

# *Ab initio NCSMC for three-cluster dynamics*

**Carolina Romero-Redondo**

Progress in Ab Initio Techniques in Nuclear Physics, TRIUMF

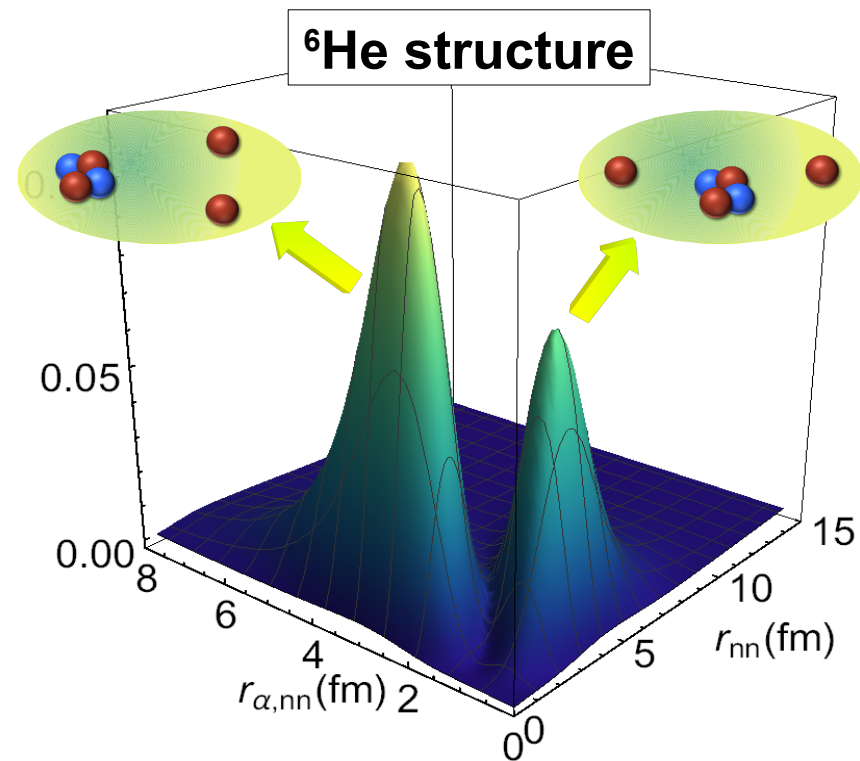
Vancouver. February 25<sup>th</sup>, 2016

 Lawrence Livermore  
National Laboratory

Collaborators:  
**S. Quaglioni, P. Navrátil, G. Hupin**

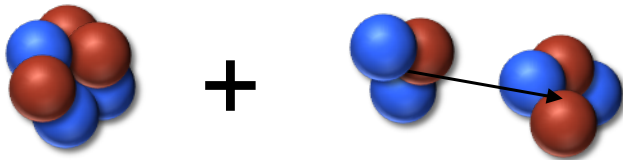
LLNL-PRES-684340

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

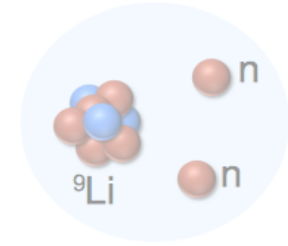


# Outline

No-core shell model with continuum

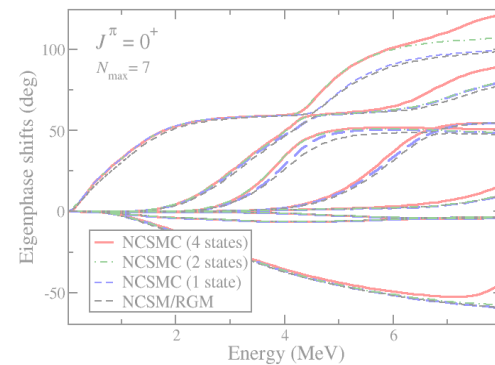
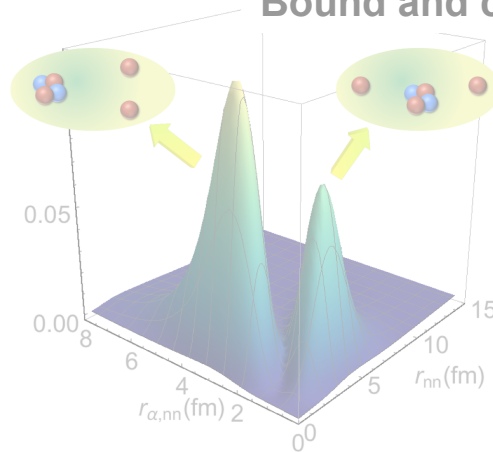


Three-cluster systems



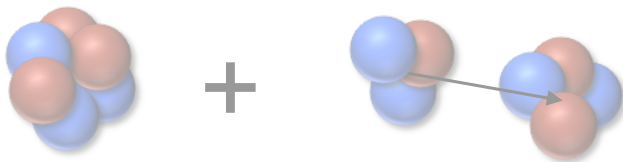
$^6\text{He}$

Bound and continuum states

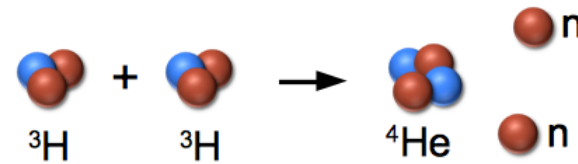
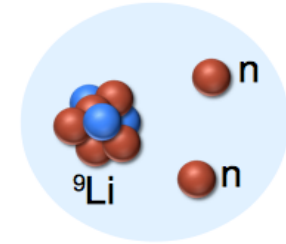


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No-core shell model with continuum

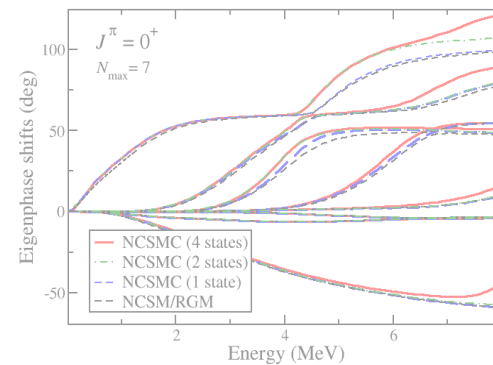
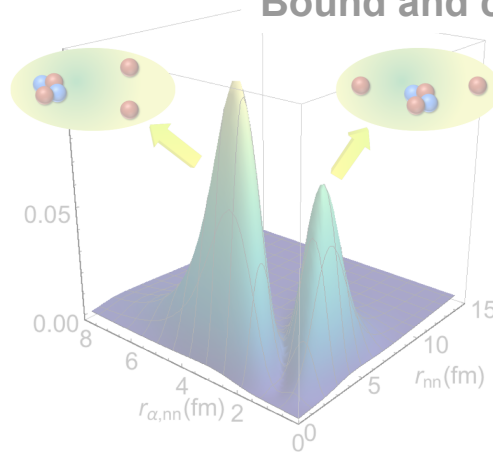


Three-cluster systems



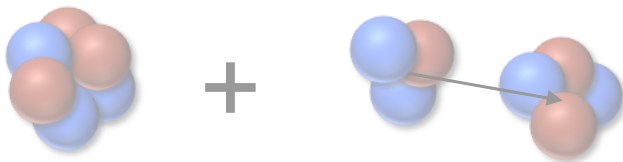
<sup>6</sup>He

Bound and continuum states

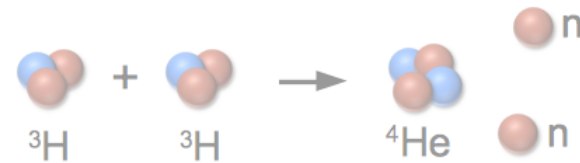
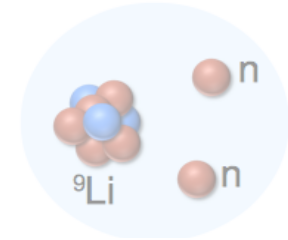


# Outline

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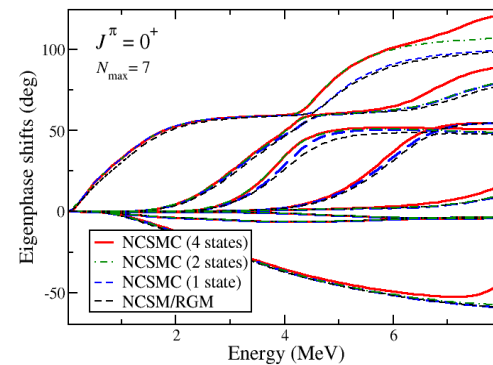
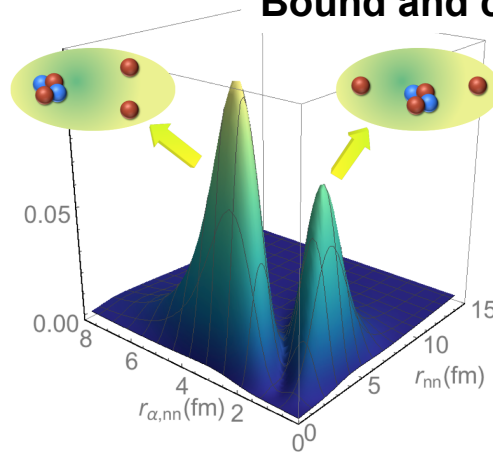


