In-Medium SRG: Recent Developments and A Look Ahead

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Outline

- The In-Medium SRG
- Ground States of Closed- and Open-Shell Nuclei
- IM-SRG + Shell Model for Excited States
- Next Steps
- Conclusions

The In-Medium SRG

S. K. Bogner, H. H., T. Morris, A. Schwenk, and K. Tuskiyama, to appear in Phys. Rept.

H. H., S. K. Bogner, S. Binder, A. Calci, J. Langhammer, R. Roth, and A. Schwenk, Phys. Rev. C **87**, 034307 (2013)

K. Tsukiyama, S. K. Bogner, and A. Schwenk, Phys. Rev. Lett. **106**, 222502 (2011)

Similarity Renormalization Group

Basic Concept

continuous unitary transformation of the Hamiltonian to band-diagonal form w.r.t. a given "uncorrelated" many-body basis

- flow equation for Hamiltonian $H(s) = U(s)HU^\dagger(s)$:

$$\frac{d}{ds}H(s) = \left[\eta(s), H(s)\right], \quad \eta(s) = \frac{dU(s)}{ds}U^\dagger(s) = -\eta^\dagger(s)$$

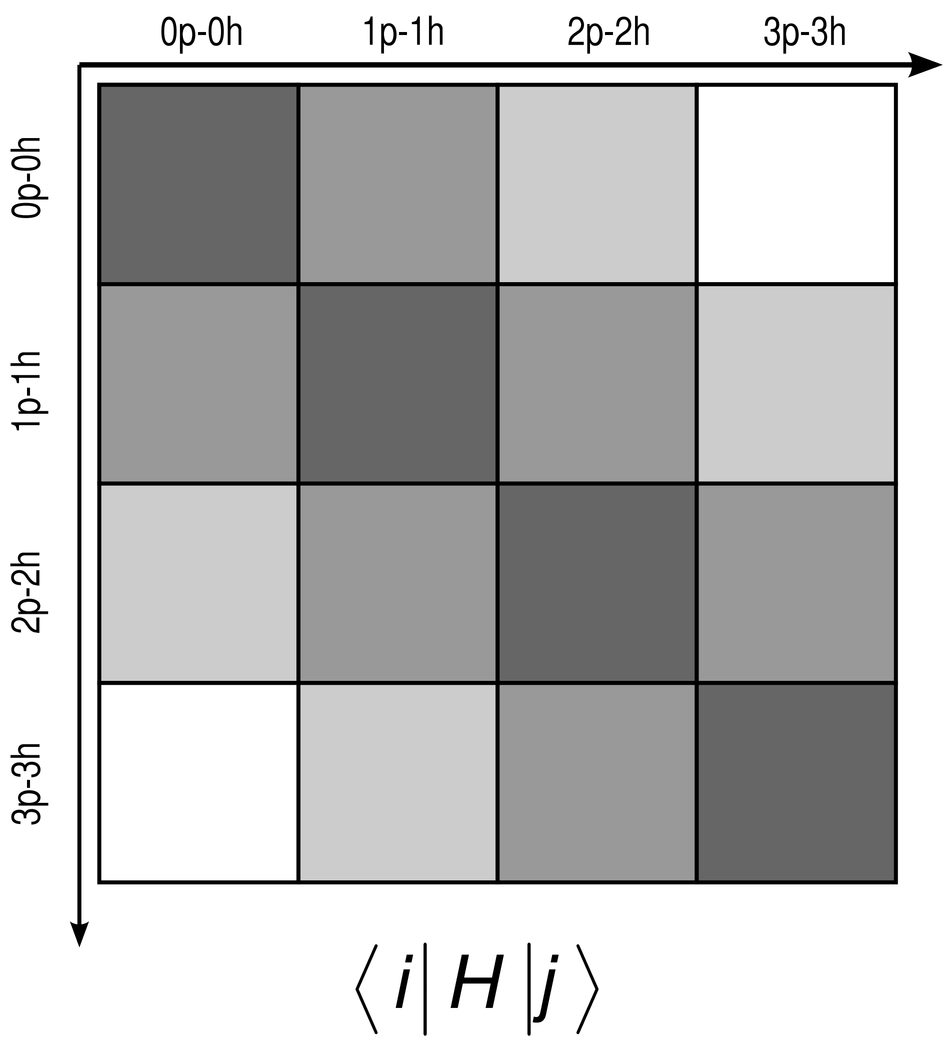
- choose $\eta(s)$ to achieve desired behavior, e.g.,

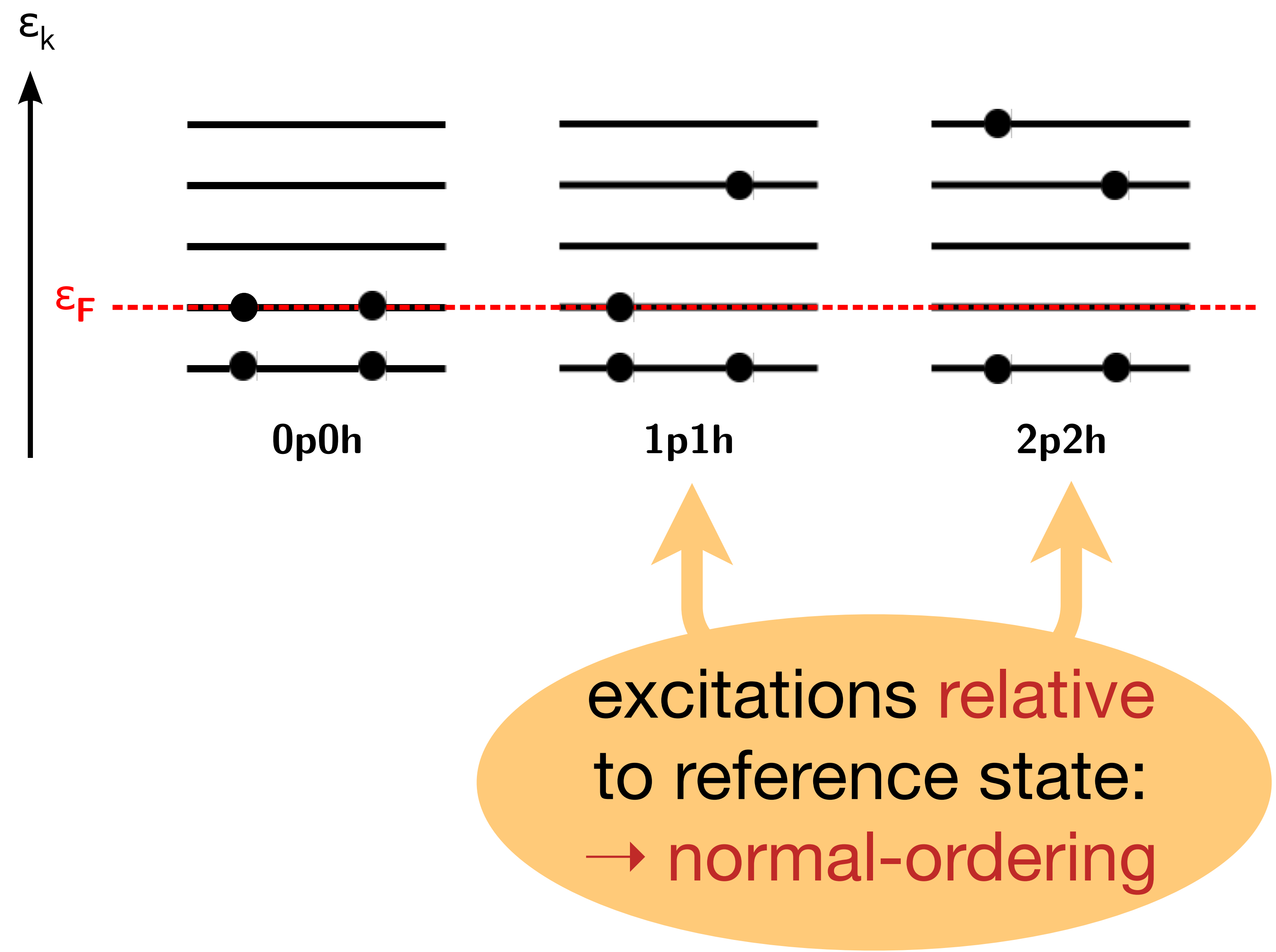
$$\eta(s) = \left[H_d(s), H_{od}(s)\right]$$

  to suppress (suitably defined) off-diagonal Hamiltonian

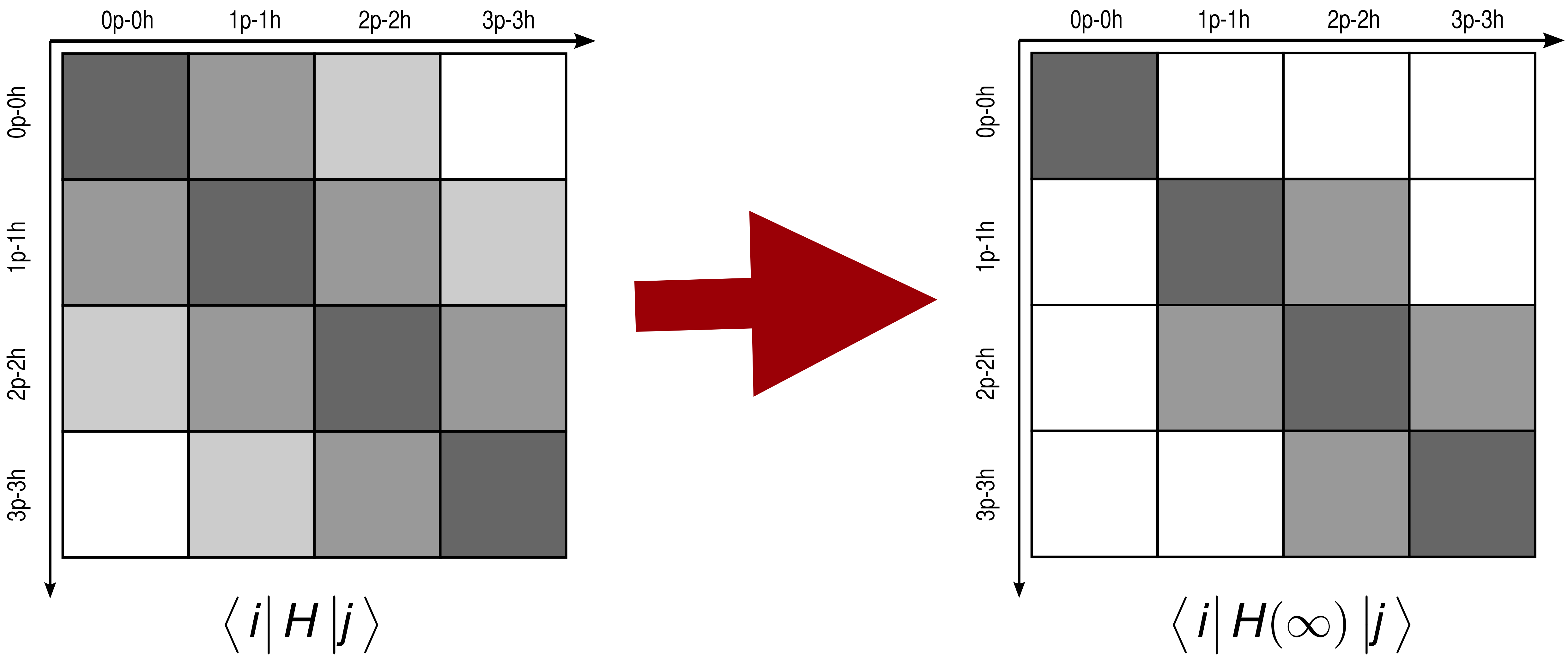
- consistent evolution for all observables of interest

Decoupling in A-Body Space





Decoupling in A-Body Space



aim: decouple reference state $|\Phi\rangle$ (0p-0h) from excitations

Normal Ordering

- second quantization: $A^{k_1 \dots k_N}_{l_1 \dots l_N} = a^\dagger_{k_1} \dots a^\dagger_{k_N} a_{l_N} \dots a_{l_1}$

- particle- and hole density matrices:

$$\lambda^k_l = \left\langle \Phi \right| A^k_l \left| \Phi \right\rangle \longrightarrow n_k \delta^k_l \,, \quad n_k \in \{0,1\}$$
$$\xi^k_l = \lambda^k_l - \delta^k_l \qquad \longrightarrow -\bar{n}_k \delta^k_l \equiv -(1-n_k)\delta^k_l$$

- define normal-ordered operators recursively:

$$A^{k_1 \dots k_N}_{l_1 \dots l_N} = :A^{k_1 \dots k_N}_{l_1 \dots l_N}: + \lambda^{k_1}_{l_1} :A^{k_2 \dots k_N}_{l_2 \dots l_N}: + \text{singles}$$
$$+ \left(\lambda^{k_1}_{l_1}\lambda^{k_2}_{l_2} - \lambda^{k_1}_{l_2}\lambda^{k_2}_{l_1}\right) :A^{k_3 \dots k_N}_{l_3 \dots l_N}: + \text{doubles} + \dots$$

- algebra is simplified significantly because

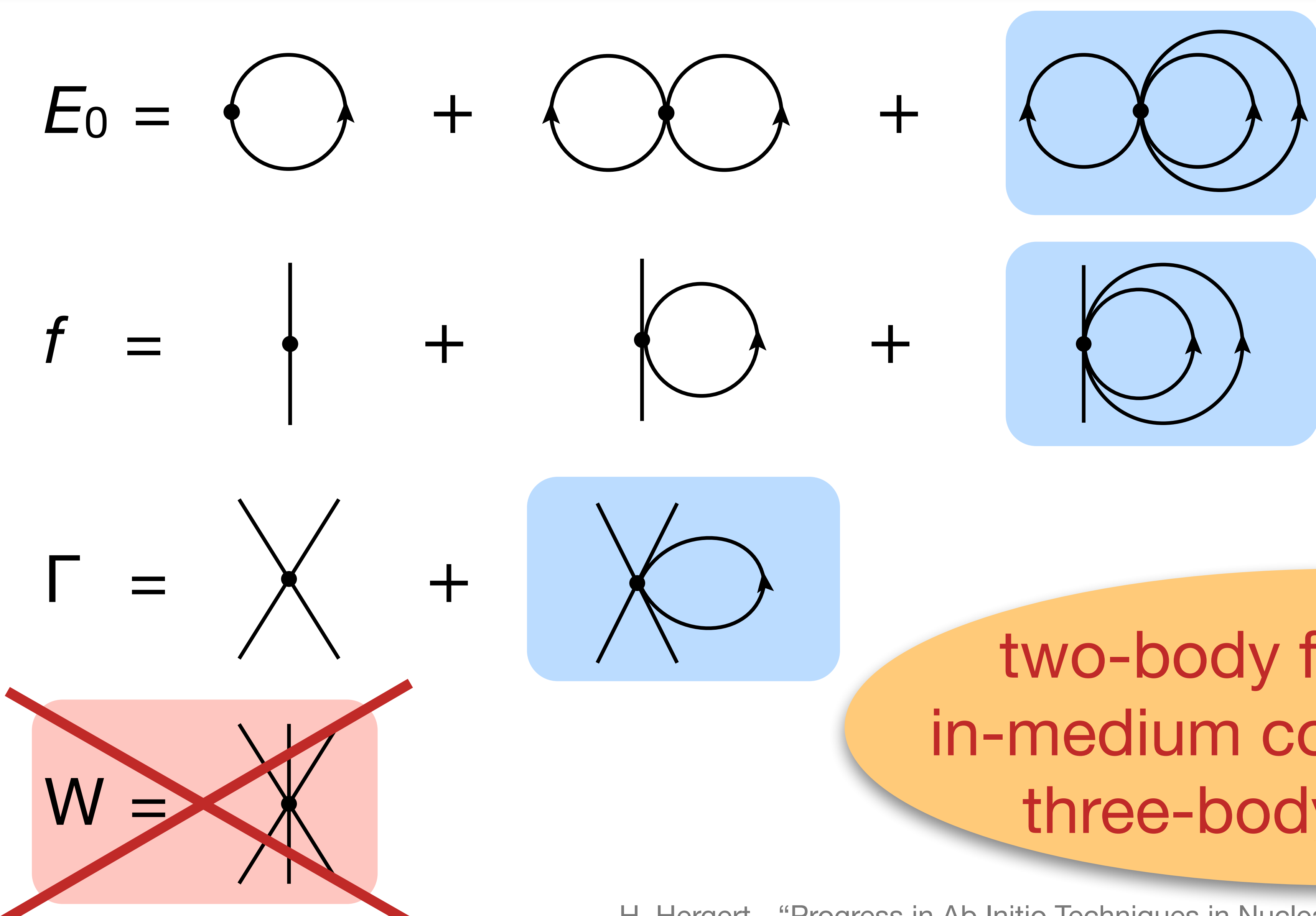
$$\left\langle \Phi \right| :A^{k_1 \dots k_N}_{l_1 \dots l_N}: \left| \Phi \right\rangle = 0$$

- Wick's theorem gives simplified expansions (fewer terms!) for products of normal-ordered operators

