

# Operator evolution for *ab initio* theory of light nuclei

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Theory Workshop 2015



# Introduction

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**But, rigorous calculations require operator evolution!**

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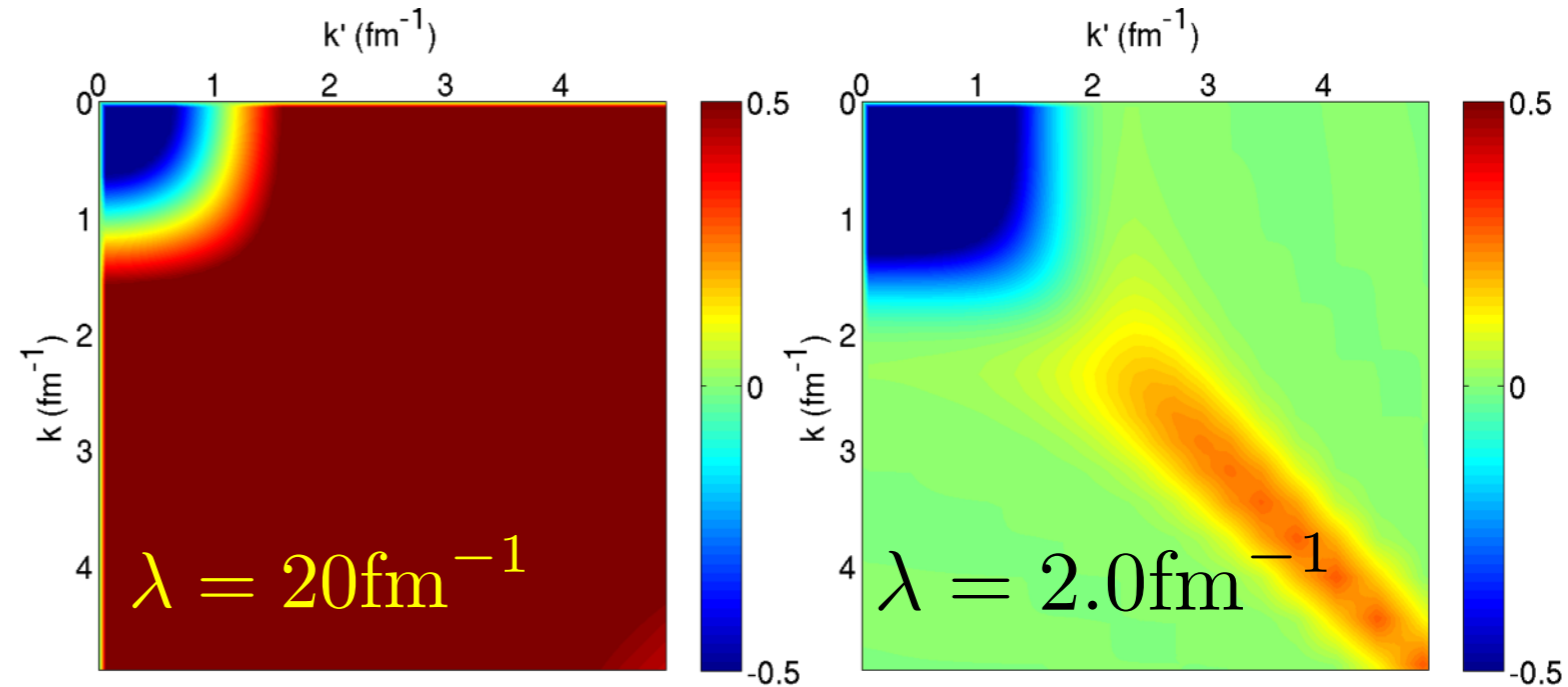
Previous work on operator evolution:

- I. Stetcu *et al.*, PRC **71**, 044325 (2005)  
One and two-body effective operator via OLS
- E. Anderson *et al.*, PRC **82**, 054001 (2010)  
Two-body evolution for the  $A=2$  system.
- M. Schuster *et al.*, PRC **90**, 011301(R) (2014)  
Two- and three-body operator evolution.  
Apply them to the  $A=3$  and  $A=4$  system.

For this work we use the NCSM in Jacobi coordinates with chiral interactions at N3LO for NN and N2LO for NNN