Three-cluster systems

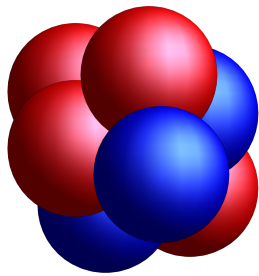


<sup>6</sup>He

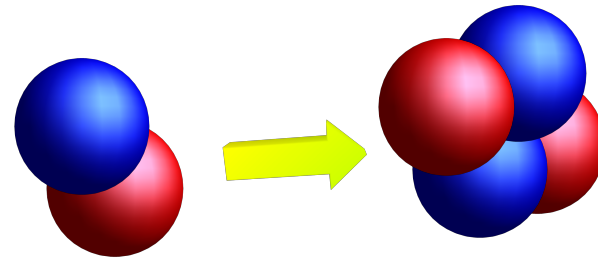
Bound and continuum states



The **No core shell model with continuum (NCSMC)** provides a unified description of bound and continuum states



Bound states



Continuum

S. Baroni, P. Navrátil and S. Quaglioni  
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013)

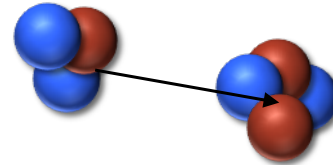
The **No core shell model with continuum (NCSMC)** provides a unified description of bound and continuum states

**NCSM**

Short range description



+



**NCSM/RGM**

Long range description

The **No core shell model with continuum (NCSMC)** provides a unified description of bound and continuum states

**NCSM**

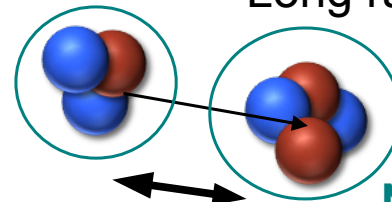
Short range description



+

**NCSM/RGM**

Long range description

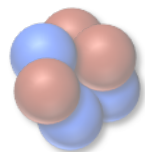


RGM

**NCSM**  
wave functions

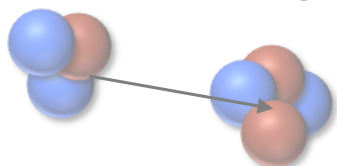
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Short range description



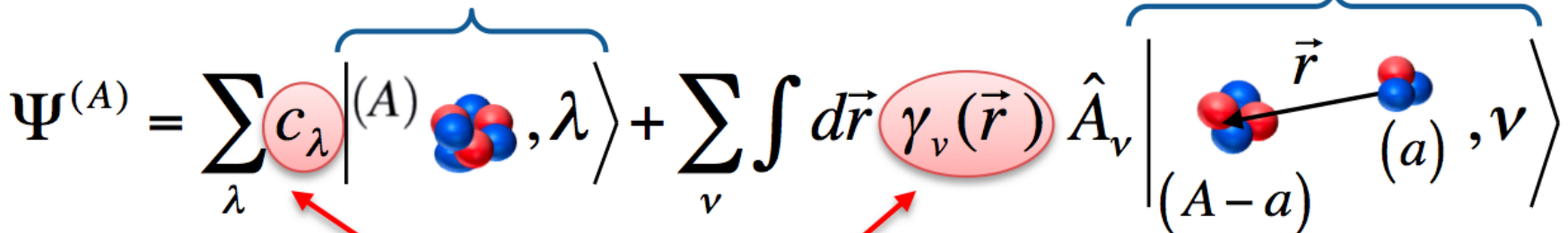
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**NCSM/RGM**  
Long range description

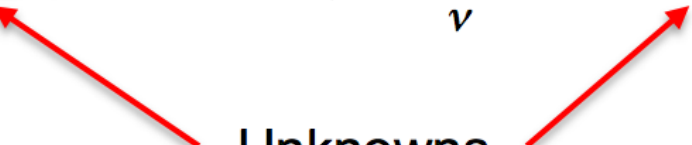


**NCSM  
eigenstates**

**NCSM/RGM  
channel states**

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} (A) \\ \text{Nucleus} \\ \lambda \end{array} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} (A-a) \quad (a) \\ \text{Nucleus} \quad \text{Nucleon} \\ \nu \end{array} \right\rangle$$


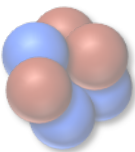
Unknowns





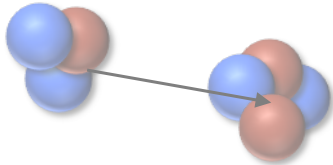
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Short range description



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**NCSM  
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**NCSM/RGM  
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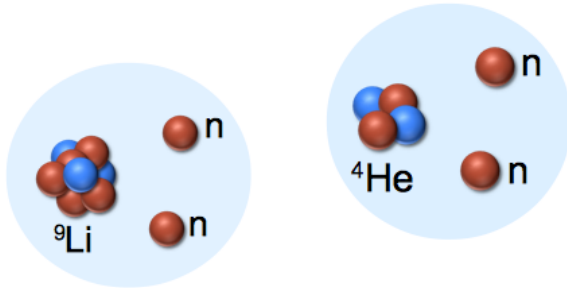
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} (A) \\ \text{cluster} \\ \lambda \end{array} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} (A-a) \quad (a) \\ \text{clusters} \\ \nu \end{array} \right\rangle$$

Unknowns
Intercluster antisymmetrizer

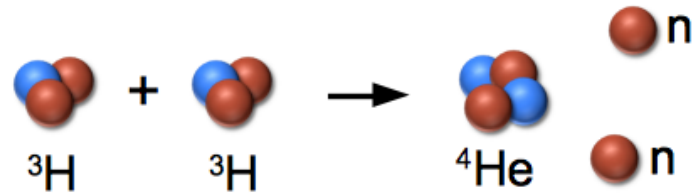
Three-cluster structures appear in many nuclear systems

# Three-cluster structures appear in many nuclear systems

Bound and resonant states:  
2n Halo nuclei



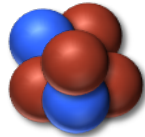
3-body continuum states:  
Reactions



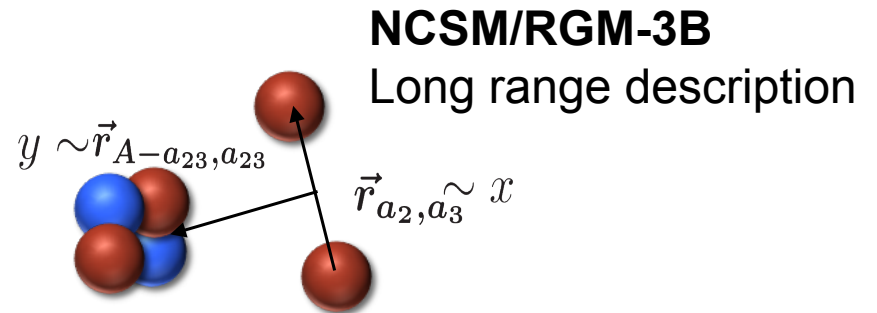
The **NCSMC** can be applied to three-cluster systems, providing a unified description of bound and continuum states

**NCSM**

Short range description



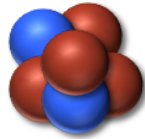
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The **NCSMC** can be applied to three-cluster systems, providing a unified description of bound and continuum states

**NCSM**

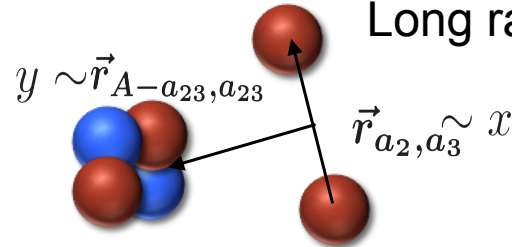
Short range description



+

**NCSM/RGM-3B**

Long range description



$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} \text{three clusters} \\ \lambda \end{array} \right\rangle + \sum_{\nu} \int d\vec{x} d\vec{y} \gamma_{\nu}(\vec{x}, \vec{y}) \hat{A}_{\nu} \left| \begin{array}{c} \text{three clusters} \\ \nu \end{array} \right\rangle$$

Unknowns

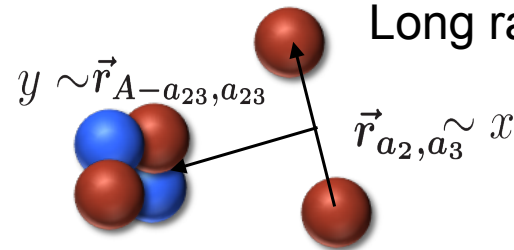
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**NCSM**