Normal-Ordered Hamiltonian

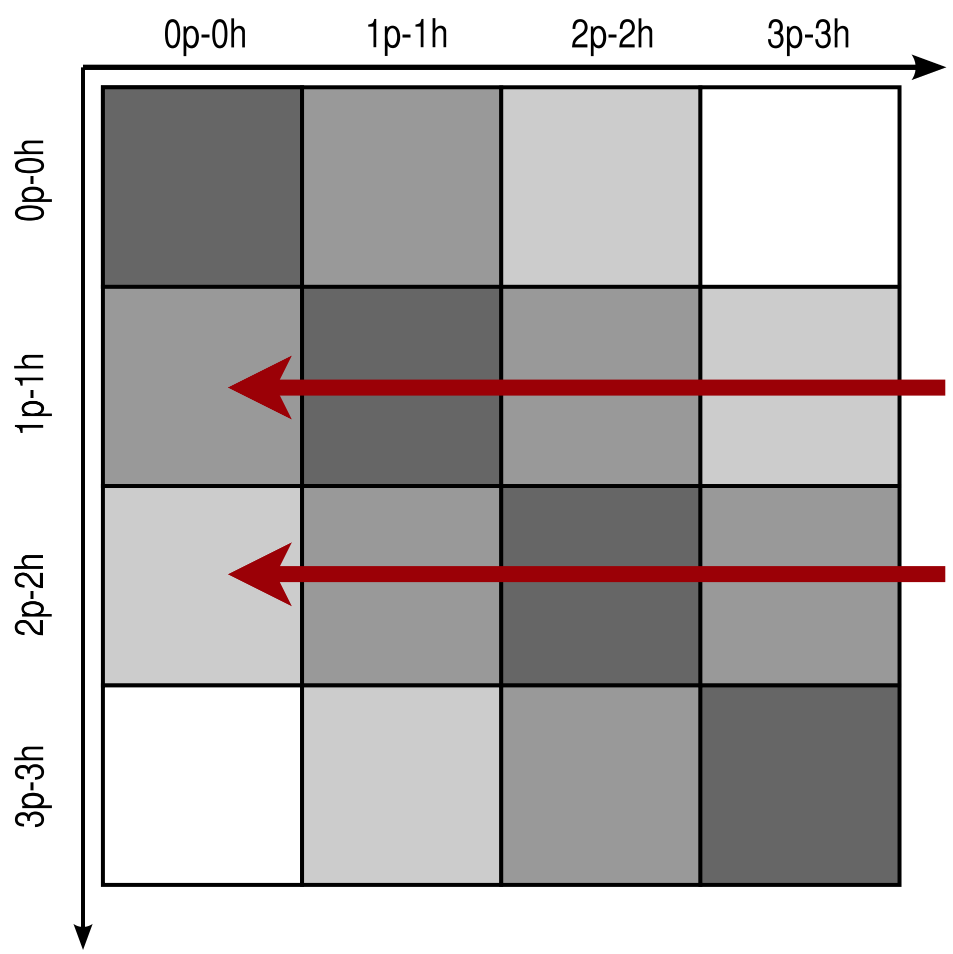
Normal-Ordered Hamiltonian

$$H = E_0 + \sum_{kl} f^k_l : A^k_l : + \frac{1}{4} \sum_{klmn} \Gamma^{kl}_{mn} : A^{kl}_{mn} : + \frac{1}{36} \sum_{ijklmn} W^{ijk}_{lmn} : A^{ijk}_{lmn} :$$



two-body formalism with in-medium contributions from three-body interactions

Choice of Generator



$$\left\langle {}^{p}_{h} \middle| H \middle| \Psi \right\rangle = \sum_{kl} f^k_l \left\langle \Psi \middle| : A^h_p :: A^k_l : \middle| \Psi \right\rangle = -n_h \bar{n}_p f^p_h$$

$$\left\langle {}^{pp'}_{hh'} \middle| H \middle| \Psi \right\rangle = \sum_{klmn} \Gamma^{kl}_{mn} \left\langle \Psi \middle| : A^{hh'}_{pp'} :: A^{kl}_{mn} : \middle| \Psi \right\rangle \sim \Gamma^{pp'}_{hh'}$$

- define off-diagonal Hamiltonian (suppressed by IM-SRG flow):

$$H^{od} \equiv f^{od} + \Gamma^{od}\,, \quad f^{od} \equiv \sum_{ph} f^p_h : A^p_h : + \text{H.c.}\,, \quad \Gamma^{od} \equiv \sum_{pp'hh'} \Gamma^{pp'}_{hh'} : A^{pp'}_{hh'} : + \text{H.c.}$$

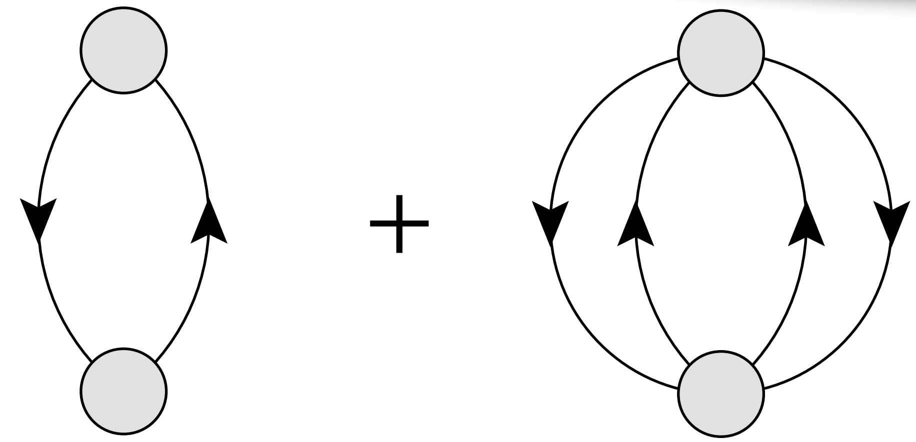
- construct generator, e.g., $\eta^I = \left[H^d, H^{od}\right]$ (Wegner-type)

IM-SRG(2) Flow Equations

0-body Flow

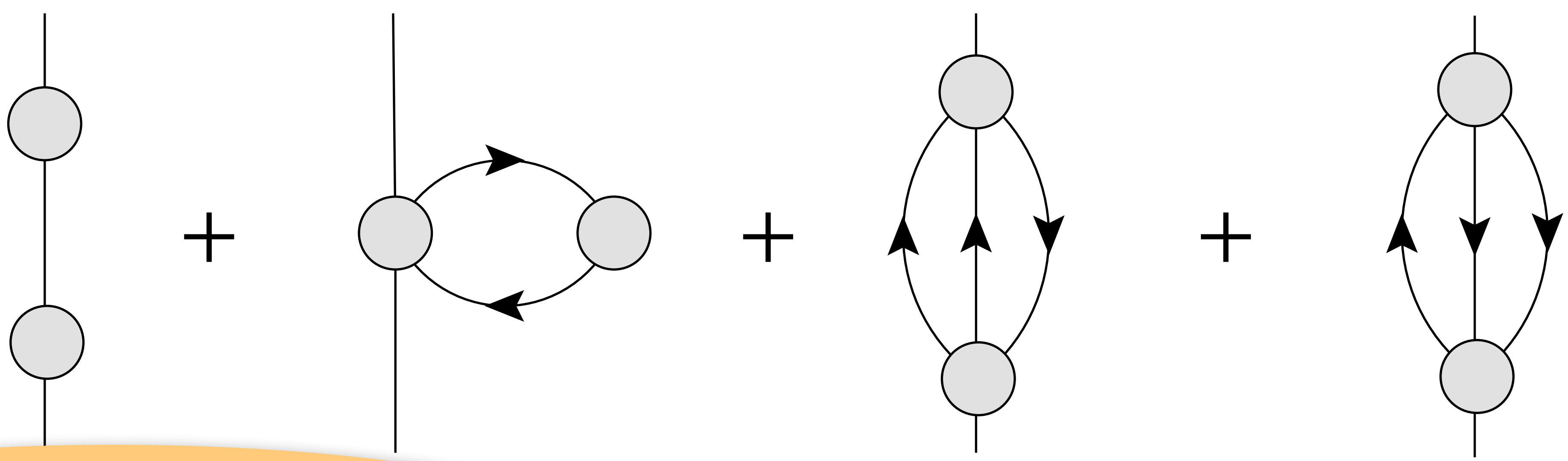
~ 2nd order MBPT for *H(s)*

$$\frac{dE}{ds} =$$



1-body Flow

$$\frac{df}{ds} =$$



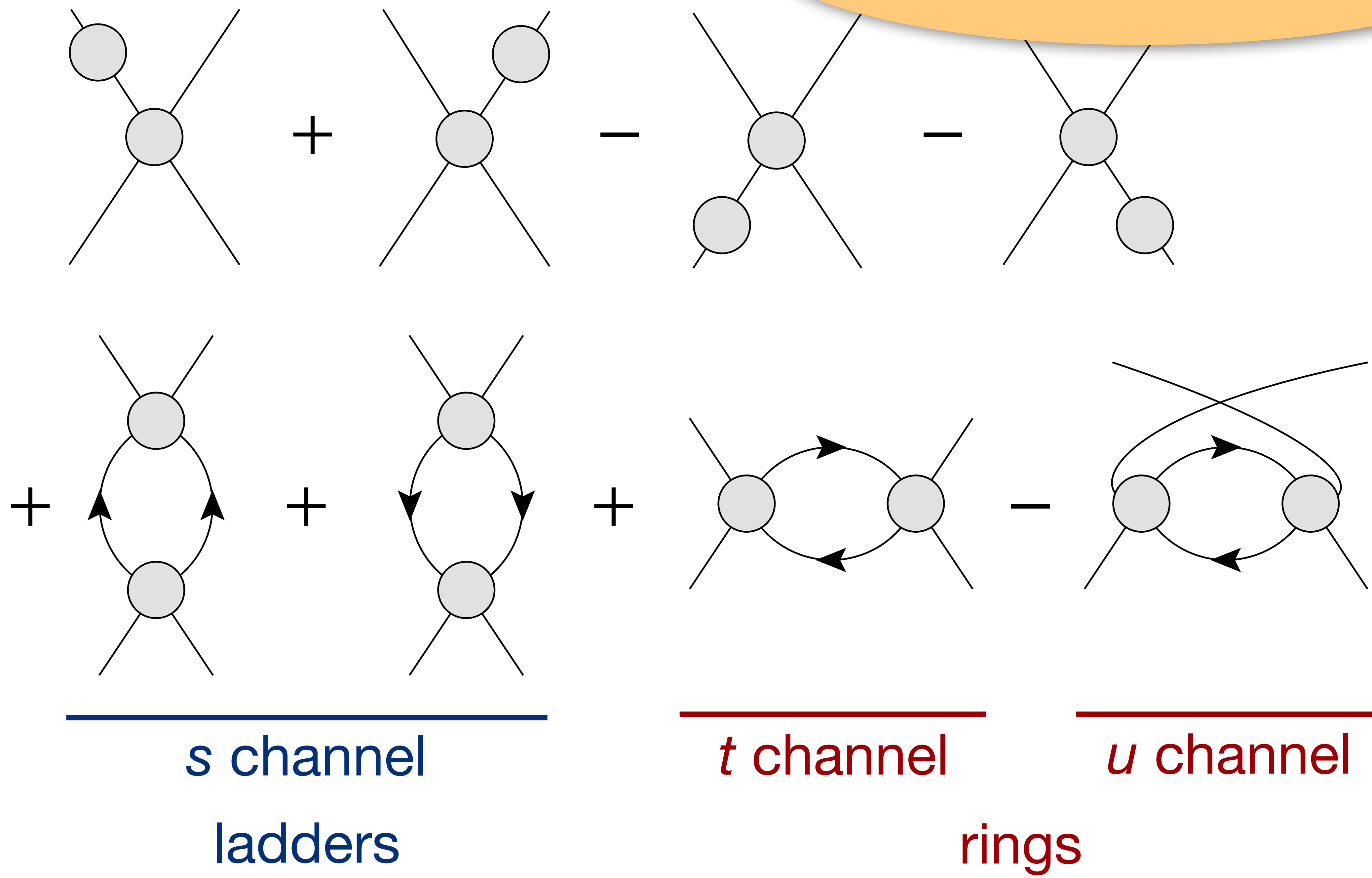
IM-SRG(2): truncate ops.
at two-body level

IM-SRG(2) Flow Equations

2-body Flow

$O(N^6)$ scaling
(before particle/hole distinction)

$$\frac{d\Gamma}{ds} =$$



Multi-Reference IM-SRG

- generalized Wick's theorem for arbitrary reference states (Kutzelnigg & Mukherjee)
- define irreducible n-body density matrices of reference state:

$$\rho^{kl}_{mn} = \lambda^{kl}_{mn} + \lambda^k_m \lambda^l_n - \lambda^k_n \lambda^l_m$$

$$\rho^{ijk}_{lmn} = \lambda^{ijk}_{lmn} + \lambda^i_l \lambda^{jk}_{mn} + \lambda^i_l \lambda^j_m \lambda^k_n + \text{permutations}$$

$$\cdots$$

- irreducible densities give rise to additional contractions:

$$:A^{ab\ldots}_{cd\ldots} :: A^{kl\ldots}_{mn\ldots}: \quad \longrightarrow \quad \lambda^{ab}_{mn}$$

$$:A^{ab\ldots}_{cd\ldots} :: A^{kl\ldots}_{mn\ldots}: \quad \longrightarrow$$

$$\cdots$$

two-body flow unchanged, $O(N^6)$ scaling preserved

Ground States of Closed and Open-Shell Nuclei

H. H., in preparation

H. H., S. Bogner, T. Morris, S. Binder, A. Calci, J. Langhammer, R. Roth, Phys. Rev. C **90**, 041302 (2014)

H. H., S. Binder, A. Calci, J. Langhammer, and R. Roth, Phys. Rev. Lett **110**, 242501 (2013)

H. H., S. K. Bogner, S. Binder, A. Calci, J. Langhammer, R. Roth, and A. Schwenk, Phys. Rev. C **87**, 034307 (2013)

Hamiltonians

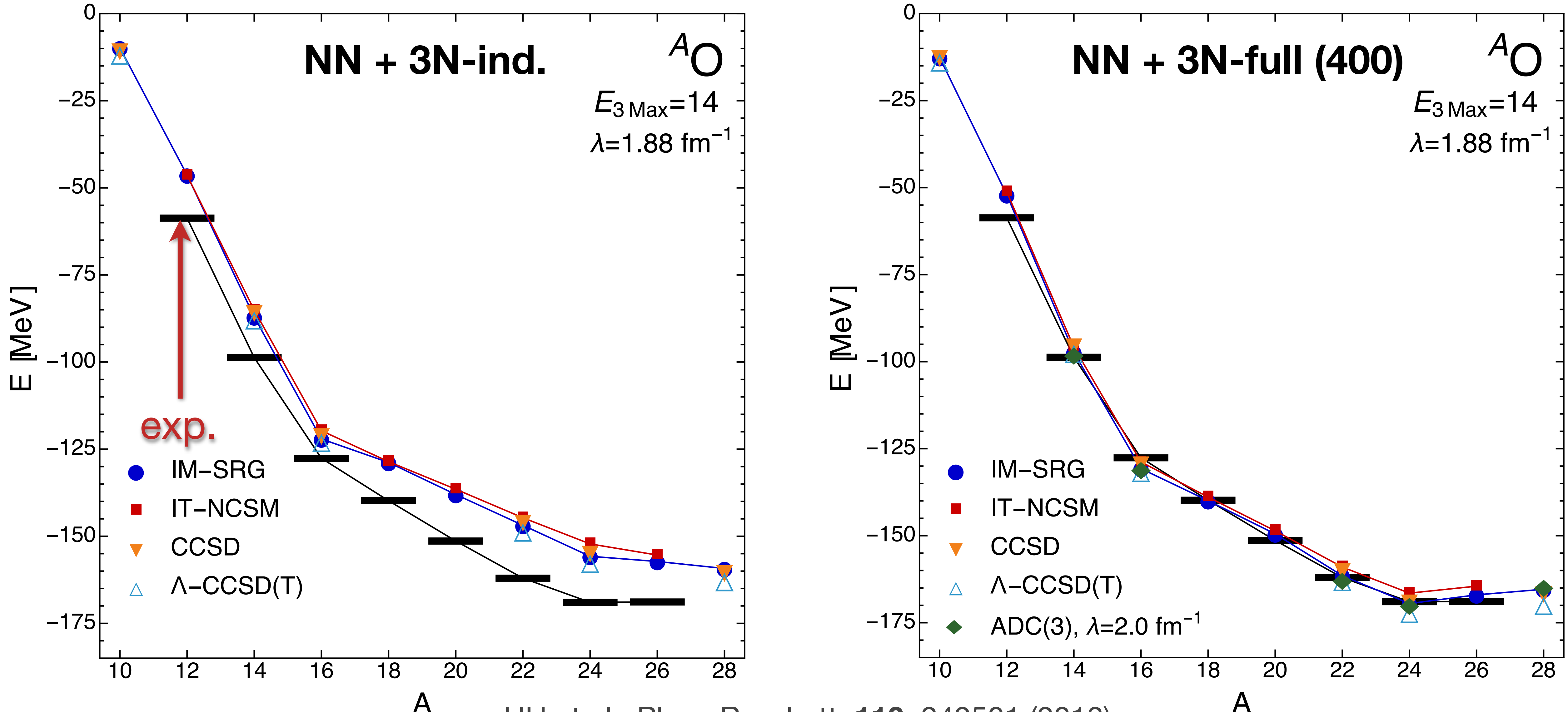
Initial Hamiltonian

- NN: chiral interaction at N^3LO (Entem & Machleidt)
- 3N: chiral interaction at N^2LO (c_D, c_E fit to ^3H, ^4He energies, β decay)

