THE OHIO STATE UNIVERSITY

Two Posters from Ohio State

Revisiting Many-Body Power Counting in Nuclear Matter

Alex Dyhdalo

(collaborators S. Bogner [MSU], K. Hebeler [TU Darmstadt], rjf)

Recipe for EFT Uncertainty Quantification in Nuclear Physics

Sarah Wesolowski

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Bayesian Uncertainty Collaboration: Knowing Errors in Your EFT (BUCKEYE)

U.S. DEPARTMENT OF ENERGY

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NUCLEI Nuclear Computational Low-Energy Initiative

LENPIC

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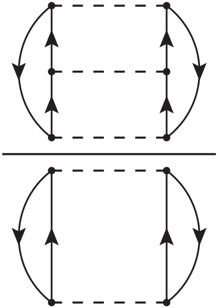
Overview

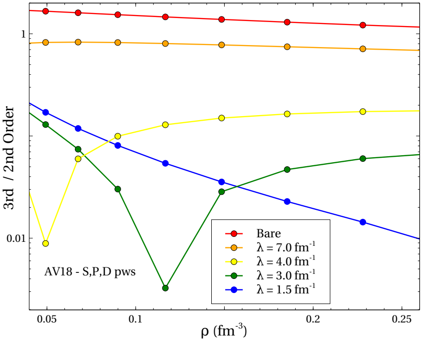
- Goal: Develop power counting estimates and test MBPT for uniform nuclear matter using low-momentum interactions
- (Some of the) Questions and issues
  - How does power counting vary with RG evolution?
  - How do we count many-body forces? What about density-dependent 2-body forces from 3-body terms?
  - How does the many-body power counting relate to the power counting for effective field theory (EFT) in free space?
  - What is the dependence on EFT regulators (e.g., scale and scheme, local vs. nonlocal)?
- Plan: Revisit Brueckner-Bethe-Goldstone (BBG) theory
  - Reference state: $\widehat{H} = \widehat{H}_0 + \widehat{H}_I \Longrightarrow |\Phi\rangle : \widehat{H}_0|\Phi\rangle = E_0|\Phi\rangle$
  - Energy of the interacting system from Goldstone's Theorem

$$E = E_0 + \langle\Phi|\, \widehat{H}_I \sum_{n=0}^{\infty} \left(\frac{1}{E_0 - \widehat{H}_0} \widehat{H}_I \right)^n |\Phi\rangle_{\text{Connected}}$$

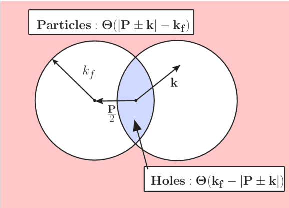
  - Develop basic estimates for Goldstone diagrams

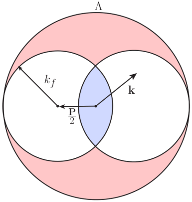
Look at ratio of Goldstone diagrams





- Large cutoff (e.g., AV18) requires G-matrix (ladder) resummation
- Understand power counting from phase space with different cutoffs





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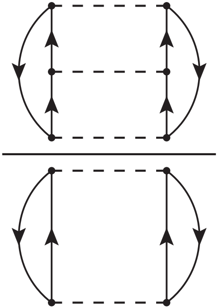

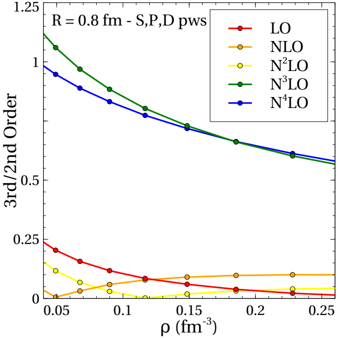

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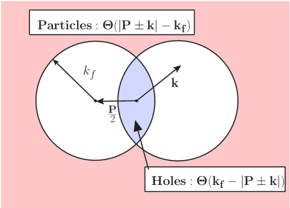