Short range description



+



**NCSM/RGM-3B**

Long range description

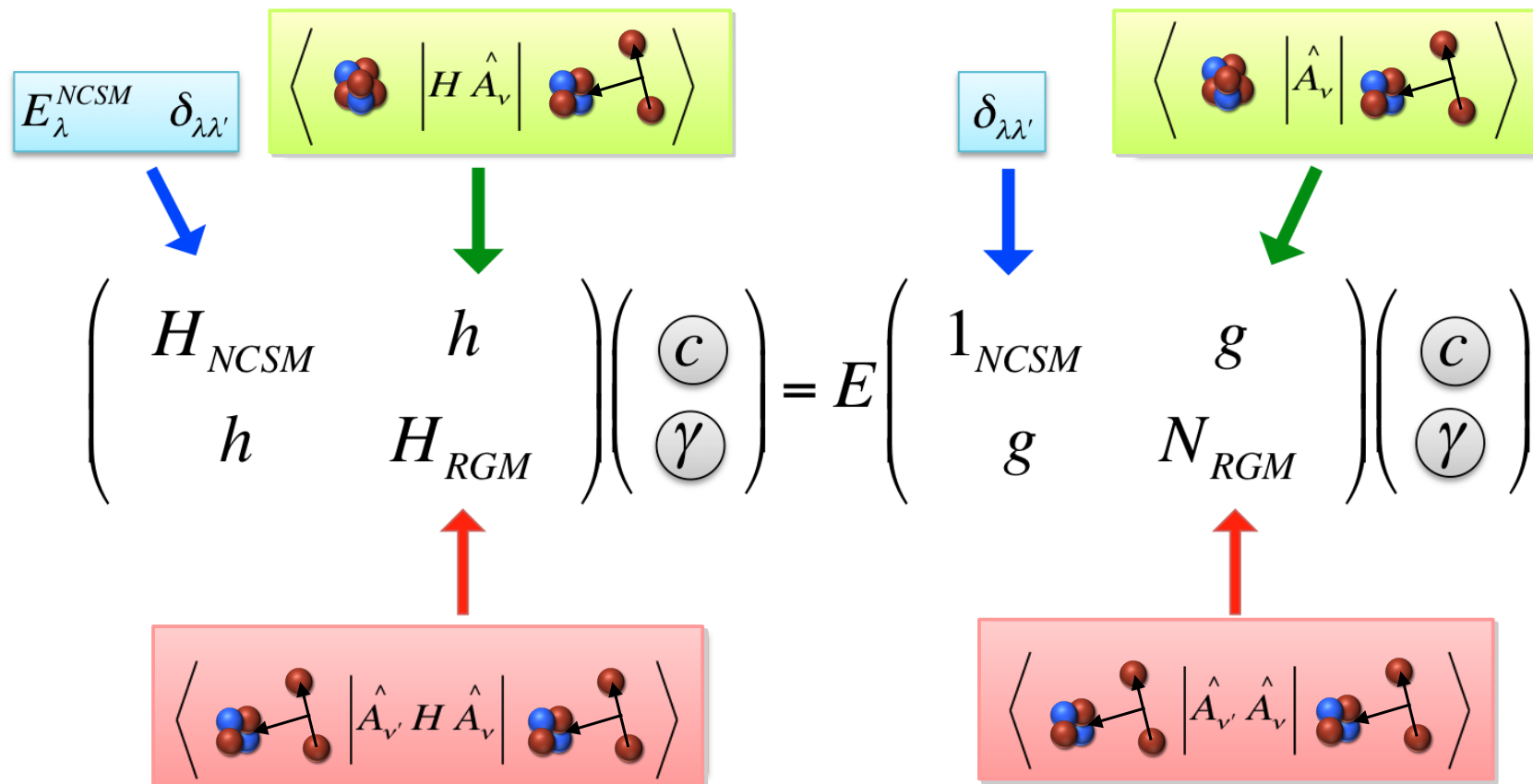
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} \text{Three clusters} \\ \lambda \end{array} \right\rangle + \sum_{\nu} \int d\vec{x} d\vec{y} \gamma_{\nu}(\vec{x}, \vec{y}) \hat{A}_{\nu} \left| \begin{array}{c} \text{Three clusters} \\ \nu \end{array} \right\rangle$$

← Unknowns
← Intercluster antisymmetrizer

The **NCSMC** can be applied to three-cluster systems, providing a unified description of bound and continuum states

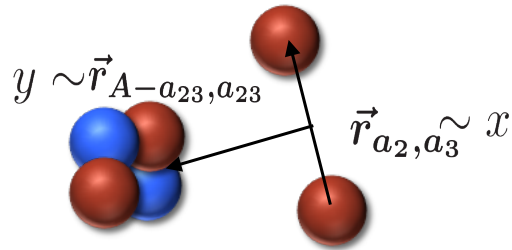
$$\begin{pmatrix} H_{NCSM} & h \\ h & H_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix} = E \begin{pmatrix} 1_{NCSM} & g \\ g & N_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix}$$

The **NCSMC** can be applied to three-cluster systems, providing a unified description of bound and continuum states





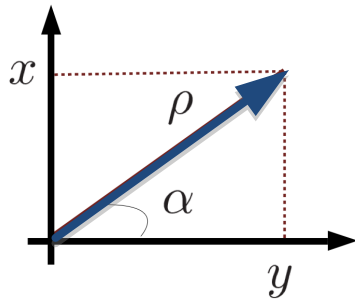
# Solving the NCSMC equations for three-cluster systems



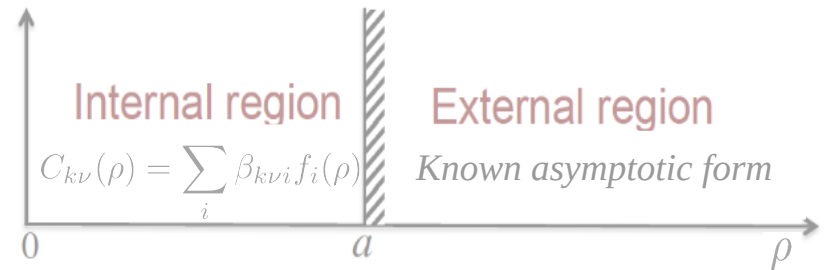
Expansion in hyperspherical basis

$$\chi_{\nu}^{J^{\pi} T}(x, y) = \sum_k C_{k\nu}(\rho) \phi_k^{\ell_x \ell_y}(\alpha)$$

Hyperspherical coordinates



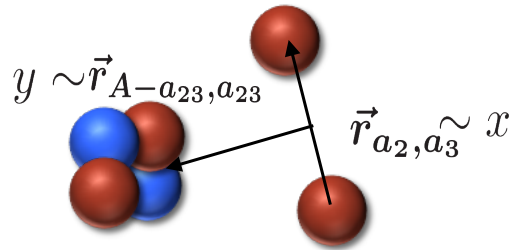
$$(x, y) \rightarrow (\rho, \alpha)$$



R-matrix on Lagrange mesh\*

\*M. Hesse, J.-M. Sparenberg, E Van Raemdonck, D. Baye. Nuclear Physics A 640 (1998) 37-51

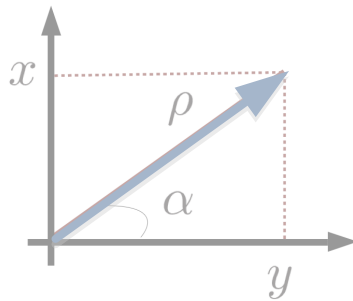
# Solving the NCSMC equations for three-cluster systems



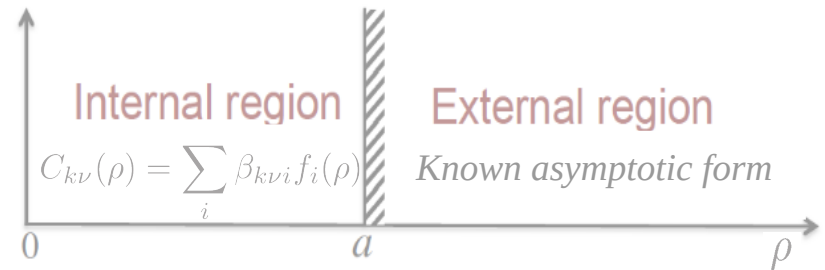
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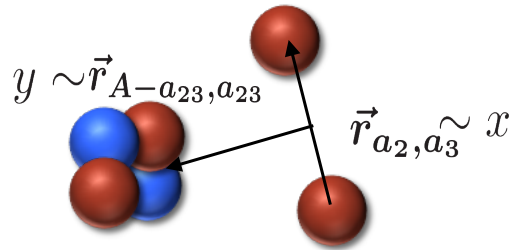
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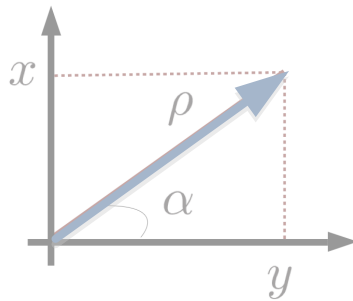
# Solving the NCSMC equations for three-cluster systems