SRG-Evolved Hamiltonians

- **NN + 3N-induced:** start with initial NN Hamiltonian, keep two- and three-body terms
- **NN + 3N-full:** start with initial NN + 3N Hamiltonian, keep two- and three-body terms

Results: Oxygen Chain



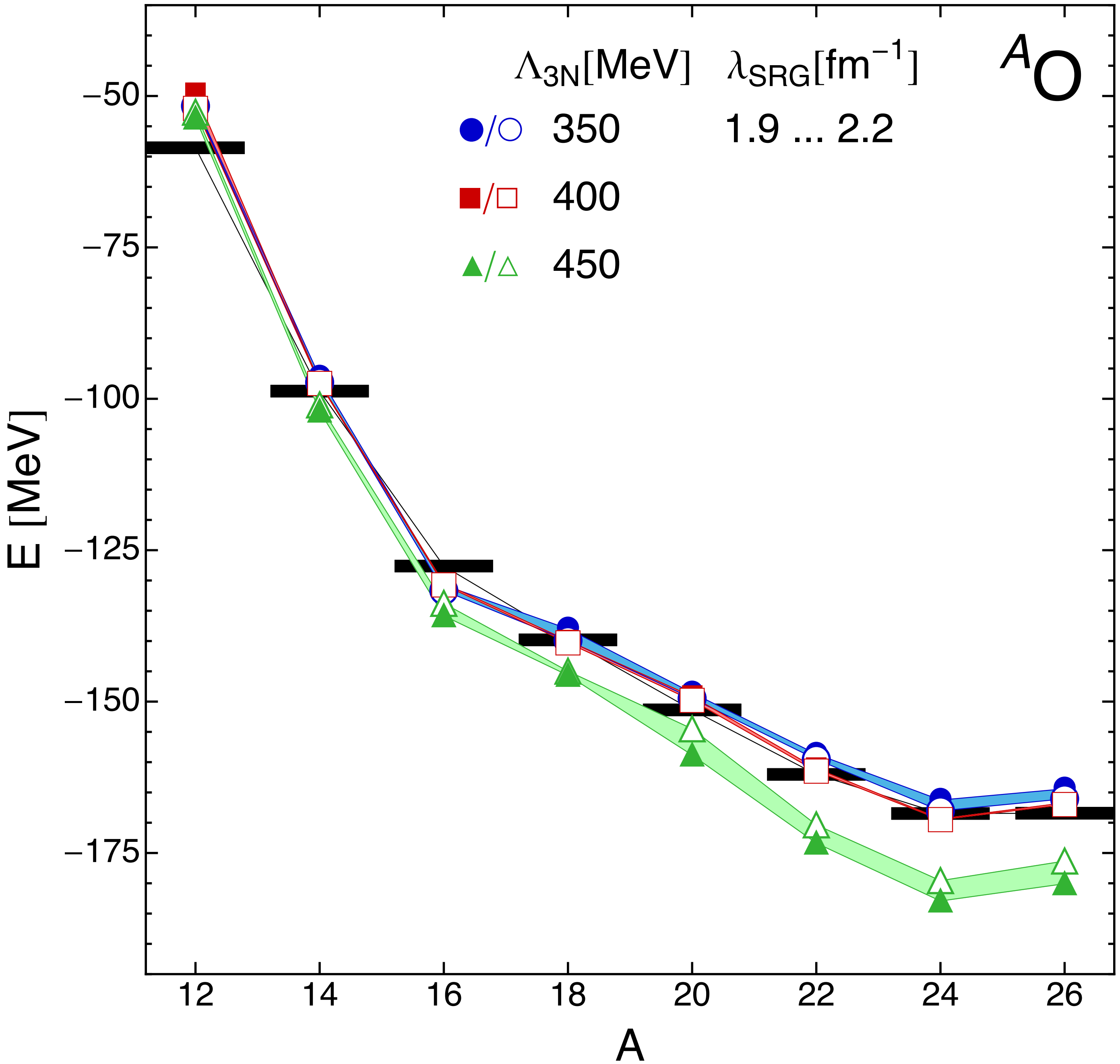
HH et al., Phys. Rev. Lett. **110**, 242501 (2013)

ADC(3): A. Cipollone et al., Phys. Rev. Lett. **111**, 242501 (2013)

- Multi-Reference IM-SRG with number-projected Hartree-Fock-Bogoliubov as reference state (pairing correlations)
- consistent results from different many-body methods

Variation of Scales

NN + 3N-full

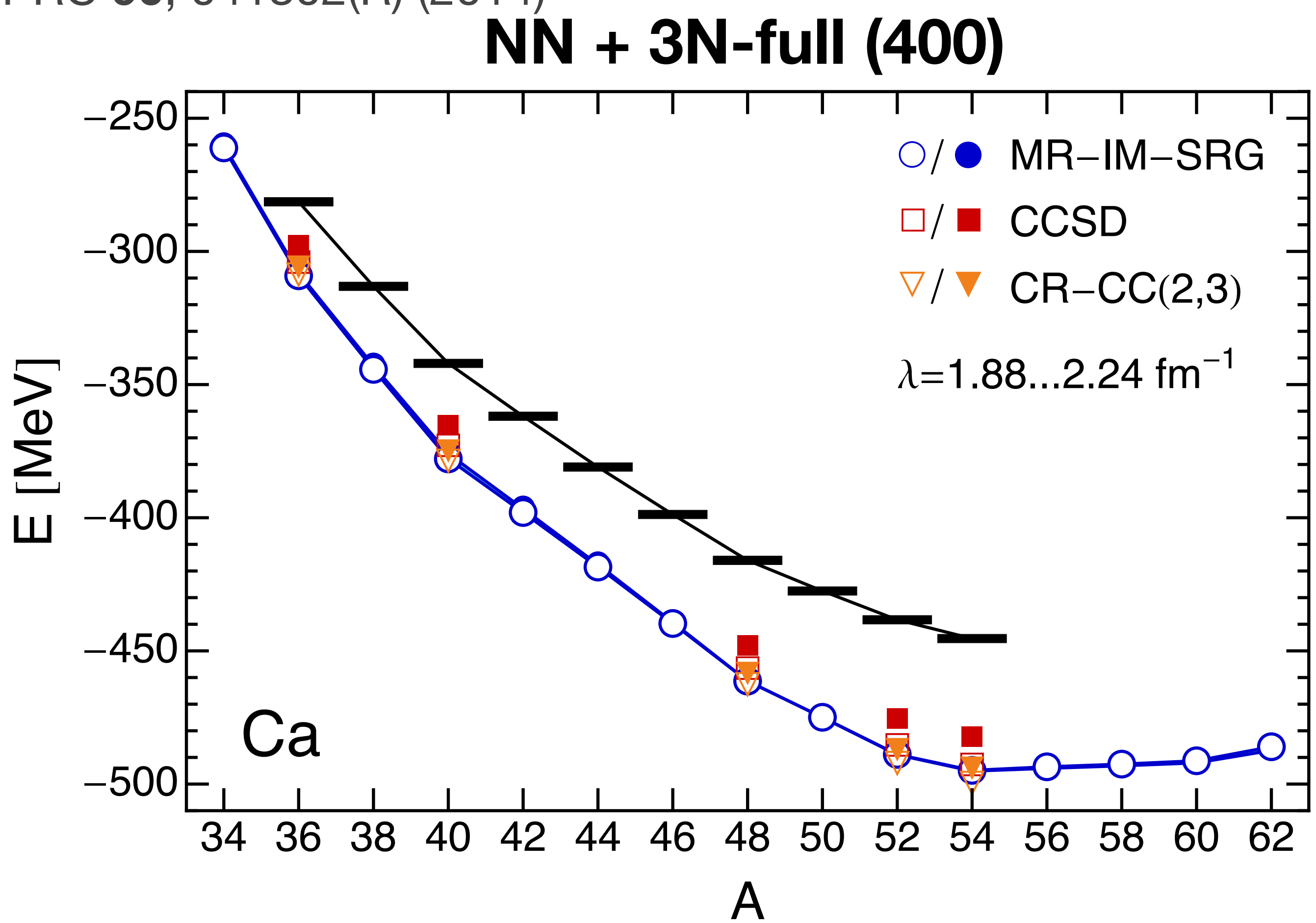


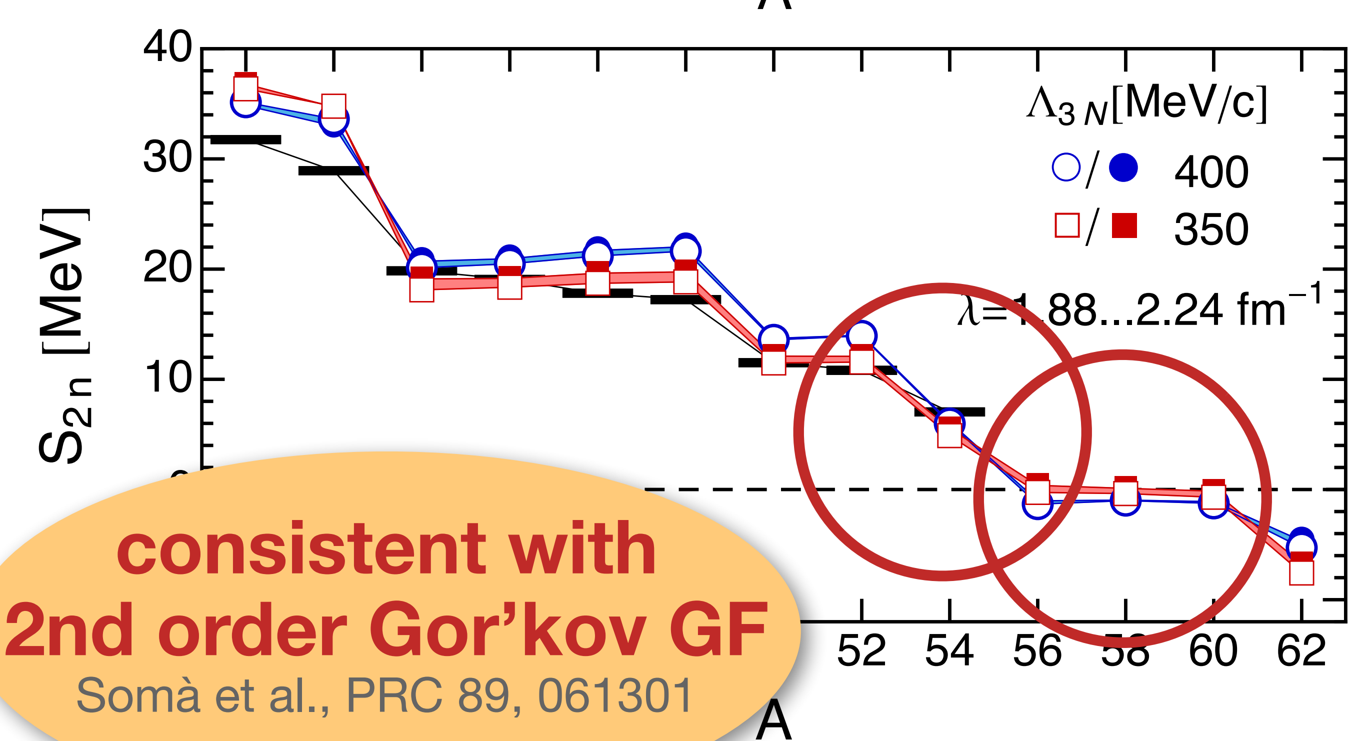
Phys. Rev. Lett. **110**, 242501 (2013)

- variation of initial 3N cutoff only
- diagnostics for chiral interactions
- dripline at A=24 is robust under variations
- (leading) continuum effects too small to bind ^{26}O

Two-Neutron Separation Energies

PRC **90**, 041302(R) (2014)





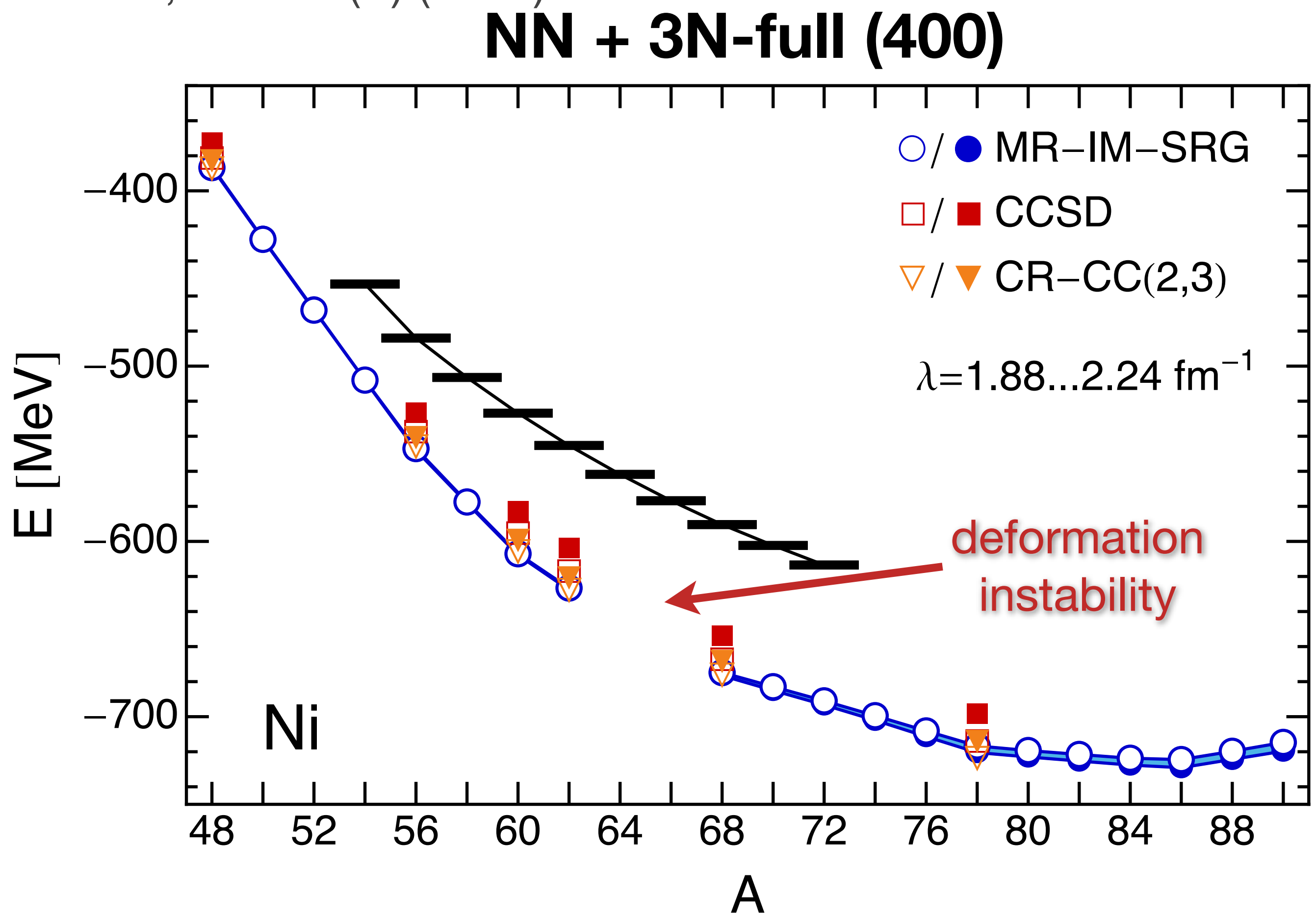
consistent with 2nd order Gor’kov GF

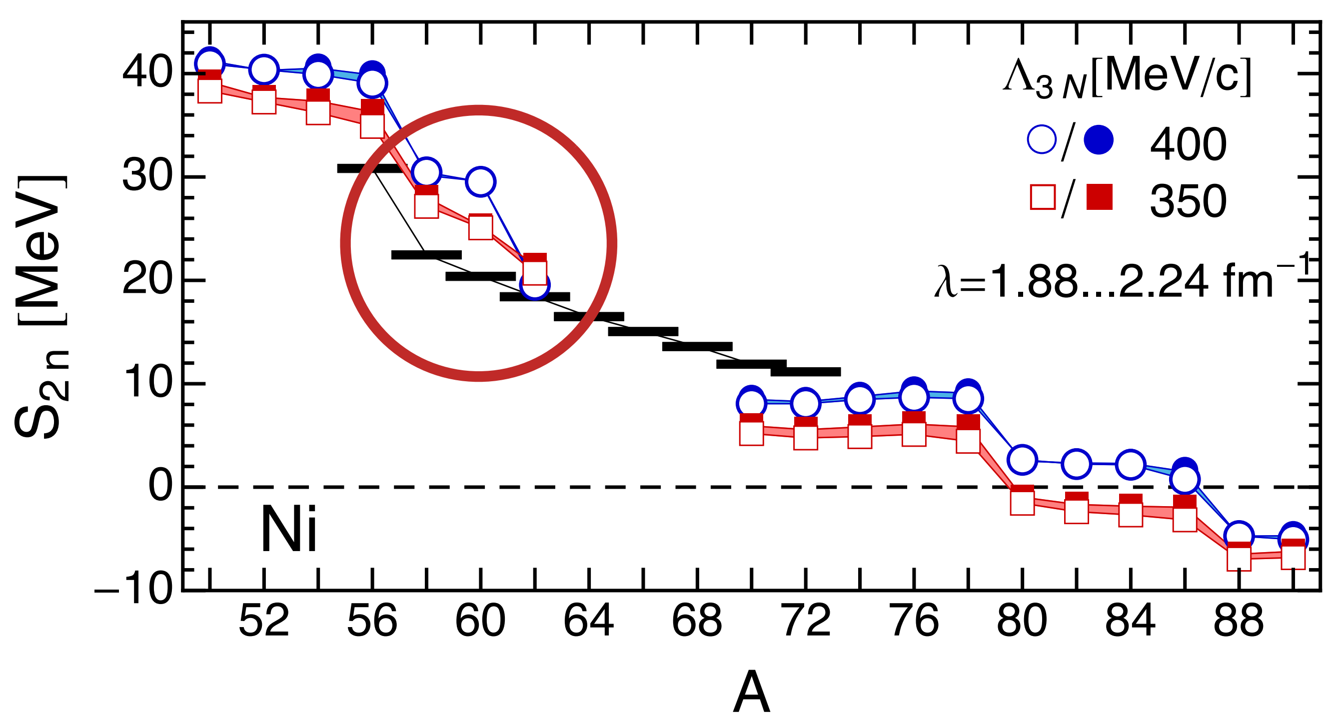
Somà et al., PRC 89, 061301

- differential observables (S_{2n}, spectra,...) filter out interaction components that cause overbinding
- predict flat trends for g.s. energies/S_{2n} beyond ^{54}Ca - await experimental data
- ^{52}Ca, ^{54}Ca robustly magic due to 3N interaction
- no continuum coupling yet, other S_{2n} uncertainties < 1MeV

