

# *Ab initio nuclear physics with chiral EFT*

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*Andreas Ekström (UT/ORNL)*



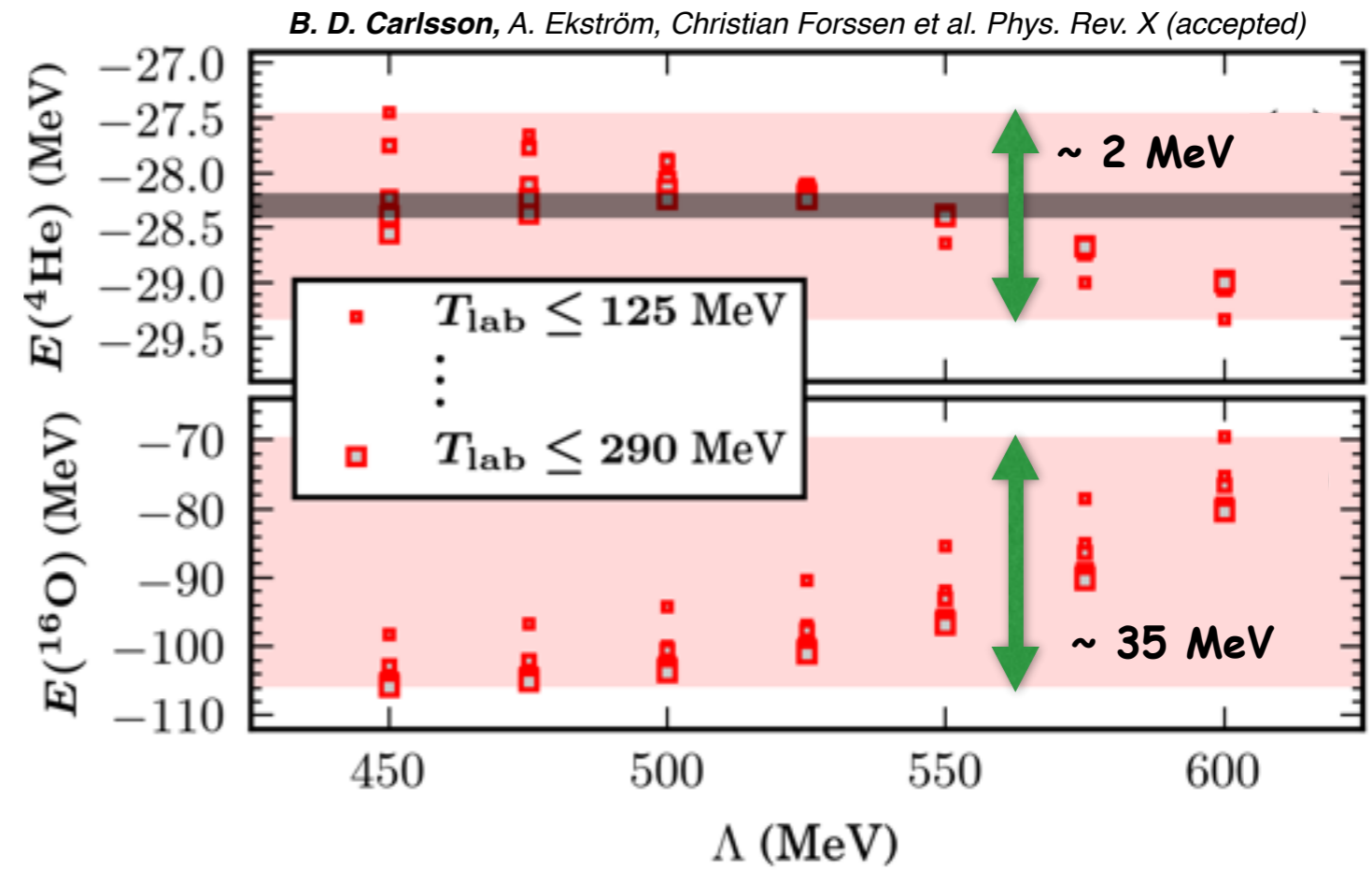
*Progress in Ab Initio Techniques in Nuclear Physics  
February 23-26, 2016, TRIUMF, Vancouver, BC, Canada*

# Overview

- *Simultaneous optimization*
- *UQ applied to proton-proton fusion*
- *Chiral EFT tailored to the HO-basis*
- *Optimizing the 3NF at N3LO*

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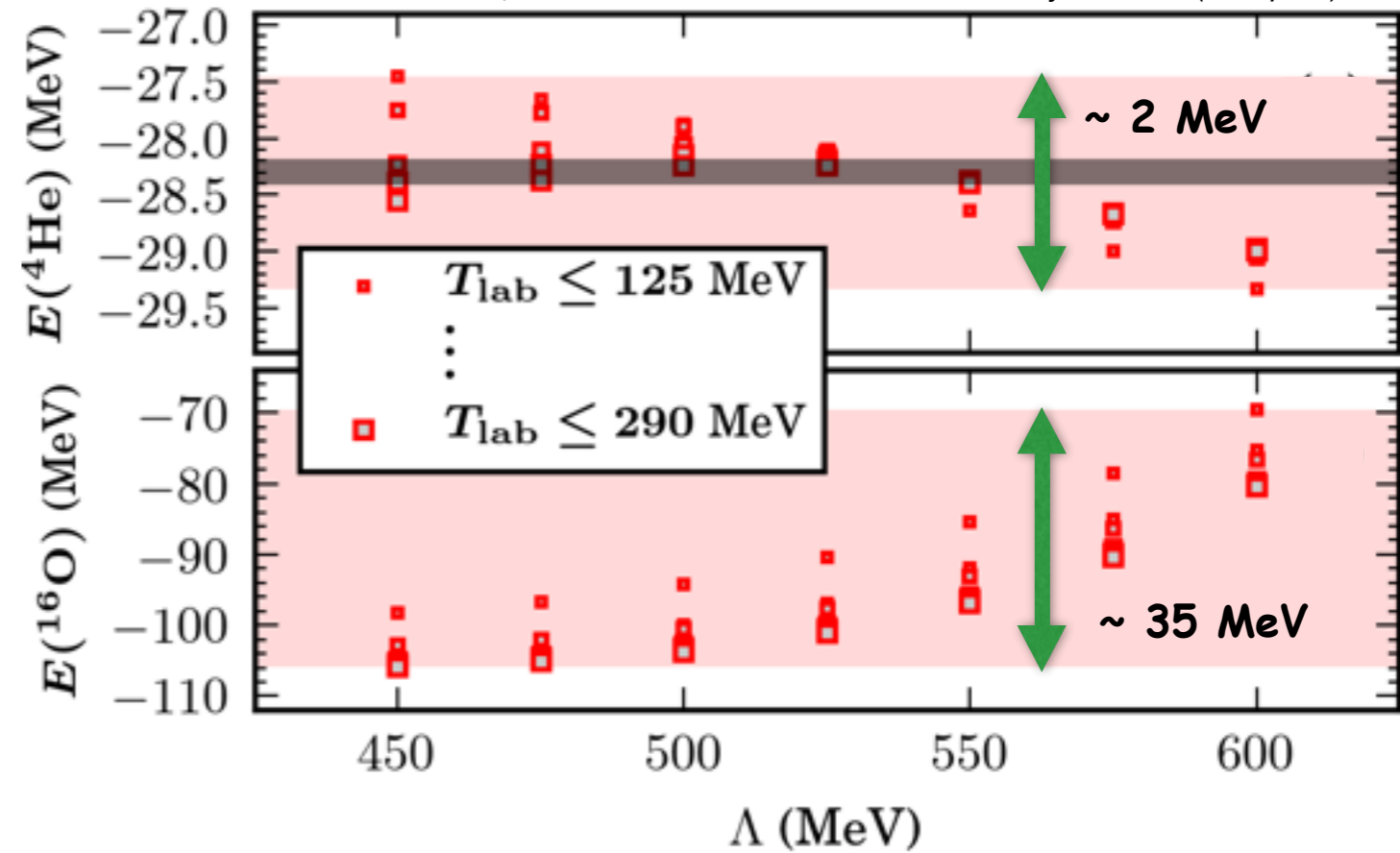
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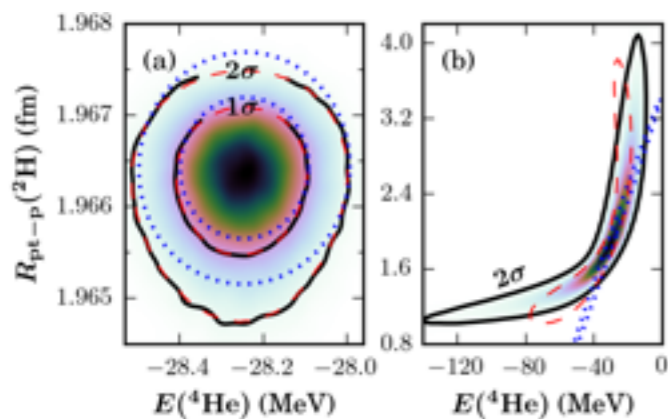
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B. D. Carlsson, A. Ekström, Christian Forssen et al. Phys. Rev. X (accepted)



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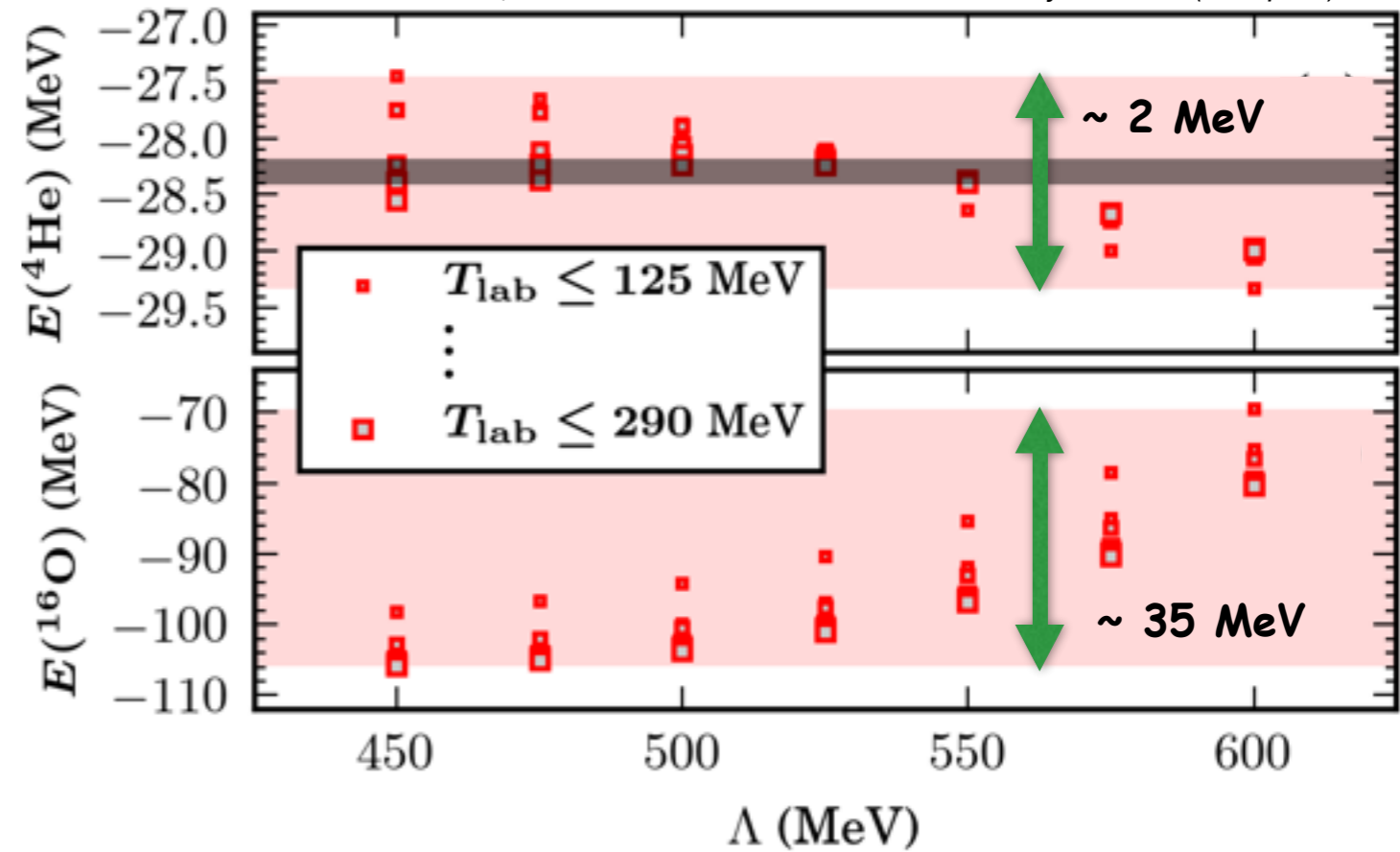
Robust parameter estimation  
Three-nucleon scattering data.  
The information content of heavy nuclei.

.....

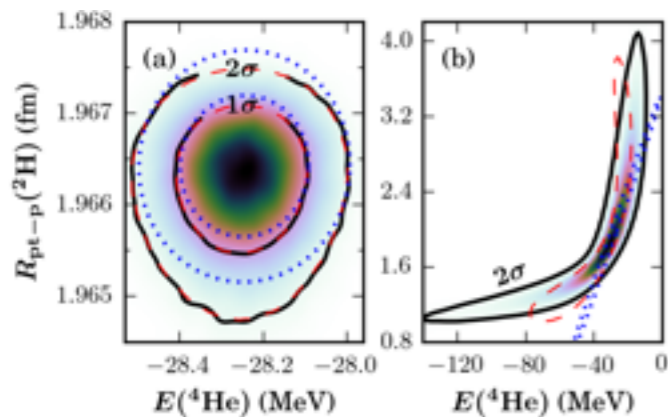
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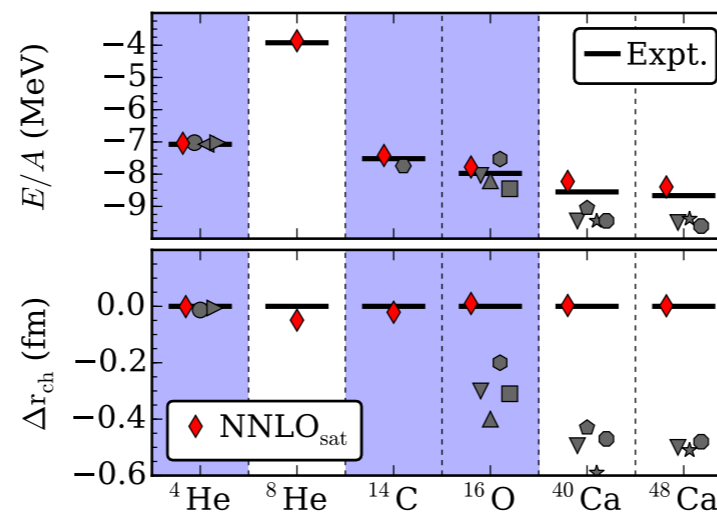


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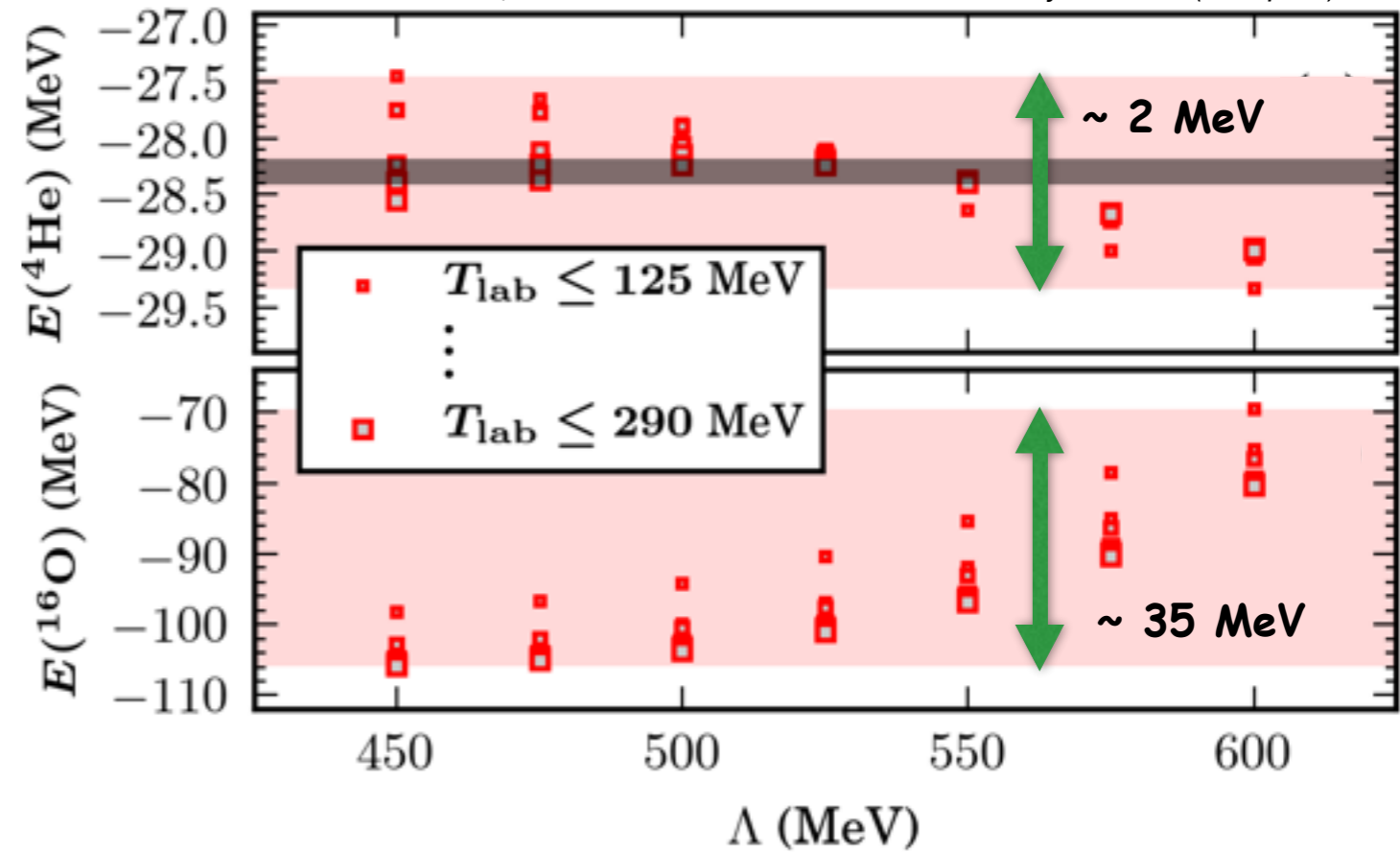
2. Explore alternative strategies of informing the model about low-energy many-body observables.



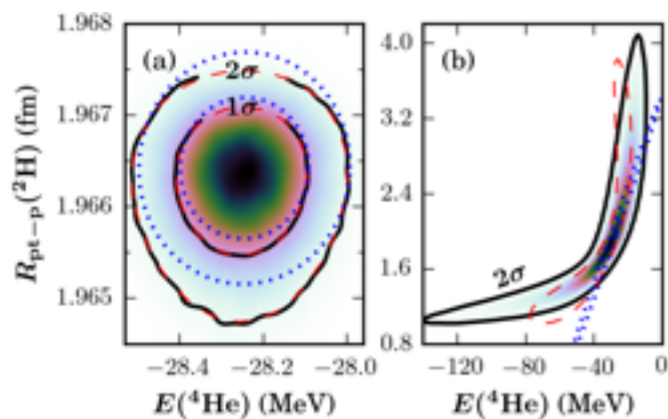
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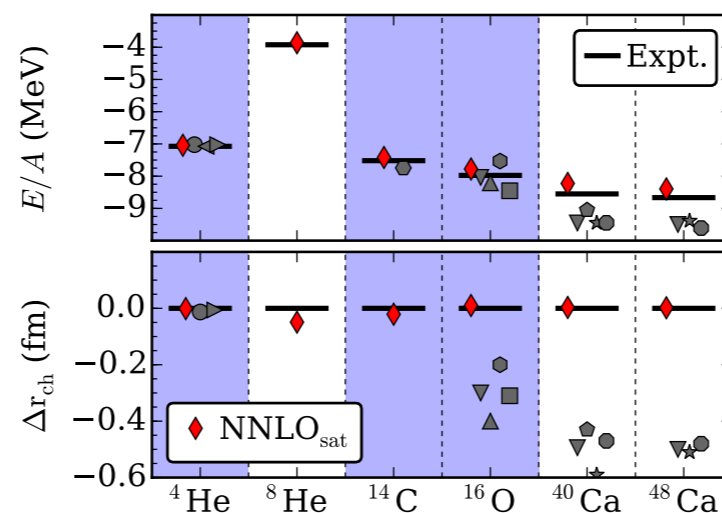


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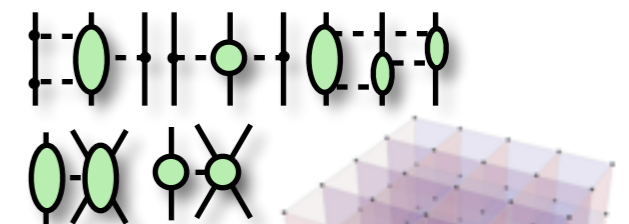


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2. Explore alternative strategies of informing the model about low-energy many-body observables.



3. Continue efforts towards higher orders of the chiral expansion, and possibly revisit the power counting.



Delta resonances  
Other regulators  
Lattice QCD data  
Modified PC

...