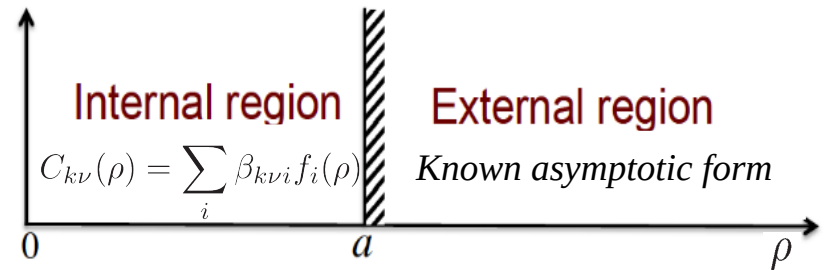
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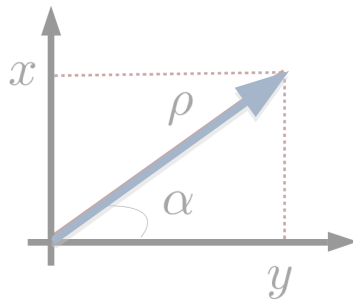
# Solving the **NCSMC** equations for three-cluster systems

**External region:** known asymptotic behavior ( $\rho > a$ )

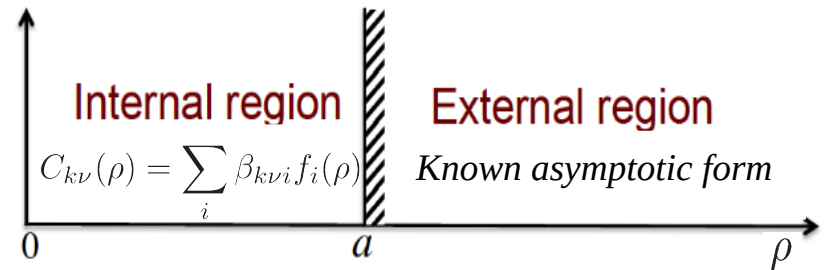
\* Bound state: 
$$C_{k\nu}(\rho) = A_{k\nu} \sqrt{\kappa\rho} K_{k+2}(\kappa\rho)$$

\* Continuum state: 
$$C_{k\nu}(\rho) = A_{k\nu} \left[ H_k^-(\kappa\rho) \delta_{\nu,\nu'} \delta_{k,k'} - S_{\nu k, \nu' k'} H_k^+(\kappa\rho) \right]$$

Hyperspherical coordinates



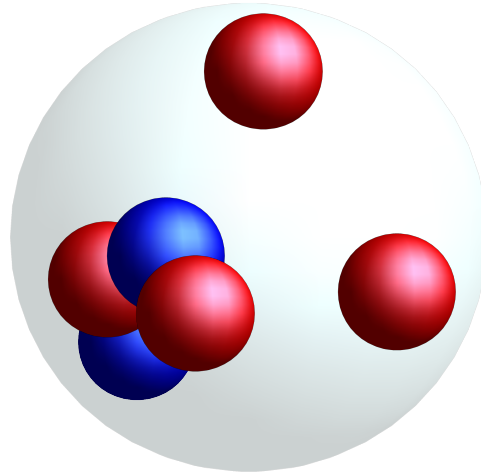
$$(x,y) \rightarrow (\rho,\alpha)$$



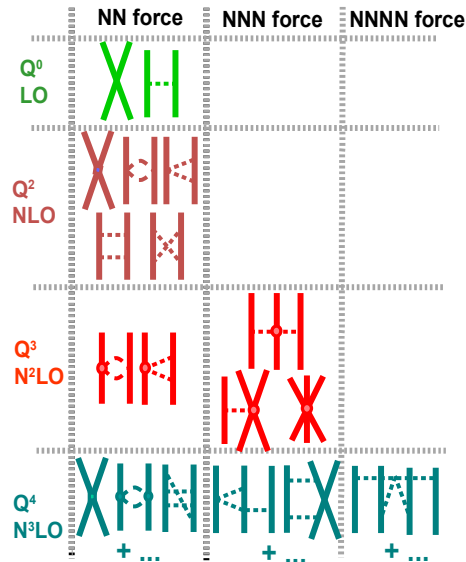
R-matrix on Lagrange mesh\*

\*M. Hesse, J.-M. Sparenberg 1, E Van Raemdonck, D. Baye. Nuclear Physics A 640 (1998) 37-51

${}^6\text{He}$  is a two-neutron halo, therefore presenting an extended three-cluster configuration

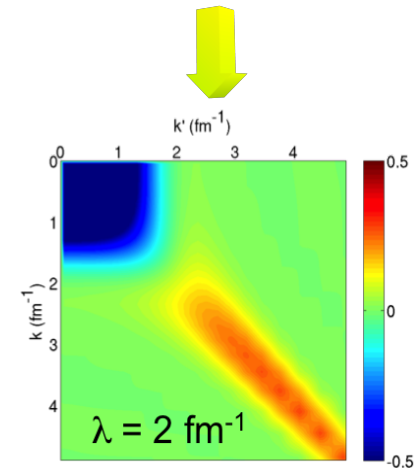
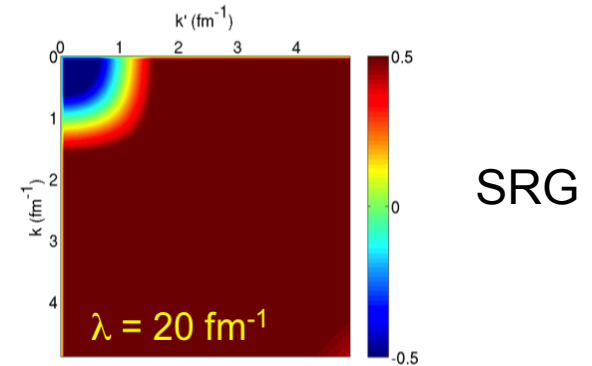


# Interaction used as input



Realistic interactions

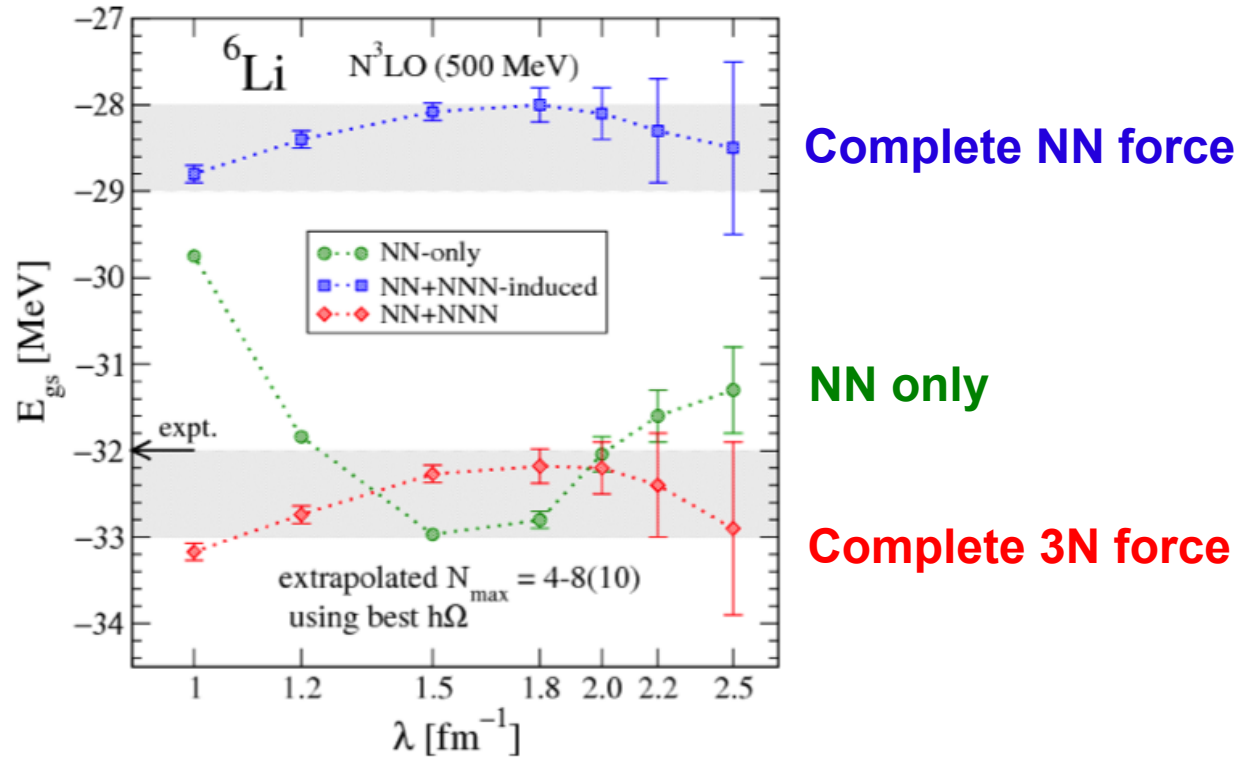
chiral  $N^3$ LO NN (Entem-Machleidt)