Two-Neutron Separation Energies

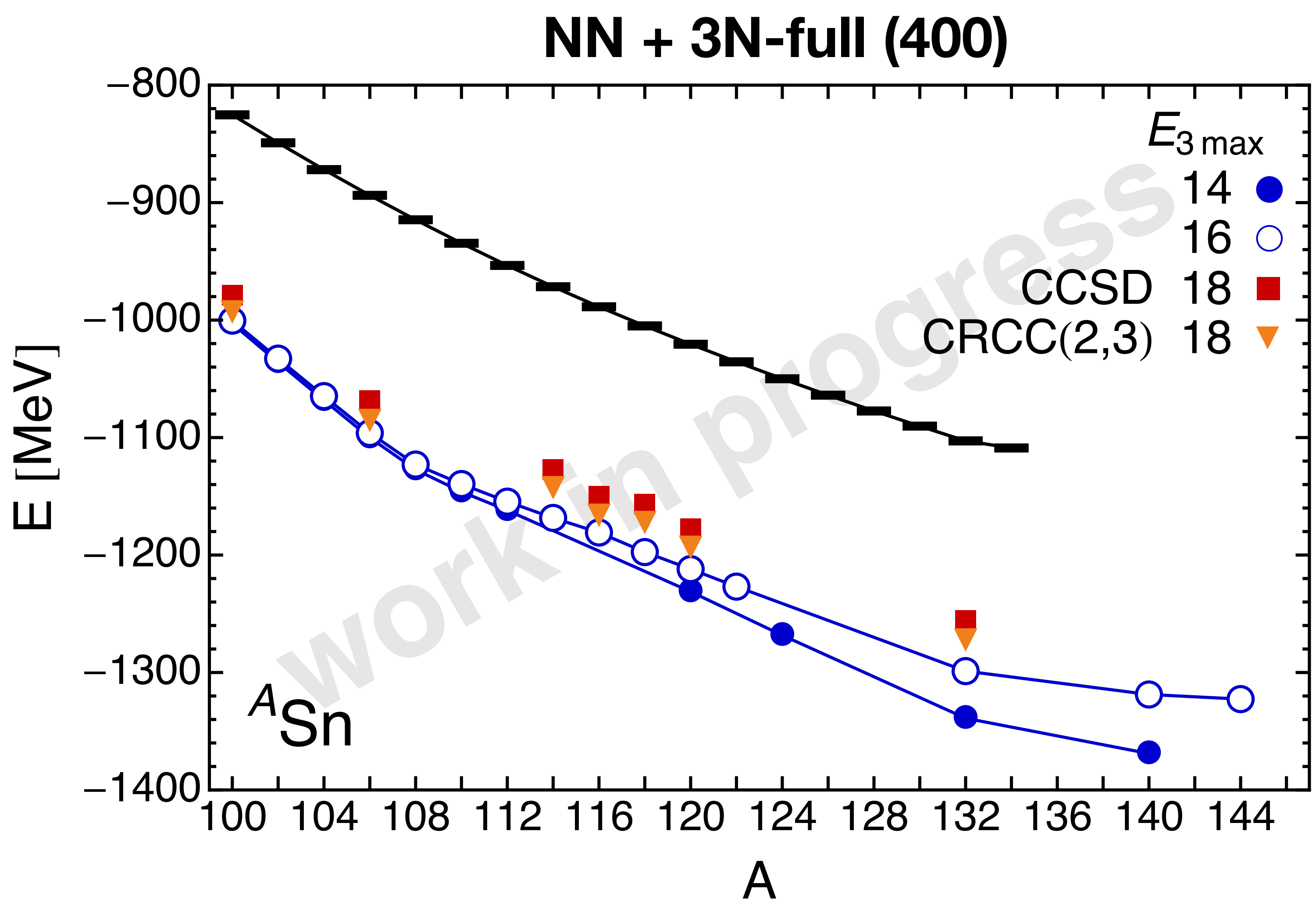
PRC **90**, 041302(R) (2014)





- flat trends for g. s. energies and S_{2n} (similar to Ca)
- deformation instability in ^{64,66}Ni calculations - issue with “shell” structure
- further evidence from 3N cutoff variation
- no continuum coupling yet, other S_{2n} uncertain-ties < 1MeV

The *Ab Initio* Mass Frontier: Tin



E_{3max}	memory (float) [GB]
14	5
16	~20
18	100+

- systematics of overbinding similar to Ca/Ni
- not converged with respect to 3N matrix element truncation:

$$e_1 + e_2 + e_3 \leq E_{3max}$$

($e_{1,2,3}$: SHO energy quantum numbers)

- need technical improvements to go further

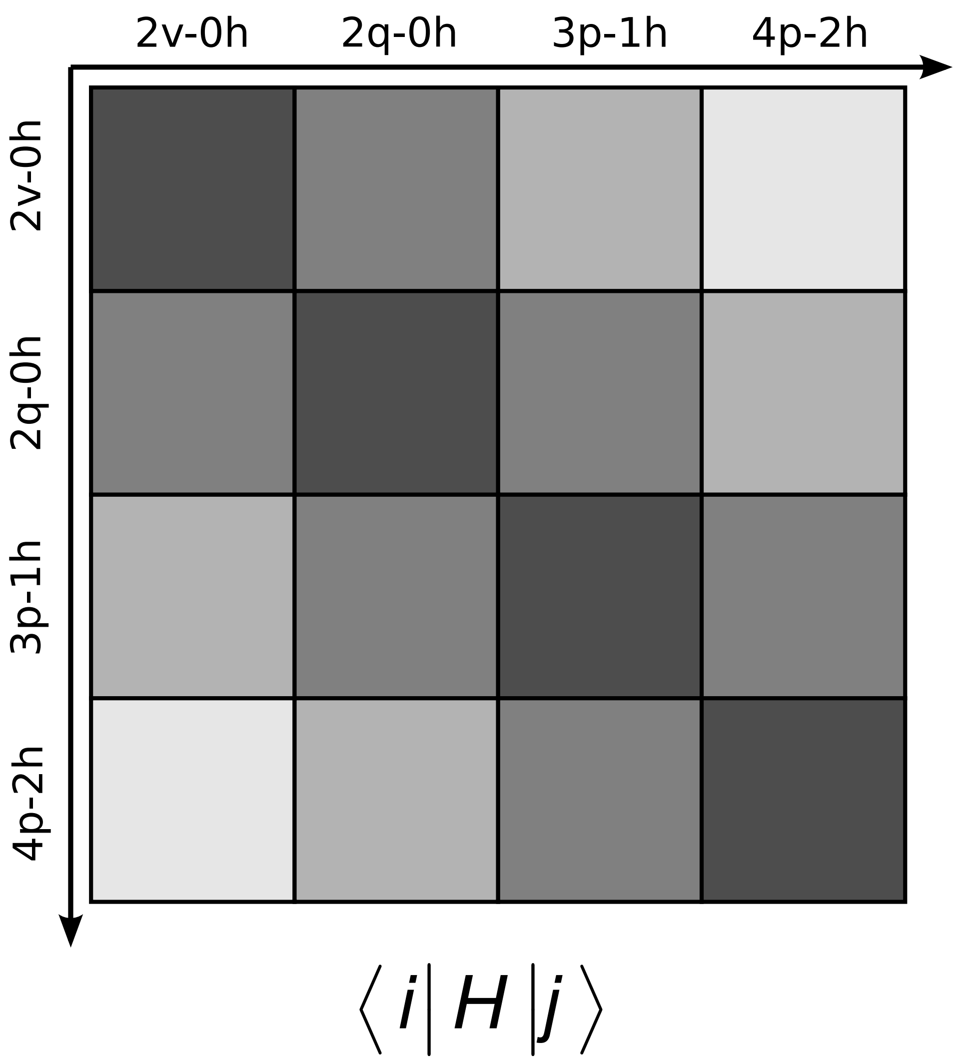
IM-SRG + Shell Model for Excited States

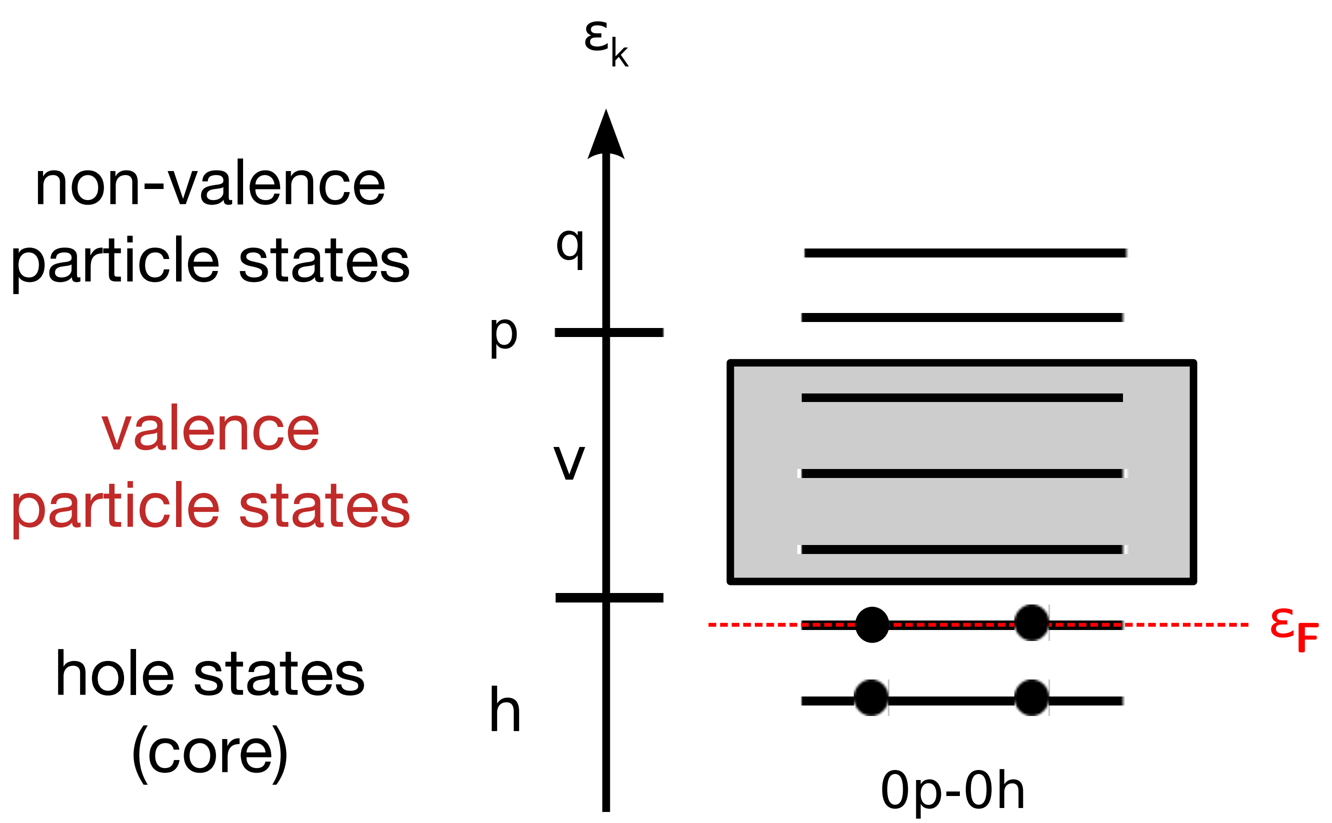
S. K. Bogner, H. H., J. D. Holt, A. Schwenk, in preparation

S. K. Bogner, H. H., J. D. Holt, A. Schwenk, S. Binder, A. Calci, J. Langhammer, R. Roth, Phys. Rev. Lett. 113, 142501 (2014)

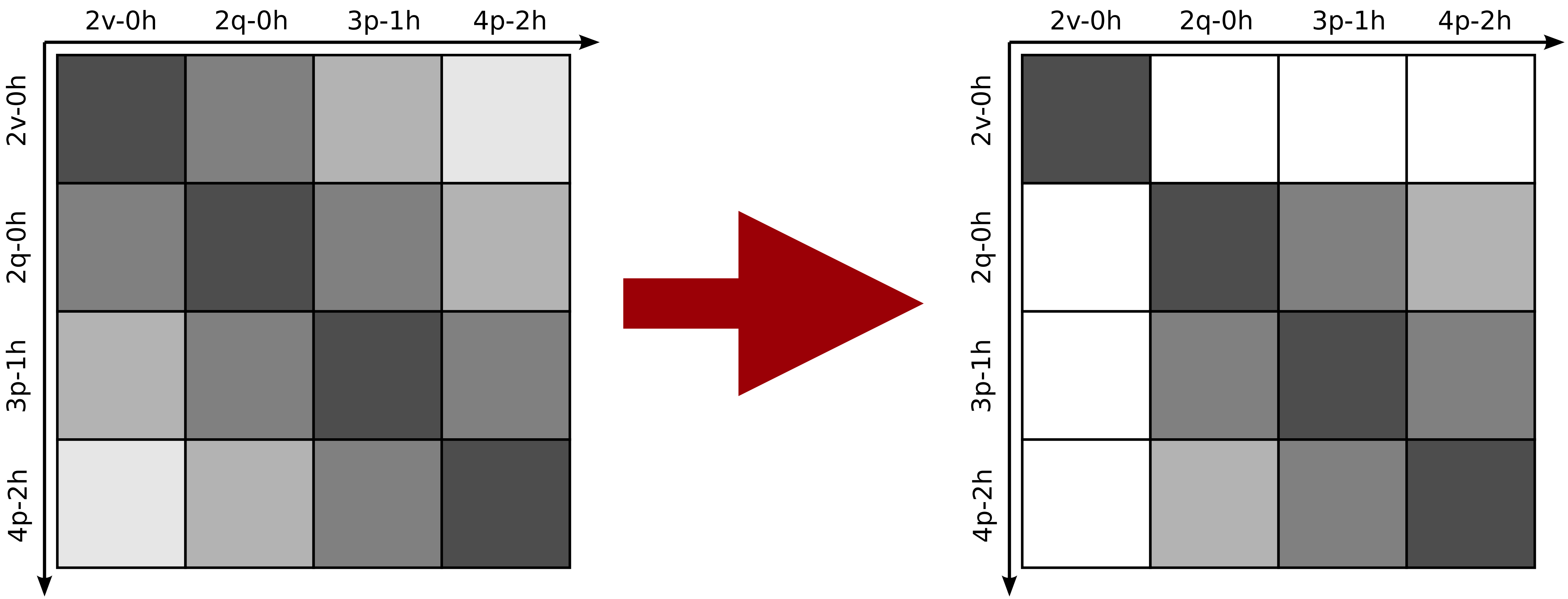
K. Tsukiyama, S. K. Bogner, and A. Schwenk, Phys. Rev. C **85**, 061304(R) (2012)