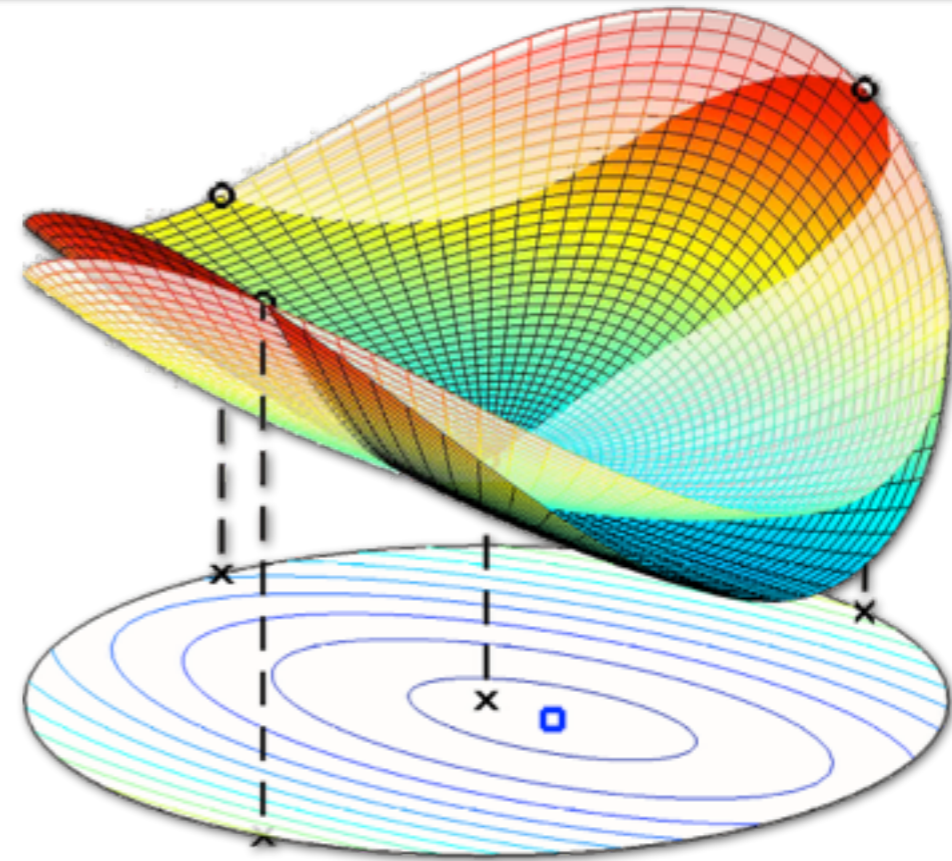


# Simultaneous optimization

chiral EFT is our tool to analyze the nuclear interaction

	NN	NNN	NNNN
LO ( $\nu=0$ )			
NLO ( $\nu=2$ )			
NNLO ( $\nu=3$ )			
N3LO ( $\nu=4$ )			
N4LO ( $\nu=5$ )			

E. Epelbaum et al. *Rev. Mod. Phys.* 81, 1773 (2009)  
 R. Machleidt et al. *Phys. Rep.* 503, 1 (2011)



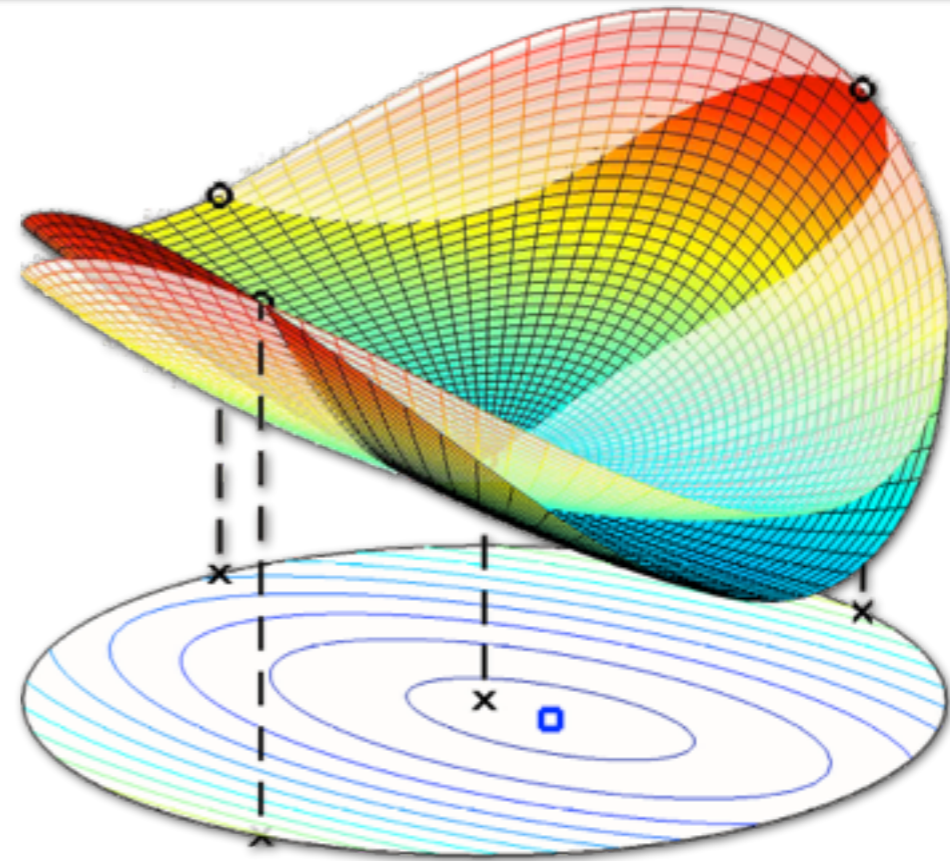
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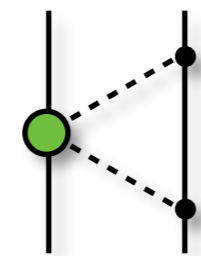
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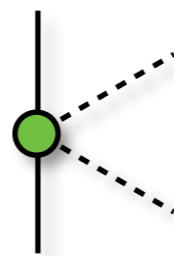


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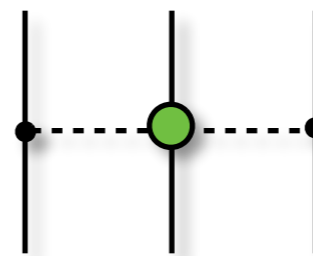
the same LECs appear in the expressions for various low-energy processes e.g. the  $c_i$  (green dot) and  $c_D$  (blue square)



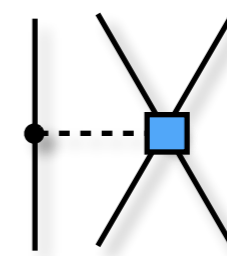
two-nucleon interaction



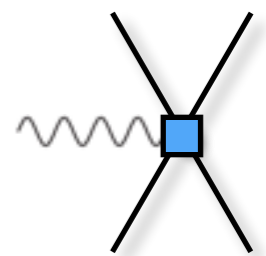
pion-nucleon scattering



three-nucleon interaction



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external probe current

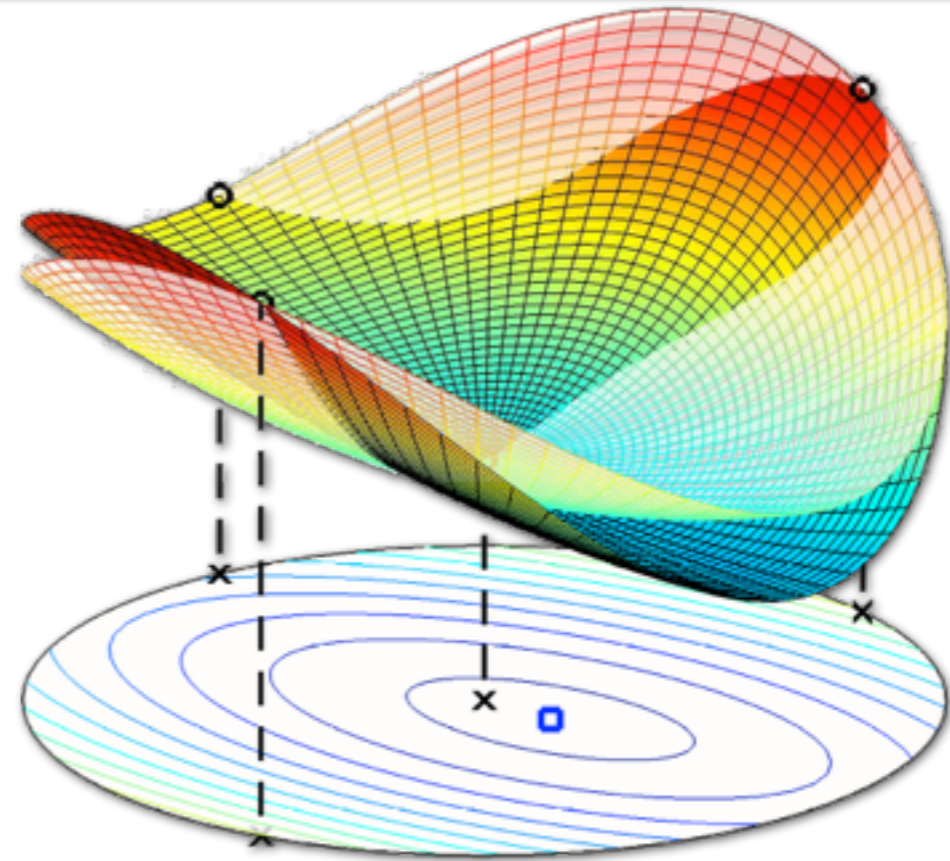


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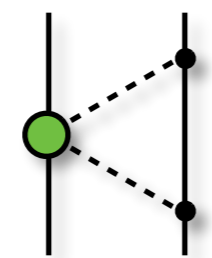
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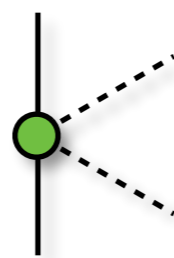


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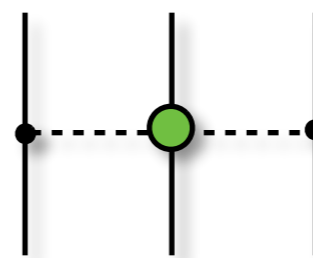
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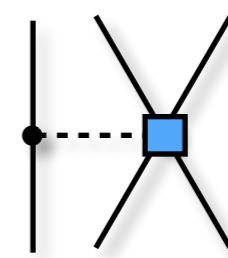
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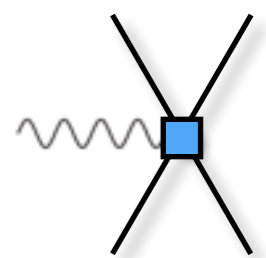
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external probe current

$$\chi^2(\mathbf{p}) \equiv \sum_{i \in \mathcal{M}} R_i^2(\mathbf{p}) = \sum_{k \in \pi\mathcal{N}} R_k^2(\mathbf{p}) + \sum_{j \in \mathcal{N}\mathcal{N}} R_j^2(\mathbf{p}) + \sum_{l \in \mathcal{N}\mathcal{N}\mathcal{N}} R_l^2(\mathbf{p})$$

**Simultaneous** optimization is critical in order to

- find the optimal set of LECs.
- capture all relevant correlations between them.
- reduce the statistical uncertainty.
- attain order-by-order convergence.

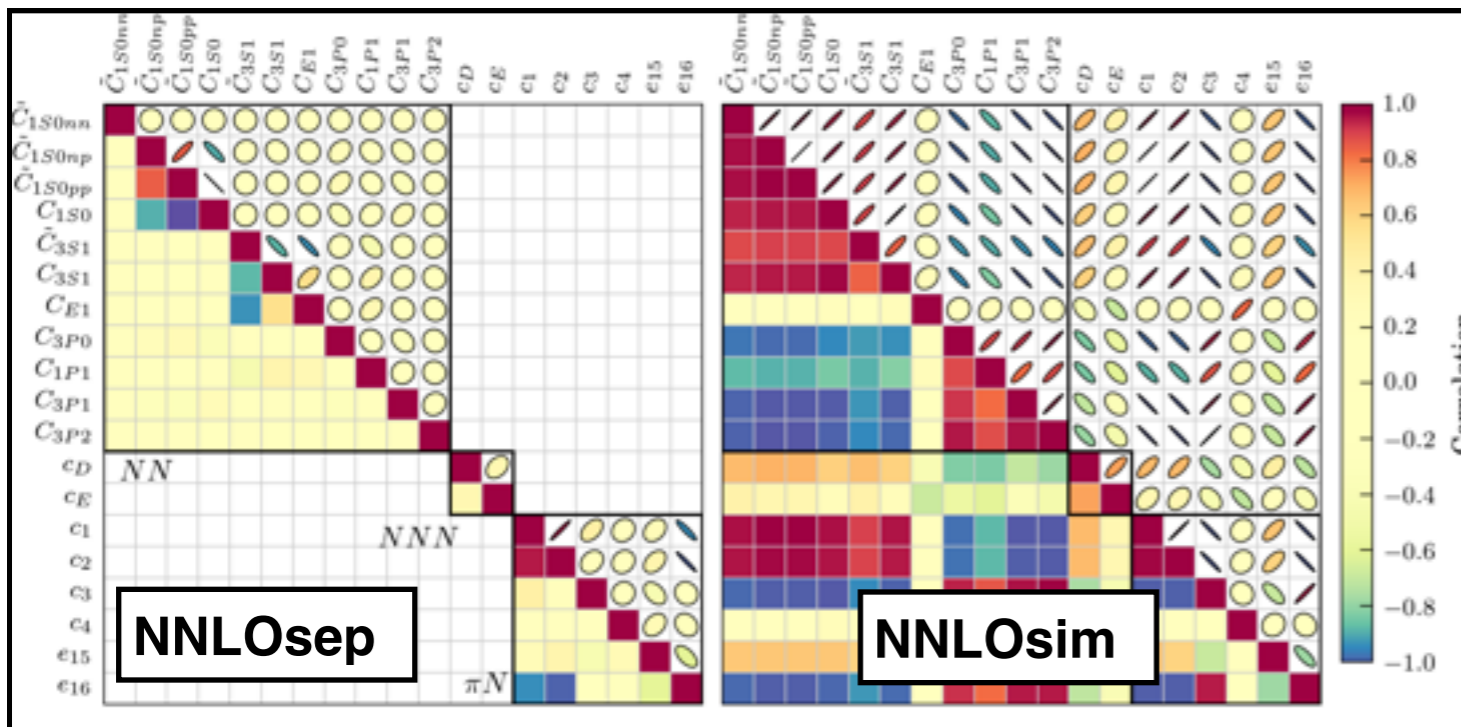
Within such an approach we find that statistical errors are, in general, small, and that the total error budget is dominated by systematic errors.

# UQ with $N^2LO_{sim}$

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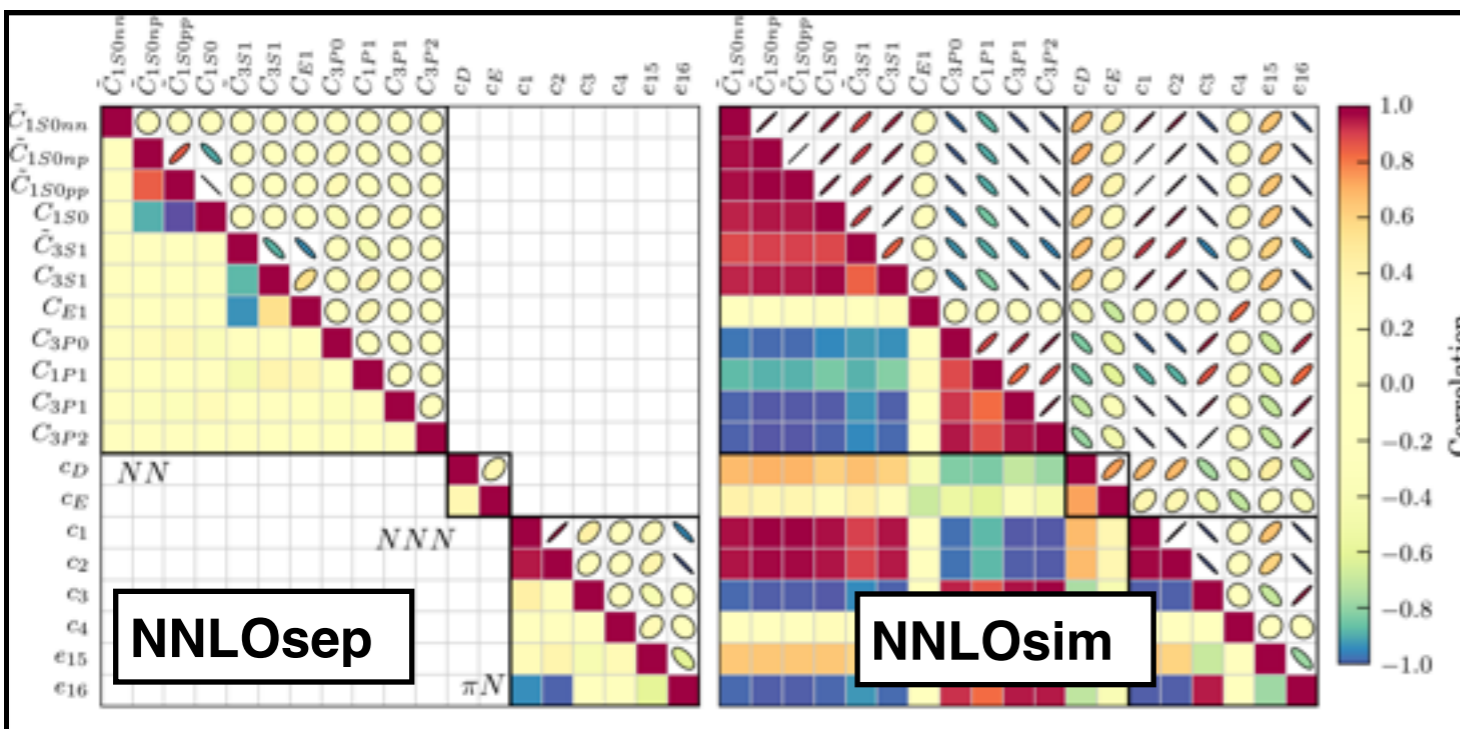
- All 42 different sim/sep potentials, as well as the respective covariance matrices are available as supplemental material.

LO-NLO-NNLO

7 different cutoffs: 450,475,...,600 MeV

6 different NN-scattering datasets

$$\begin{aligned} \text{Cov}(A, B) &\equiv \mathbb{E}[(\mathcal{O}_A(\mathbf{p}) - \mathbb{E}[\mathcal{O}_A(\mathbf{p})])(\mathcal{O}_B(\mathbf{p}) - \mathbb{E}[\mathcal{O}_B(\mathbf{p})])] \\ &\approx \mathbb{E}\left[\left(\tilde{J}_{A,i}x_i + \frac{1}{2}\tilde{H}_{A,ij}x_ix_j - \frac{1}{2}\tilde{H}_{A,ii}\sigma_i^2\right)\right. \\ &\quad \left.\times \left(\tilde{J}_{B,k}x_k + \frac{1}{2}\tilde{H}_{B,kl}x_kx_l - \frac{1}{2}\tilde{H}_{B,kk}\sigma_k^2\right)\right] \\ &= \tilde{\mathbf{J}}_A^T \Sigma \tilde{\mathbf{J}}_B + \frac{1}{2}(\boldsymbol{\sigma}^2)^T (\tilde{\mathbf{H}}_A \circ \tilde{\mathbf{H}}_B) \boldsymbol{\sigma}^2, \end{aligned}$$



**compute the derivatives of your own observables wrt LECs, then explore:**

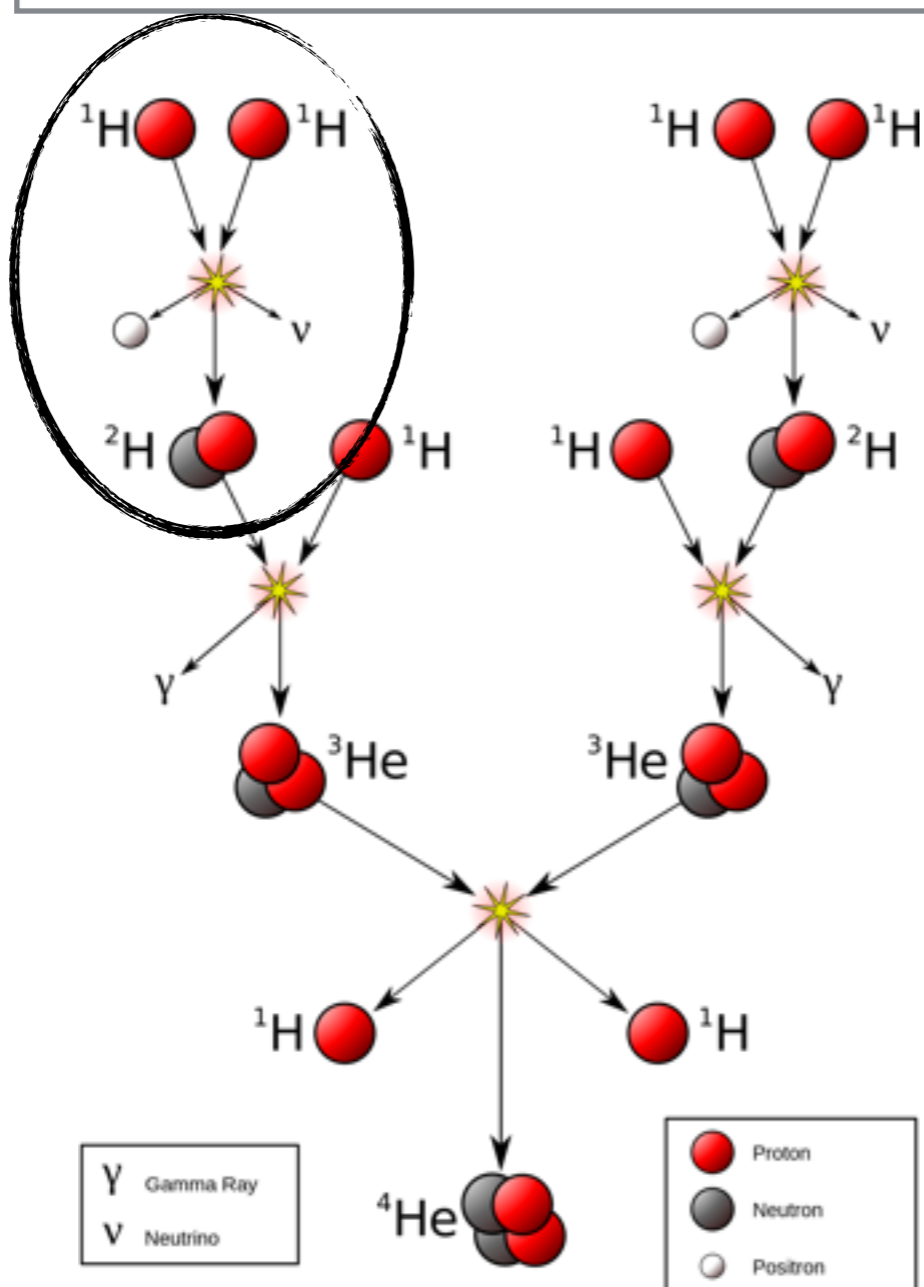
- **cutoff variations**
- **order-by-order evolution**
- **LEC UQ/correlations**