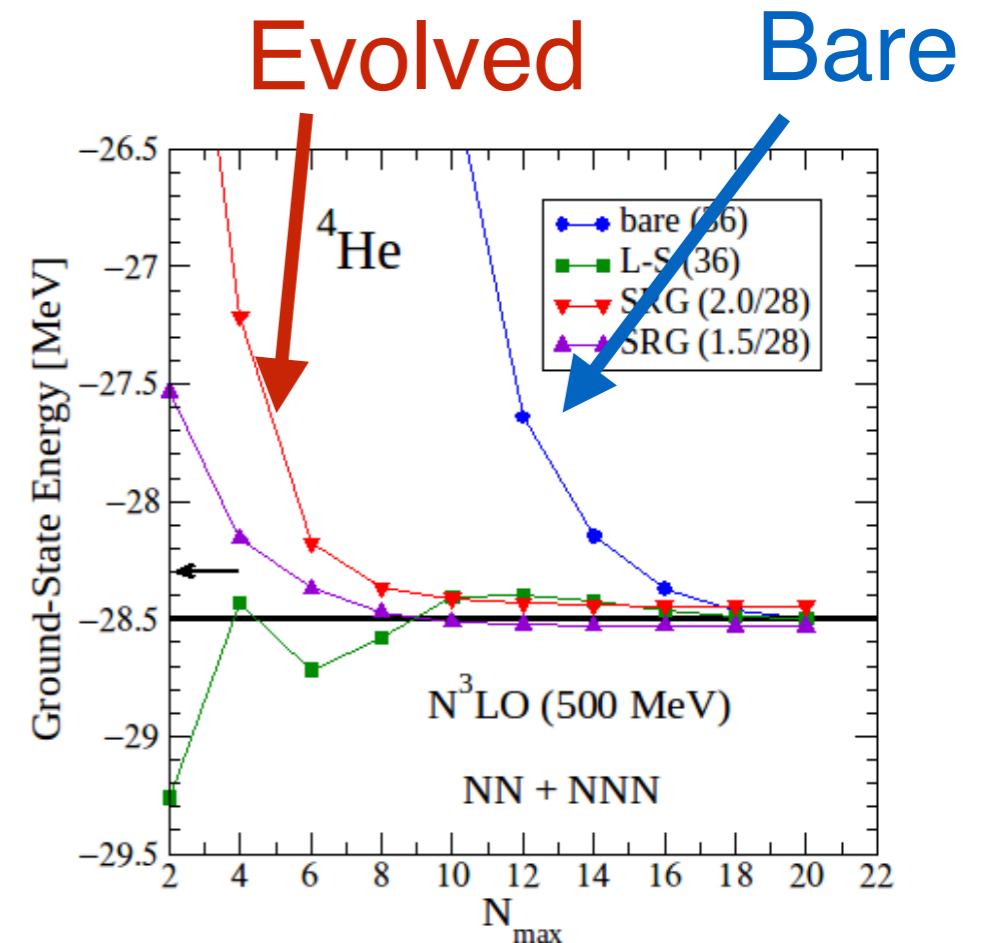
# The similarity renormalization group

Series of unitary transformations on the Hamiltonian that decouple high- and low-momentum parts of the interaction



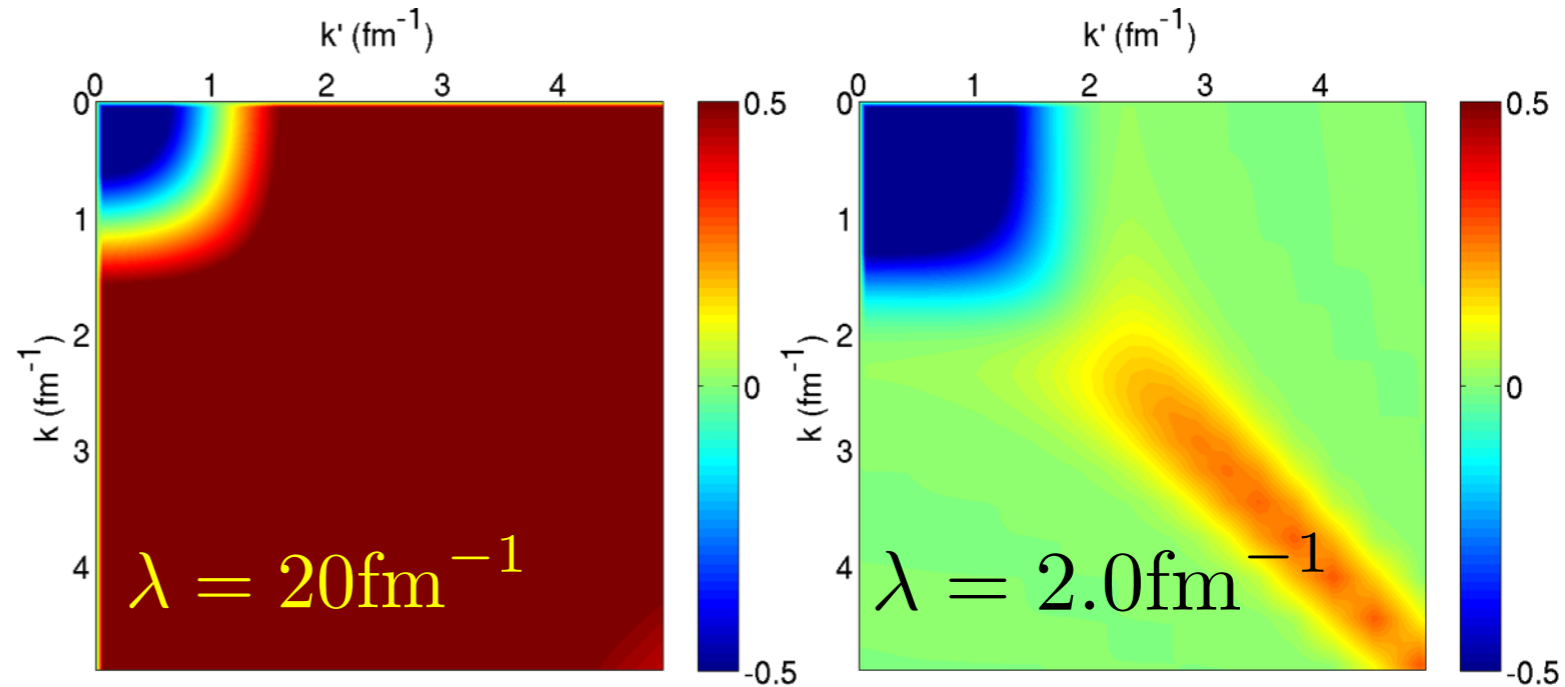
$$\frac{dH_s}{ds} = [[T, H_s], H_s] \quad \lambda = s^{-1/4}$$