Valence Space Decoupling





Valence Space Decoupling

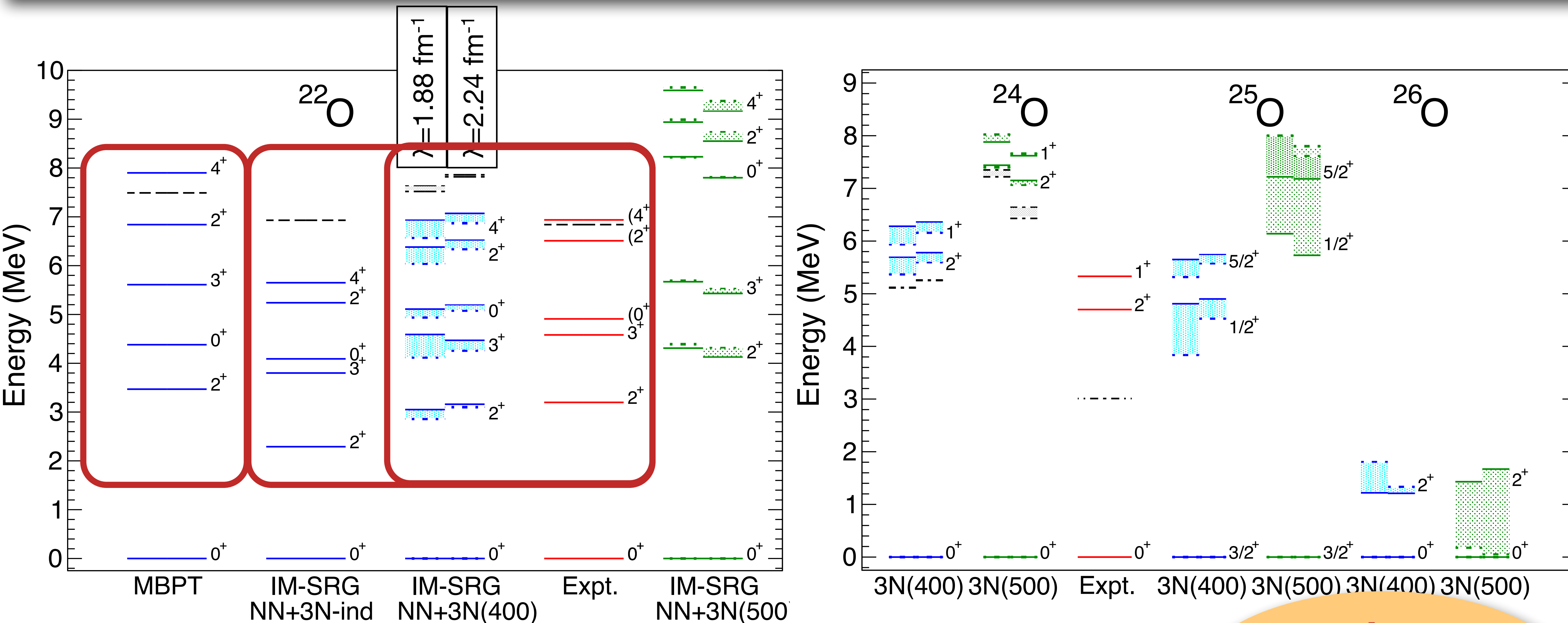


$\langle i | H | j \rangle$ $\qquad$ $\langle i | H(\infty) | j \rangle$

- construct generator from off-diagonal Hamiltonian

$$\left\{H^{od}\right\} = \{f^{h}_{h'}, f^{p}_{p'}, f^{p}_{h}, f^{q}_{v}, \Gamma^{pp'}_{hh'}, \Gamma^{pp'}_{hv}, \Gamma^{pq}_{vv'}\} \text{ \& H.c.}$$

From Oxygen...



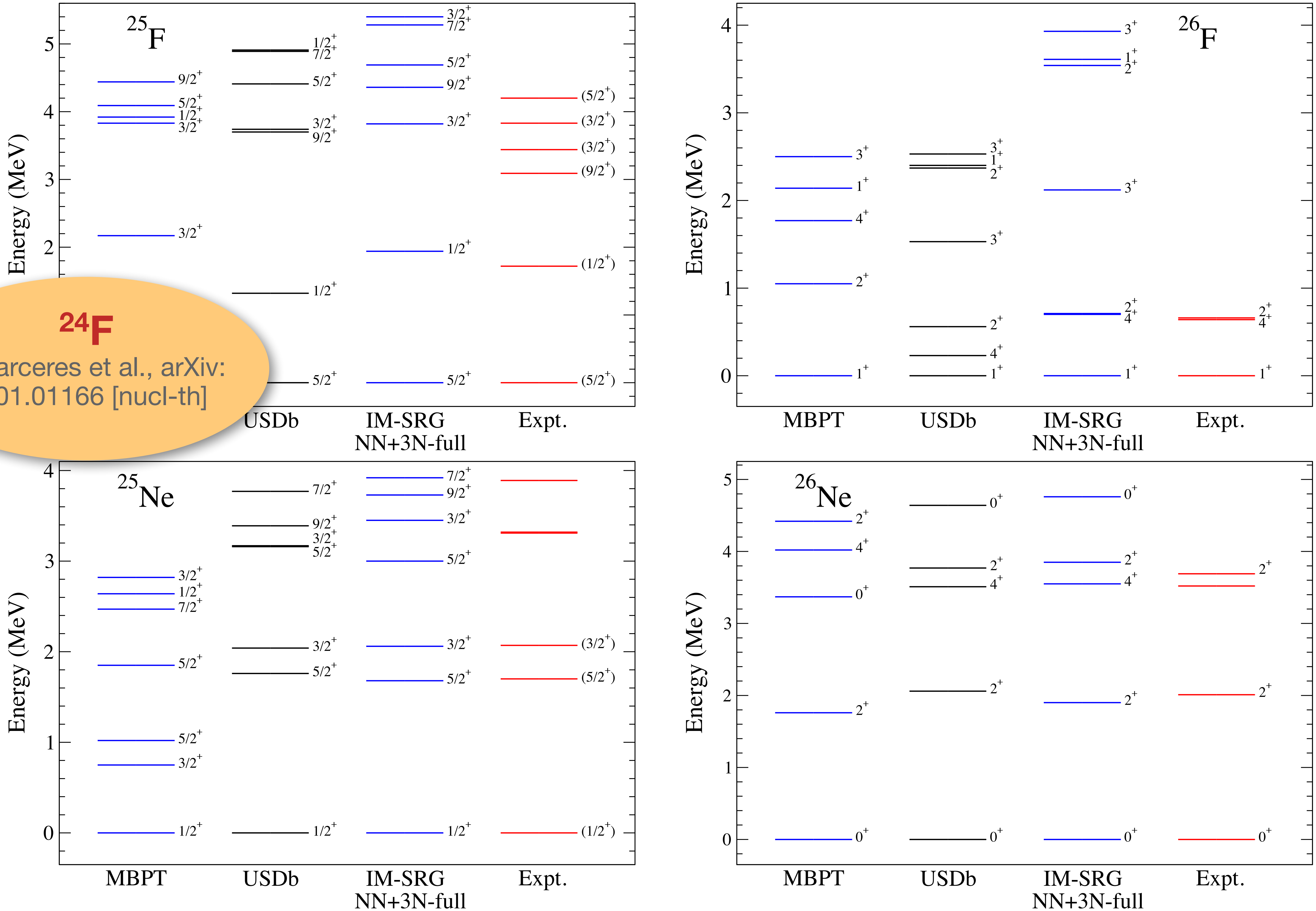
shading: $\hbar\Omega$ variation

Phys. Rev. Lett. **113**, 142501 (2014)

continuum lowers states by <1 MeV

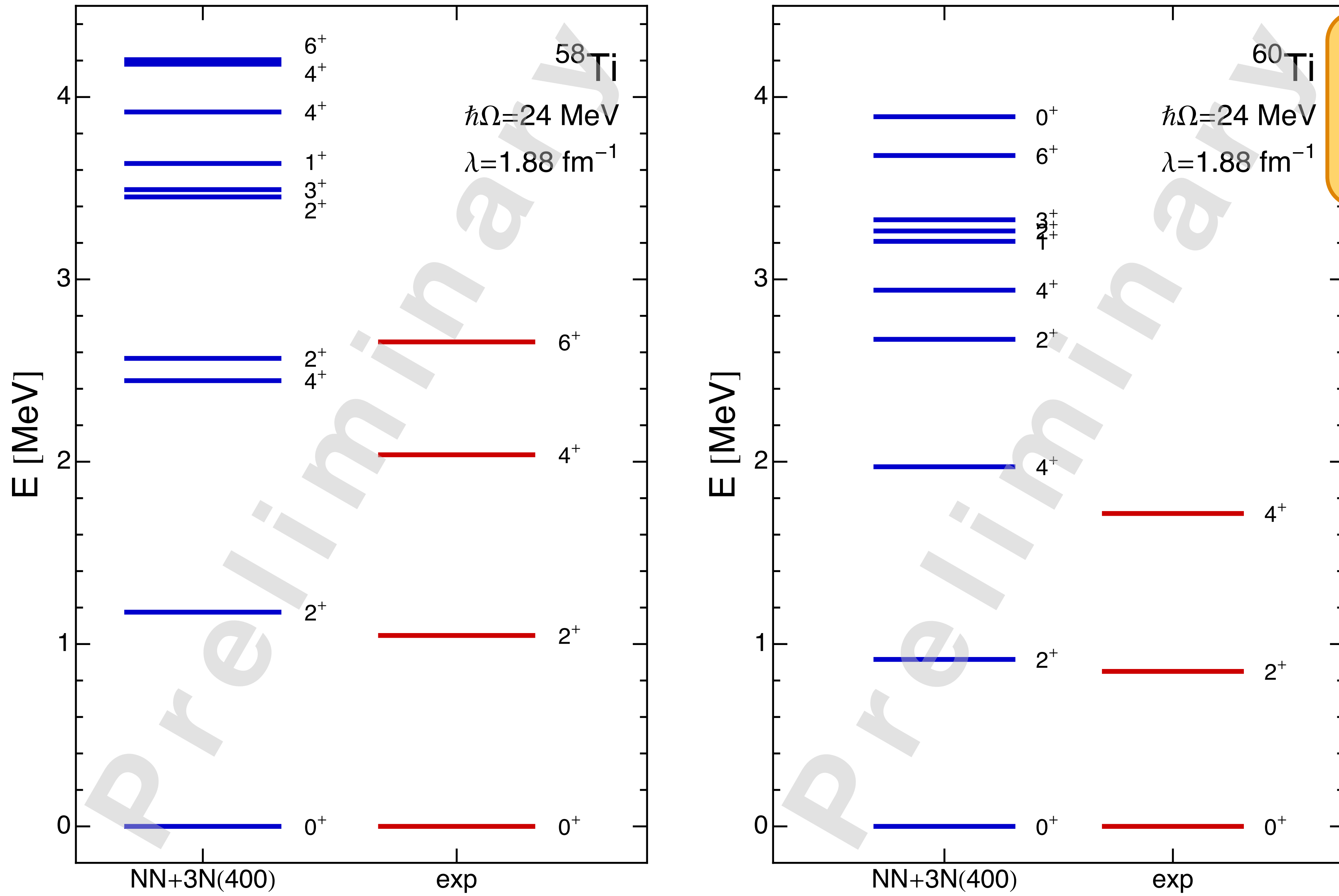
- **3N forces crucial**
- IM-SRG improves on finite-order MBPT effective interaction
- competitive with phenomenological calculations

... Into the *sd*-Shell...



24F
L. Carceres et al., arXiv: 1501.01166 [nucl-th]

… And Beyond



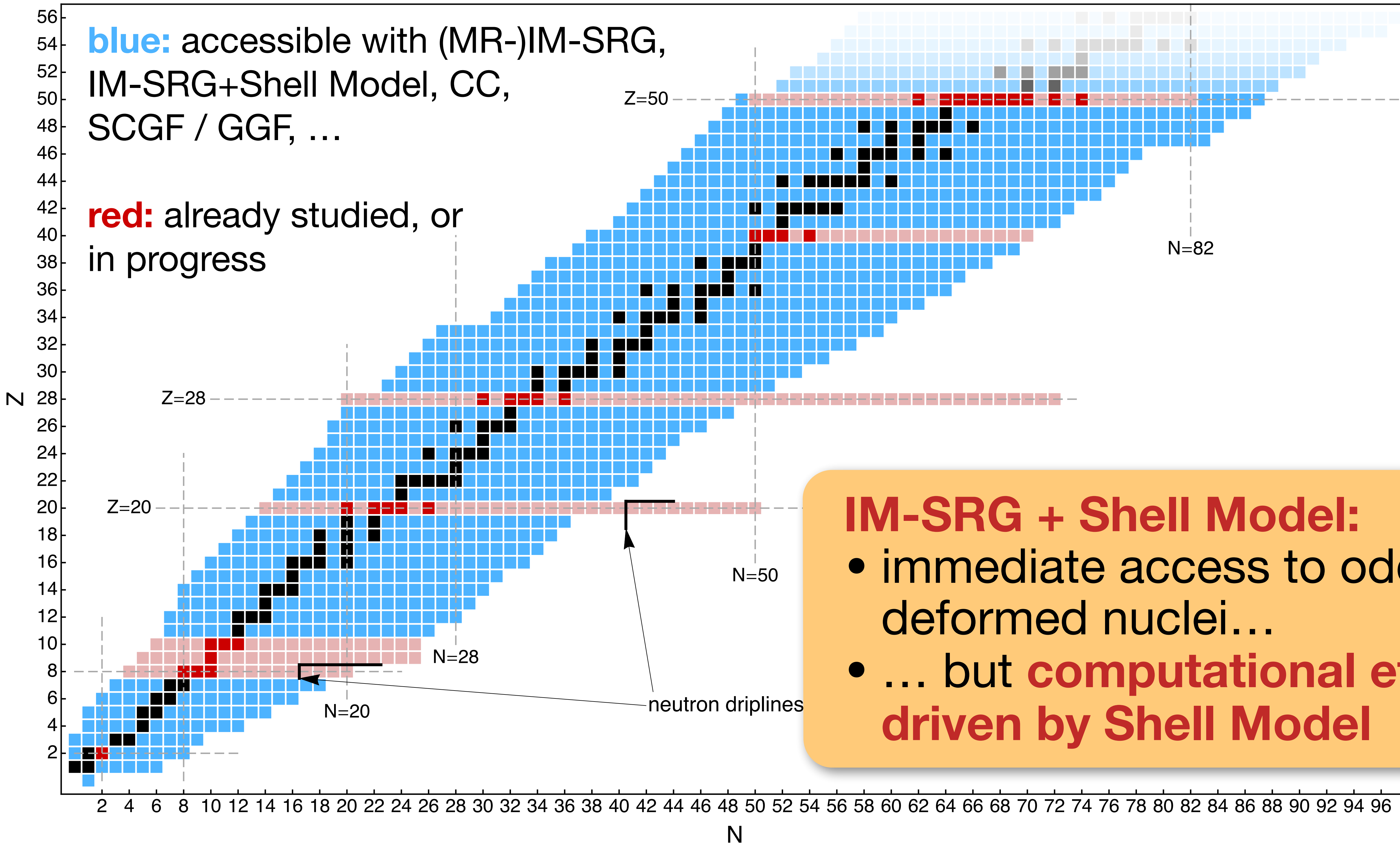
^{48}Ca core, $\nu pf_{5/2}$, πpf valence space

experimental data: A. Gade et al., Phys. Rev. Lett. **112**, 112503 (2014) and NNDC

➡ theoretical level scheme similar to empirical interactions (LNPS, GXPF1A)

Next Steps

Reach of Ab Initio Methods



IM-SRG + Shell Model:

- immediate access to odd & deformed nuclei…
- … but **computational effort driven by Shell Model**

Equations-of-Motion for Excitations

- describe "excited states" based on reference state:

$$\left|\Psi_k\right\rangle \equiv R_k \left|\Psi_0\right\rangle$$

- **(MR-)IM-SRG effective Hamiltonian** in EOM approach:

$$\left[H(\infty), R_k\right] = \omega_k R_k\,, \quad \omega_k = E_k - E_0$$

- computational effort scales **polynomially**, vs. factorial scaling of Shell Model

- can exploit Multi-Reference capabilities (commutator formulation identical to flow equations)

➡ **complementary** to Shell Model

EOM Applications

- particle-hole excitations (TDA, RPA, Second RPA, ...)

$$R_k = \sum_{ph} R^{(k)}_{ph} :a^\dagger_p a_h: + \sum_{pp'hh'} R^{(k)}_{pp'hh'} :a^\dagger_p a^\dagger_{p'} a_{h'} a_h: + \ldots$$