# UQ applied to proton-proton fusion

*In the core of the Sun, energy is released through sequences of nuclear reactions that convert hydrogen into helium. The primary reaction is thought to be the fusion of two protons with the emission of a low-energy neutrino and a positron.*



$$p + p \rightarrow d + e^+ + \nu_e$$

$$S(E) = \sigma(E) E e^{2\pi\eta}$$

$$\sigma(E) = \int \frac{d^3 p_e}{(2\pi)^3} \frac{d^3 p_\nu}{(2\pi)^3} \frac{1}{2E_e} \frac{1}{2E_\nu} \times$$

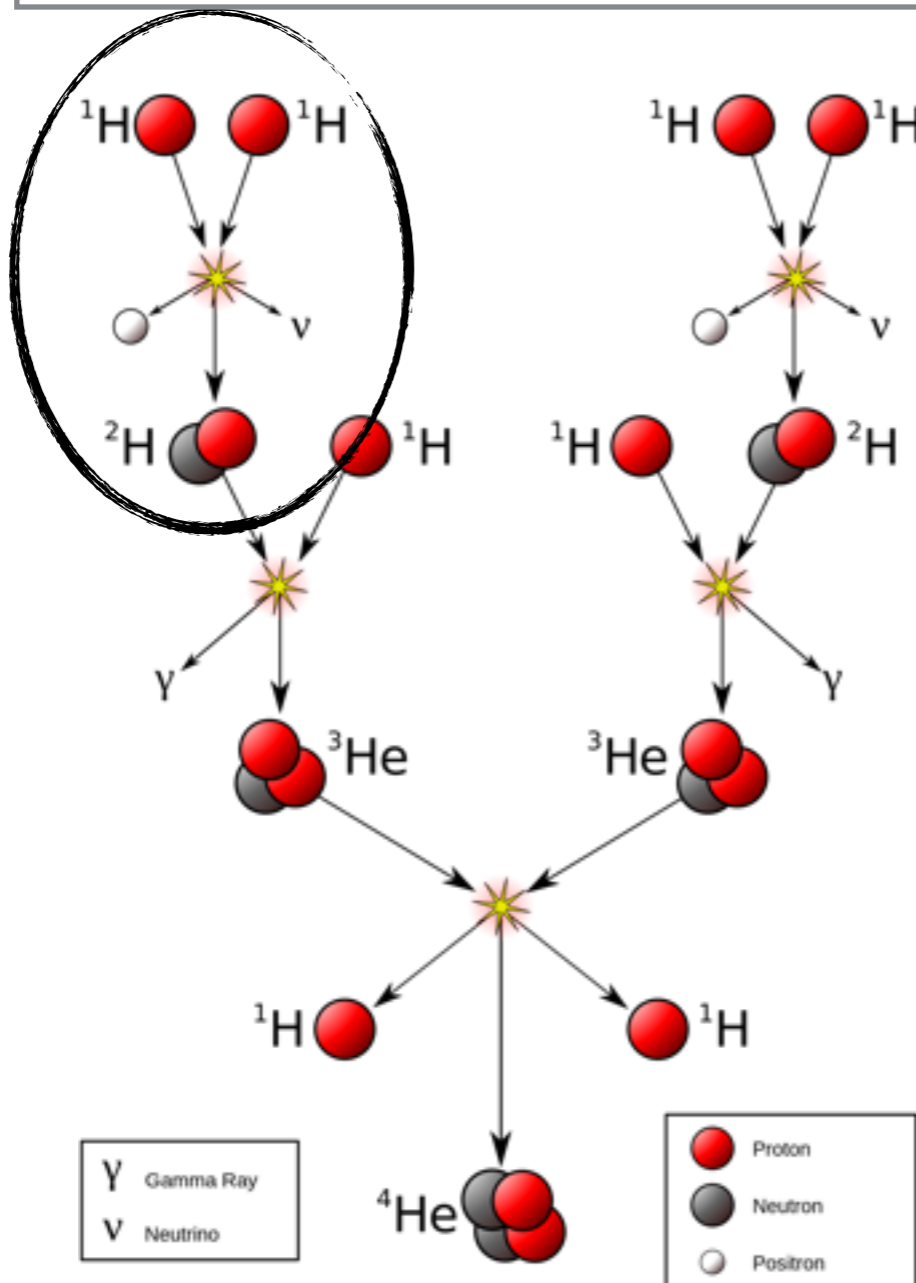
$$2\pi \delta \left( E + 2m_p - m_d - \frac{q^2}{2m_d} - E_e - E_\nu \right)$$

$$\frac{1}{v_{rel}} F(Z, E_e) \frac{1}{4} \sum |\langle f | \hat{H}_W | i \rangle|^2$$



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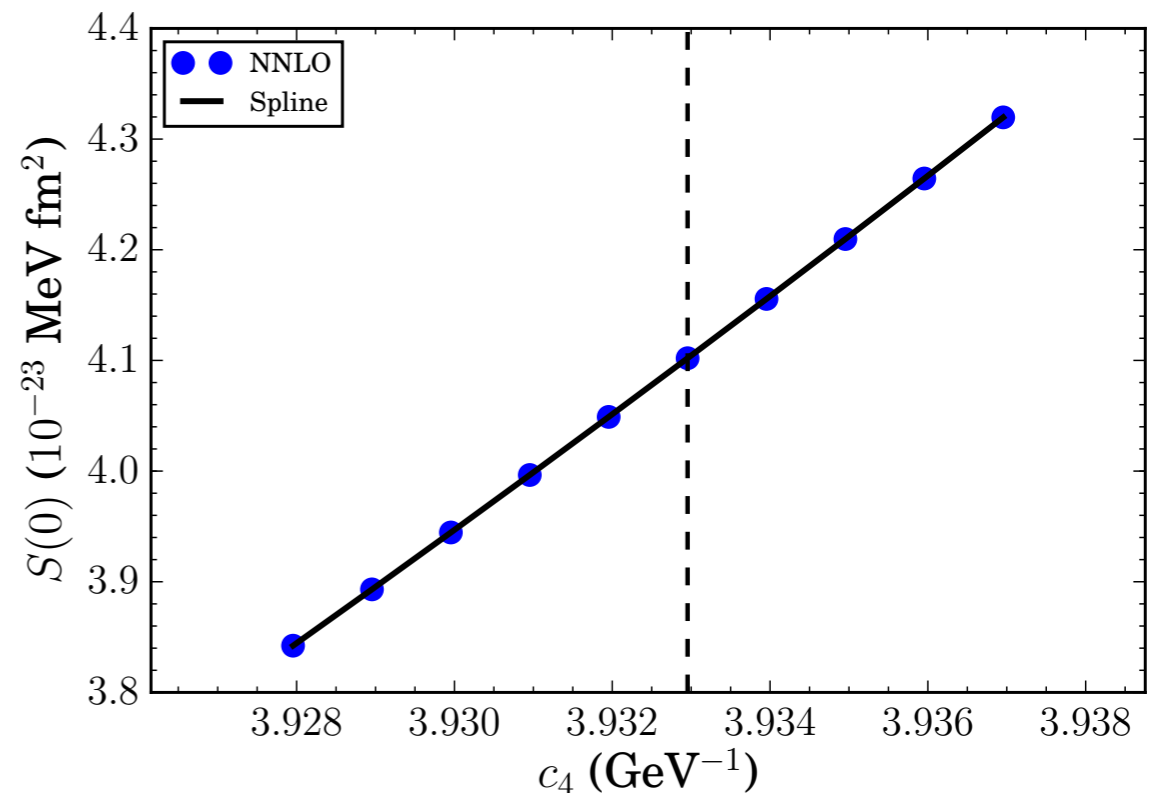
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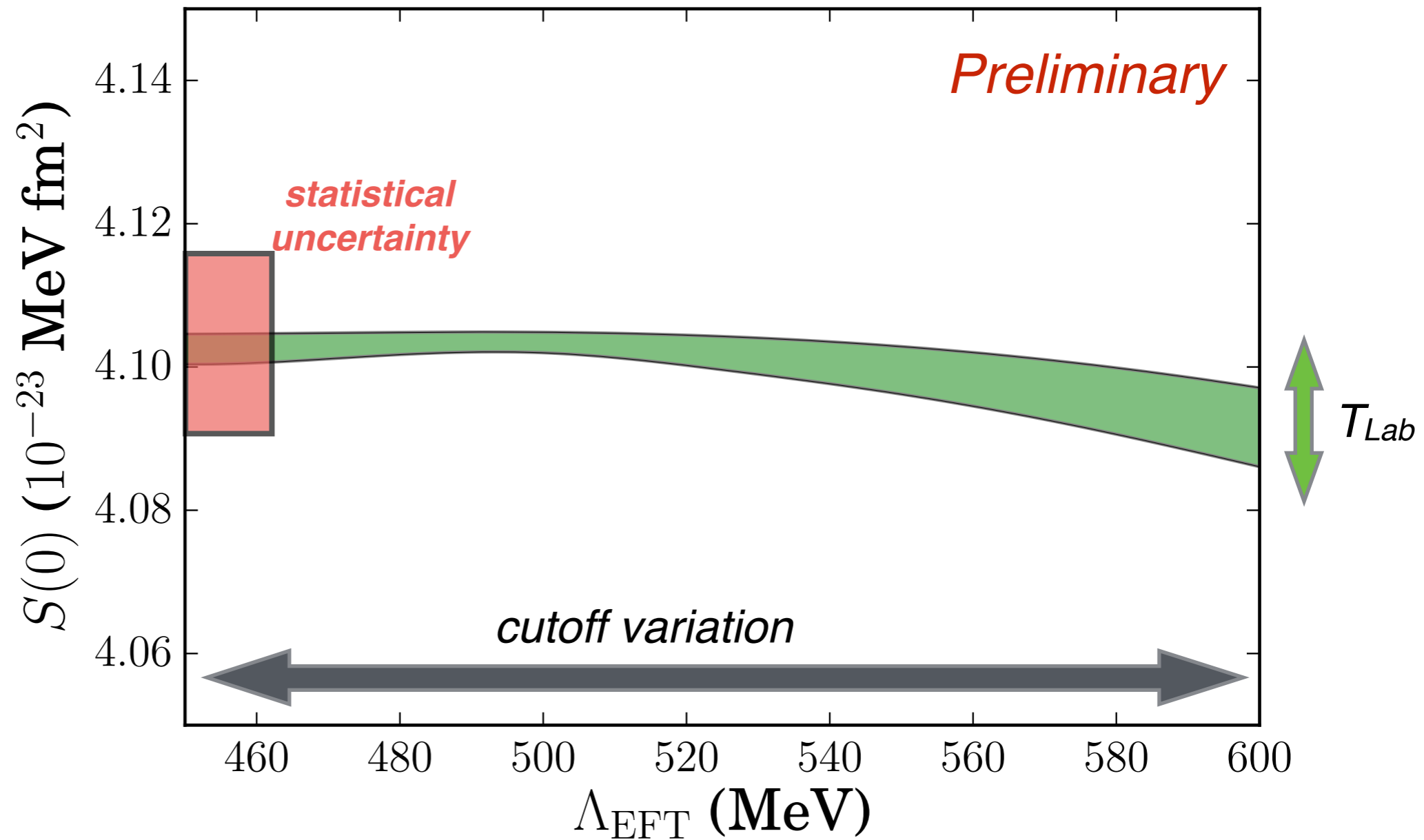
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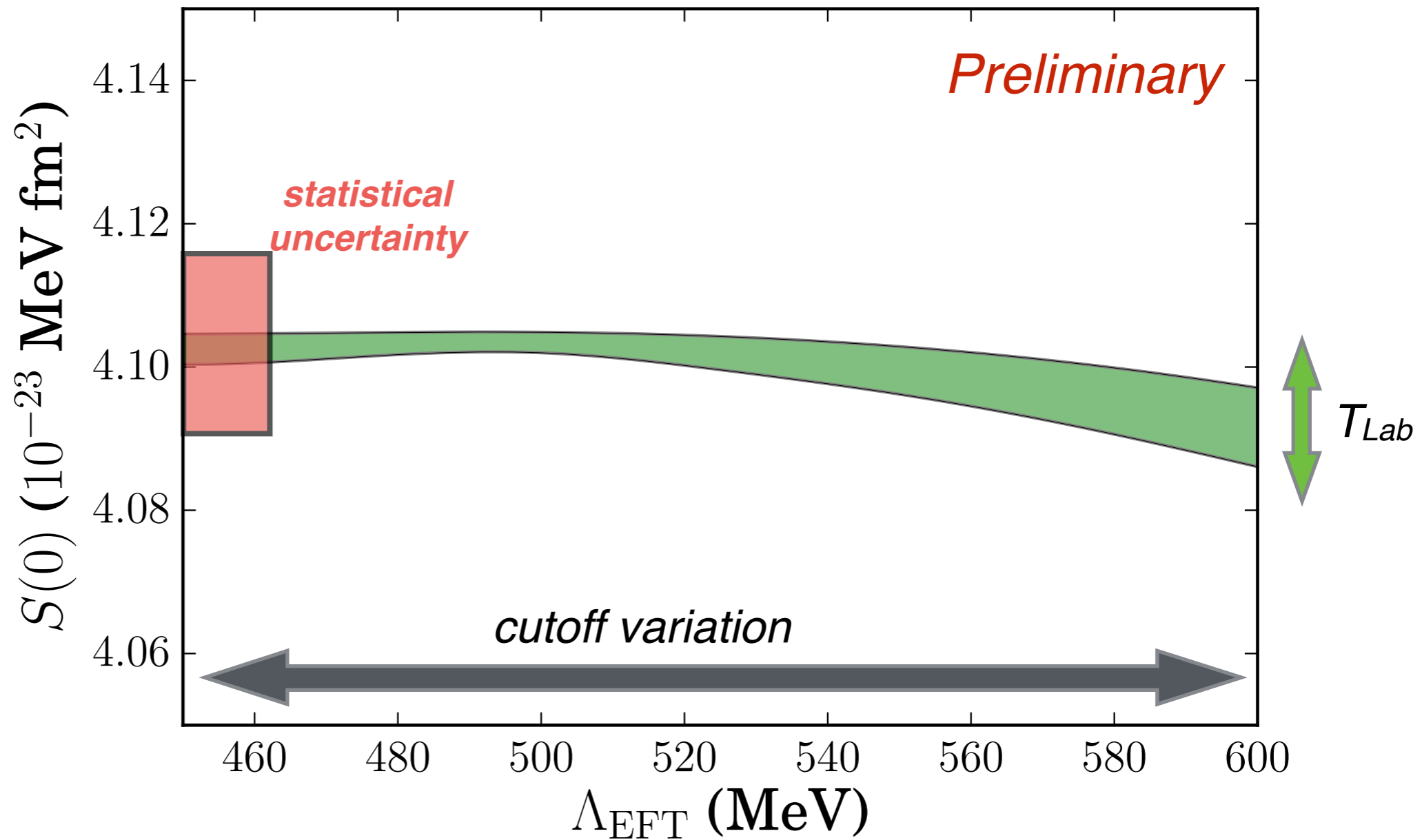
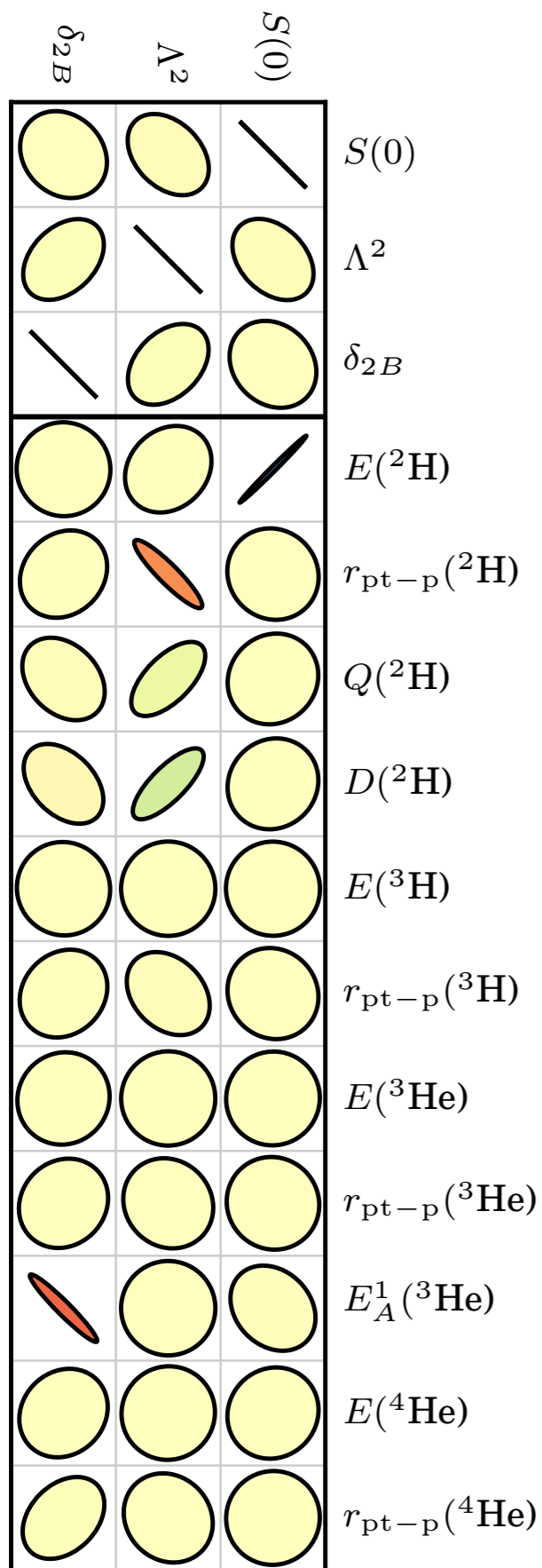
L. E. Marcucci et al PRL 110, 192503 (2013)  
 R. Schiavilla et al PRC 58, 1263 (1998)  
 J-W. Chen et al. PLB 720, 385 (2013)

*In collaboration with B. Acharya, L. Platter, B. D. Carlsson, and C. Forssen*

# UQ applied to proton-proton fusion



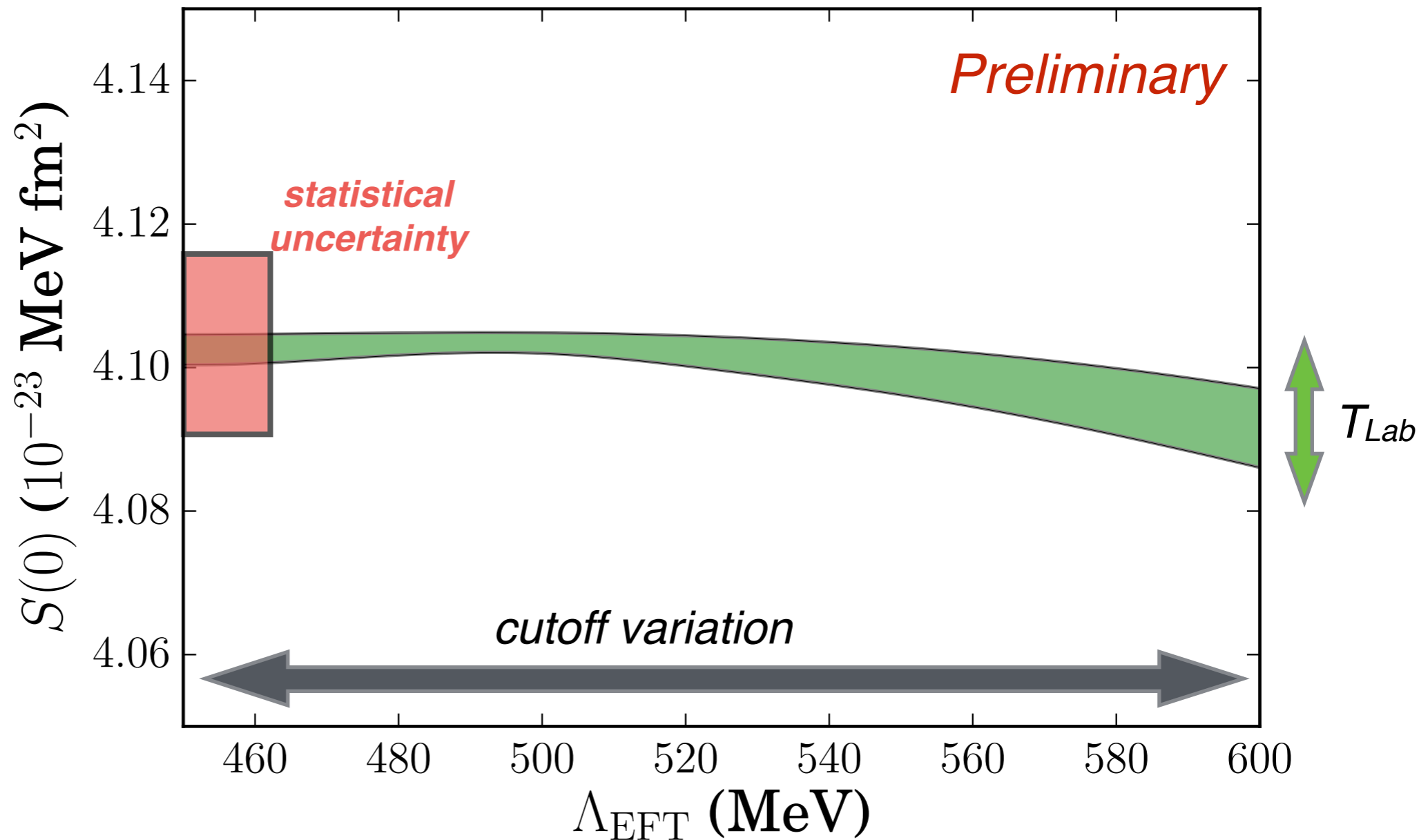
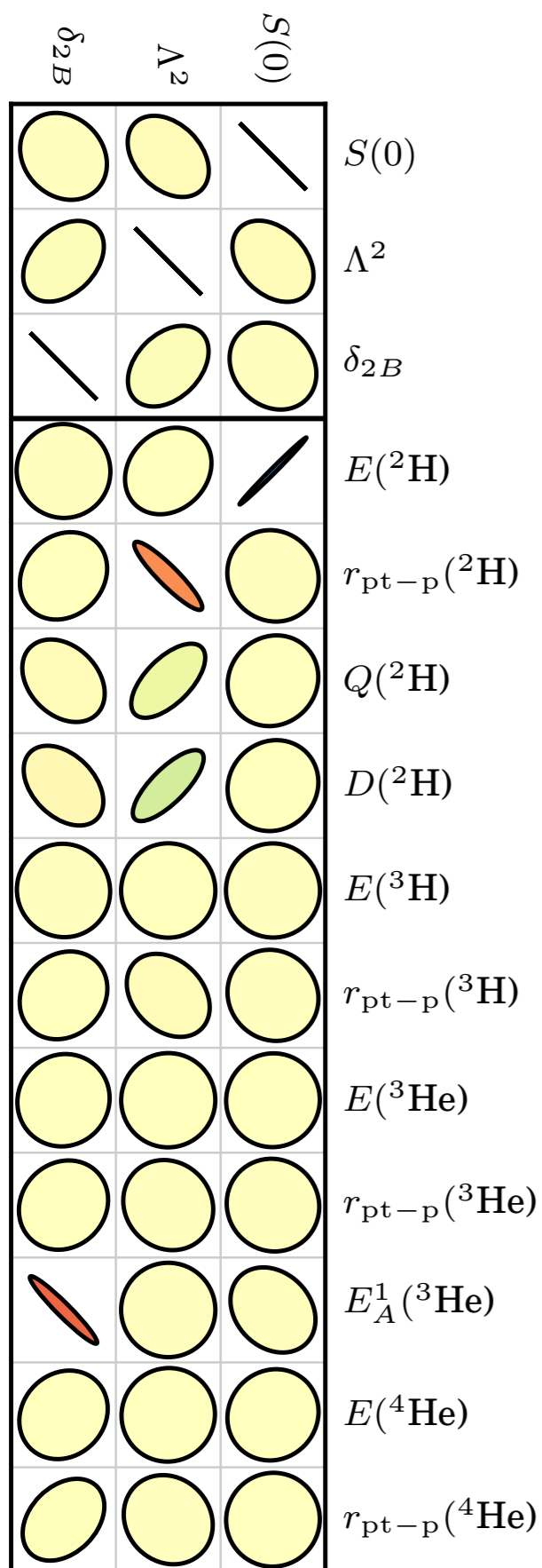
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**correlation analysis:**