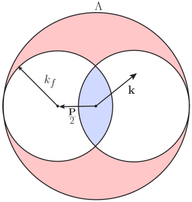

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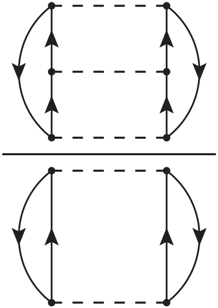

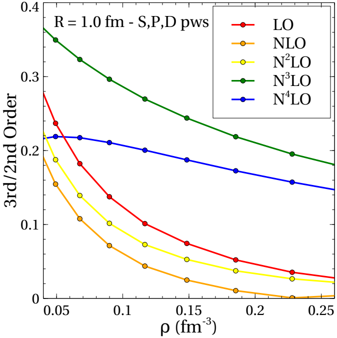

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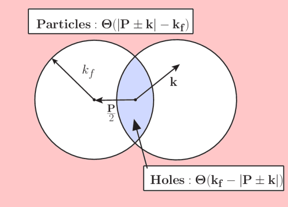

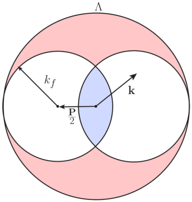

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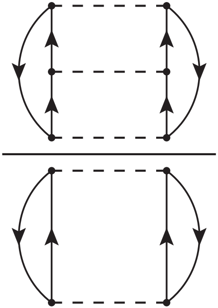

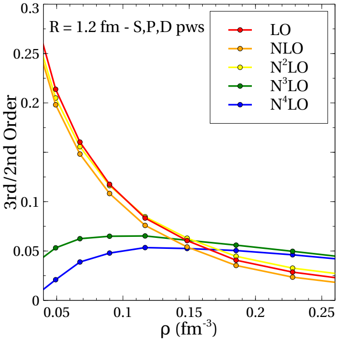

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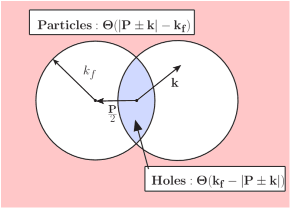
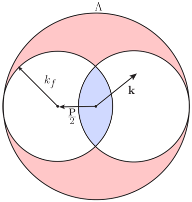
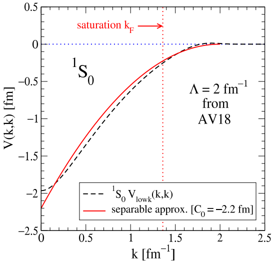
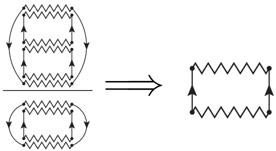




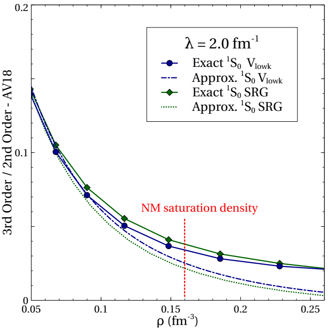
Making rough estimates

- Approximate potential as separable, average Pauli and momenta



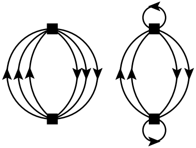


$$\frac{E_{\mathrm{pp}}^{(n+1)}}{E_{\mathrm{pp}}^{(n)}} \approx \frac{m^*}{m} \int \frac{d^3k}{(2\pi)^3}\, \overline{Q}(P_{\mathrm{av}}, k)\, \frac{\langle \mathbf{k}|V|\mathbf{k}\rangle}{k_{\mathrm{av}}^2 - k^2}$$



- For evolved potentials, estimates at about the 20% level at ρ_{sat}
- Unitary gas exception to suppression: $E_{\mathrm{pp}}^{(n+1)}/E_{\mathrm{pp}}^{(n)} \sim -1$
- Hole-line expansion
  - Old story: resum G-matrix
  - New story: particle *and* hole lines suppressed