Decouples low and high momenta

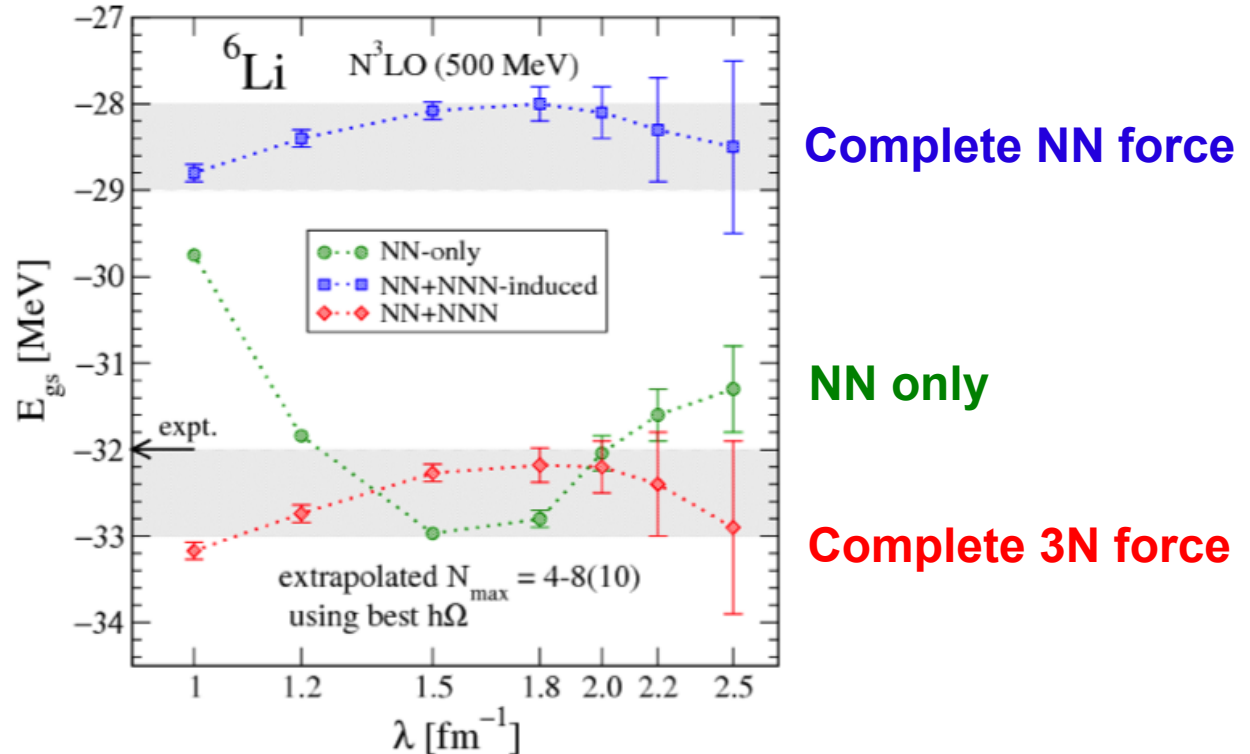
Induces many-body forces of higher order

There are special values  $\lambda$  for which omitted induced 3N forces “compensate” for the lack of inclusion of real 3N forces



PRL 103, 082501 (2009)

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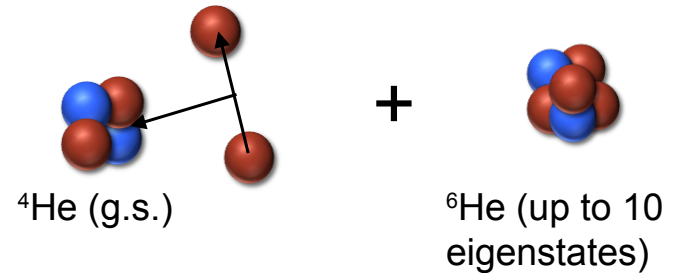
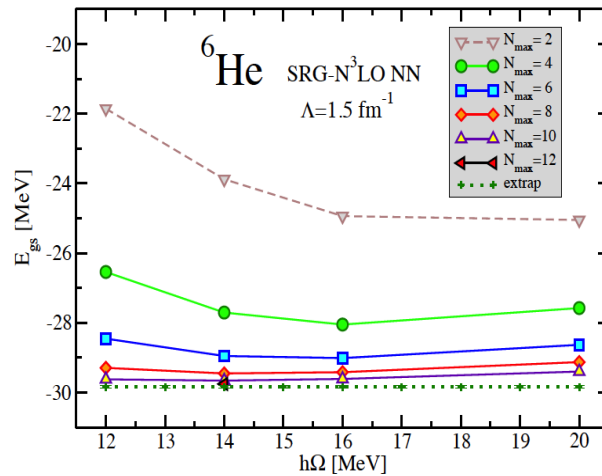


We will use both  $\lambda=1.5$  and  $2.0 \text{ fm}^{-1}$



${}^6\text{He}$  is a two-neutron halo, therefore presenting an extended three-cluster configuration

### ${}^6\text{He}$ ground state, NCSM

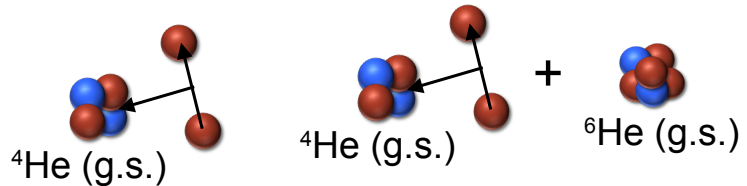


# The NCSM 6-nucleon eigenstate compensates for the missing many-body correlations

Experimental value  
-29.269 MeV

$\lambda=1.5 \text{ fm}^{-1}$

Energy of  $0^+$  g.s.



$N_{\text{max}}$	NCSM	NCSM/RGM	NCSMC ( $0^+_{11}$ )
4	-27.70	-27.14	-28.29
6	-27.98	-28.91	-30.02
8	-28.95	-28.61	-29.69
10	-29.45	-28.70	-29.86
12	-29.66	-28.70	-29.86
Extrapolation	-29.84(4)	---	---

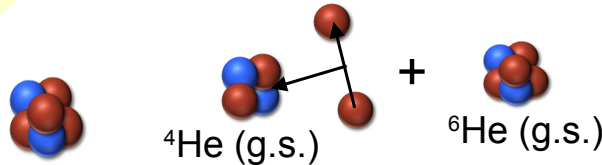
CRR, S. Quaglioni, P. Navrátil. *In progress*

# The **NCSM** 6-nucleon eigenstate compensates for the missing many-body correlations

Experimental value  
-29.269 MeV

$\lambda = 2.0 \text{ fm}^{-1}$

Energy of  $0^+$  g.s.



$N_{\text{max}}$	NCSM	NCSMC ( $0^+_{11}$ )
8	-26.44	-28.81
10	-27.70	-28.97
12	-28.37	-29.17
Extrapolation	-29.20(11)*	---

\*D. Sääf, C. Forssén, PRC **89** 011303 (2014)

CRR, S. Quaglioni, P. Navrátil. *In progress*

The **NCSM** 6-nucleon eigenstate compensates for the missing many-body correlations

### Matter radius

$N_{\max}$	$\Lambda_{\text{SRG}} = 1.5 \text{ fm}^{-1}$		$\Lambda_{\text{SRG}} = 2.0 \text{ fm}^{-1}$	
	NCSM	NCSMC ( $0^+_{11}$ )	NCSM	NCSMC ( $0^+_{11}$ )
8	2.18	2.28	2.06	2.30
10	2.22	2.33	2.10	2.35
12	2.25	2.34	2.15	2.36

### Experiments

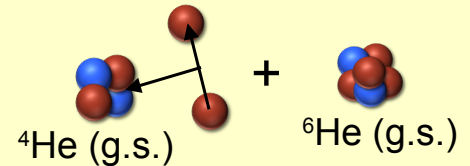
$2.33 \pm 0.04 \text{ fm}$ , PLB **289**, 261 (1992)

$2.30 \pm 0.07 \text{ fm}$ , PRL **78**, 2313 (1997)

$2.37 \pm 0.05 \text{ fm}$ , EurPhys J A**25** 215 (2005)