- $S(0) - E(^2\text{H})$ : phase space
- $\Lambda^2 - r(^2\text{H})$ : radial overlap
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- $\delta_{2B} - E1A$ : 2B-current operator

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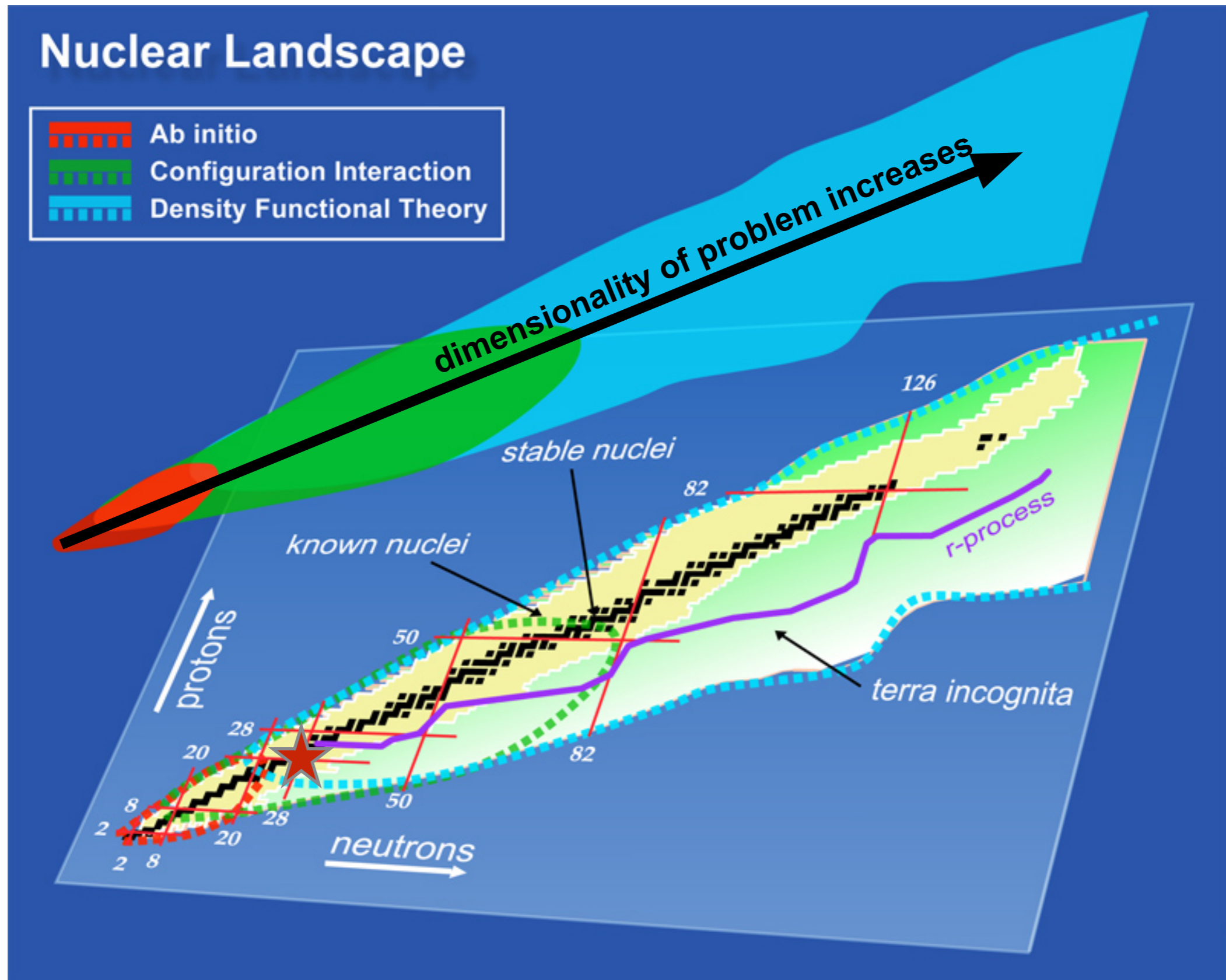
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- Insofar most consistent xEFT-study of this reaction
- Correlation study indicates sound statistical analysis
- Cutoff variation not large source of error
- Statistical error in  $S(0)$  is 3 times larger than what was previously thought
- Central value is most likely also larger due to previously neglected systematic uncertainties.

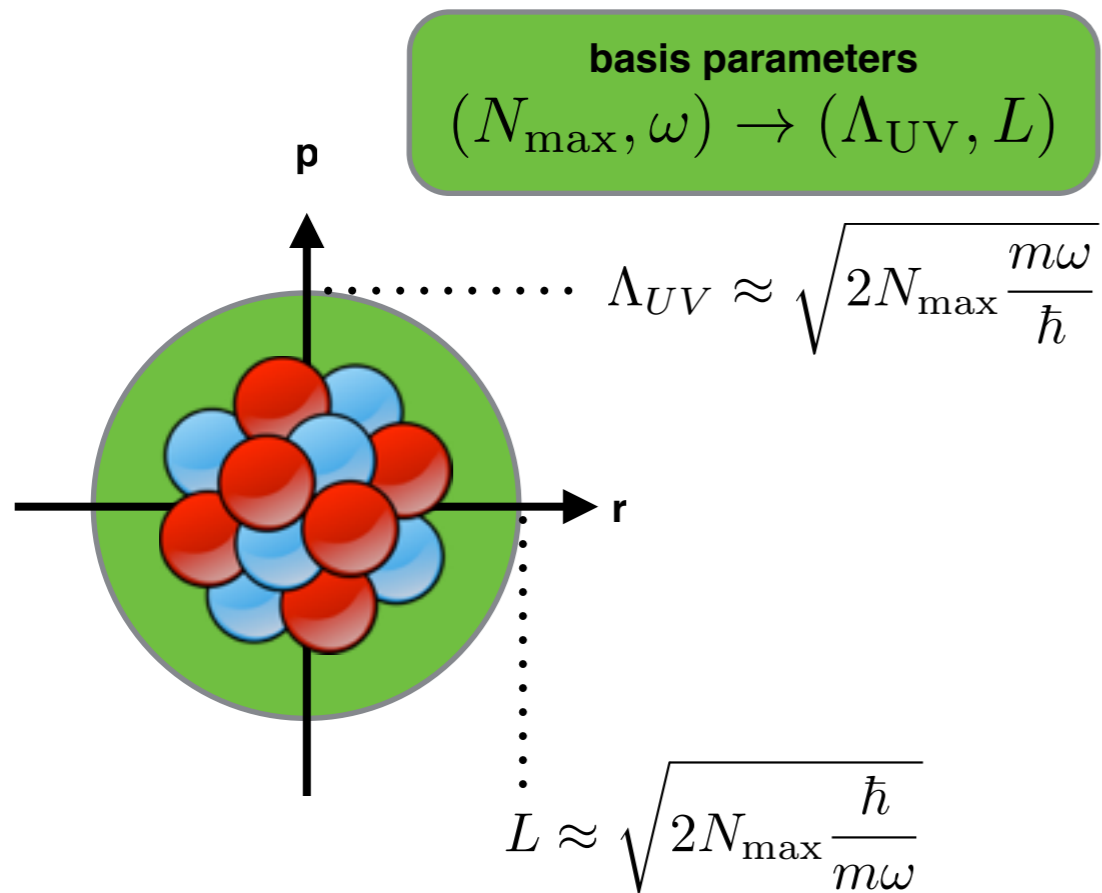


# converging heavy nuclei with *ab initio* methods





# Nuclei in the Harmonic Oscillator basis



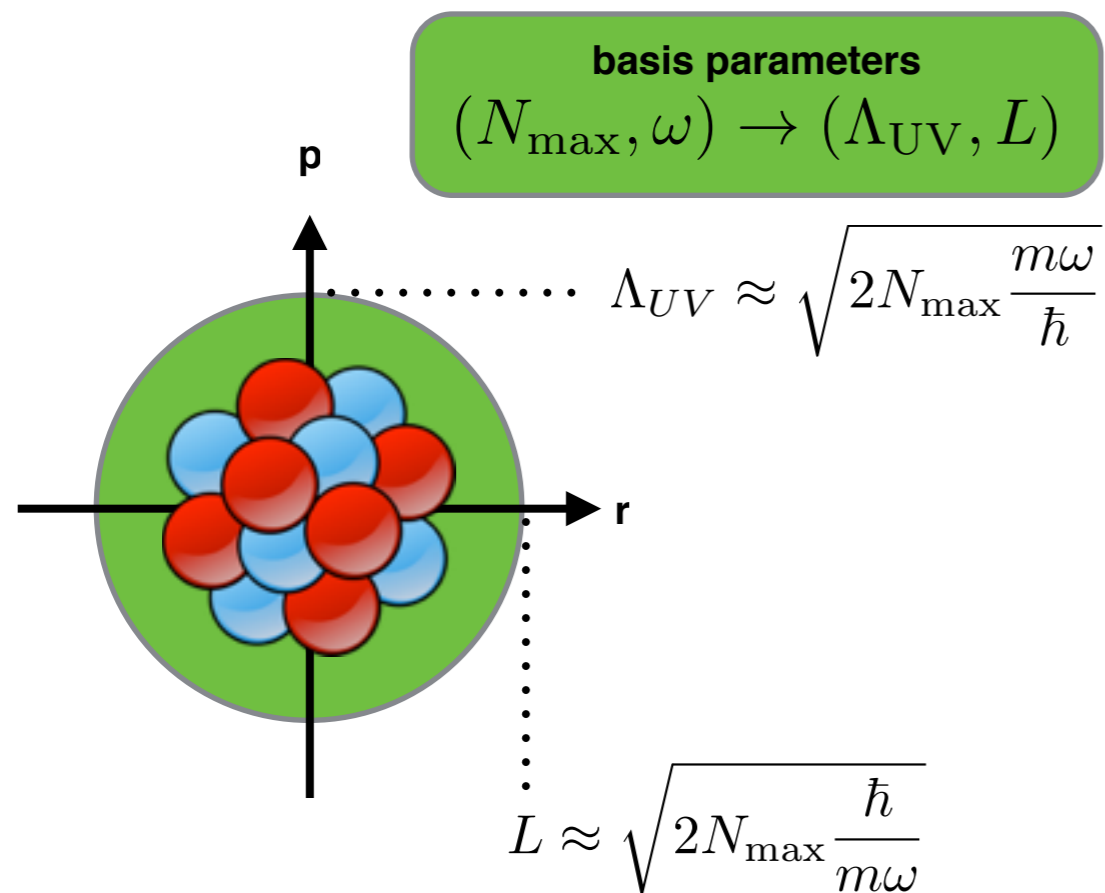
**Nucleus needs to fit into the modelspace**

$$L > R_{\text{nucleus}}$$

**Interaction must be captured**

$$\Lambda_{UV} > \Lambda_{\chi}$$

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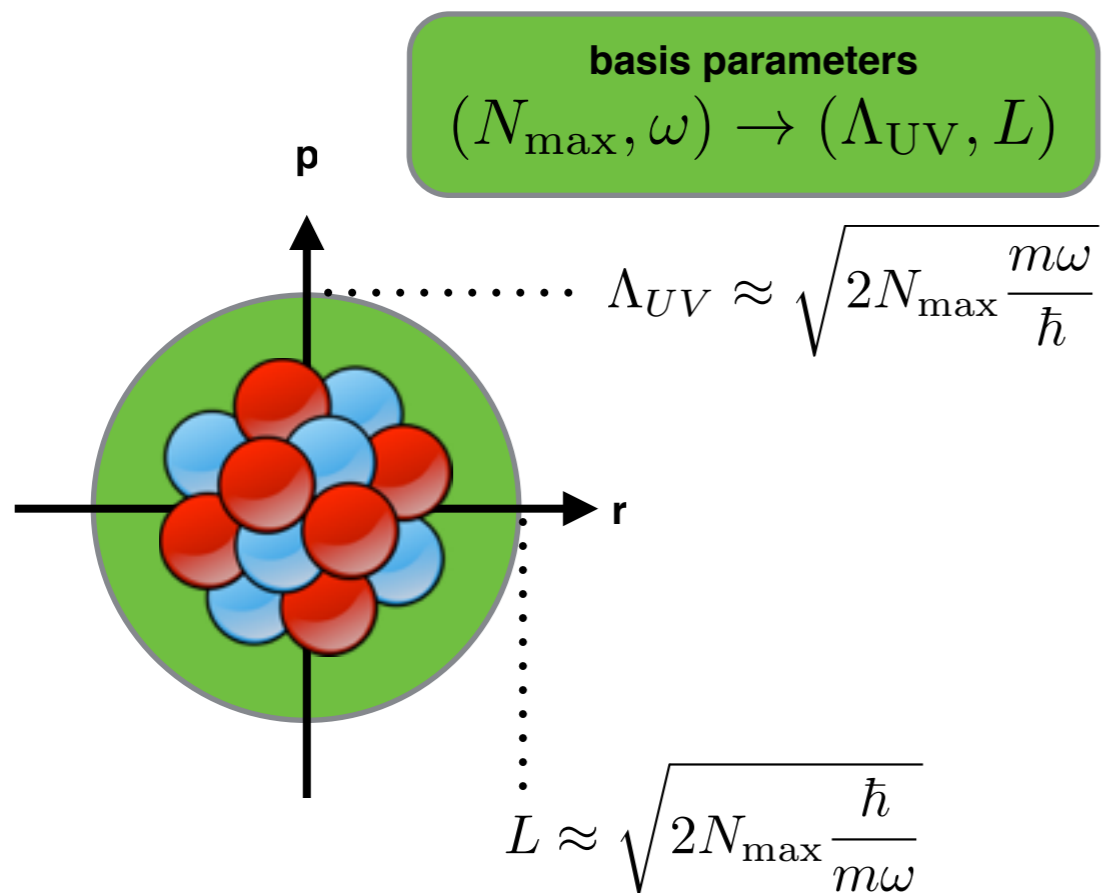
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- To facilitate calculations in heavy nuclei we propose to ***tailor the EFT interaction to a finite HO basis***

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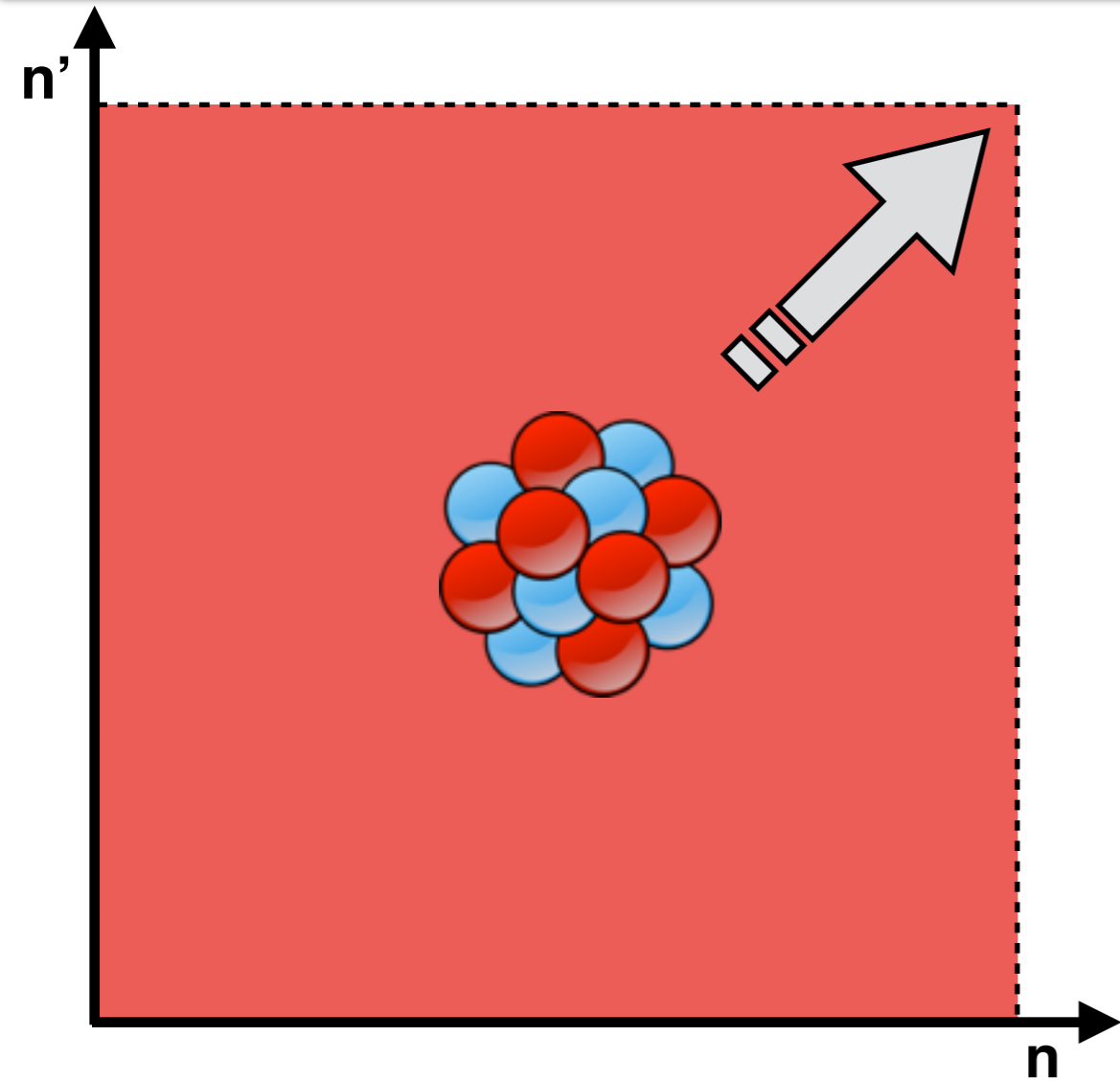
**Similar ideas in nuclear physics already exist:**

**Arizona group** developed pionless-EFT in HO basis and studied UV/IR cutoff dependencies. Coupling constants depend on the size of the basis.