- Makes the many-body problem more tractable



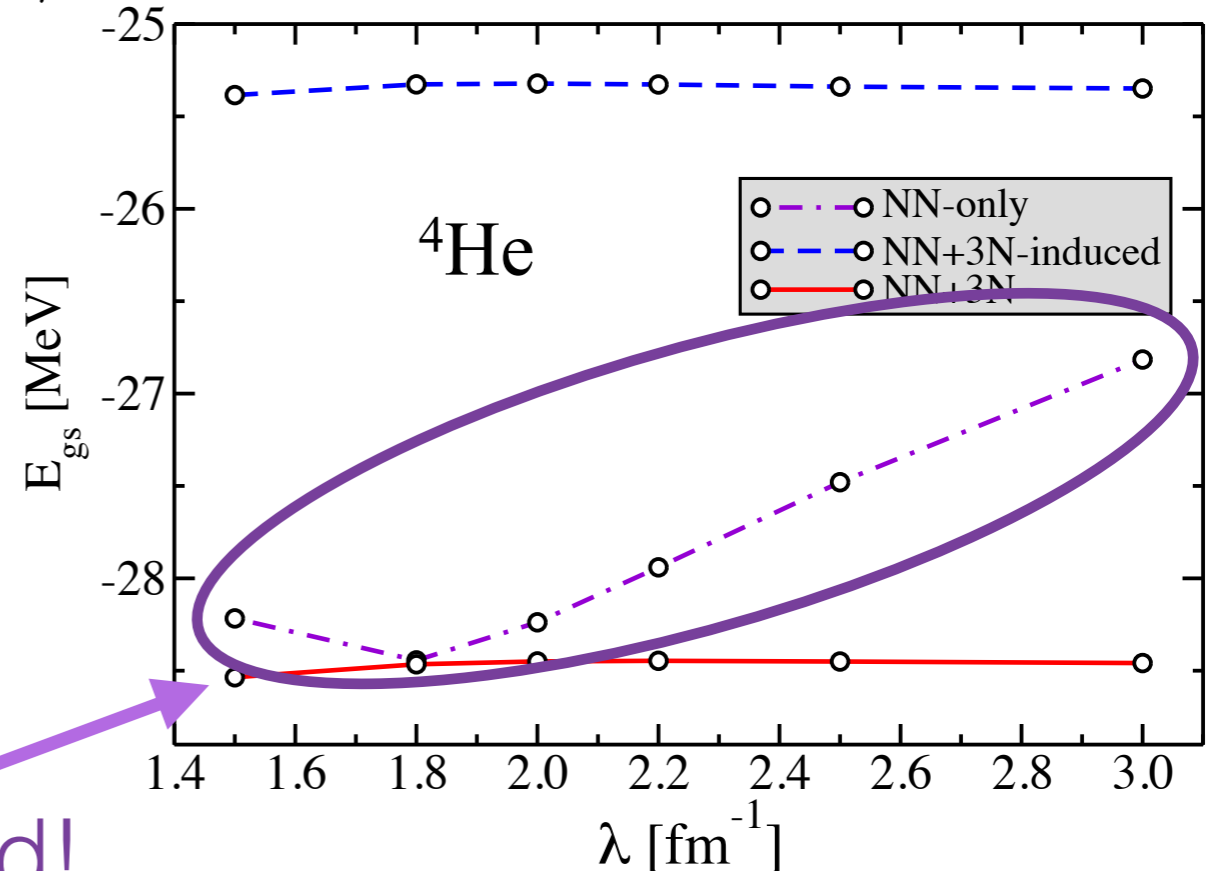
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$$\frac{dH_s}{ds} = [[T, H_s], H_s] \quad \lambda = s^{-1/4}$$

- Makes the many-body problem more tractable
- Induces higher-order terms into the Hamiltonian



No induced terms included!

# Evolving operators

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Hamiltonian eigenvector **before** and **after** transformation

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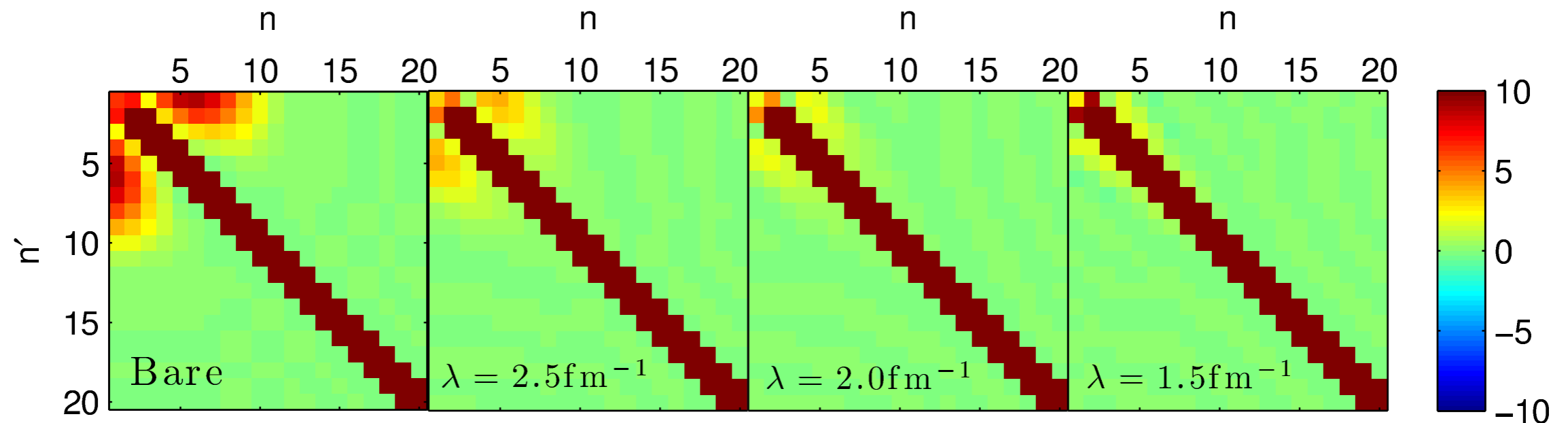
And apply via a matrix product:

$$O_s = U_s O_{s=0} U_s^{\dagger}$$

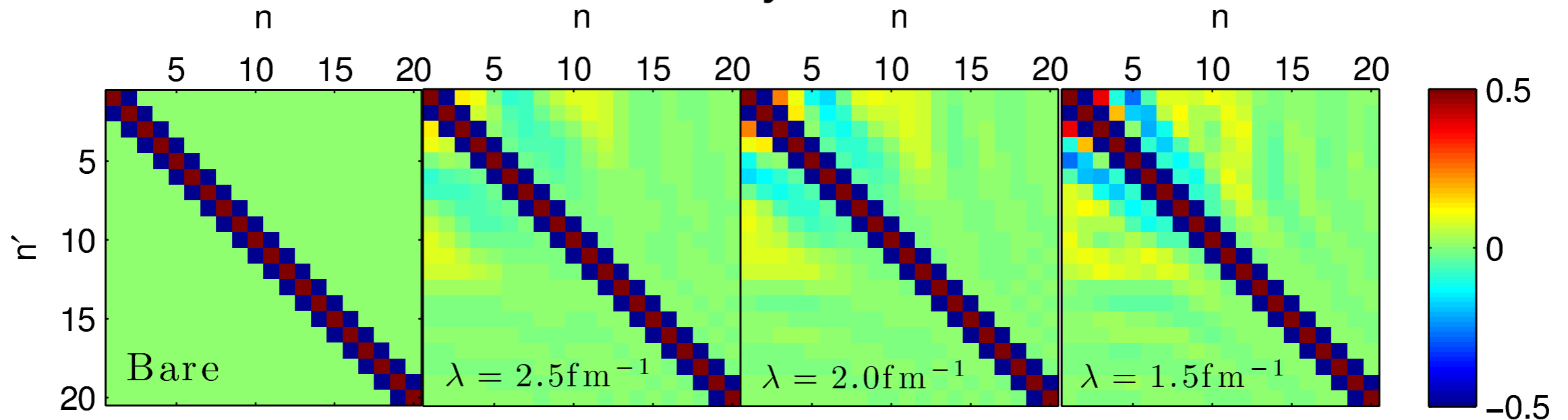


# A look in harmonic oscillator space

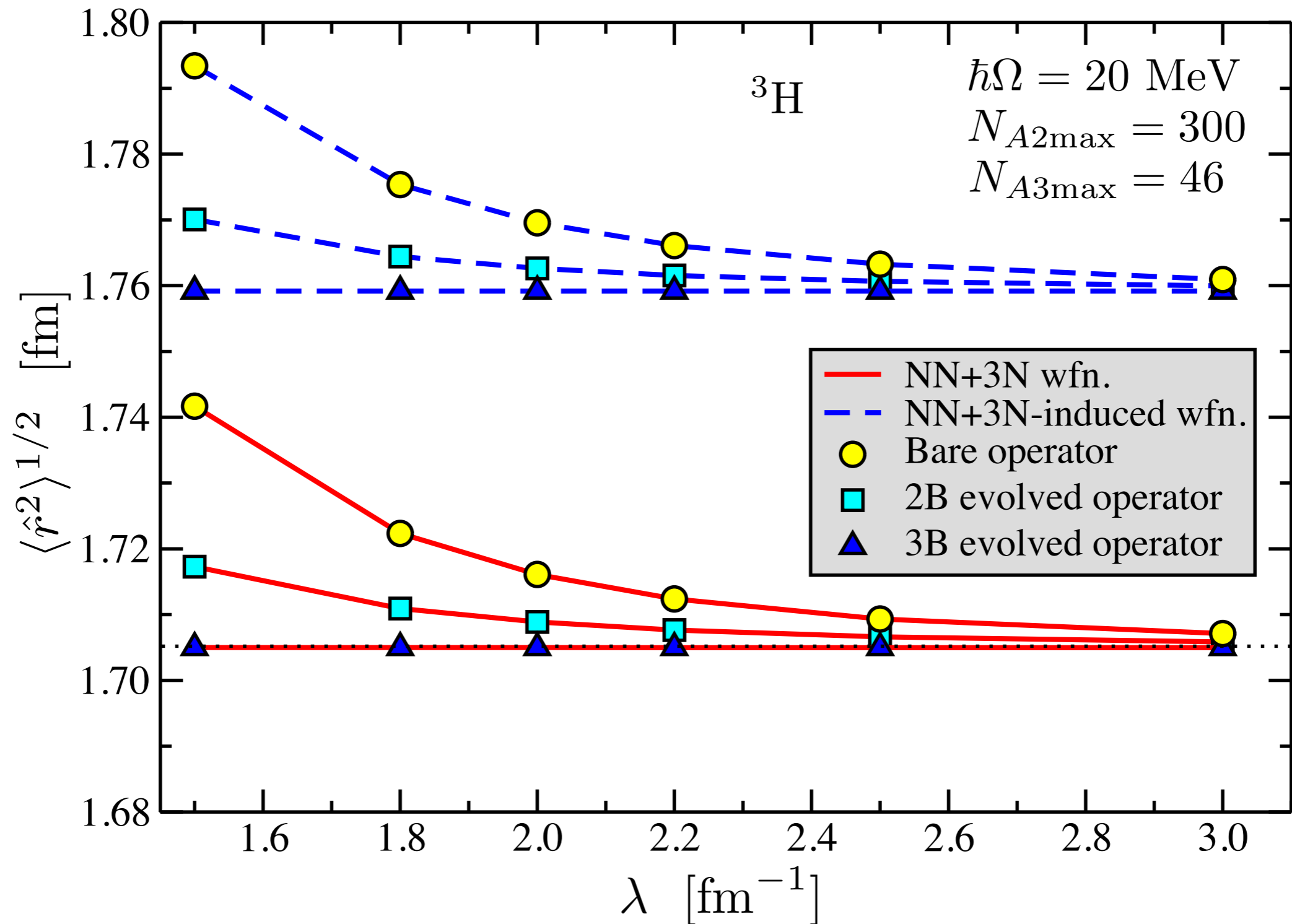
$^3S_1$  Hamiltonian evolution:



$^3S_1$  two-body  $r^2$  evolution:



# Hydrogen-3 rms radius



MS, Sofia Quaglioni, Calvin Johnson, Eric Jurgenson, Petr Navrátil.,  
Phys. Rev. C **90**, 011301(R) (2014)

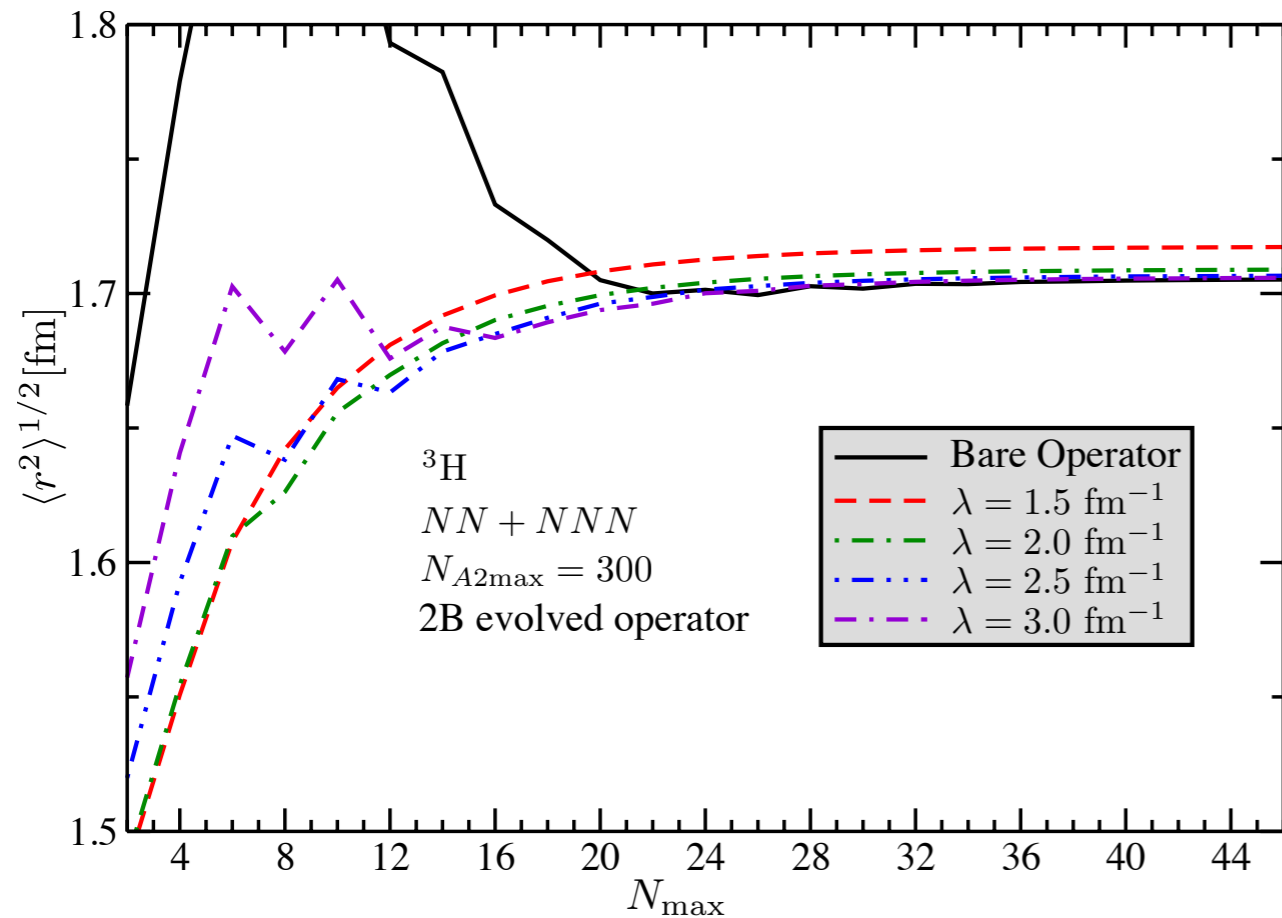
# Hydrogen-3 rms radius

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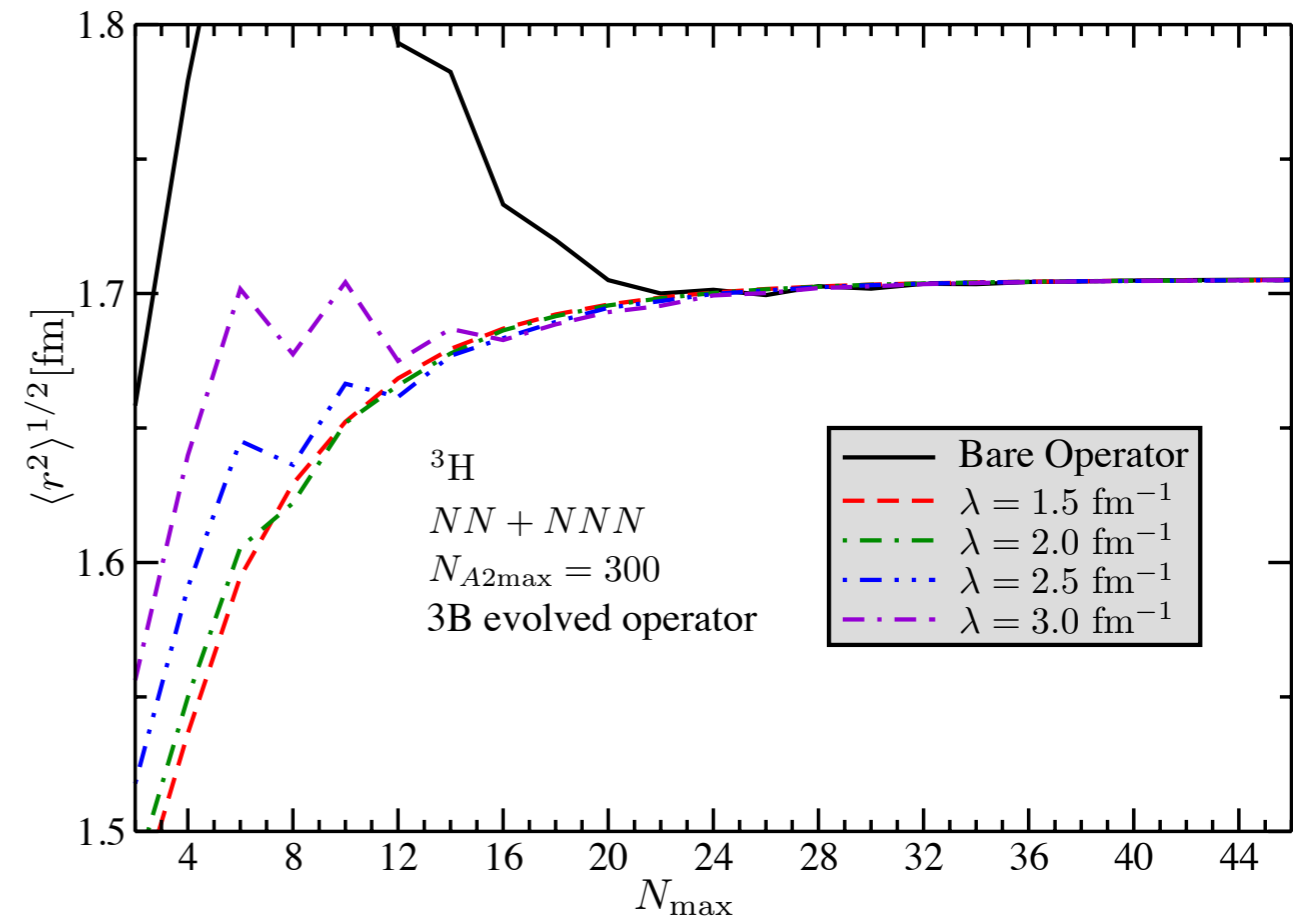
What about convergence?