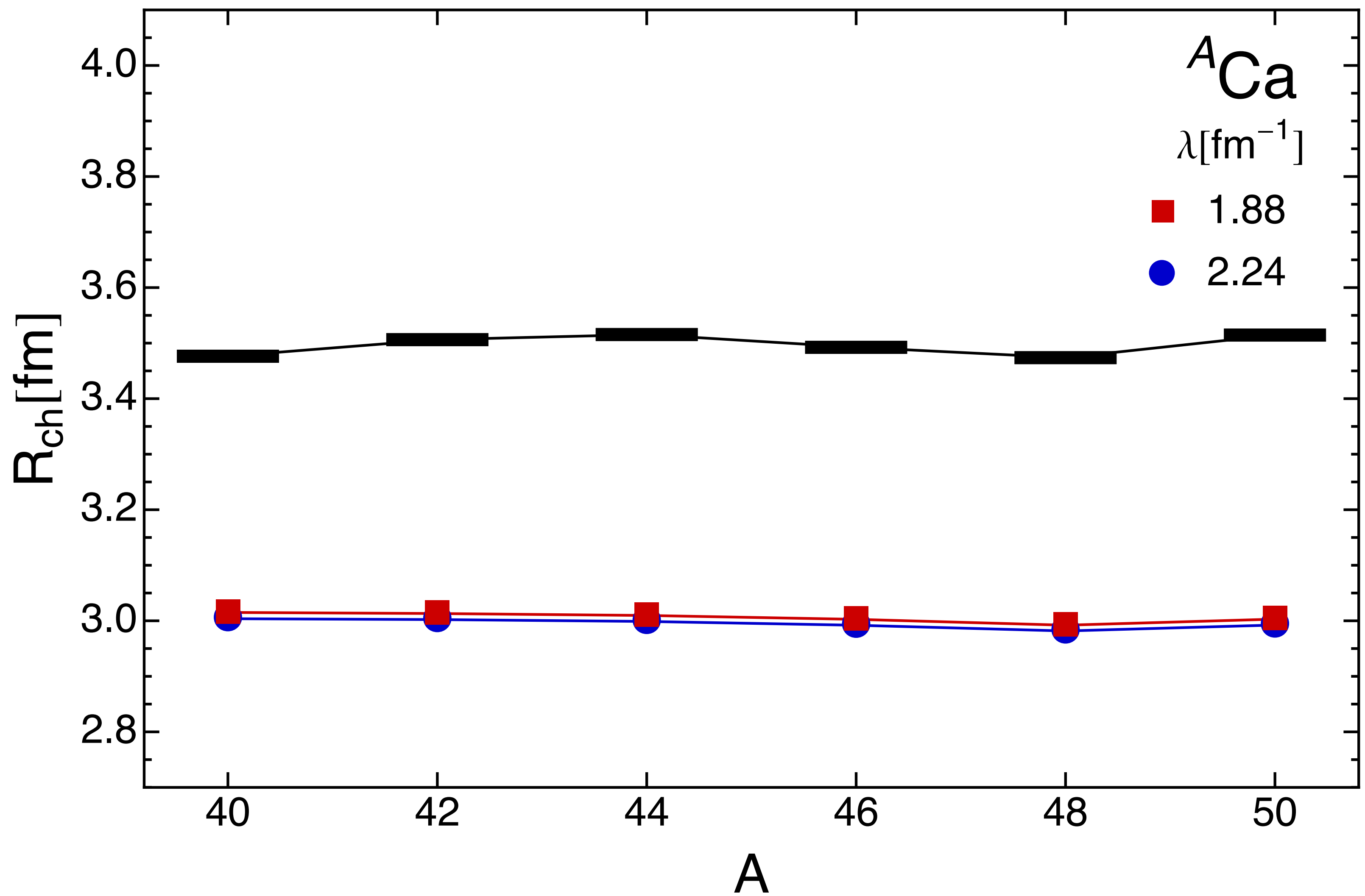
➡ **giant resonances**

- particle attachment (analogous for removal):

$$R_k = \sum_{ph} R^{(k)}_{p} :a^\dagger_p: + \sum_{pp'h} R^{(k)}_{pp'h} :a^\dagger_p a^\dagger_{p'} a_h: + \ldots$$

➡ **ground and excited states in odd nuclei**

Effective Operators



- small radii: interaction issue (power counting, regulators, LECs, …), also consider currents?
- implementation of electromagnetic & weak transition operators in progress; aim for consistent treatment: chiral EFT, SRG, IM-SRG (& Shell Model code !)

Magnus Series Formulation

also see talk by R. Stroberg

- construct unitary transformation explicitly:

$$U(s) = \mathcal{S} \exp \int_0^s ds' \eta(s') \equiv \exp \Omega(s)$$

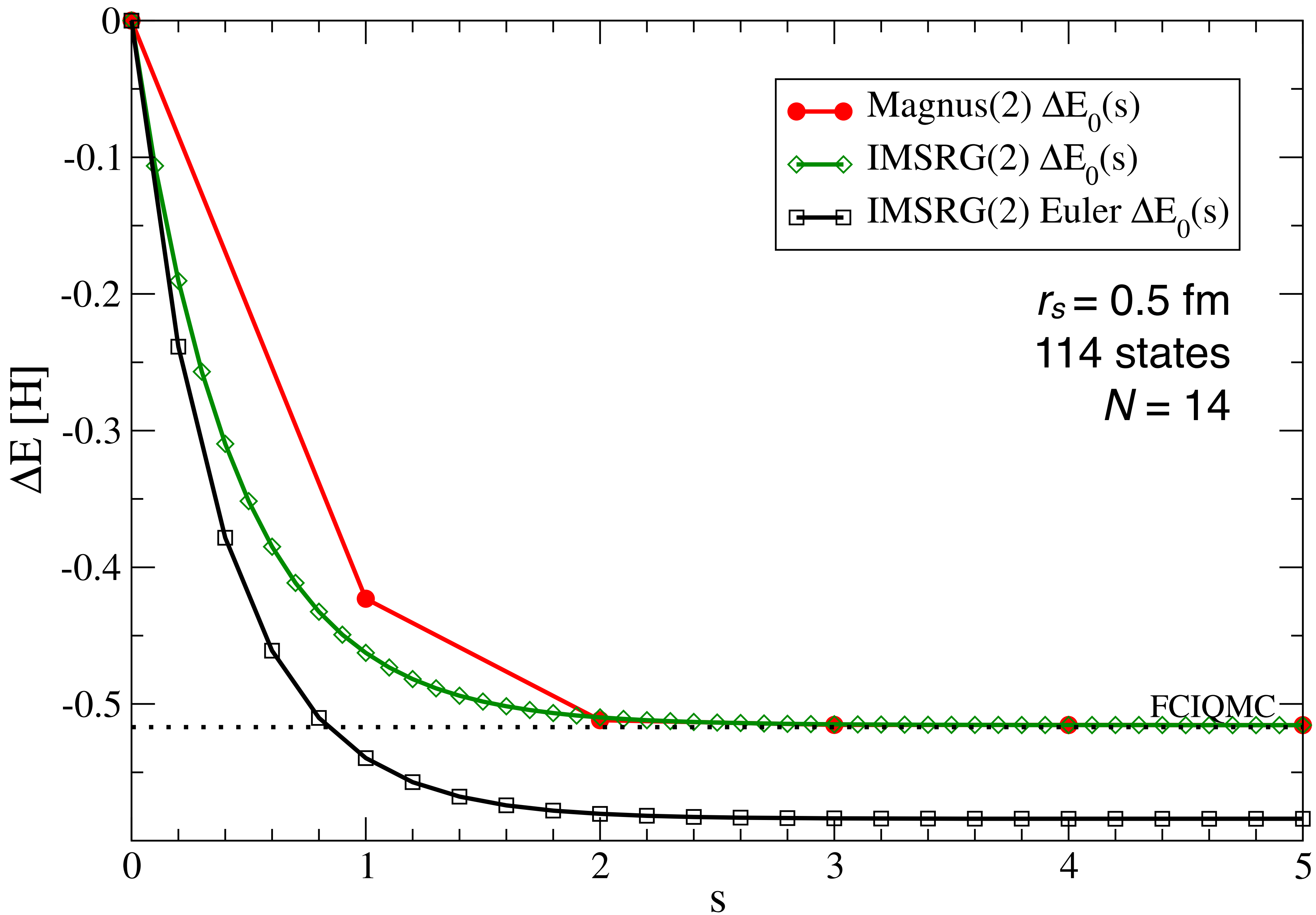
- flow equation for Magnus operator :

$$\frac{d}{ds}\Omega = \sum_{k=0}^{\infty} \frac{B_k}{k!} \mathrm{ad}_{\Omega}^{k}(\eta) \,, \quad \mathrm{ad}_{\Omega}(O) = [\Omega, O]$$

(Bk: Bernoulli numbers)

- construct $O(s) = U(s) O_0 U^\dagger(s)$ using Baker-Campbell-Hausdorff expansion (Hamiltonian + effective operators)
- generate systematic approximations to (MR-)IM-SRG(3)
- simple integrator sufficient (Euler!) - **unitarity built in**

Example: Homogenous Electron Gas



T. D. Morris, S. K. Bogner, in preparation

Conclusions

Conclusions

- IM-SRG is a powerful *ab initio* framework for closed- and open-shell, medium-mass & (heavy) nuclei
- derivation of Shell-Model interactions
  ➡ immediate access to spectra, odd nuclei, intrinsic deformation (at Shell Model numerical cost)
- soon:
  - EOM for excited states
  - effective transition operators (see R. Stroberg's talk)
  - triples (everywhere)
- new perspectives for old (?) problems: evolution of long-range correlations, construction of density functionals...

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Ohio Supercomputer Center
ICER

Supplements

- SRG is a unitary transformation in A-body space
- up to A-body interactions are induced during the flow:

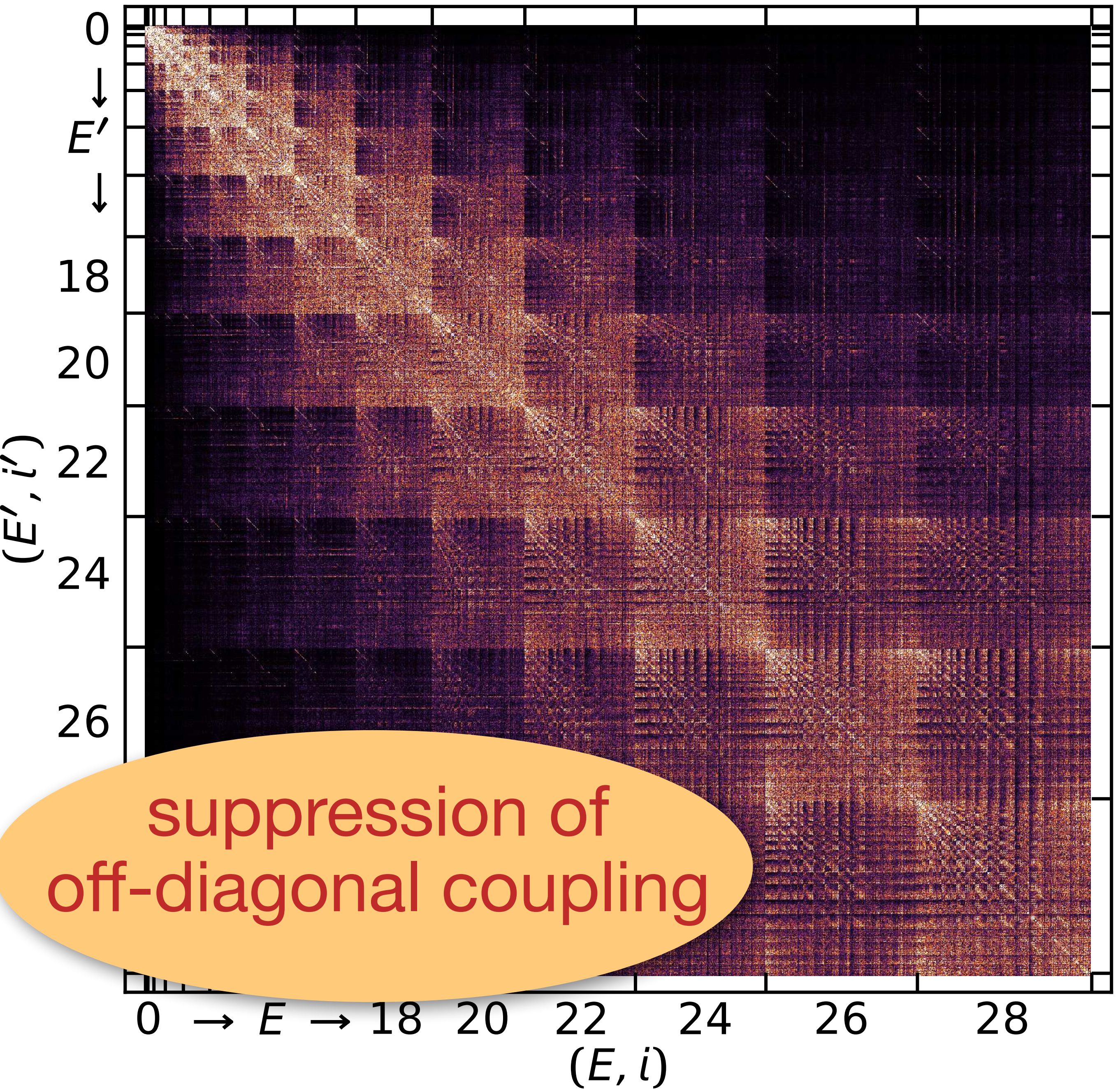
$$\frac{dH}{d\lambda} = [[\sum a^\dagger a, \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}], \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}] = \ldots + \sum \underbrace{a^\dagger a^\dagger a^\dagger aaa}_{\text{3-body}} + \ldots$$

- state-of-the-art: evolve in three-body space, truncate induced four- and higher many-body forces
(Jurgenson, Furnstahl, Navratil, PRL 103, 082501; Hebeler, PRC 85, 021002; Wendt, PRC 87, 061001)
- λ-dependence of eigenvalues is a diagnostic for size of omitted induced interactions

SRG in Three-Body Space

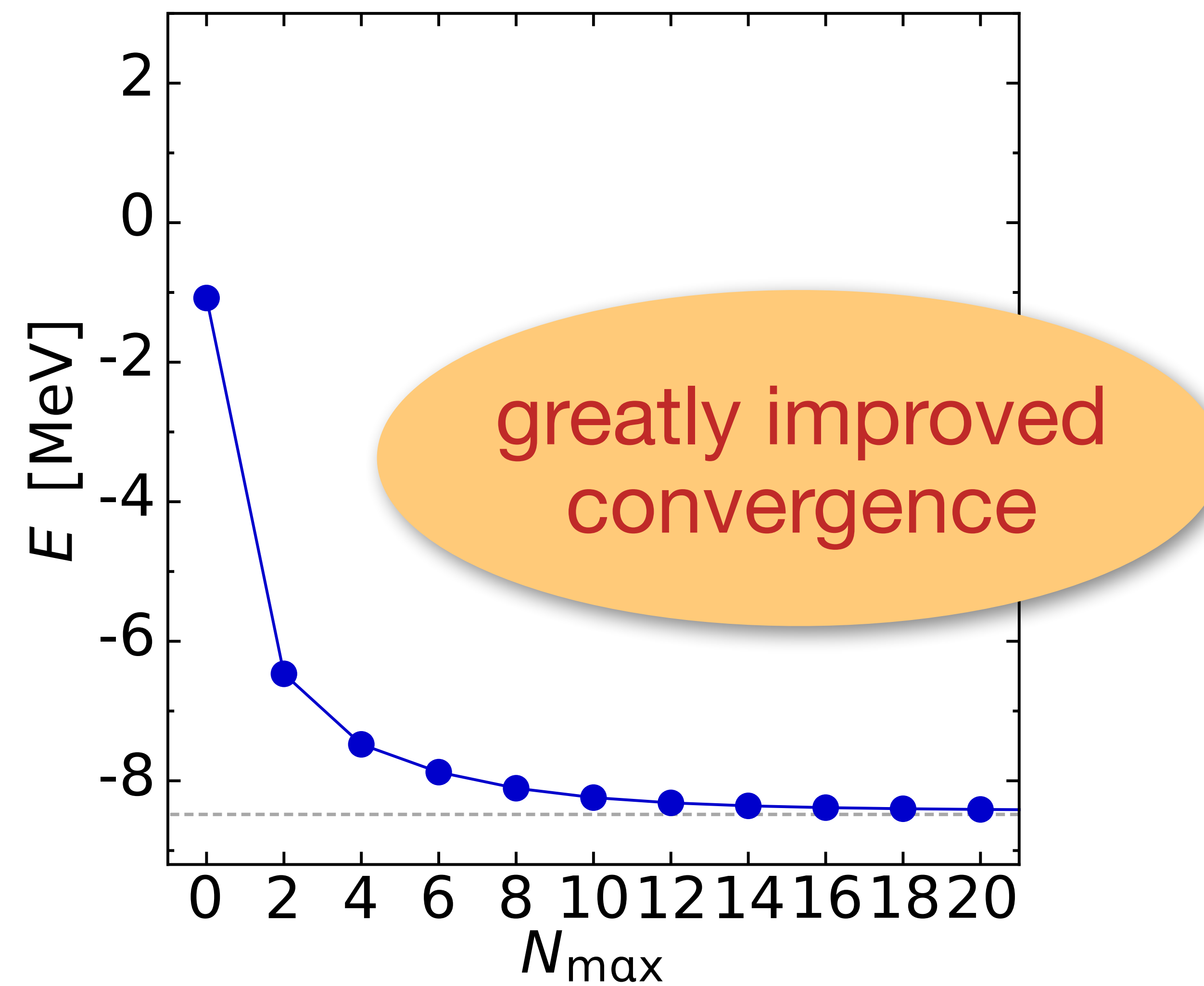
3B Jacobi-HO Matrix Elements

$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}, \hbar\Omega = 28$ MeV



λ = 1.33 fm^{-1}

^{3}H ground-state (NCSM)



[figures by R. Roth, A. Calci, J. Langhammer]

Normal-Ordering & Wick's Theorem

- define elementary contractions of a one-body operator w.r.t. a given reference state as

$$A^k_l \equiv a^\dagger_k a_l\,, \quad \lambda^k_l \equiv \left\langle \Psi \middle| A^k_l \middle| \Psi \right\rangle, \quad \xi^k_l \equiv \lambda^k_l - \delta^k_l$$

- define normal-ordered operators recursively through all possible internal contractions:

$$A^{k_1\dots k_N}_{l_1\dots l_N} = :A^{k_1\dots k_N}_{l_1\dots l_N}: + \lambda^{k_1}_{l_1} :A^{k_2\dots k_N}_{l_2\dots l_N}: + \text{singles} \\ + \left(\lambda^{k_1}_{l_1}\lambda^{k_2}_{l_2} - \lambda^{k_1}_{l_2}\lambda^{k_2}_{l_1}\right) :A^{k_3\dots k_N}_{l_3\dots l_N}: + \text{doubles} + \dots$$