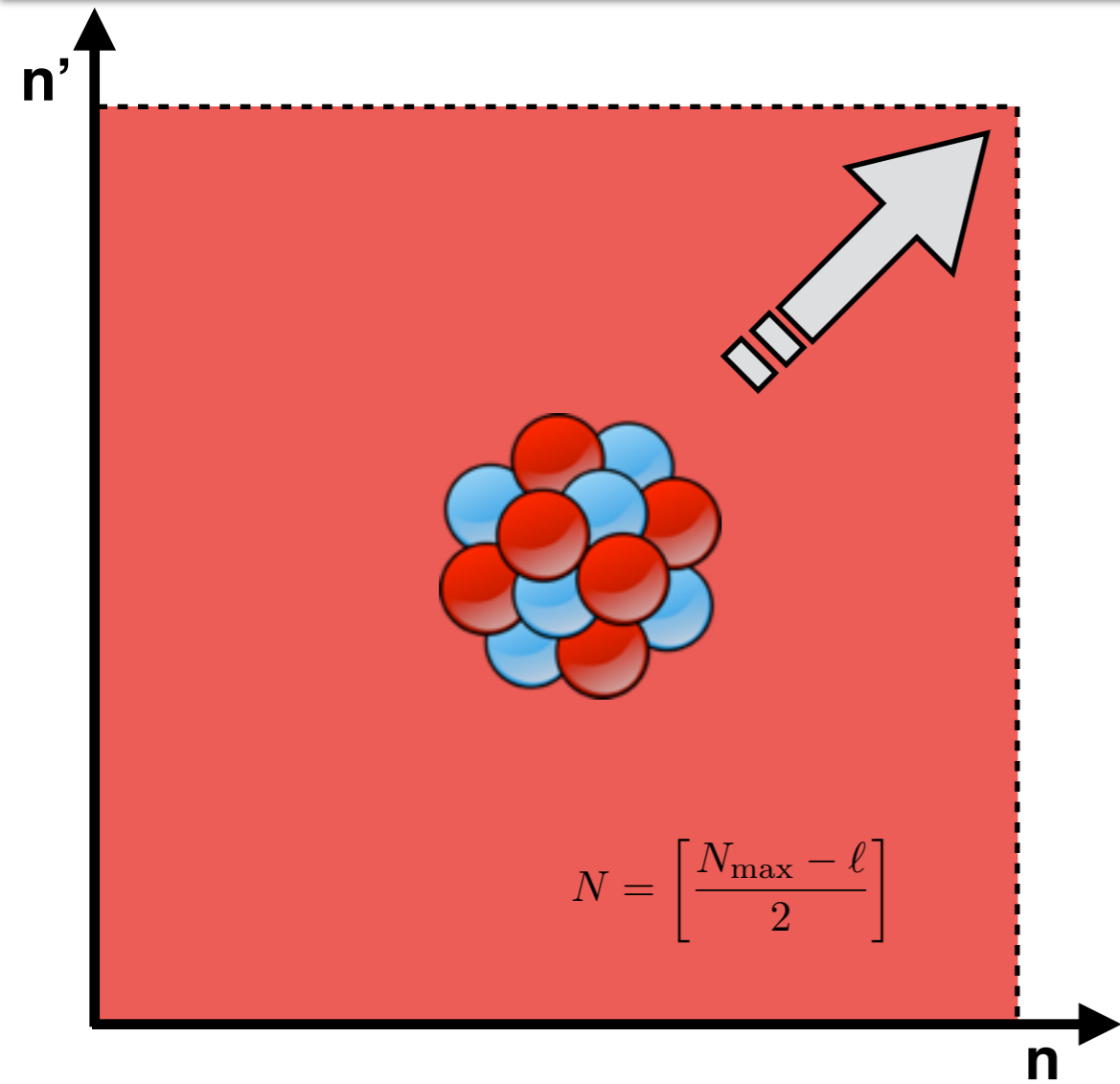
**Haxton et al.** proposed HOBET (HO-based effective theory). “Shell-Model” (Bloch-Horowitz) plus resummed kinetic energy and physics beyond a cutoff absorbed by contact-gradient expansion (like EFT contact potential)

**We propose** to choose (and fix) an oscillator space and evaluate the *existing* chiral EFT interaction operators in this space. This *projection* requires us to refit the LECs of chiral EFT

# EFT interaction in a truncated HO basis

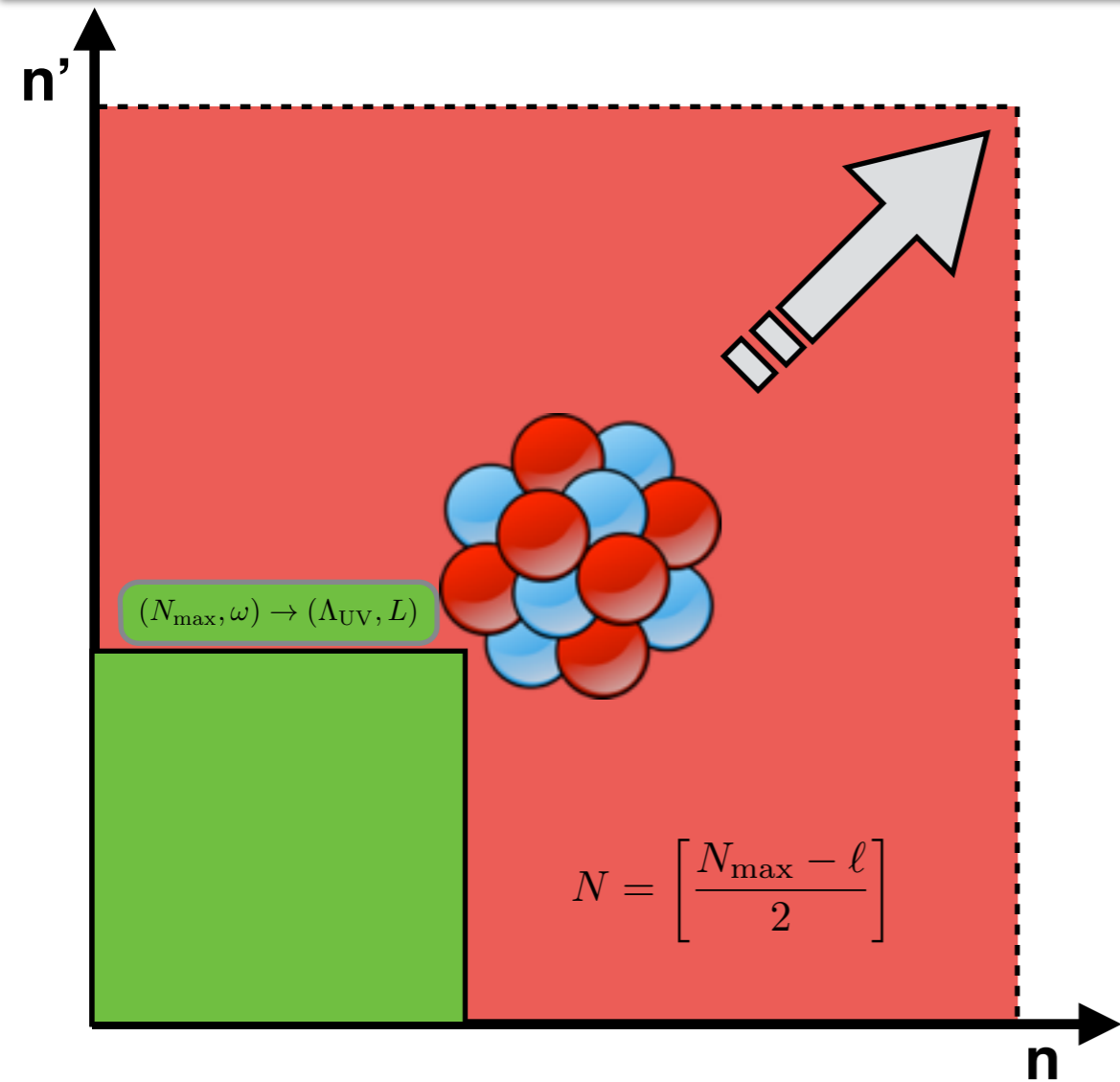


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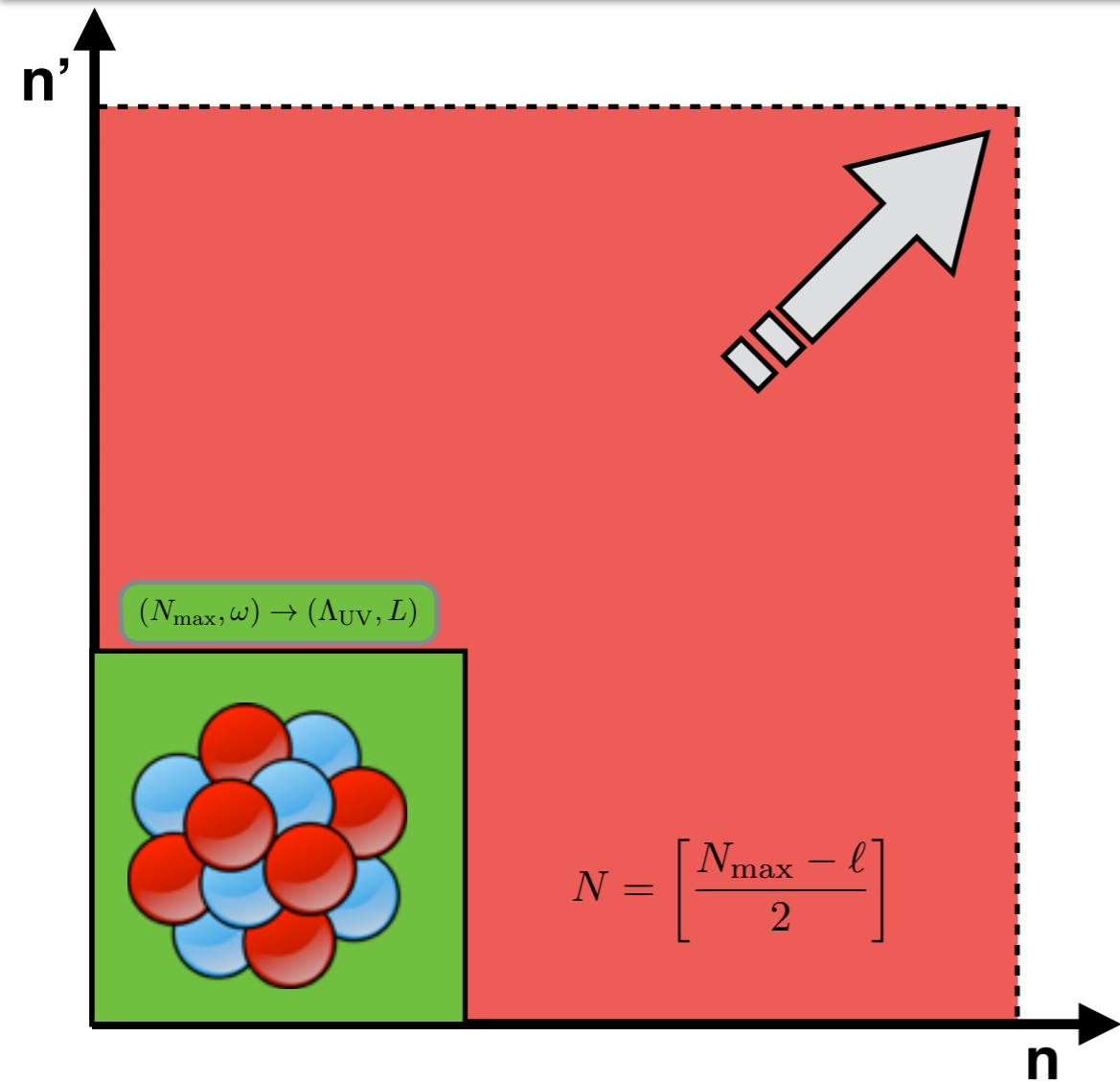




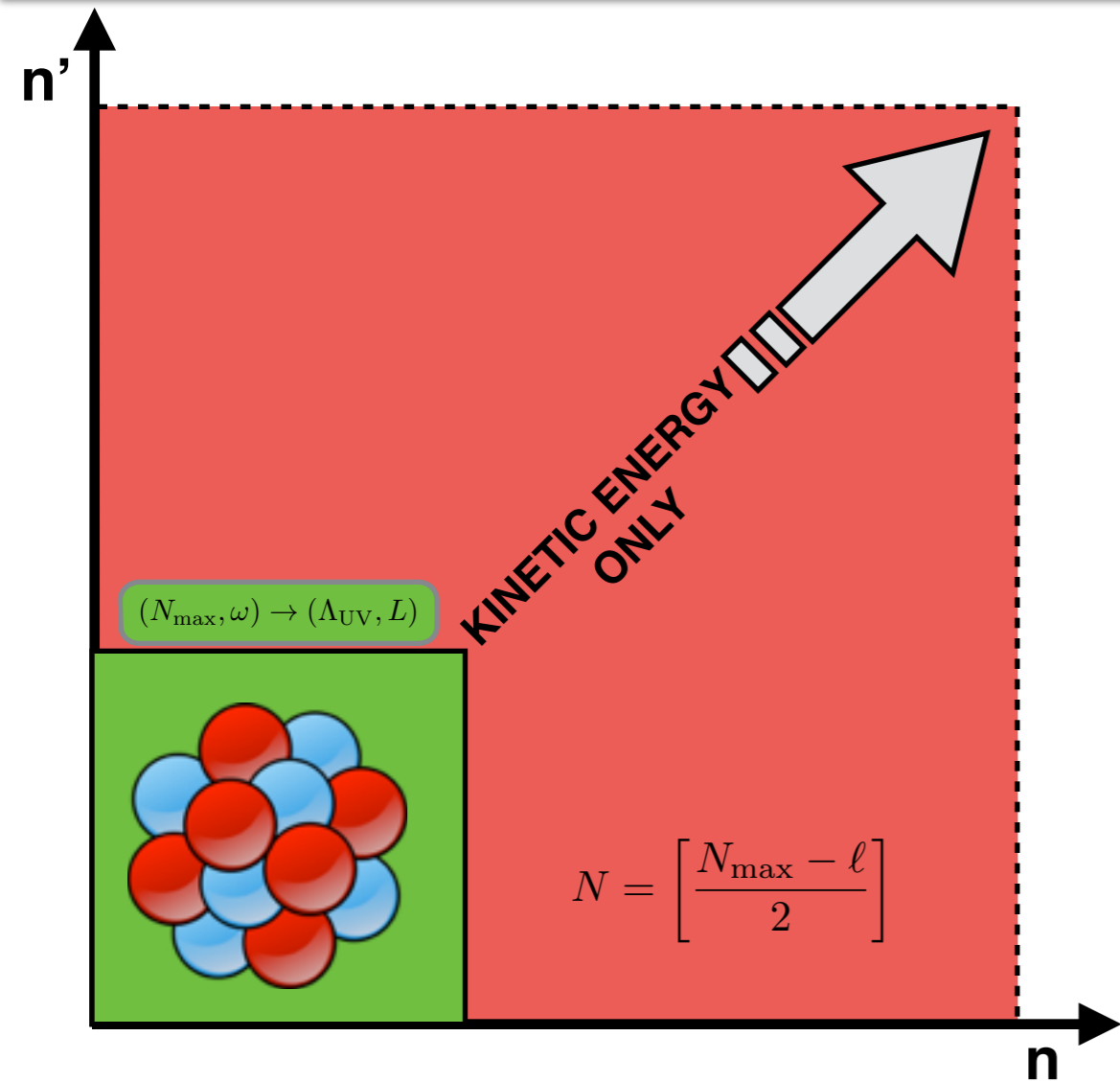
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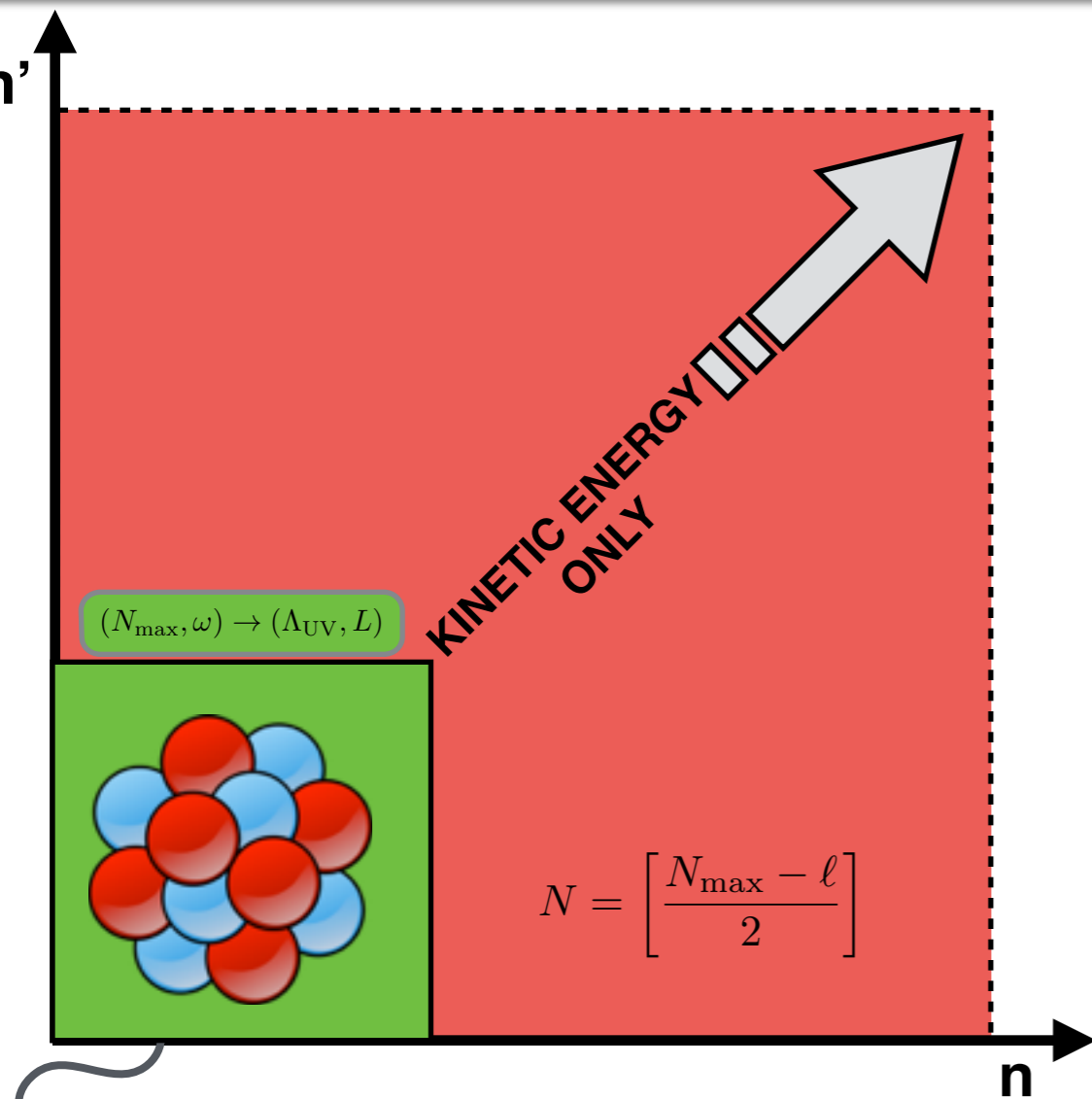


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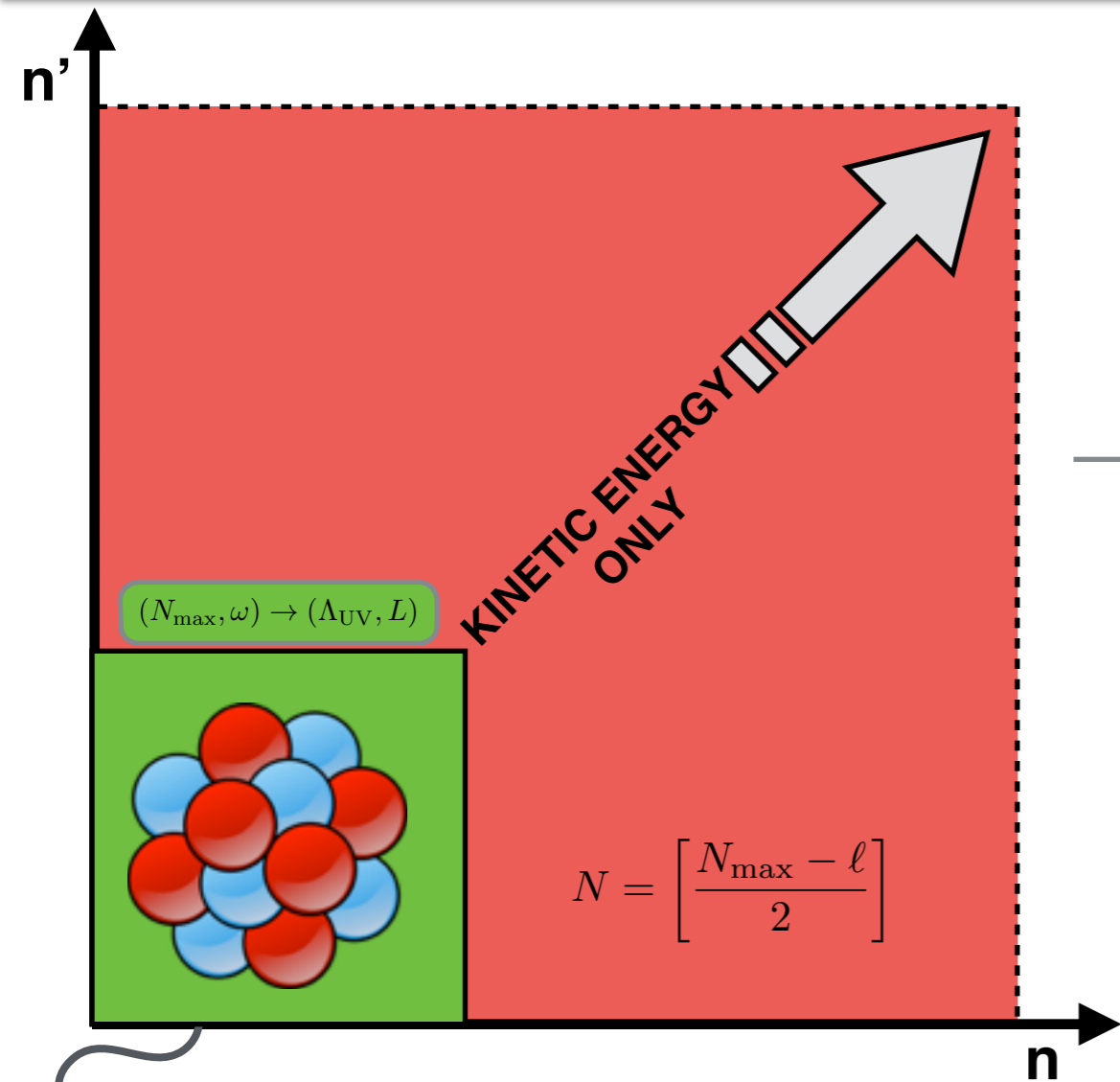
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simple picture: choose e.g.  $N_{\max} = 10$ ,  $\hbar\omega = 40$  MeV  
set  $\langle n|V|n'\rangle$  to zero outside  $N_{\max}$



*"thinking inside the box"*

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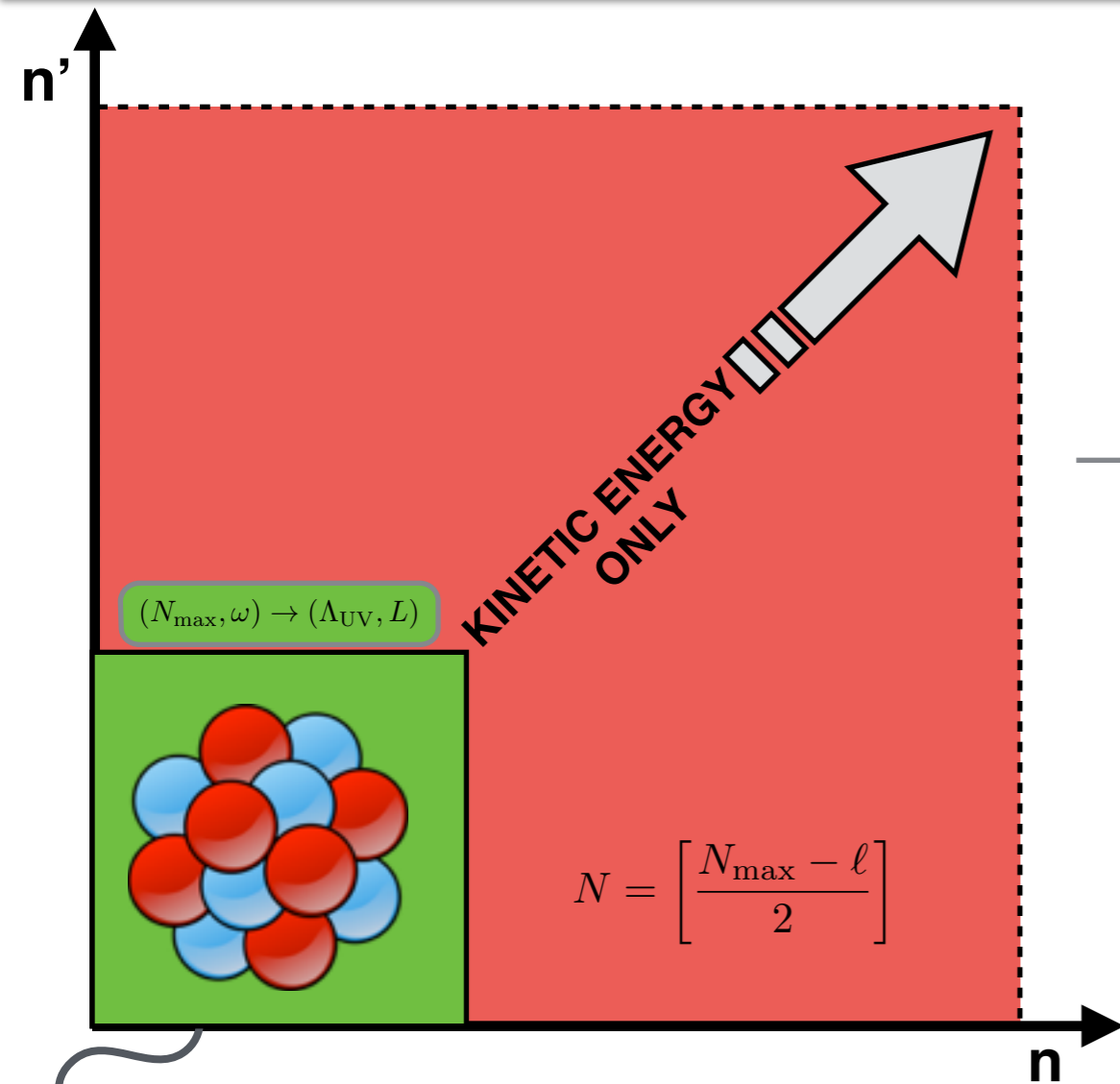
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Optimize the LECs to reproduce selected fit-data  
we can compute phase shifts using the J-matrix formalism,  
and finite nuclei using e.g. NCSM, CC, IM-SRG, .....