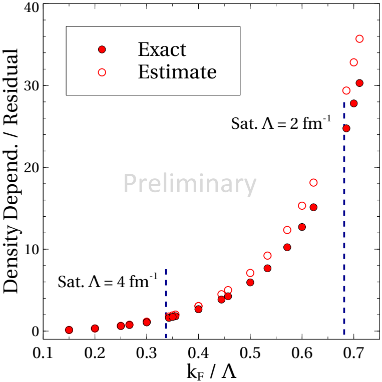
3-Body Example: N2LO Contact Interaction



- Look at ratio of 2nd-order diagrams (cf. NO2B approximation)
- Energy contributions from individual diagrams will be scale/scheme dependent
- Test non-local regulator:

$$f(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \equiv \exp(-[(p_1^2 + p_2^2 + p_3^2 - \mathbf{p}_1 \cdot \mathbf{p}_2 - \mathbf{p}_2 \cdot \mathbf{p}_3 - \mathbf{p}_1 \cdot \mathbf{p}_3)/3\Lambda_{3N}^2]^n)$$

- For estimate: average momenta for energy denominators, treat regulators and theta functions as geometric phase-space integral
- For different cutoffs, ratio is a pure function of k_{F}/Λ
- Cutoff squeezes phase space for full 3-body interactions
- How do these ratios change for a local regulator? (And many other ?'s)



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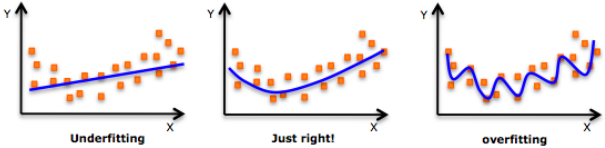
Why is a Bayesian framework well suited to EFT errors?



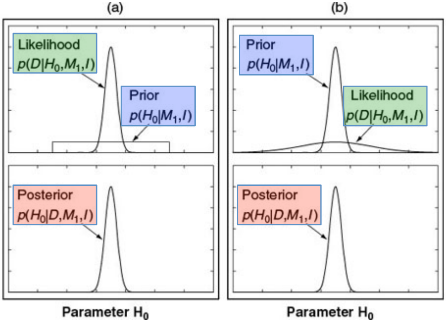
- What can happen in an EFT fit? What are the complications?
  - More statistical power if larger energy range included, but EFT is less accurate approaching breakdown scale $\Longrightarrow$ Where to fit?
  - How do we combine data and theory uncertainties?
  - Is the EFT working? Are there too few or too many LECs?
- Bayesian probabilities: pdf is a measure of state of our knowledge
  - Ideal for treating systematic errors (such as theory errors!)
  - Assumptions (or expectations) about EFT encoded in prior pdfs
  - Can predict values of observables with credibility intervals (errors)
  - Incorporates usual statistical tools (e.g., covariance analysis)
- For EFT, makes explicit what is usually implicit, allowing assumptions to be applied consistently, tested, and modified given new information

Limiting cases in applying Bayes' theorem

Suppose we are fitting a parameter H_0 to some data D given a model M_1 and some information (e.g., about the data or the parameter)

Bayes' theorem tells us how to find the posterior distribution of H_0:

$$\mathrm{pr}(H_0|D, M_1, I) = \frac{\mathrm{pr}(D|H_0, M_1, I) \times \mathrm{pr}(H_0|M_1, I)}{\mathrm{pr}(D|I)}$$

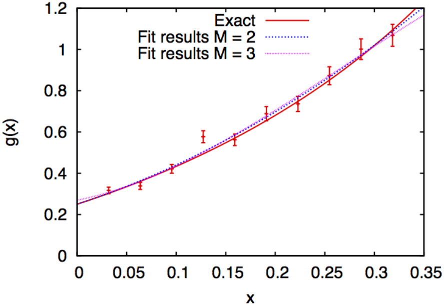


[From P. Gregory, "Bayesian Logical Data Analysis for the Physical Sciences"]

Special cases:
(a) If the data is overwhelming, the prior has no effect on the posterior
(b) If the likelihood is unrestrictive, the posterior returns the prior

Diagnostic tools (applications to toy models)