# Hydrogen-3 rms radius

## 2-Body Evolved Operator



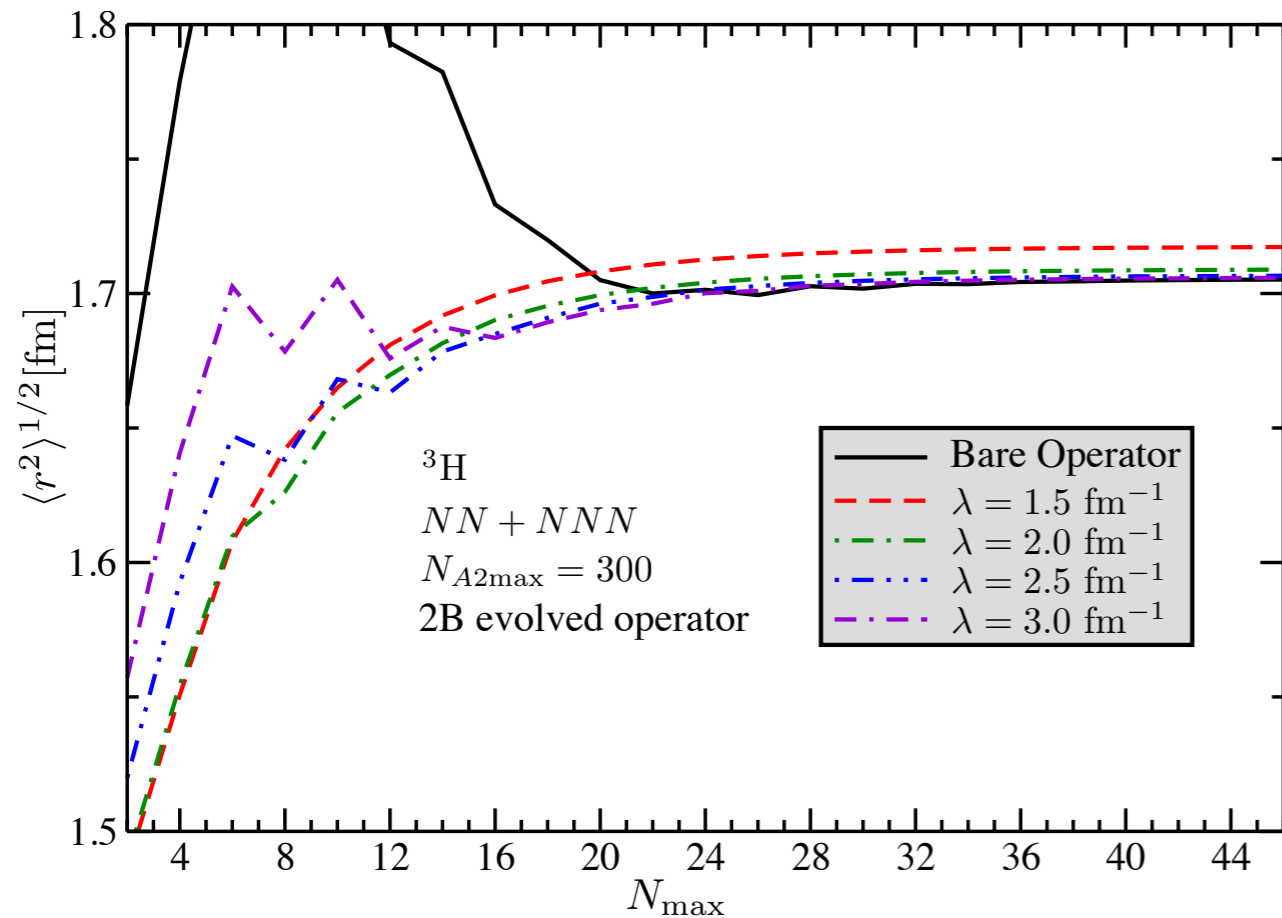
## 3-Body Evolved Operator



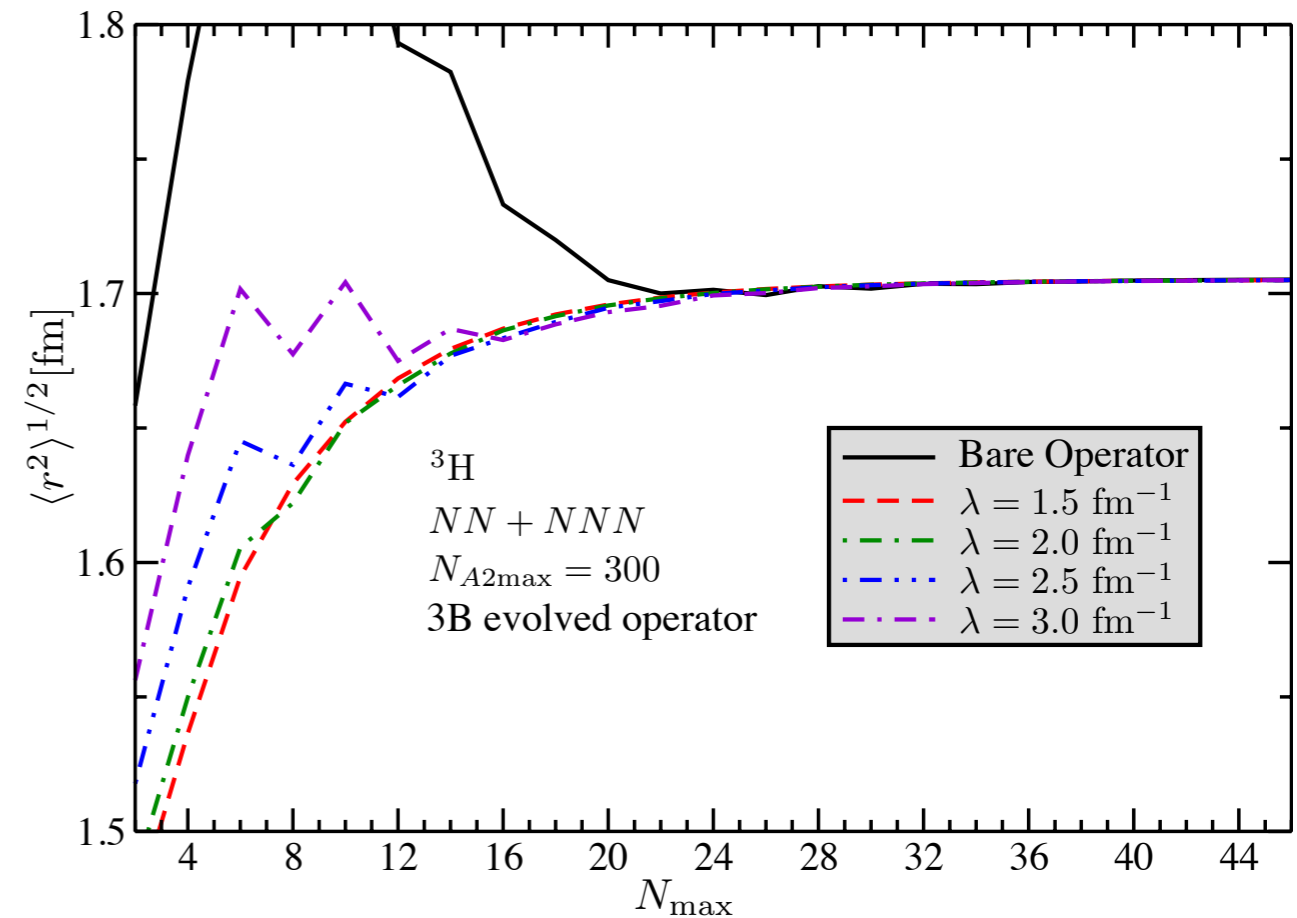
Faster convergence?