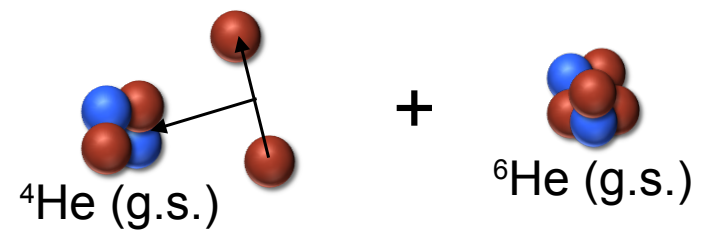
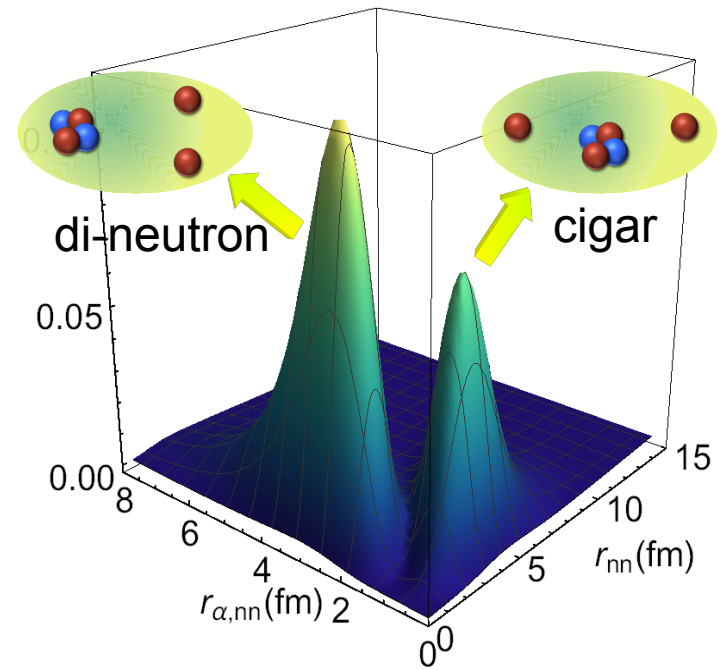
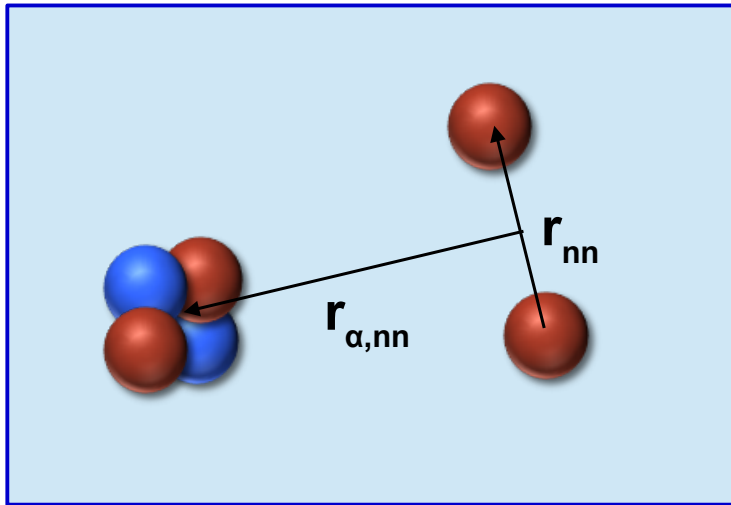
**NCSM**



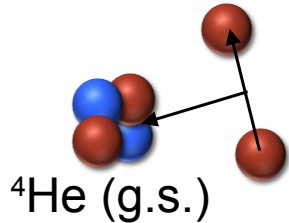
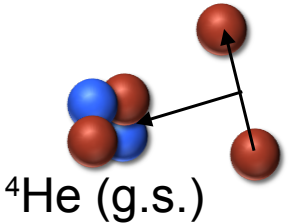
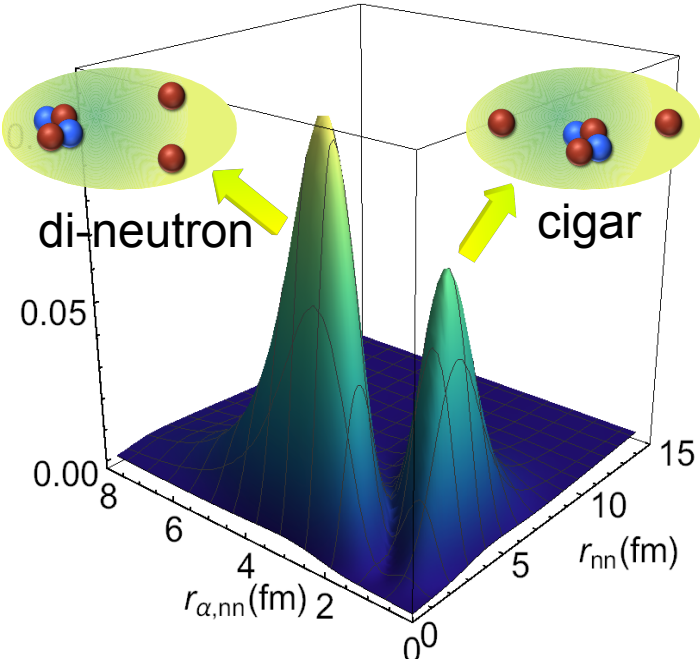
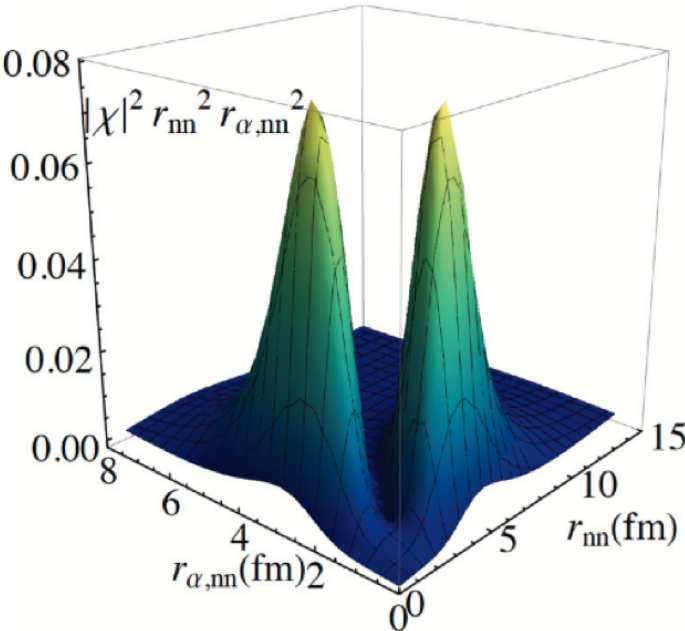
**NCSMC**

CRR, S. Quaglioni, P. Navrátil. *In progress*

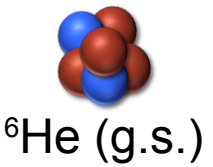
The probability distribution of the  ${}^6\text{He}$  ground state presents two peaks corresponding to the di-neutron and cigar configurations



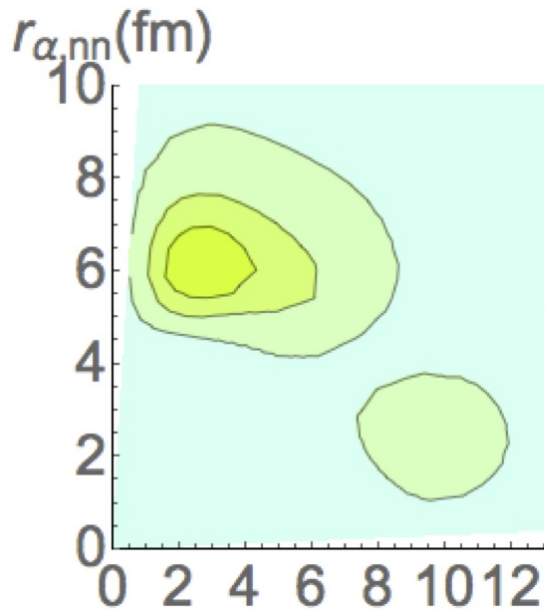
The probability distribution of the  ${}^6\text{He}$  ground state presents two peaks corresponding to the di-neutron and cigar configurations



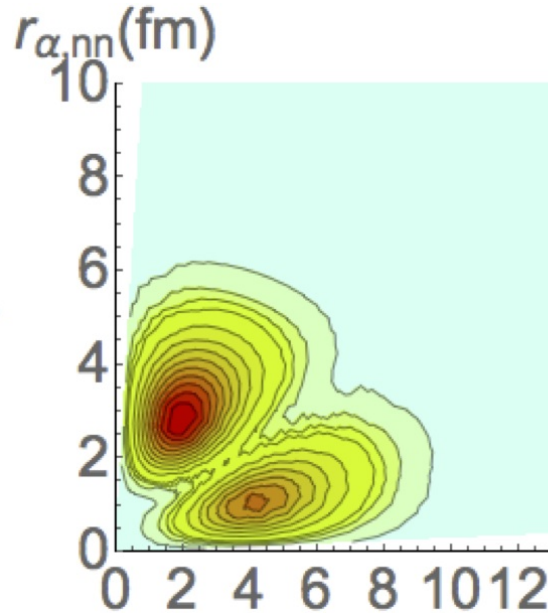
+



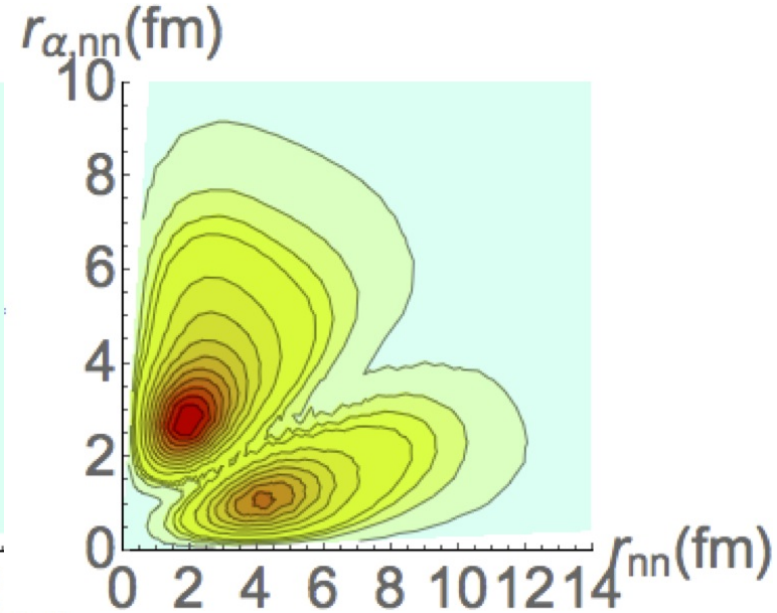
Within the NCSMC, we can see how the RGM part of the basis contributes to the tail of the probability distribution