- Example: $g(x) = (1/2 + \tan(\pi x/2))^2 \Longrightarrow$ "model" $\approx a_0 + a_1 x + a_2 x^2 + \mathcal{O}(x^3)$
- Likelihood $\propto e^{-\chi^2}$ as usual; prior could be uniform or for natural coefficients



Without prior

M	χ^2/dof	a_0	a_1
true		0.25	1.57
1	2.24	0.203 ± 0.01	2.55 ± 0.11
2	1.64	0.25 ± 0.02	1.6 ± 0.4
3	1.85	0.27 ± 0.04	0.95 ± 1.1
4	1.96	0.33 ± 0.07	-1.9 ± 2.7
5	1.39	0.57 ± 0.3	-14.8 ± 6.9

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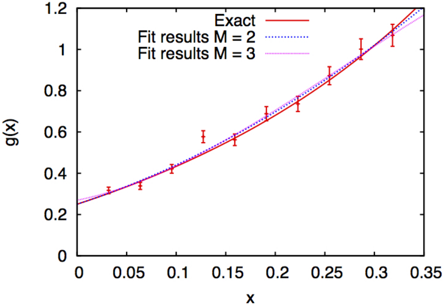

With Gaussian prior ($R = 5$)

M	a_0	a_1	a_2
true	0.25	1.57	2.47
2	0.25 ± 0.02	1.63 ± 0.4	3.2 ± 1.3
3	0.25 ± 0.02	1.65 ± 0.5	3 ± 2.3
4	0.25 ± 0.02	1.64 ± 0.5	3 ± 2.4
5	0.25 ± 0.02	1.64 ± 0.5	3 ± 2.4

- Marginalize over omitted higher-order terms $\Longrightarrow$ use all data stably
- Prior on naturalness suppresses overfitting by limiting parameter play-offs

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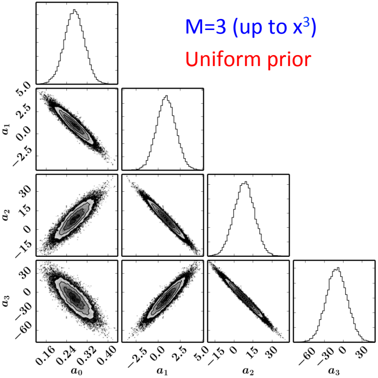

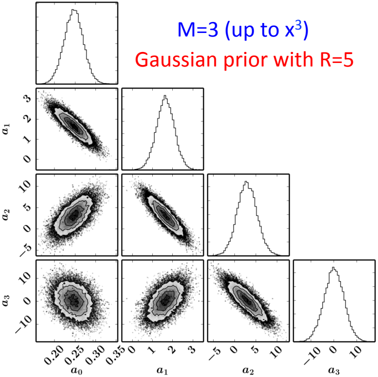

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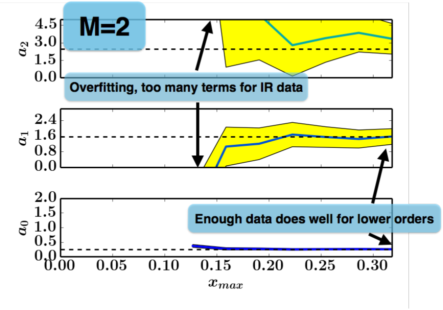

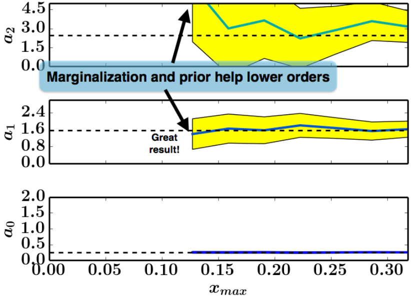

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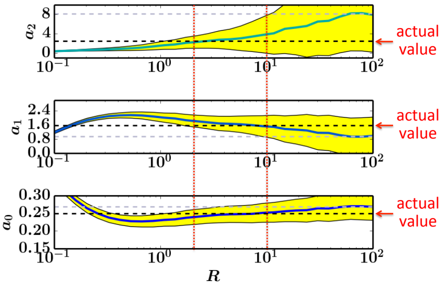