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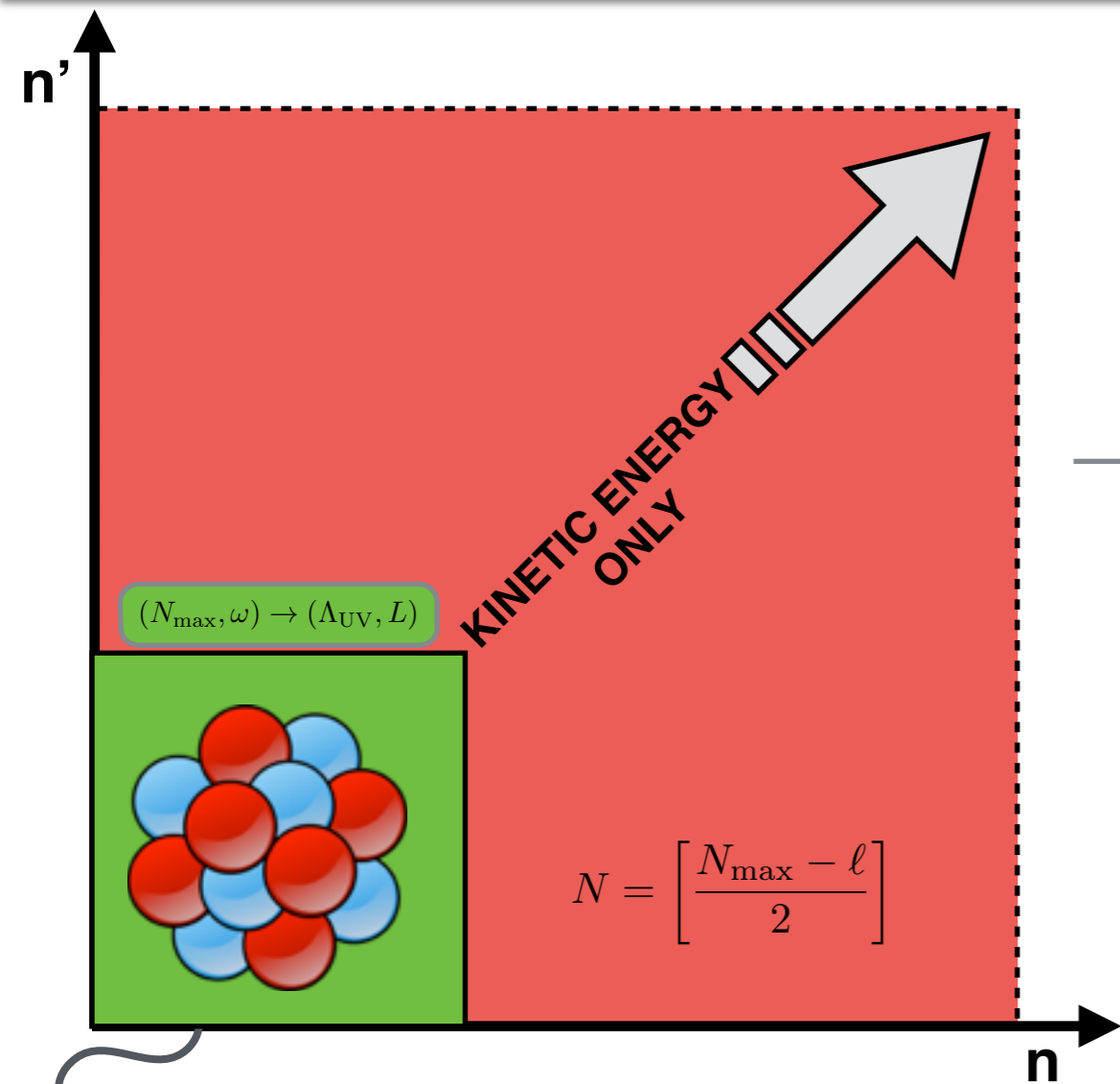
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**However**, in the longer perspective, we seek to develop an EFT. So it is important to understand the squared momentum operator

$$\langle n\ell | \hat{p}^2 | n'\ell' \rangle$$



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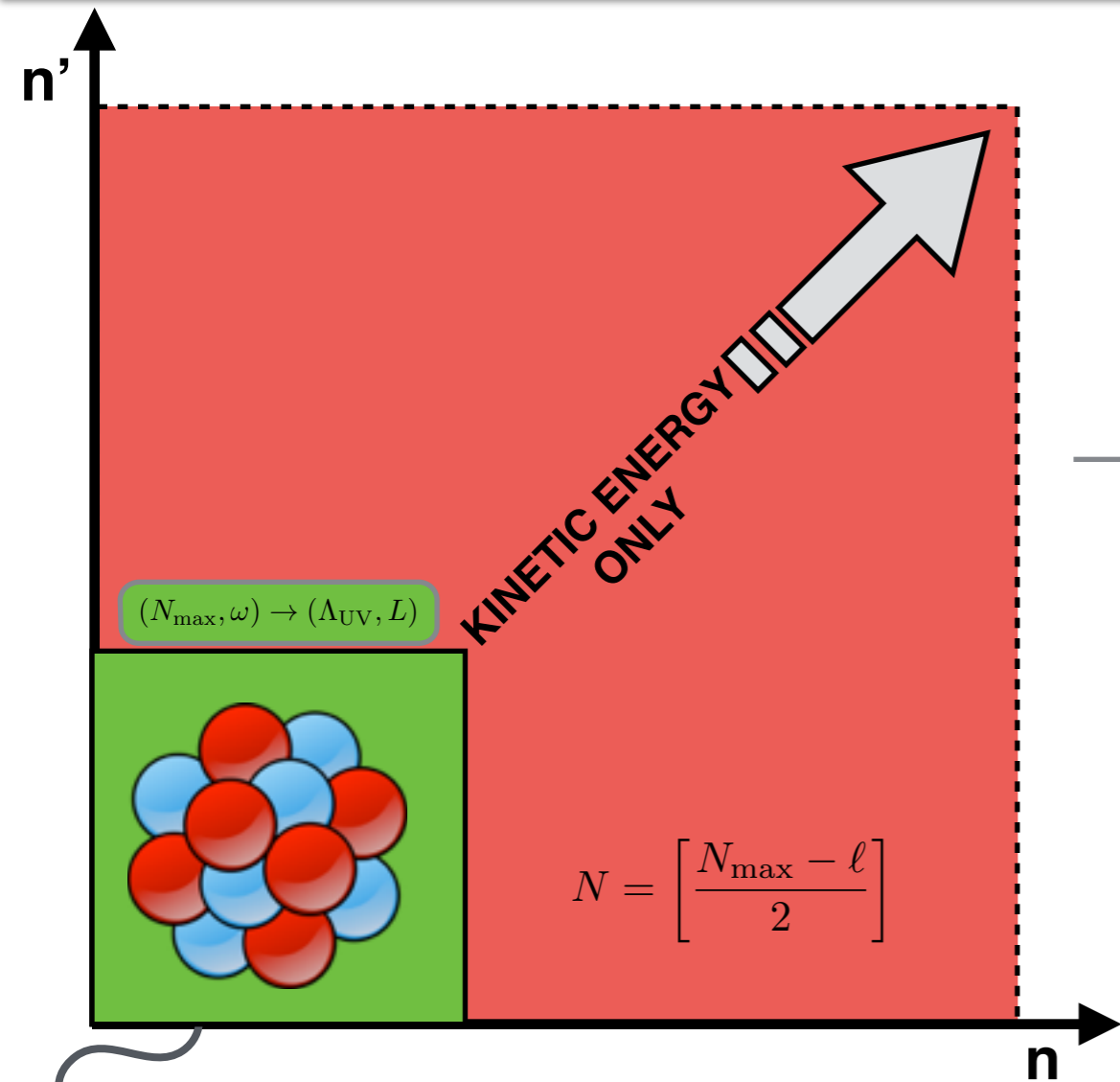
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$$\langle n\ell|\hat{p}^2|n'\ell'\rangle$$

We start by solving the eigenvalue problem of the squared momentum operator in a finite oscillator space truncated at an energy  $(N_{\max} + 3/2)\hbar\omega$ . It turns out that momenta  $k=k_\mu$  such that  $\psi_{N+1}(k)=0$  solve this eigenvalue problem.

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 set  $\langle n|V|n' \rangle$  to zero outside  $N_{\max}$

Optimize the LECs to reproduce selected fit-data  
 we can compute phase shifts using the J-matrix formalism,  
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**However**, in the longer perspective, we seek to develop an EFT. So it is important to understand the squared momentum operator

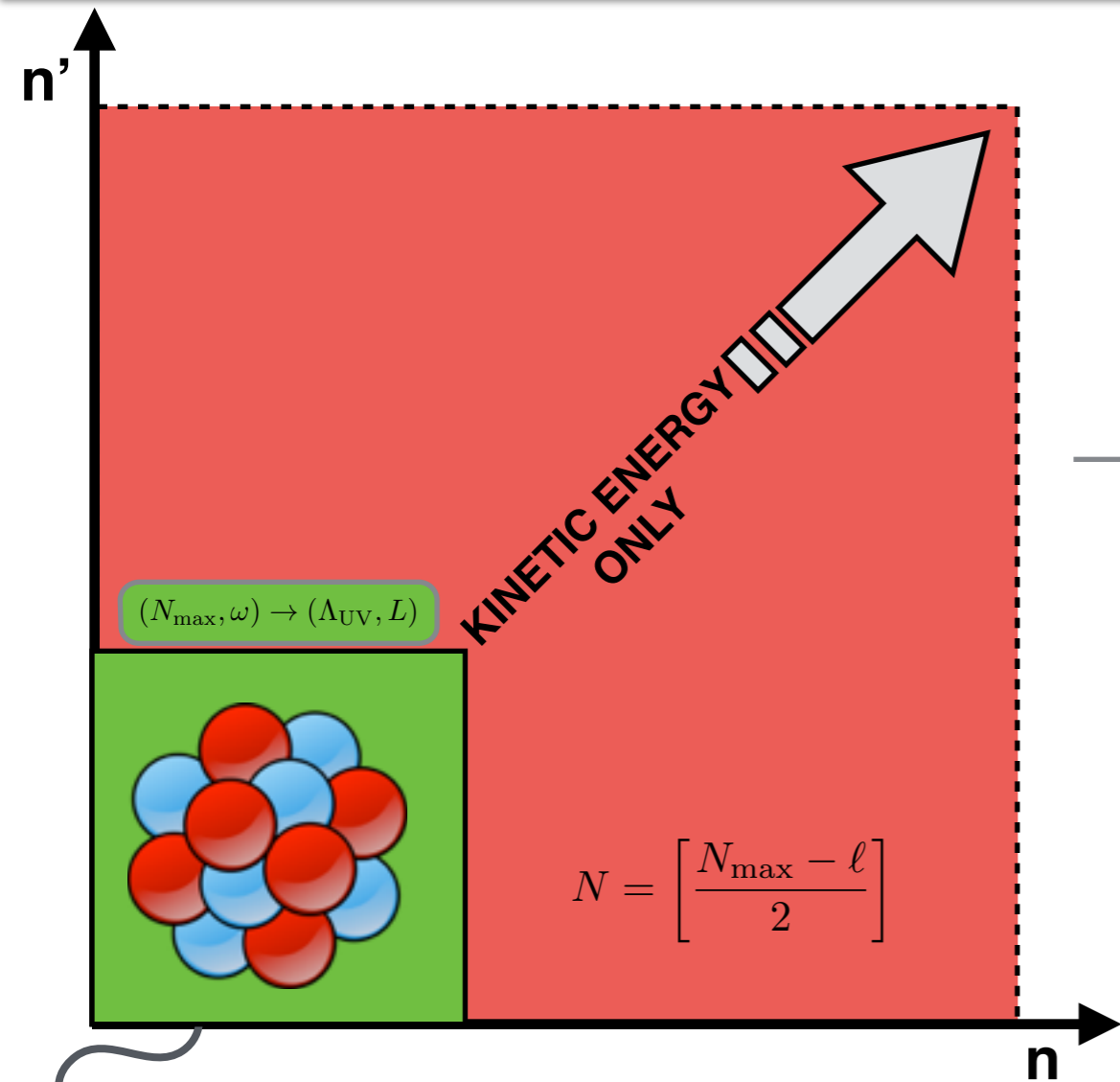
$$\langle n\ell | \hat{p}^2 | n'\ell' \rangle$$

We start by solving the eigenvalue problem of the squared momentum operator in a finite oscillator space truncated at an energy  $(N_{\max} + 3/2)\hbar\omega$ . It turns out that momenta  $k=k_\mu$  such that  $\psi_{N+1}(k)=0$  solve this eigenvalue problem.

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**This has pleasant consequences for most analytical and numerical evaluations!**

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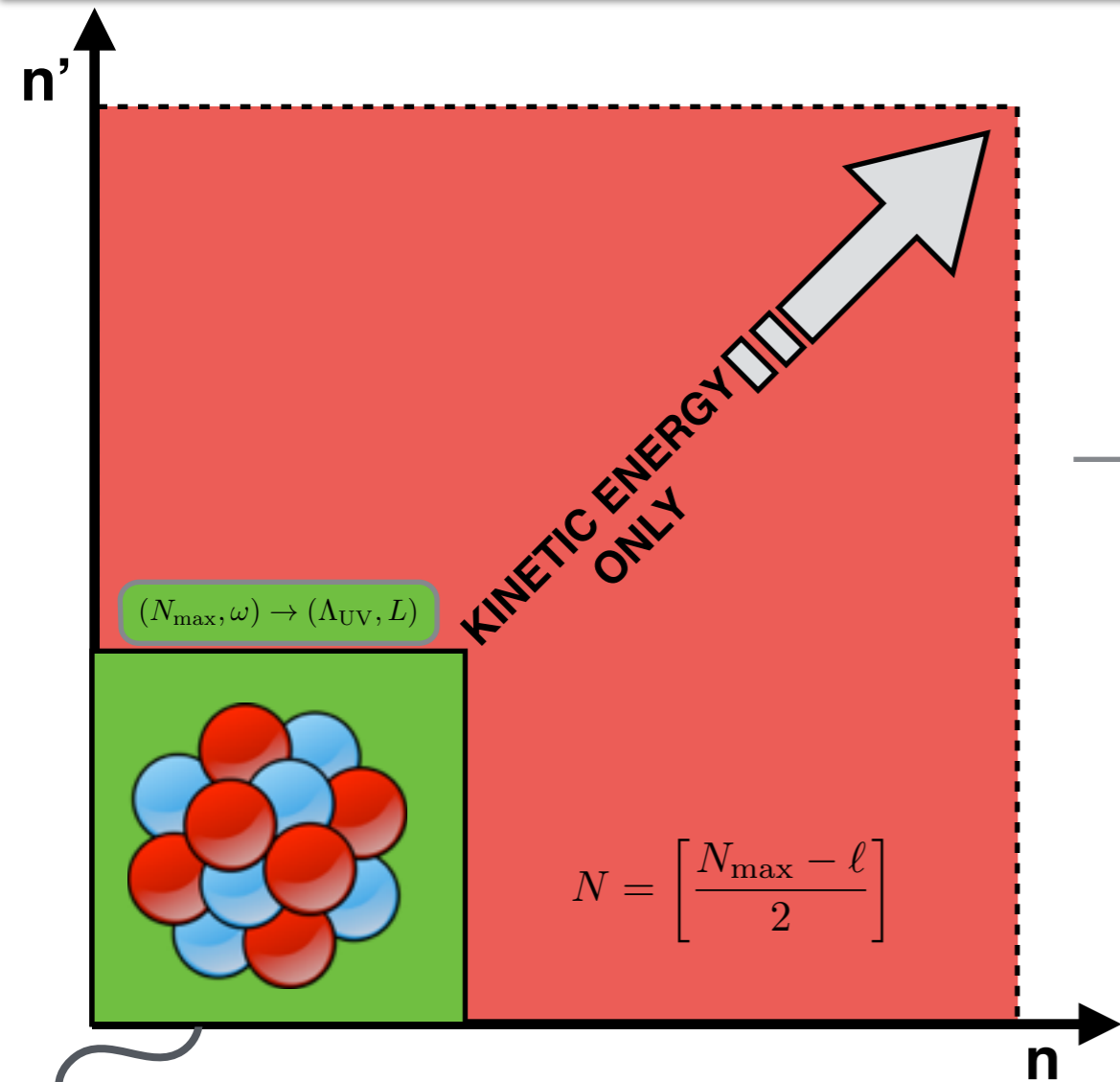
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# Idaho-N3LO in the Harmonic Oscillator basis