RGM  
contribution

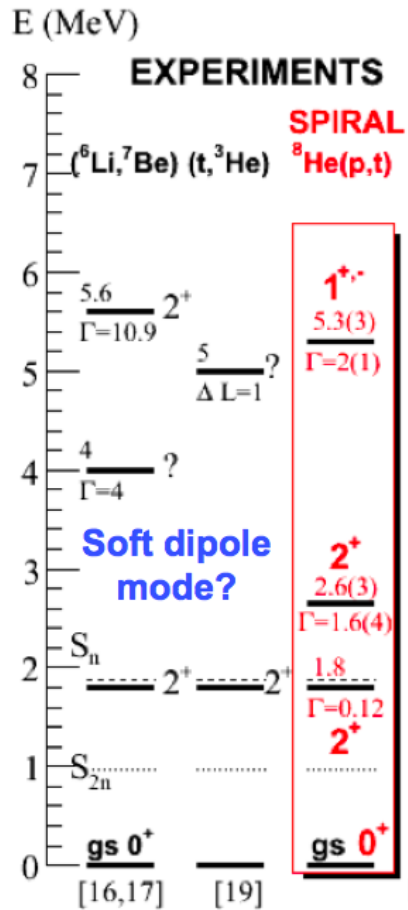


NCSM  
contribution

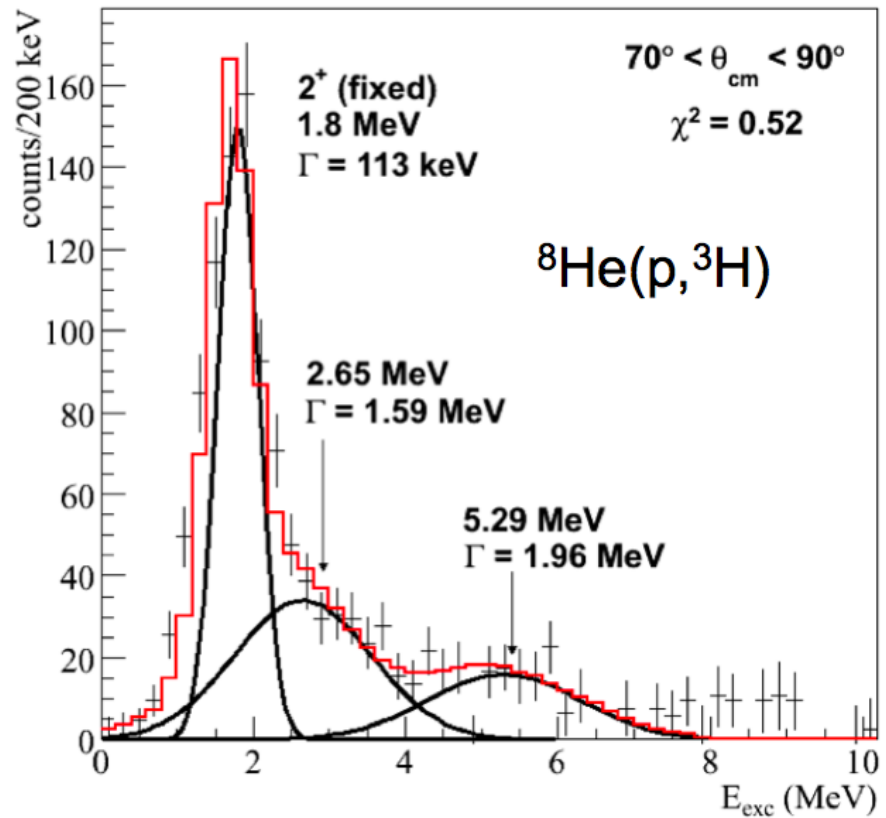


Total NCSMC

# The spectrum of ${}^6\text{He}$ has been remeasured in 2012

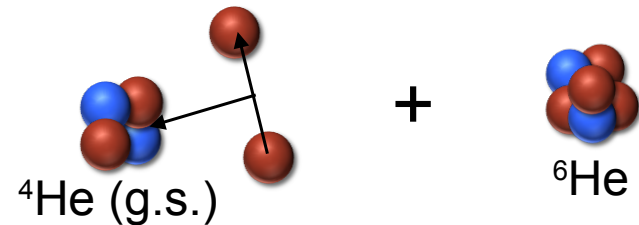
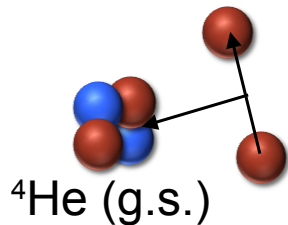
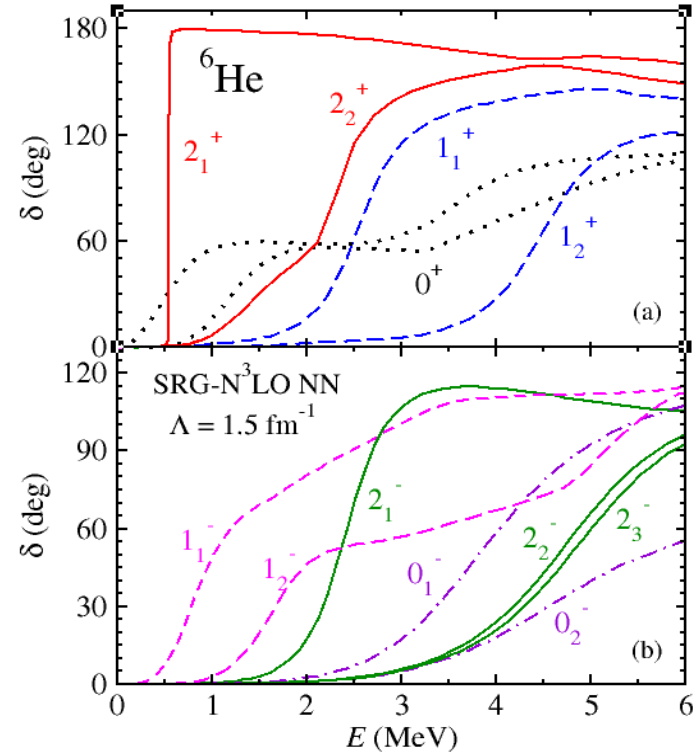
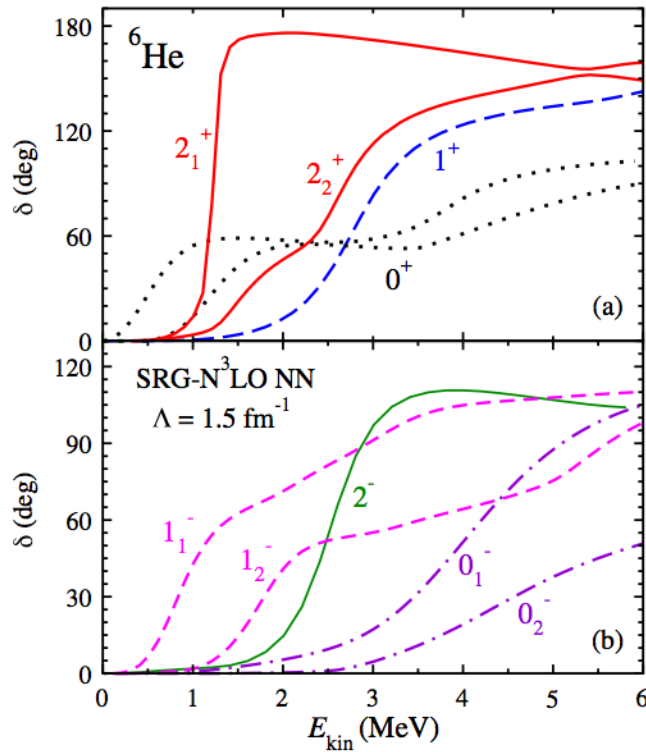