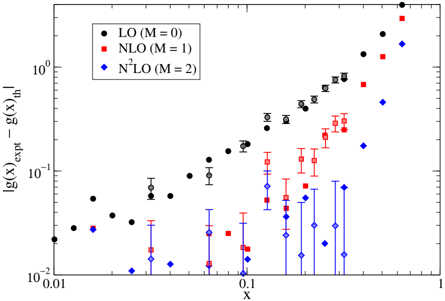

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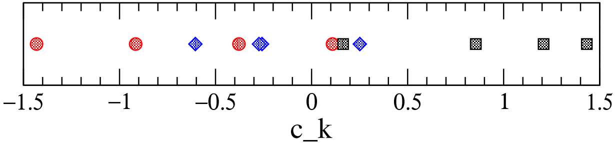
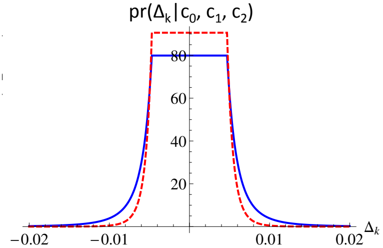
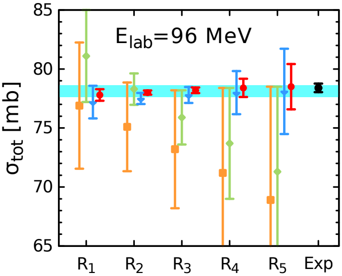
Can we justify EKM errors in Bayesian framework?

- Basic assumption: c_n's the same size in
  $\sigma_{np} \approx \sigma_0(1 + c_2 Q^2 + c_3 Q^3 + \cdots)$ with $Q = \{p, m_\pi\}/600\,\text{MeV}$



- Yes! So apply set A (left plot blue) to σ_{np} at $E_{\text{lab}} = 96\,\text{MeV} \Longrightarrow Q \approx 1/3$





- 68% credibility interval widths are 4.0, 1.2, 0.40, 0.13 mb $\Longrightarrow$ agrees!

Summary: Goals of theory UQ for EFT calculations

- Reflect *all* sources of uncertainty in an EFT prediction
  $\Longrightarrow$ likelihood or prior for each $\Longrightarrow$ Next: apply to NN fits
- Compare theory predictions and experimental results statistically
  $\Longrightarrow$ error bands as credibility intervals $\Longrightarrow$ Next: model problem blind tests; then apply to $A > 2$ nuclear observables
- Distinguish uncertainties from IR vs. UV physics
  $\Longrightarrow$ separate priors
- Guidance on how to extract EFT parameters (LECs)
  $\Longrightarrow$ Bayes propagates new info (e.g., will an additional or better measurement or lattice calculation help?)
  $\Longrightarrow$ Next: M_{N} from lattice
- Test whether EFT is working as advertised— do our predictions exhibit the anticipated systematic improvement?
  $\Longrightarrow$ trends of credibility interval $\Longrightarrow$ Future: model selection

The Bayesian framework lets us consistently achieve our UQ goals!

Advertisement: INT Workshop in 2016

Bayesian Methods in Nuclear Physics (INT-16-2a)

June 13 to July 8, 2016

R.J. Furnstahl, D. Higdon, N. Schunck, A.W. Steiner

A four-week workshop to explore how Bayesian inference can enable progress on the frontiers of nuclear physics and open up new directions for the field.

Among our goals are to

- facilitate cross communication, fertilization, and collaboration on Bayesian applications among the nuclear sub-fields;
- provide the opportunity for nuclear physicists who are unfamiliar with Bayesian methods to start applying them to new problems;
- learn from the experts about innovative and advanced uses of Bayesian statistics, and best practices in applying them;
- learn about advanced computational tools and methods;
- critically examine the application of Bayesian methods to particular physics problems in the various subfields.

Existing efforts using Bayesian statistics will continue to develop over the next two years, but Summer 2016 will be an opportune time to bring the statisticians and nuclear practitioners together. [See computingnuclei.org for more meetings]