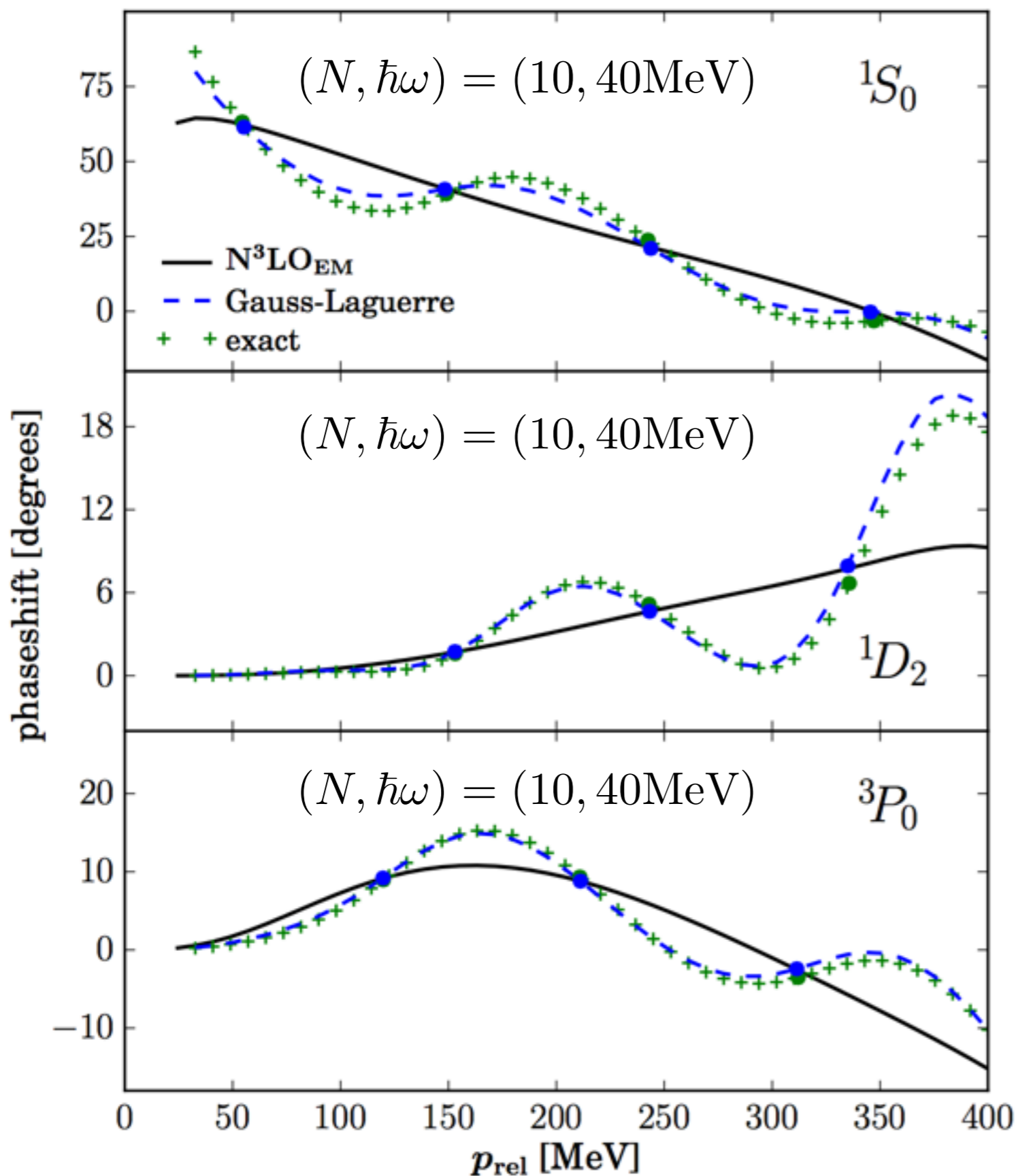


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selected proton-neutron phase shifts of Idaho-N3LO(500) projected onto an  $N_{\text{max}}=10$ ,  $hw=40$  MeV oscillator space

$$\Lambda_{UV} = 700 \text{ MeV} > \Lambda_{\chi} = 500 \text{ MeV}$$

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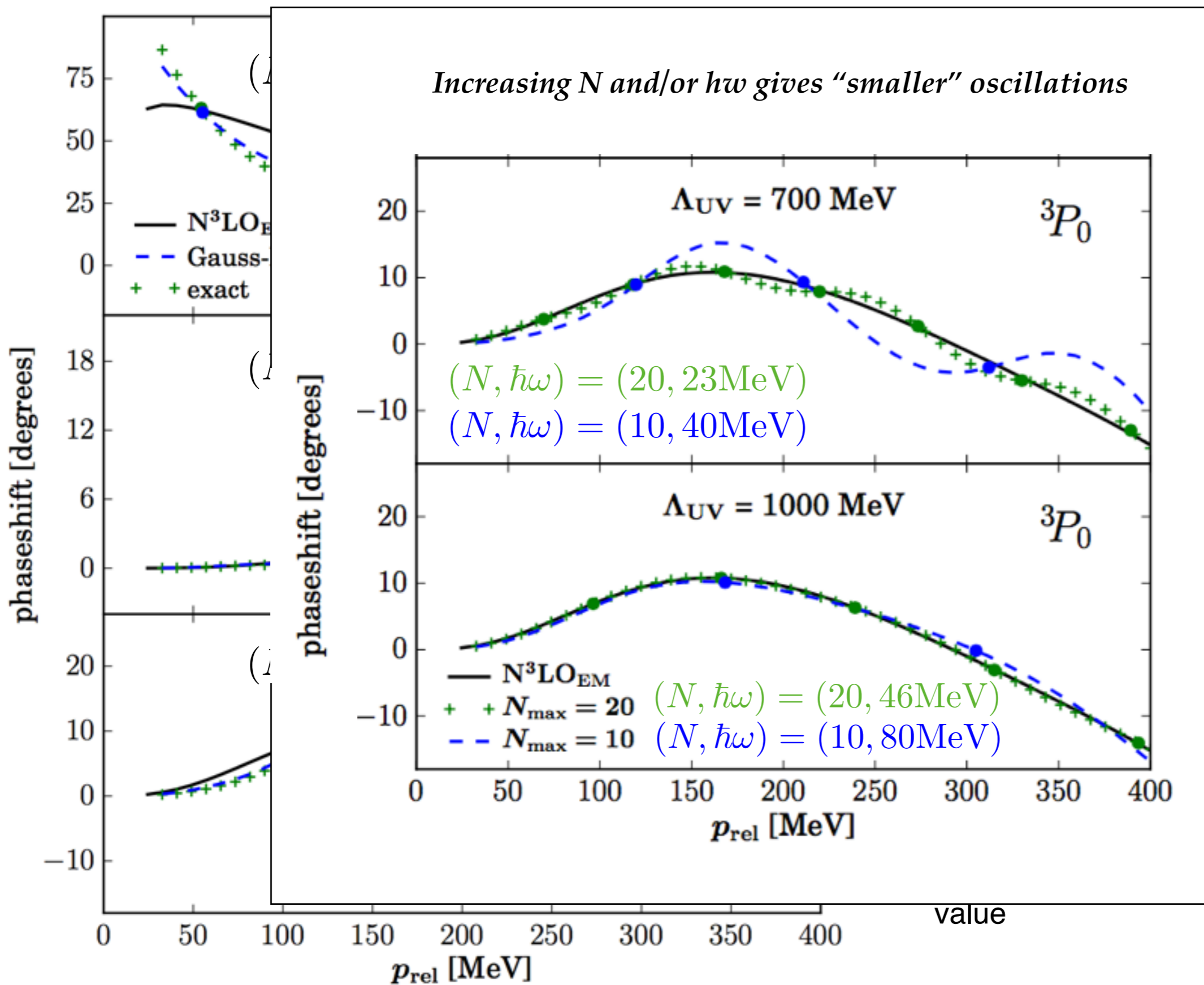
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## OBSERVATIONS:

- There are oscillations in the phase shifts
- The period of this oscillation is approximately proportional to the IR cutoff
- Gauss-Laguerre (“HO-EFT”) phases exhibits slightly smaller oscillations than the “exact” phases
- At the energies corresponding to the eigenenergies of the truncated Hamiltonian (solid dots), computed phases are closest to the true N3LO value

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**With goal of computing heavy nuclei,  
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- 😊 small number of oscillator shells  $N_{\max}=10$
- 😊 Lower frequencies, rapid IR convergence
- 😞 Lower frequencies, lower UV cutoffs

We choose  $\hbar\omega=24$  MeV, which gives:

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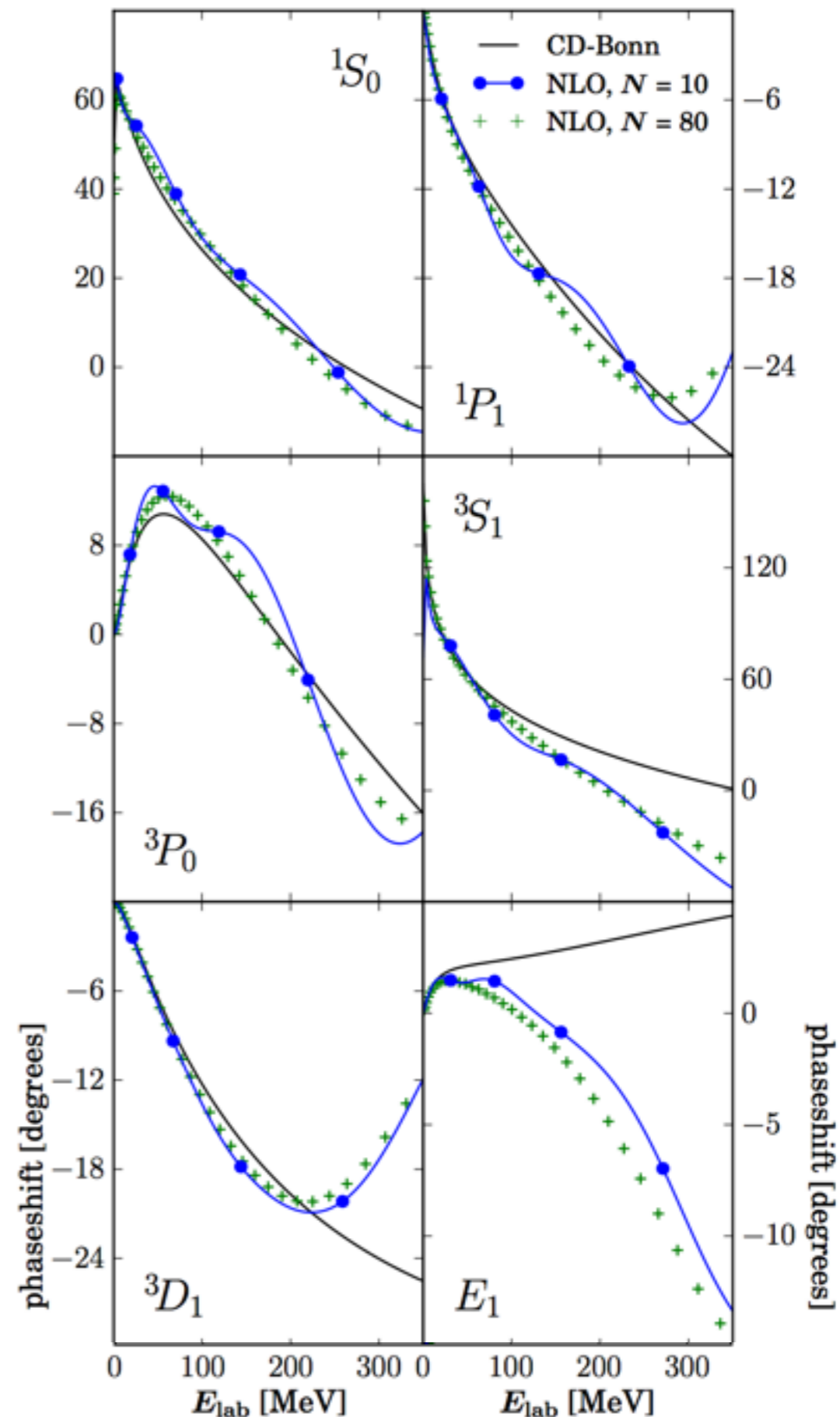
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$E_d$ [MeV]	-2.227	2.221	-2.225
$r_d$ [fm]	1.984	1.961	1.976
$Q_d$ [fm <sup>2</sup> ]	0.229	0.259	0.286
$P_d$	0.026	0.028	
$\tilde{C}_{1S_0}^{(np)}$	-0.140992	-0.139687	
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$C_{1S_0}$	1.247001	1.260132	
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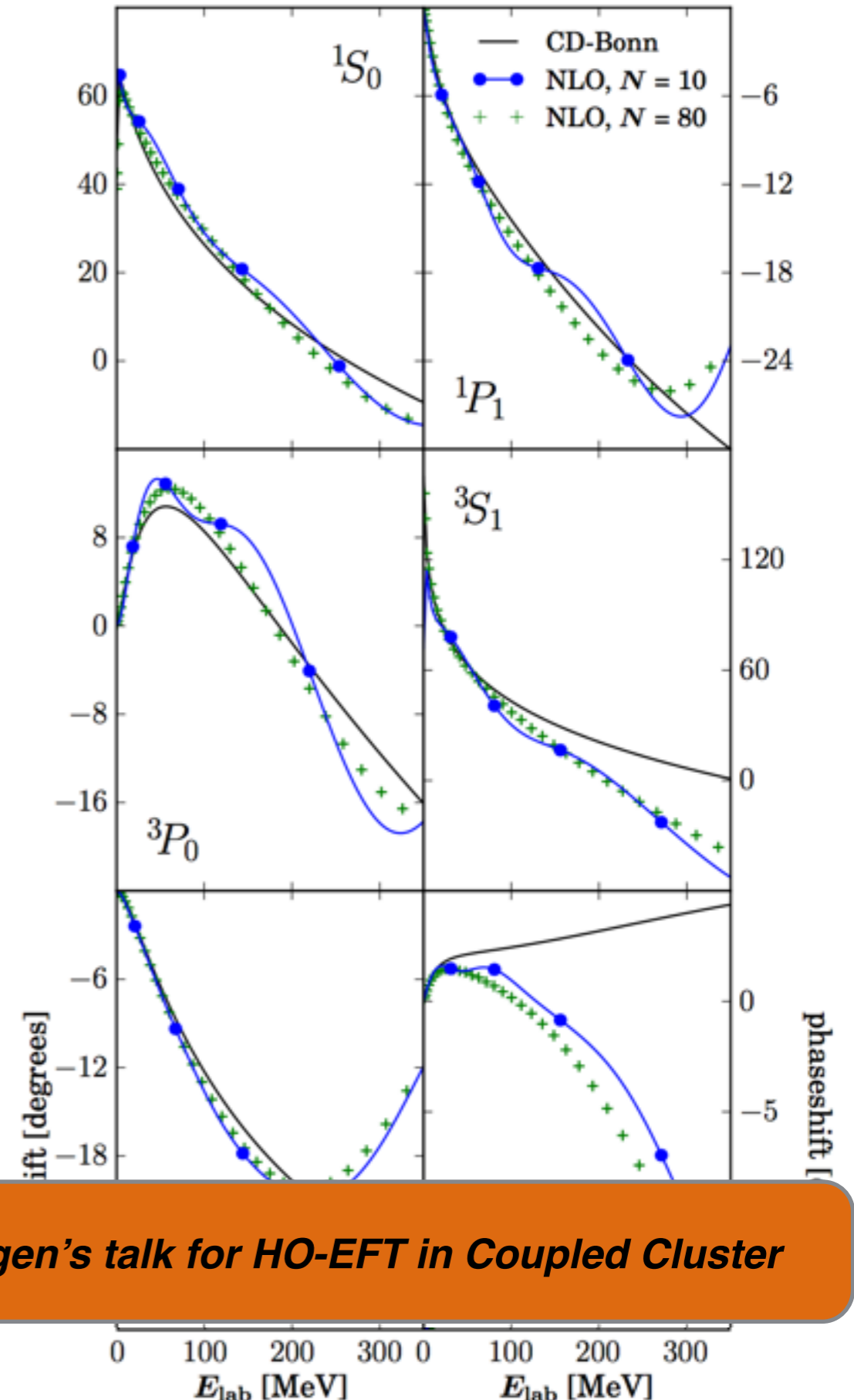
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See G. Hagen's talk for HO-EFT in Coupled Cluster

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*Or, put differently, how much uncertainty do we bring in by resolving the chiral EFT in HOEFT? Insofar, we have not observed any catastrophes when studying light or heavy nuclei with HOEFT, nor have any LECs become unnatural.*

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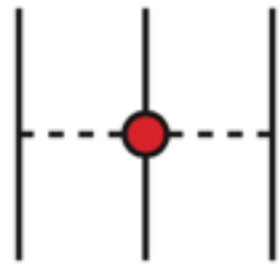
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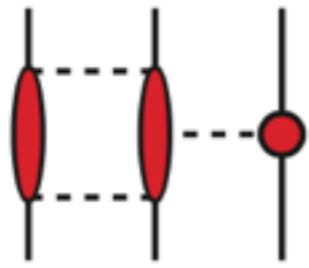
***Next step:***

**HO-NNLO<sub>sat</sub>**

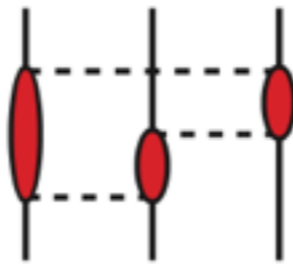
# NNN-N3LO: simple cD/cE scan with Idaho-N3LO



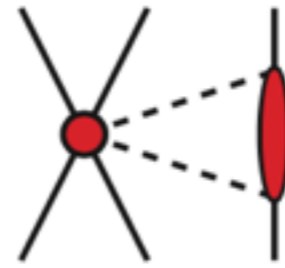
$2\pi$



$2\pi-1\pi$



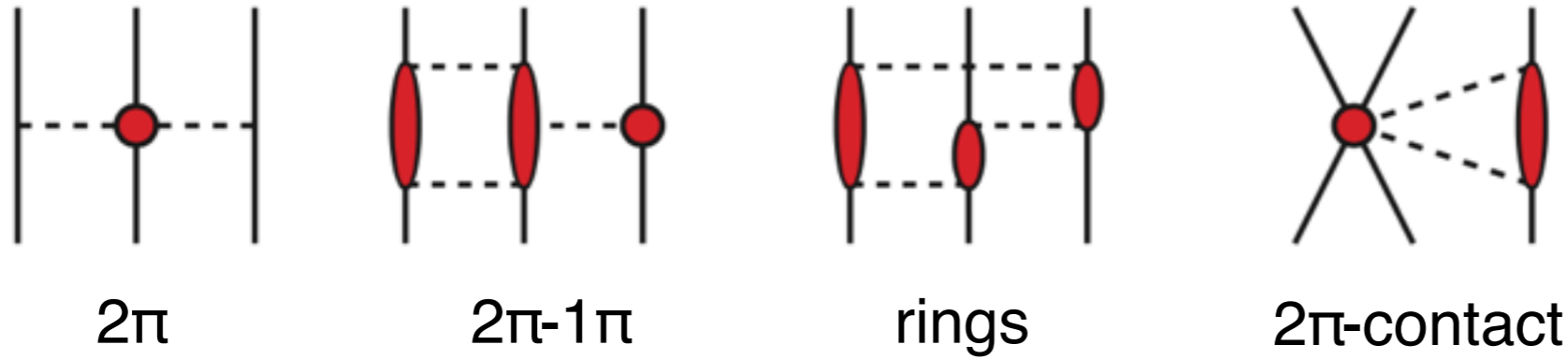
rings



$2\pi$ -contact

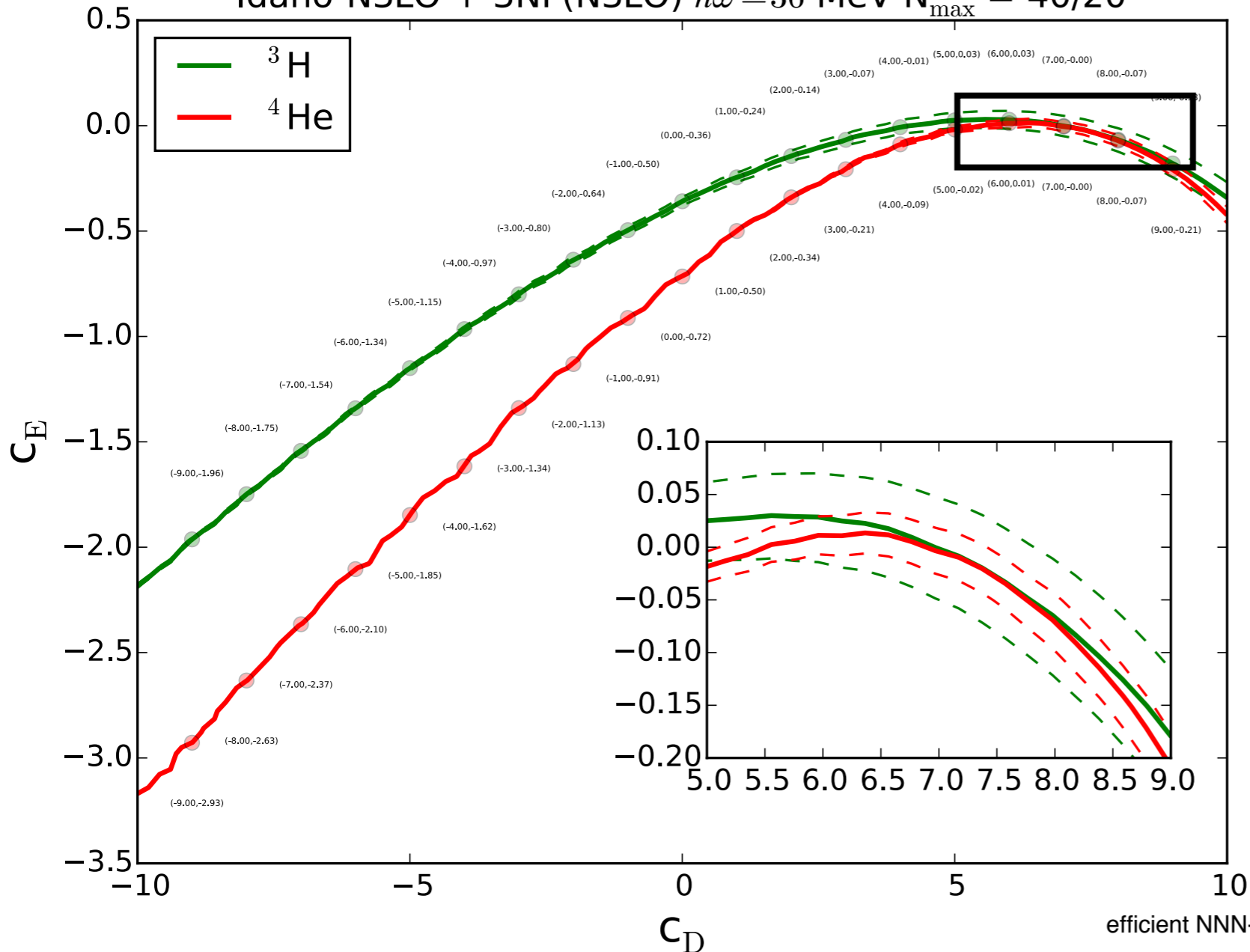
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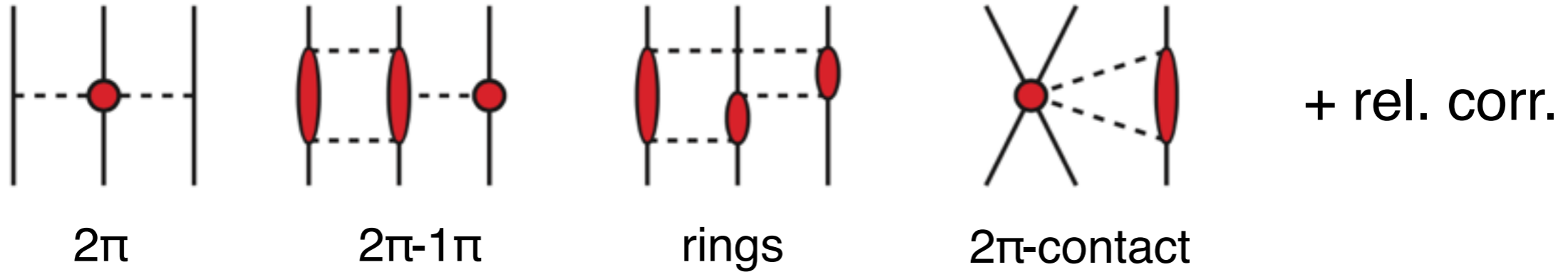


