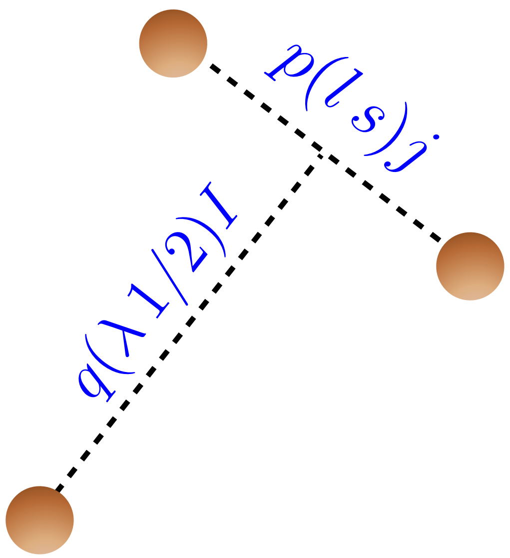
- Good convergence of chiral expansion
- Error bands are consistent with each other
- Excellent agreement with experimental data
- Similar conclusions for results based on other cutoffs which are, however, less stringent due to lower accuracy of such calculations

3NF up to N^{4}LO



Partial wave decomposition

Golak et al. Eur. Phys. J. A 43 (2010) 241



- Faddeev equation is solved in the partial wave basis

$$|p, q, \alpha\rangle \equiv |pq(ls)j(\lambda\frac{1}{2})I(jI)JM_J\rangle\,|(t\frac{1}{2})TM_T\rangle$$

- Too many terms for doing PWD by hand ⟹ Automatization

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{\text{matrix} \sim\ 10^5 \times 10^5} = \int \underbrace{d\hat{p}'\,d\hat{q}'\,d\hat{p}\,d\hat{q}}_{\text{can be reduced to 5 dim. integral}} \sum_{m_l,\dots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p})\,Y_{l',m_l'}(\hat{p}')\,\dots\Big)\underbrace{\langle m'_{s_1}m'_{s_2}m'_{s_3}|V|m_{s_1}m_{s_2}m_{s_3}\rangle}_{\text{depends on spin \& isospin}}$$

- Numerically expensive due to many channels and 5-dim. integration

- PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis

Straightforward implementation of high order 3nf's in many-body calc. within No-Core Shell Model

PWD for local forces

$$\langle m_s'|\vec\sigma\cdot\vec p\,|m_s\rangle = \sum_{\mu=-1}^{1} p\, Y^*_{1\mu}(\hat p)\sqrt{\frac{4\pi}{3}}\langle m_s'|\vec\sigma\cdot\vec e_\mu\,|m_s\rangle$$

← momentum-independent part

$$\langle m'_{s_1} m'_{s_2} m'_{s_3}|V|m_{s_1}m_{s_2}m_{s_3}\rangle = \sum_{\mu' s}\langle m'_{s_1} m'_{s_2} m'_{s_3}|\text{Spin matrices}\,\&\,\vec{\mathrm{e}}_\mu\,'\mathrm{s}|m_{s_1}m_{s_2}m_{s_3}\rangle (Y'_{1\mu}s\,)$$
$$\times\; V((\vec p\,' - \vec p\,)^2, (\vec q\,' - \vec q\,)^2, (\vec p\,' - \vec p\,)\cdot(\vec q\,' - \vec q\,))$$

can be reduced to 3 dim. integral

$$\langle p' q' \alpha'|V|p\,q\,\alpha\rangle = \sum_{m_l\ldots}(\,\text{CG coeffs.})\int d\hat p' d\hat q' d\hat p\, d\hat q\, Y^*_{l_1'm_1'}(\hat p'\,)Y^*_{l_2'm_2'}(\hat q'\,)Y^*_{l_1 m_1}(\hat p\,)Y^*_{l_2 m_2}(\hat q\,)$$
$$\times\; V((\vec p\,' - \vec p\,)^2, (\vec q\,' - \vec q\,)^2, (\vec p\,' - \vec p\,)\cdot(\vec q\,' - \vec q\,))$$

Unregularized 3NF matrix elements can be used to generate locally regularized 3NFs

$\langle p' q' \alpha'|V|p\,q\,\alpha\rangle \to \sum_n \langle p' q' \alpha'|V|n\rangle\langle n|R|p\,q\,\alpha\rangle$ with $\langle p' q' \alpha'|R|p\,q\,\alpha\rangle$ matrix element of local regulator

Hebeler, HK, Epelbaum, Golak, Skibinski arXiv:1502.02977[nucl-th]

Update on the status of matrix elements → talk by Kai Hebeler

Summary

- Chiral NN forces with local regulators are studied up to N^4LO
- NPWA data used to fit 27 LECs
- Satisfactory description of NN observables provides a strong support for chiral error estimate at fixed cutoff
- Optimized version of PWD for local 3NF's

Outlook

- N^4LO Δ-less/N^3LO-Δ calc. of shorter range part of 3NF

  → Generation of matrix-elements for 3NF's upto N^4LO Δ-less/N^3LO-Δ

  Due to optimized PWD should not cost much