# Hydrogen-3 rms radius

## 2-Body Evolved Operator



## 3-Body Evolved Operator



Faster convergence?

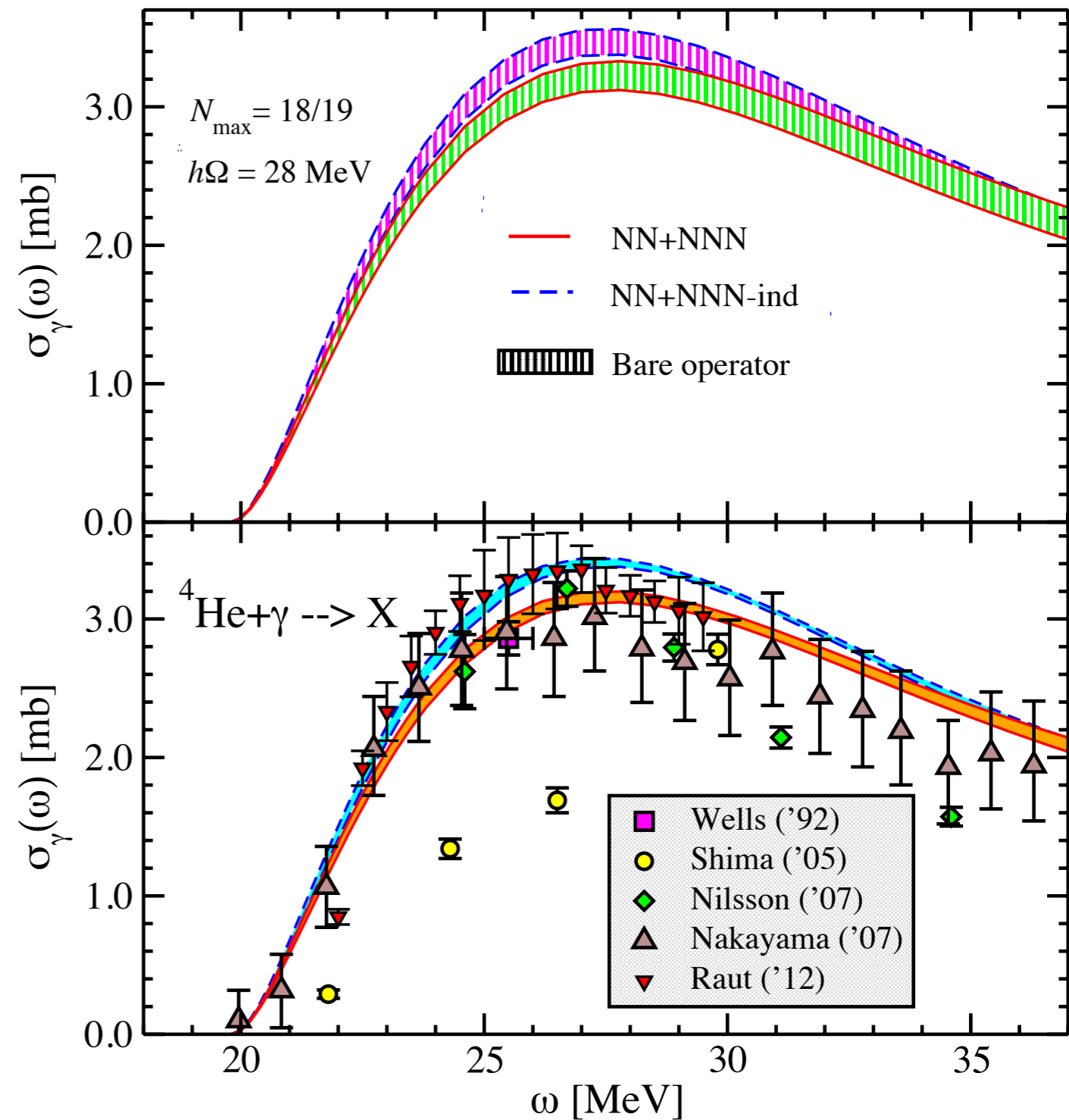
Probably not.

# Total photo-absorption cross-section of ${}^4\text{He}$

Non-scalar operator  
evolution:

$$O_s^{JT} = U_s^i O_{s=0}^{JT} U_s^{f\dagger}$$

Requires the unitary  
transformation from both  
the initial and final states.



# Current work and prospects

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Our goal is to expand this work to larger nuclei and to evolve additional operators.

- ${}^6\text{Li}$  rms radius - On the poster!
- ${}^6\text{He}$  rms radius - Very slow convergence

More work for three-body evolution:

- Applying in larger nuclei
- Non-scalar operators

# Acknowledgements

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This work was performed under the auspices of the US Department of Energy by Lawrence Livermore National Laboratory under Contract No. DE-AC52-07NA27344. Support came from the US DOE/SC/NP (Work Proposal SCW1158), US Department of Energy Grants No. DE-FG02-96ER40985 and No. DE-FC02-07ER41457, and the Natural Sciences and Engineering Research Council of Canada (NSERC) Grant No. 401945-2011. TRIUMF receives funding through the National Research Council Canada. Computing support came from the LLNL institutional Computing Grand Challenge program.

**Thank You!**