- Wick's Theorem: products of normal-ordered operators can be expanded in terms of external contractions alone

$$:A^{k_1\dots k_N}_{m_1\dots m_N}::A^{l_1\dots l_N}_{n_1\dots n_N}: = (-1)^{N-1}\lambda^{k_1}_{n_1} :A^{k_2\dots k_N l_1\dots l_N}_{m_1\dots m_N n_2\dots n_N}: \\ + (-1)^{N-1}\xi^{l_1}_{m_1} :A^{k_1\dots k_N l_2\dots l_N}_{m_2\dots m_N n_1\dots n_N}: + \dots$$

Choice of Generator

- Wegner: $$\eta^I = [H^d, H^{od}]$$
- White: (J. Chem. Phys. 117, 7472)

$$\eta^{II} = \sum_{ph} \frac{f_h^p}{\Delta_h^p} :A_h^p: + \sum_{pp'hh'} \frac{\Gamma_{hh'}^{pp'}}{\Delta_{hh'}^{pp'}} :A_{hh'}^{pp'}: + \text{H.c.}$$

$\Delta_h^p, \Delta_{hh'}^{pp'}$: approx. 1p1h, 2p2h excitation energies

- "imaginary time": (Morris, Bogner)

$$\eta^{III} = \sum_{ph} \text{sgn}(\Delta_h^p) f_h^p :A_h^p: + \sum_{pp'hh'} \text{sgn}(\Delta_{hh'}^{pp'}) \Gamma_{hh'}^{pp'} :A_{hh'}^{pp'}: + \text{H.c.}$$

- off-diagonal matrix elements are suppressed like $e^{-\Delta^2 s}$ (Wegner), e^{-s} (White), and $e^{-|\Delta| s}$ (imaginary time)
- g.s. energies ($s \to \infty$) differ by $\ll 1\%$

In-Medium SRG Flow Equations

0-body Flow

$$\frac{dE}{ds} = \sum_{ab}(n_a - n_b)\left(\eta^a_b f^b_a - f^a_b \eta^b_a\right) + \frac{1}{4}\sum_{abcd}\left(\eta^{ab}_{cd}\Gamma^{cd}_{ab} - \Gamma^{ab}_{cd}\eta^{cd}_{ab}\right) n_a n_b \bar{n}_c \bar{n}_d$$

~ 2nd order MBPT for *H(s)*

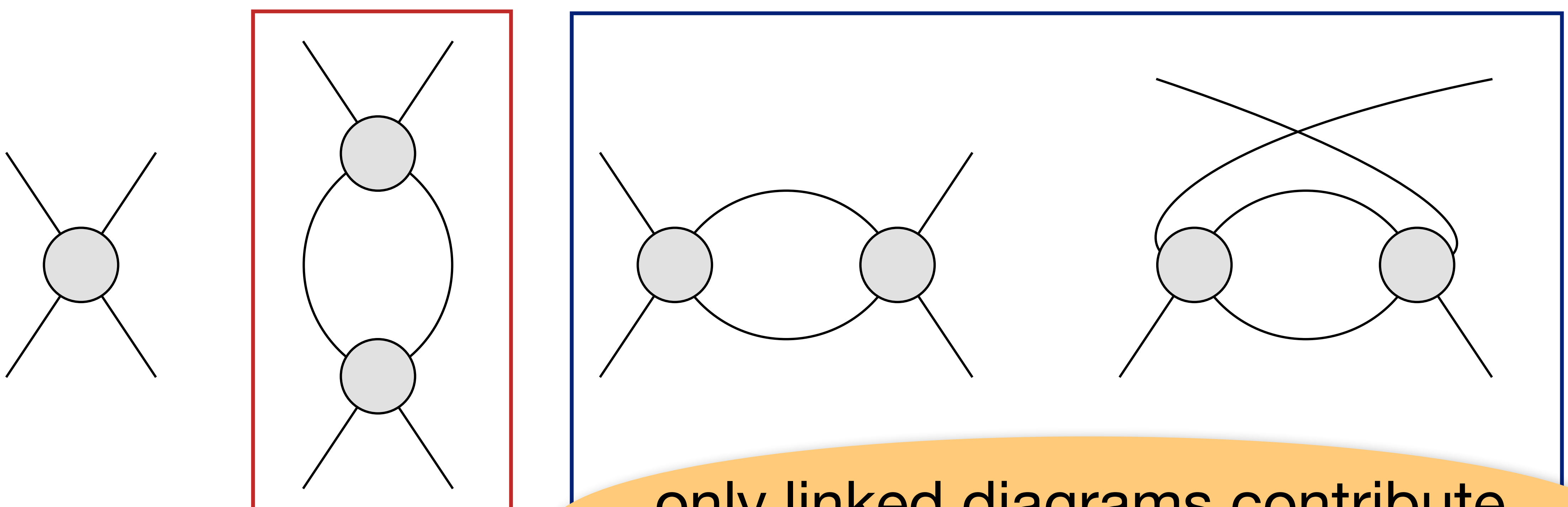
1-body Flow

$$\frac{d}{ds}f^1_2 = \sum_{a}\left(\eta^1_a f^a_2 - f^1_a \eta^a_2\right) + \sum_{ab}\left(\eta^a_b \Gamma^{b1}_{a2} - f^a_b \eta^{b1}_{a2}\right)(n_a - n_b)$$
$$+ \frac{1}{2}\sum_{abcdef}\left(\eta^{1a}_{bc}\Gamma^{bc}_{2a} - \Gamma^{1a}_{bc}\eta^{bc}_{2a}\right)(n_a \bar{n}_b \bar{n}_c + \bar{n}_a n_b n_c)$$

In-Medium SRG Flow Equations

2-body Flow

$$
\begin{aligned}
\frac{d}{ds}\Gamma^{12}_{34} = &\sum_a \left(\eta^1_a \Gamma^{a2}_{34} + \eta^2_a \Gamma^{1a}_{34} - \eta^a_3 \Gamma^{12}_{a4} - \eta^a_4 \Gamma^{12}_{3a} - f^1_a \eta^{a2}_{34} - f^2_a \eta^{1a}_{34} + f^a_3 \eta^{12}_{a4} + f^a_4 \eta^{12}_{3a}\right) \\
&+ \frac{1}{2}\sum_{ab} \underline{\left(\eta^{12}_{ab}\Gamma^{ab}_{34} - \Gamma^{12}_{ab}\eta^{ab}_{34}\right)(1 - n_a - n_b)} \\
&+ \sum_{ab} \underline{(n_a - n_b)\left(\left(\eta^{1a}_{3b}\Gamma^{2b}_{4a} - \Gamma^{1a}_{3b}\eta^{2b}_{4a}\right) - \left(\eta^{2a}_{3b}\Gamma^{1b}_{4a} - \Gamma^{2a}_{3b}\eta^{1b}_{4a}\right)\right)}
\end{aligned}
$$