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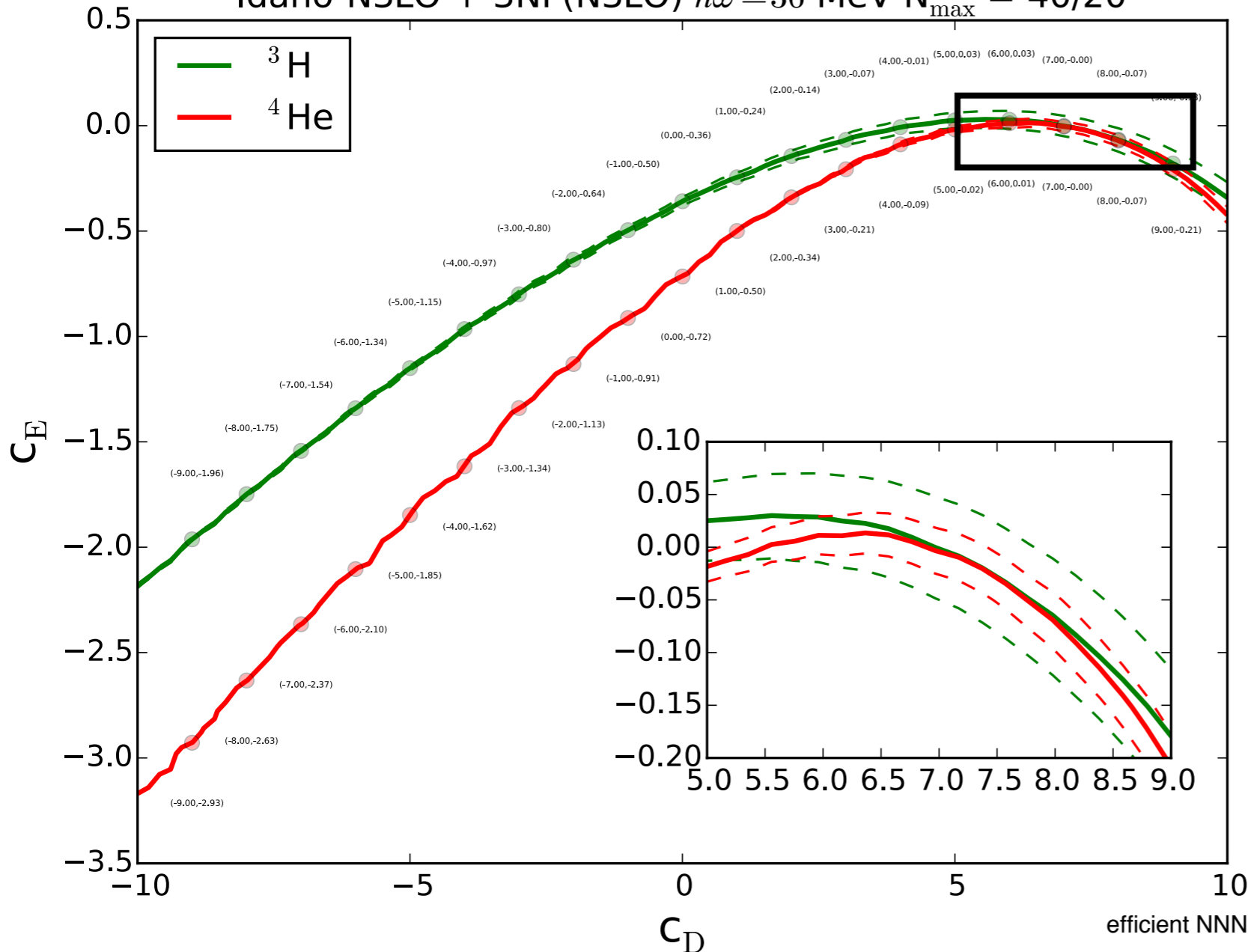
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Expectation values (in MeV and fm)

	<b>3H</b>	<b>4He</b>
<b><math>E_{gs}</math></b>	<b>-8.48</b>	<b>-28.32</b>
$r_{pt-p}$	1.61	1.49
$\langle c1 \rangle$	-0.14	-0.69
$\langle c3 \rangle$	-1.29	-6.82
$\langle c4 \rangle$	0.35	2.16
$\langle cD \rangle$	-0.39	-2.16
$\langle cE \rangle$	0.02	0.08
$\langle 2\pi \rangle$	-0.40	-2.54
$\langle 2\pi 1\pi \rangle$	1.22	6.48
$\langle \text{rings} \rangle$	-0.57	-3.38
$\langle 2\pi\text{-cont} \rangle$	0.20	1.25
$\langle \text{rel. corr} \rangle$	0.24	1.28
<b>N2LO</b>	<b>-1.45</b>	<b>-7.44</b>
<b>N3LO</b>	<b>0.68</b>	<b>3.09</b>

# N3LO optimizations are challenging

*Work led by B. D. Carlsson (Chalmers)*

Initialize by computing phase shifts for  $10^5$  random contact LEC values for each partial wave and select the  $\sim 1000$  best values and optimize. This leads to 192 different optima (for cutoff 450 MeV) with respect to phase shifts. (pi-N LECs from sep-optimization)

The A=3 observables weed out several of the S-wave minima, but many P-wave minima remain. Things improve when A=4 is included. But still, several local minima remain.

This is where we stand right now.



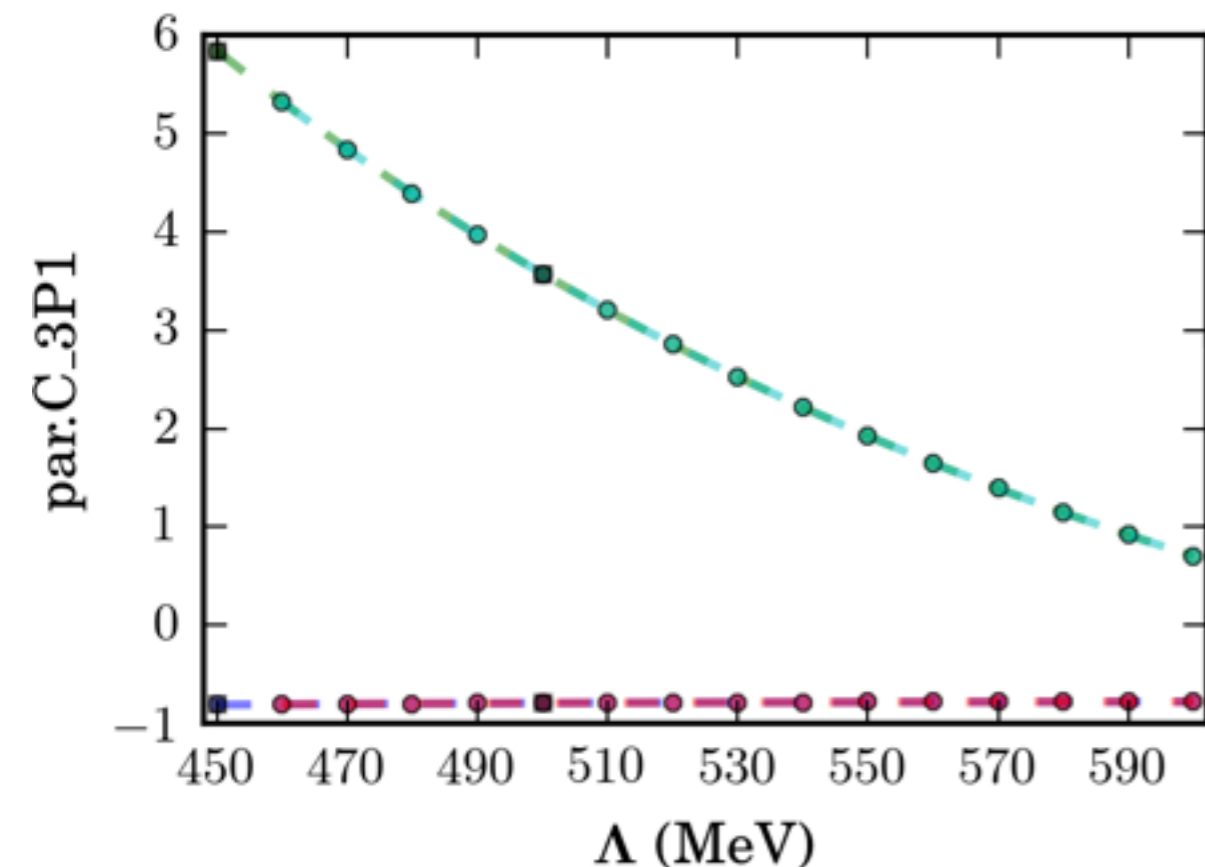
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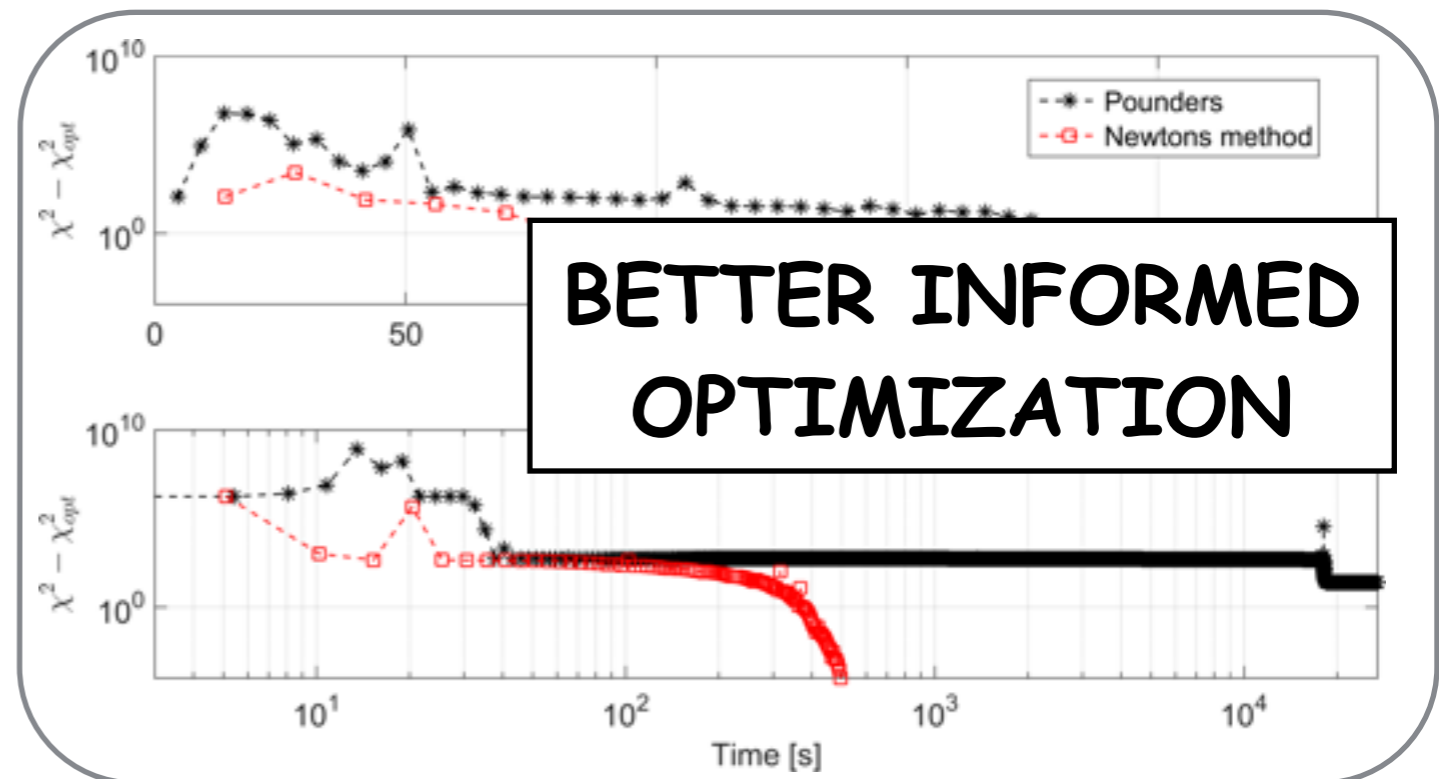
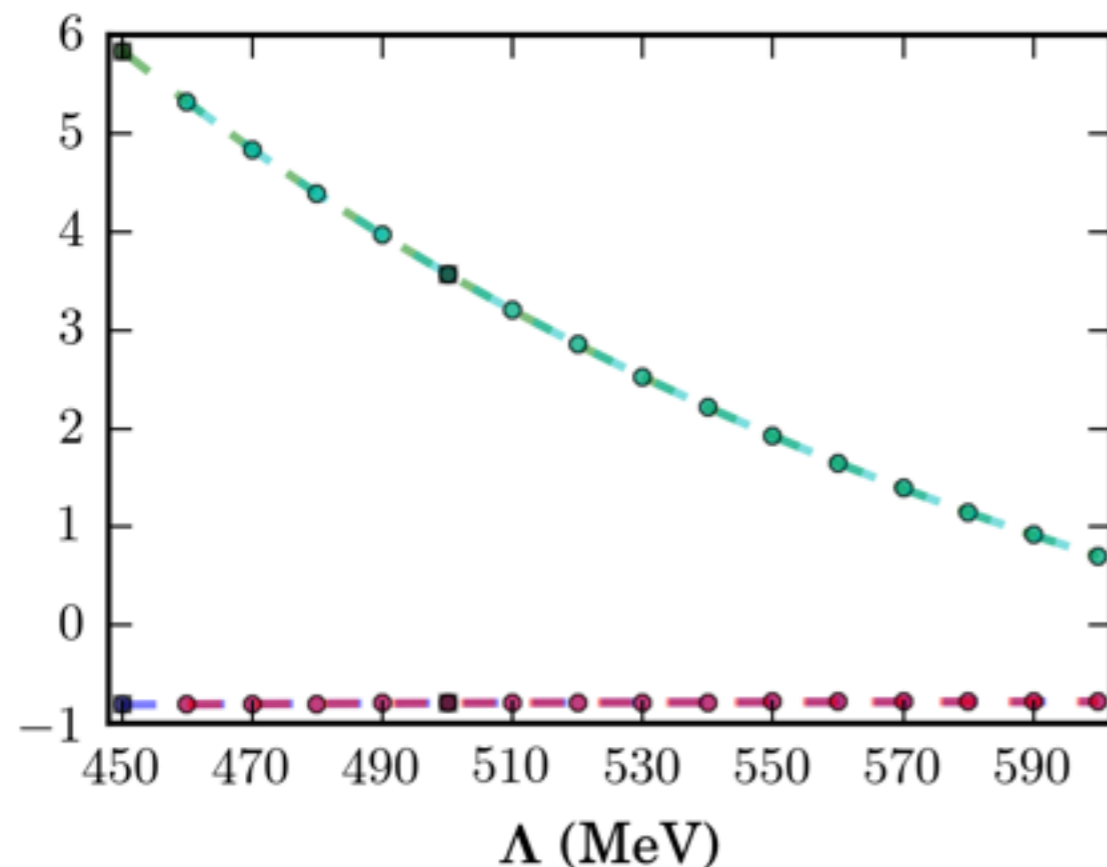
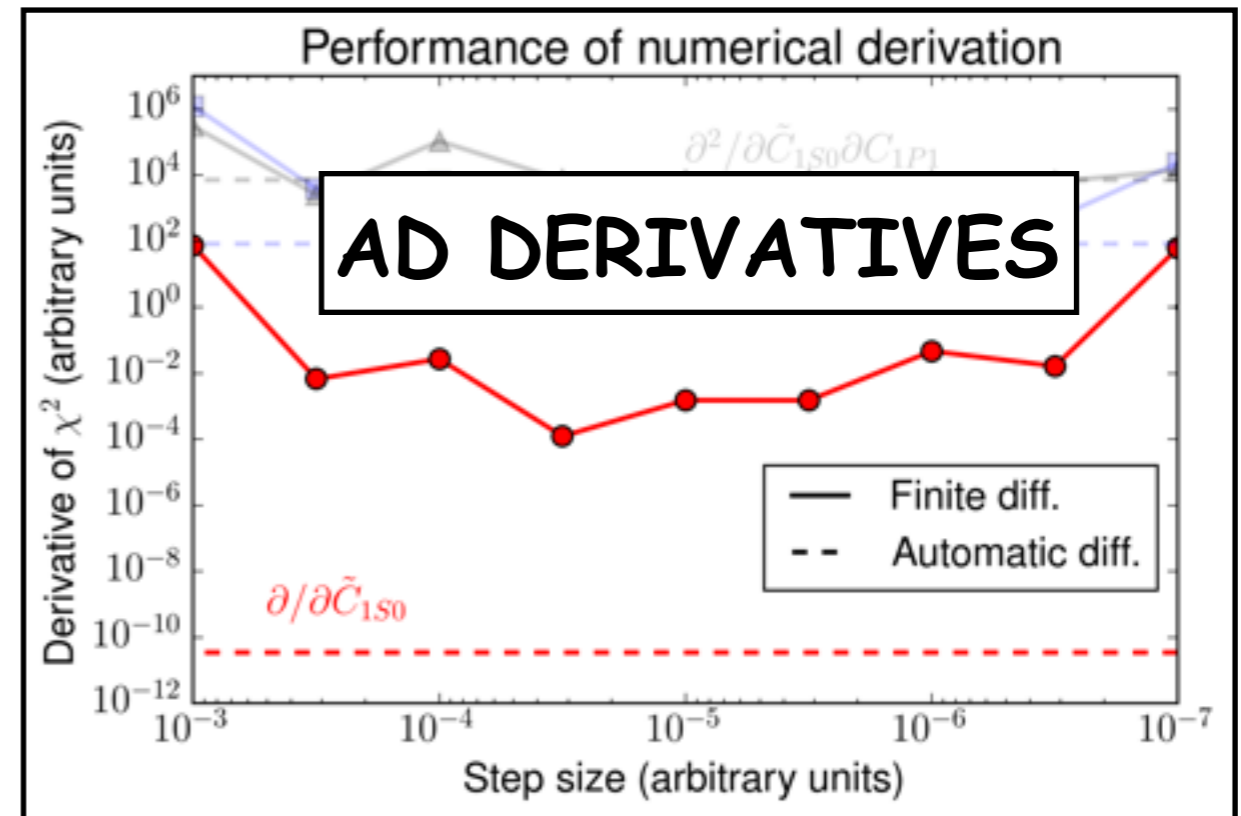
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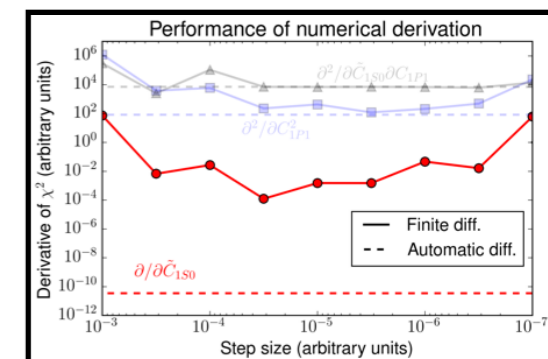
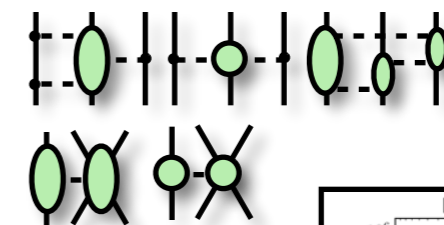
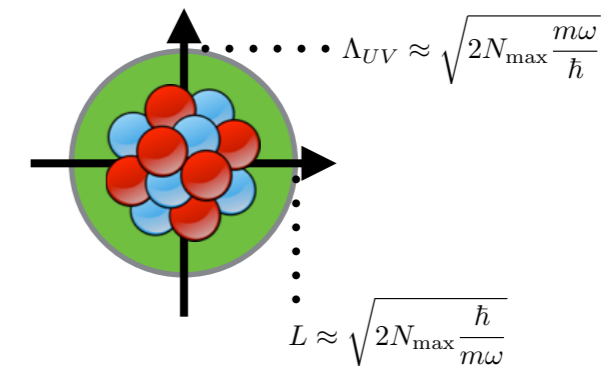
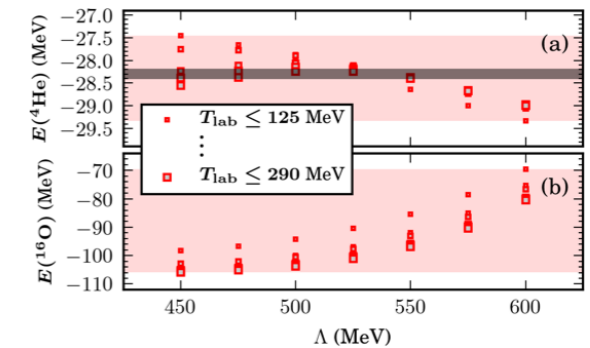
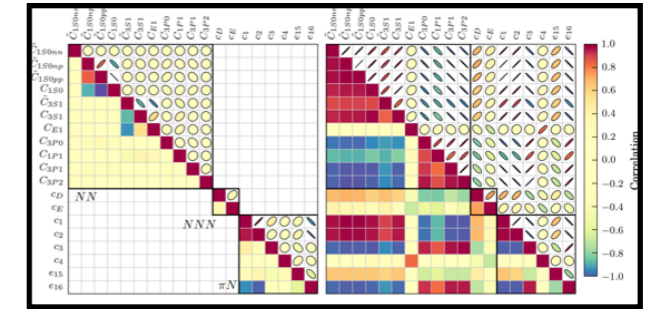
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# Summary and conclusions

- *Covariance matrices for optimized LO-NLO-NNLO potentials available for download*
- *Small variations in the nuclear interaction renders large fluctuations in predictions for heavier nuclei*
- *Harmonic Oscillator EFT could be a promising approach for ab initio studies of heavy atomic nuclei*
- *Non-local 3NF at N3LO is not constrained by  $A=2,3$  data*
- *N3LO optimizations benefit from gradients*



# Thank you for your attention

*and thanks to all collaborators!*

*Bijaya Acharya*

*Sven Binder*

*Boris D. Carlsson*

*Christian Forssen*

*Gaute Hagen*

*Gustav Jansen*

*Oskar Lilja*

*Mattias Lindby*

*Björn Mattsson*

*Thomas Papenbrock*

*Lucas Platter*

*Dag Fahlin Strömberg*

*Kyle Wendt*