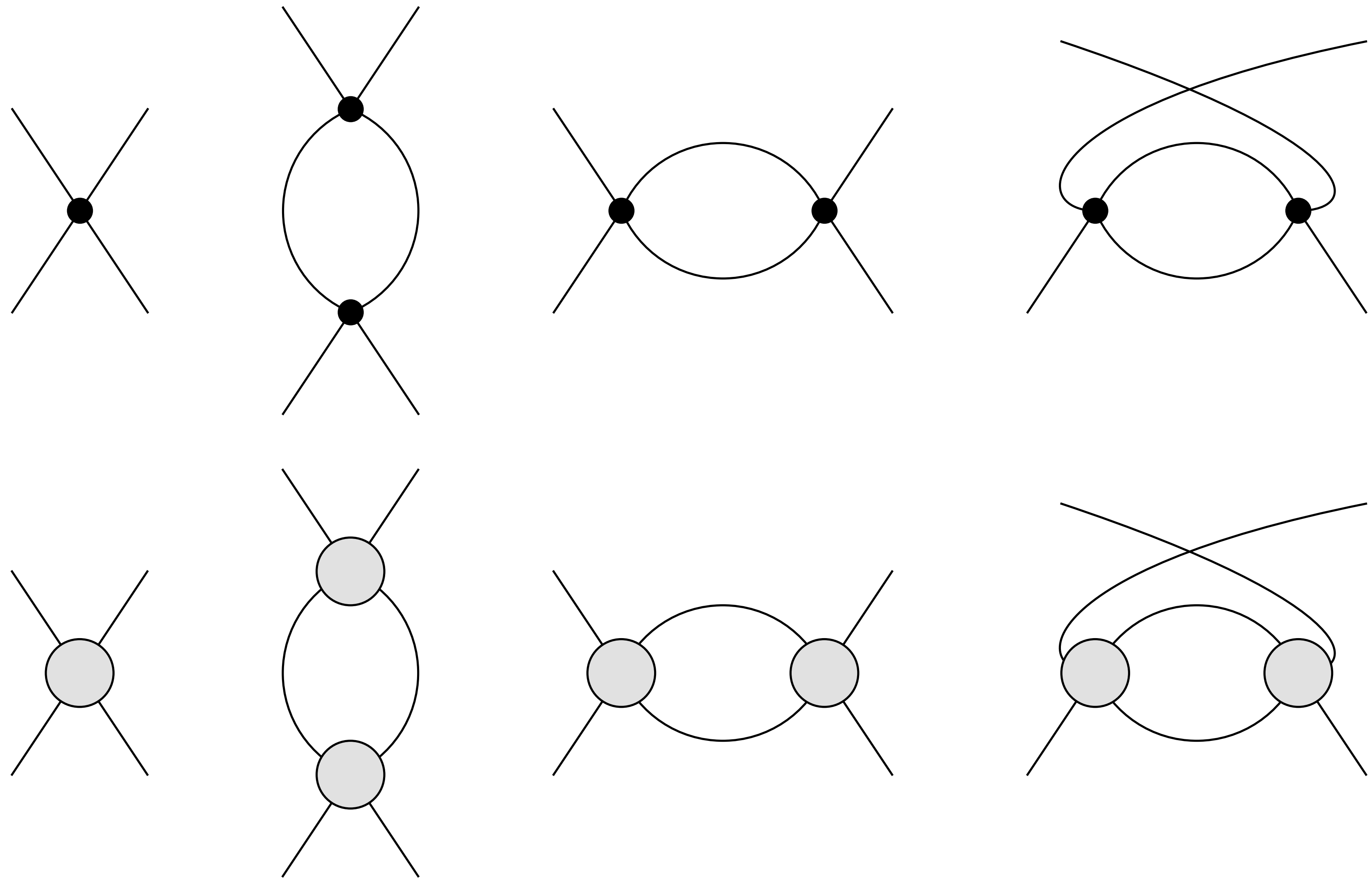


only linked diagrams contribute,
IM-SRG size-extensive

In-Medium SRG Flow: Diagrams

$\Gamma(\delta s) \sim$

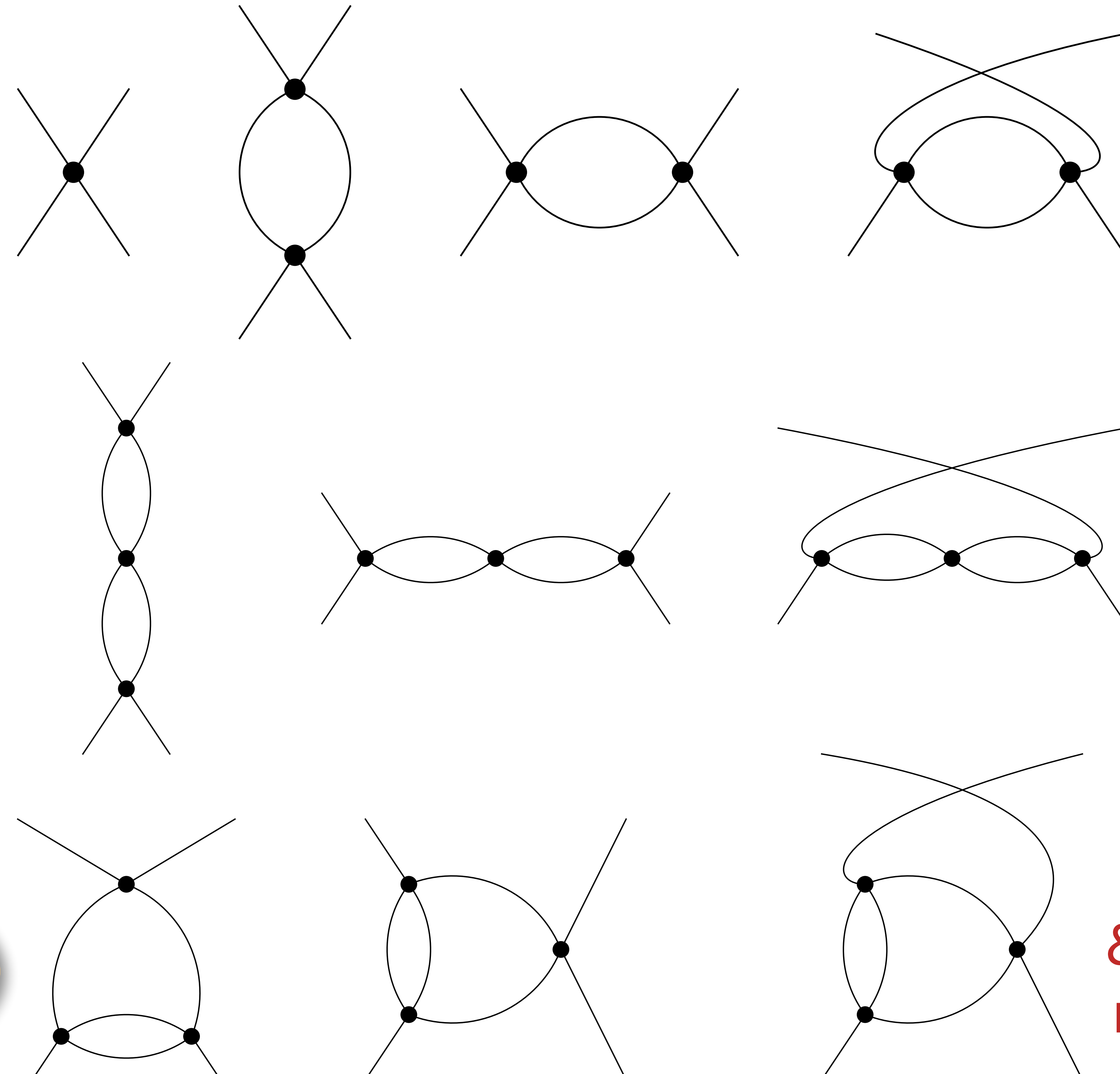
$\Gamma(2\delta s) \sim$



In-Medium SRG Flow: Diagrams

$\Gamma(\delta s) \sim$

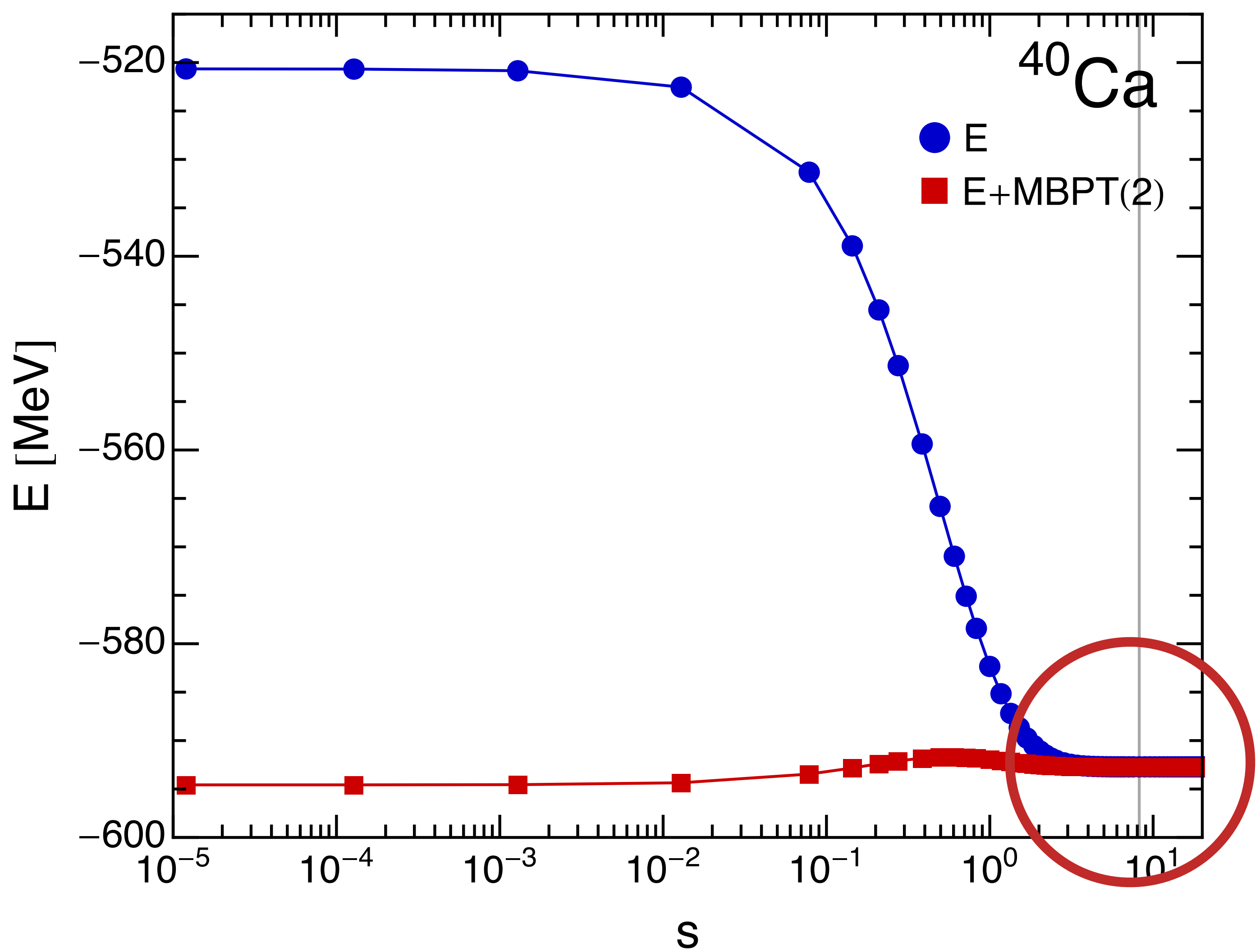
$\Gamma(2\delta s) \sim$

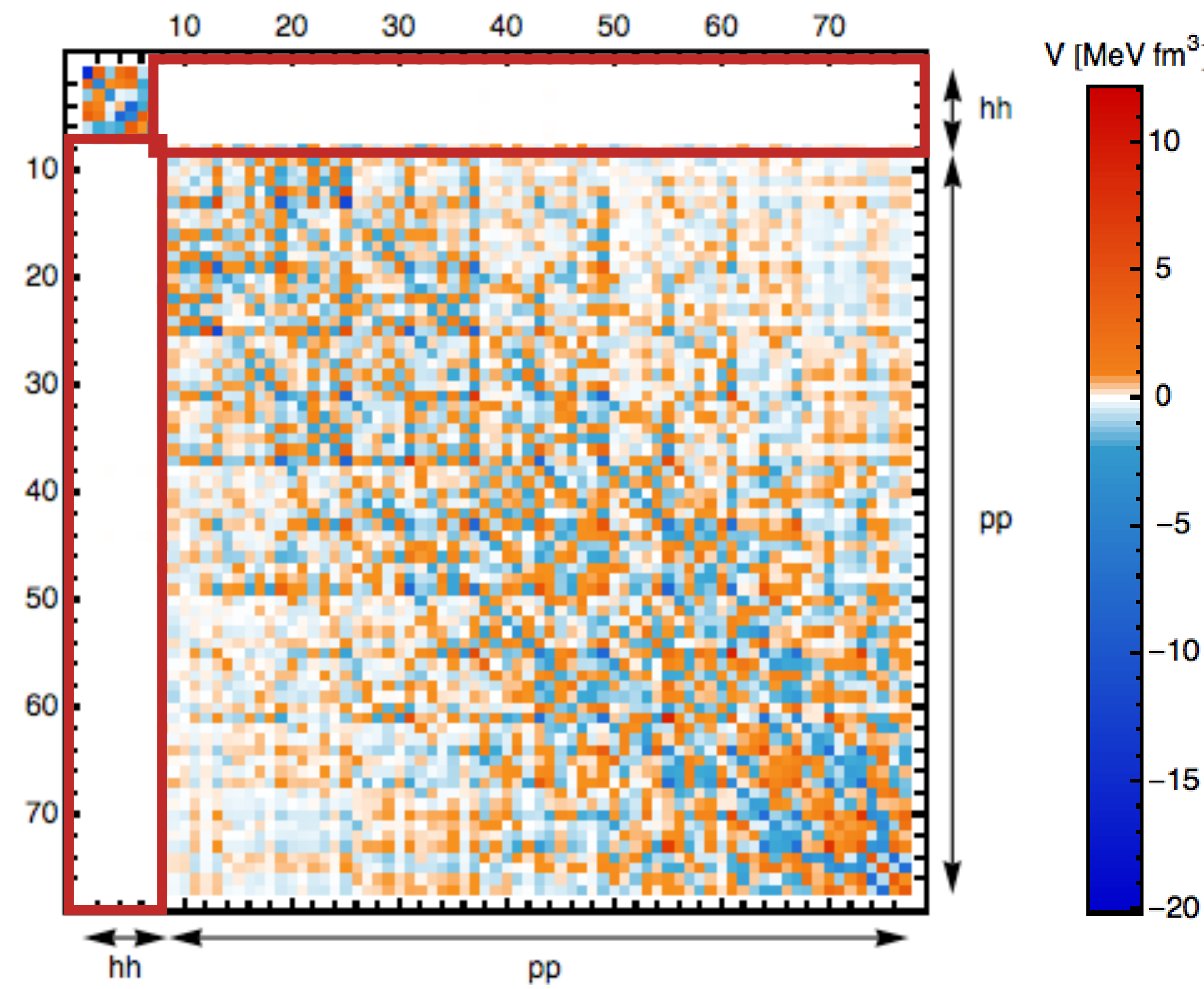


non-perturbative resummation

& many more...

Decoupling



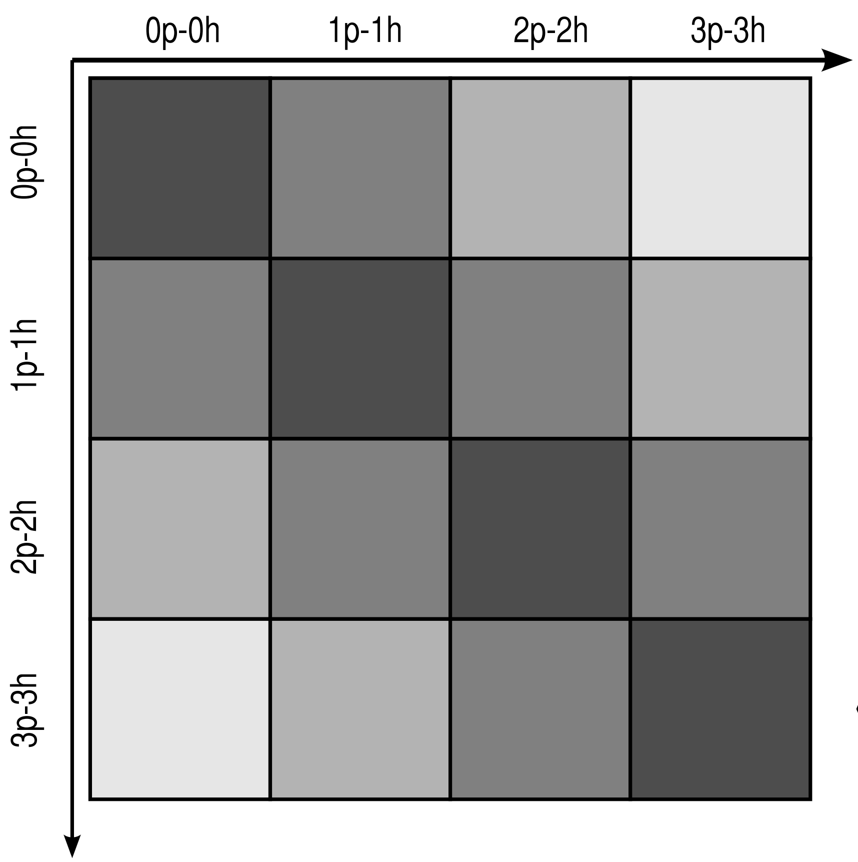


N3LO, $\lambda = 2.0\ \text{fm}^{-1}, e_{\text{Max}} = 8$

non-perturbative resummation of MBPT series (correlations)

off-diagonal couplings are rapidly driven to zero

Decoupling



$$\left\langle {}^{p}_{h} \right| H \left| \Psi \right\rangle \sim f^{p}_{h}, \sum_{kl} f^{k}_{l} \lambda^{hk}_{pl}, \sum_{klmn} \Gamma^{kl}_{mn} \lambda^{hkl}_{pmn}, \ldots$$

$$\left\langle {}^{pp'}_{hh'} \right| H \left| \Psi \right\rangle \sim \Gamma^{pp'}_{hh'}, \sum_{km} \Gamma^{pk}_{hm} \lambda^{h'k}_{p'm}, \sum_{kl} f^{k}_{l} \lambda^{hh'k}_{pp'l}, \sum_{klmn} \Gamma^{kl}_{mn} \lambda^{hh'kl}_{pp'mn}, \ldots$$

$$\left\langle {}^{pp'p''}_{hh'hh'} \right| H \left| \Psi \right\rangle \sim \ldots$$

- truncation in irreducible density matrices
  - number of correlated vs. total pairs, triples, ... (caveat: highly collective reference states)
  - perturbative analysis (e.g. for shell-model like states)
- verify for chosen multi-reference state when possible

Particle-Number Projected HFB State

- HFB ground state is a superposition of states with different particle number:

$$\left|\Psi\right\rangle = \sum_{A=N,N\pm 2,\ldots} c_A \left|\Psi_A\right\rangle\,, \quad \left|\Psi_N\right\rangle \equiv P_N\left|\Psi\right\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{i\phi(\hat{N}-N)}\left|\Psi\right\rangle$$

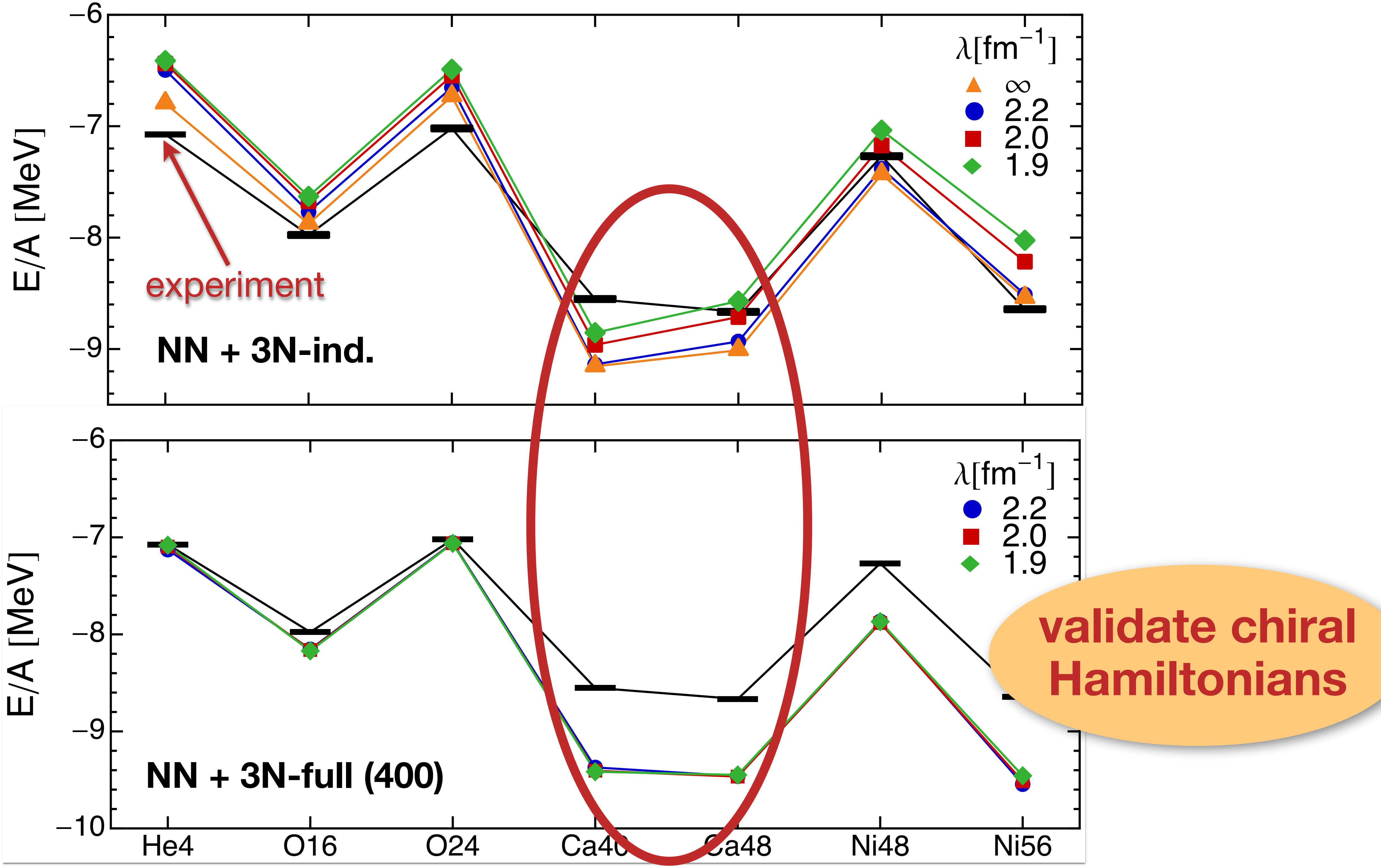
- calculate one- and two-body densities (project only once):

$$\lambda^k_l = \frac{\left\langle\Psi\right|A^k_l P_N\left|\Psi\right\rangle}{\left\langle\Psi\middle|\Psi\right\rangle}\,, \quad \lambda^{kl}_{mn} = \frac{\left\langle\Psi\right|A^{kl}_{mn} P_N\left|\Psi\right\rangle}{\left\langle\Psi\middle|\Psi\right\rangle} - \lambda^k_m\lambda^l_m + \lambda^k_n\lambda^l_m$$

- work in natural orbitals (= HFB canonical basis):

$$\lambda^k_l = n_k\delta^k_l \left(= v^2_k\delta^k_l\right)\,, \quad 0 \le n_k \le 1$$

Results: Closed-Shell Nuclei



Phys. Rev. C **87**, 034307 (2013), arXiv: 1212.1190 [nucl-th]

Multi-Reference Flow Equations

0-body flow:

$$\frac{dE}{ds} = \sum_{ab}(n_a - n_b)\left(\eta^a_b f^b_a - f^a_b \eta^b_a\right) + \frac{1}{4}\sum_{abcd}\left(\eta^{ab}_{cd}\Gamma^{cd}_{ab} - \Gamma^{ab}_{cd}\eta^{cd}_{ab}\right) n_a n_b \bar{n}_c \bar{n}_d$$

$$+ \frac{1}{4}\sum_{abcd}\left(\frac{d}{ds}\Gamma^{ab}_{cd}\right)\lambda^{ab}_{cd} + \frac{1}{4}\sum_{abcdklm}\left(\eta^{ab}_{cd}\Gamma^{kl}_{am} - \Gamma^{ab}_{cd}\eta^{kl}_{am}\right)\lambda^{bkl}_{cdm}$$

1-body flow:

$$\frac{d}{ds}f^1_2 = \sum_a \left(\eta^1_a f^a_2 - f^1_a \eta^a_2\right) + \sum_{ab}\left(\eta^a_b \Gamma^{b1}_{a2} - f^a_b \eta^{b1}_{a2}\right)(n_a - n_b)$$

$$+ \frac{1}{2}\sum_{abcdef}\left(\eta^{1a}_{bc}\Gamma^{bc}_{2a} - \Gamma^{1a}_{bc}\eta^{bc}_{2a}\right)(n_a \bar{n}_b \bar{n}_c + \bar{n}_a n_b n_c)$$

$$+ \frac{1}{4}\sum_{abcde}\left(\eta^{1a}_{bc}\Gamma^{de}_{2a} - \Gamma^{1a}_{bc}\eta^{de}_{2a}\right)\lambda^{de}_{bc} + \sum_{abcde}\left(\eta^{1a}_{bc}\Gamma^{be}_{2d} - \Gamma^{1a}_{bc}\eta^{be}_{2d}\right)\lambda^{ae}_{cd}$$

$$- \frac{1}{2}\sum_{abcde}\left(\eta^{1a}_{2b}\Gamma^{cd}_{ae} - \Gamma^{1a}_{2b}\eta^{cd}_{ae}\right)\lambda^{cd}_{be} + \frac{1}{2}\sum_{abcde}\left(\eta^{1a}_{2b}\Gamma^{bc}_{de} - \Gamma^{1a}_{2b}\eta^{bc}_{de}\right)\lambda^{ac}_{de}$$

Multi-Reference Flow Equations

2-body flow:

$$\frac{d}{ds}\Gamma^{12}_{34} = \sum_a \left(\eta^1_a \Gamma^{a2}_{34} + \eta^2_a \Gamma^{1a}_{34} - \eta^a_3 \Gamma^{12}_{a4} - \eta^a_4 \Gamma^{12}_{3a} - f^1_a \eta^{a2}_{34} - f^2_a \eta^{1a}_{34} + f^a_3 \eta^{12}_{a4} + f^a_4 \eta^{12}_{3a}\right)$$

$$+ \frac{1}{2}\sum_{ab}\left(\eta^{12}_{ab}\Gamma^{ab}_{34} - \Gamma^{12}_{ab}\eta^{ab}_{34}\right)(1 - n_a - n_b)$$

$$+ \sum_{ab}(n_a - n_b)\left(\left(\eta^{1a}_{3b}\Gamma^{2b}_{4a} - \Gamma^{1a}_{3b}\eta^{2b}_{4a}\right) - \left(\eta^{2a}_{3b}\Gamma^{1b}_{4a} - \Gamma^{2a}_{3b}\eta^{1b}_{4a}\right)\right)$$

2-body flow unchanged