Recent expt. @SPIRAL, GANIL: PLB 718 (2012) 441

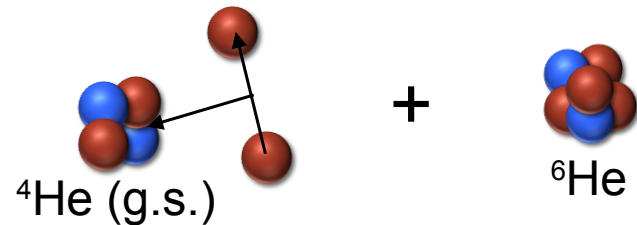
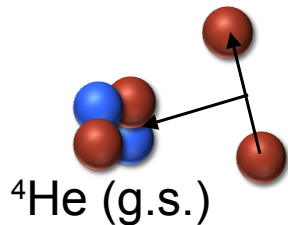
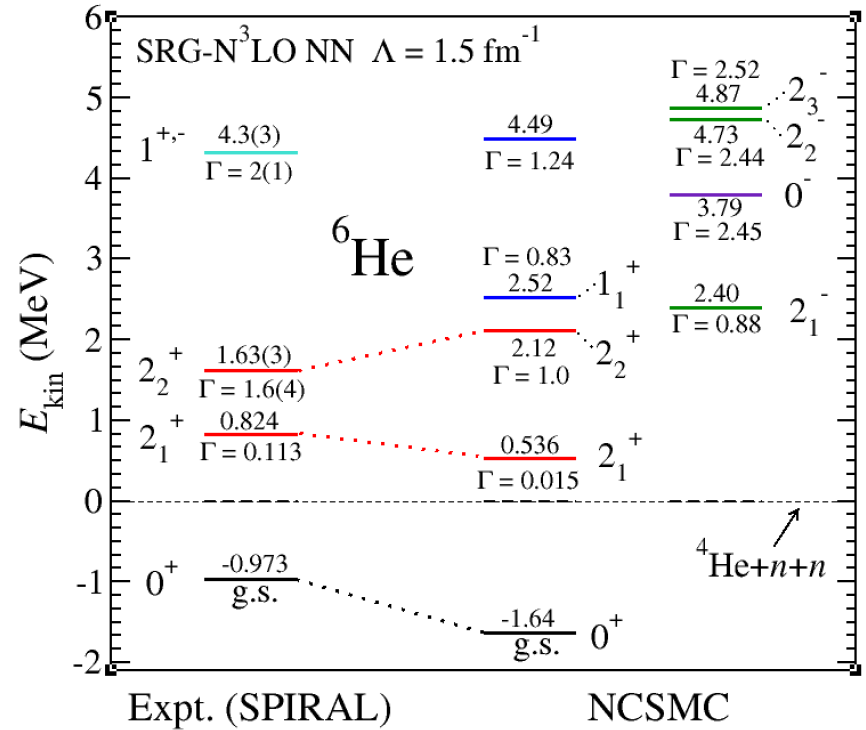
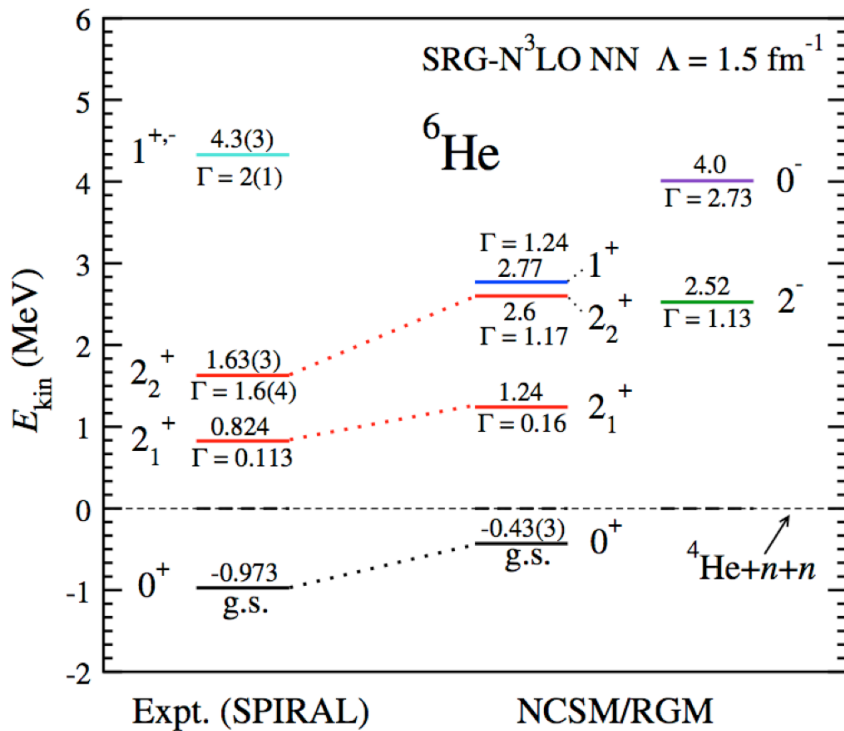




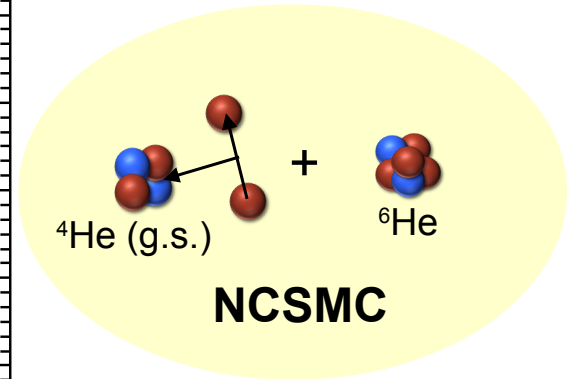
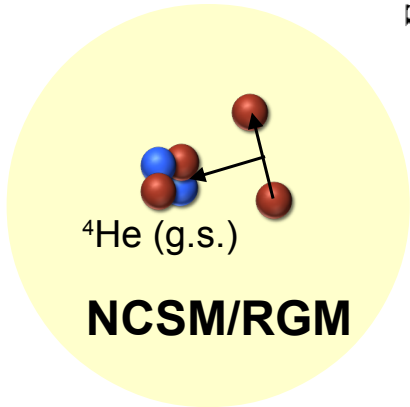
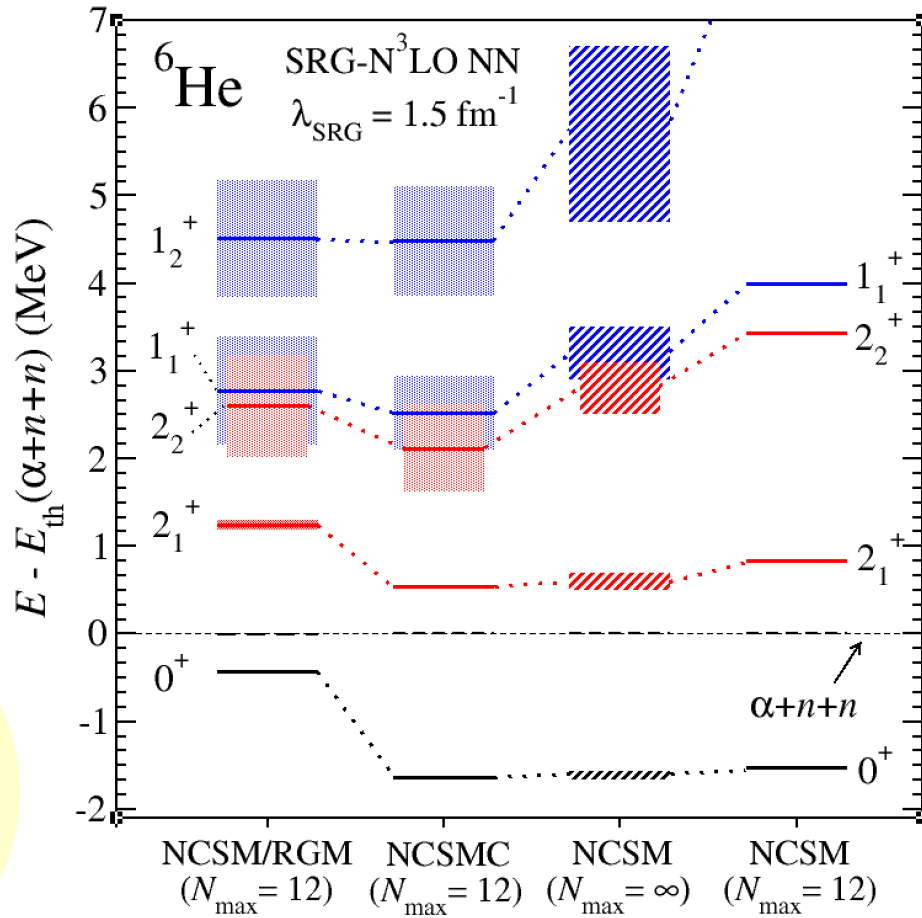
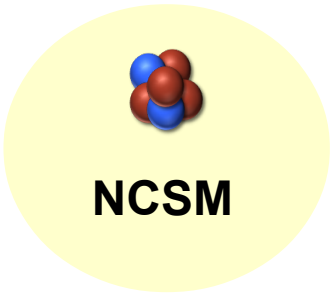
# The continuum spectrum can be extracted from the phase shifts



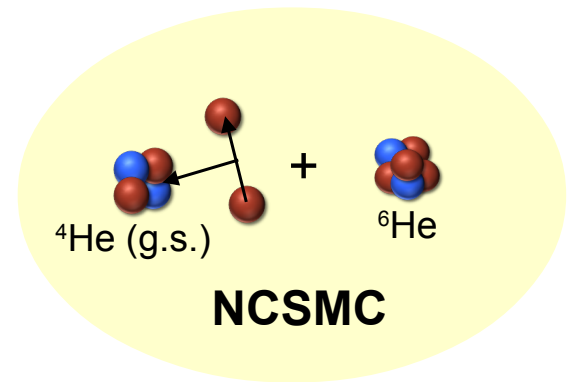
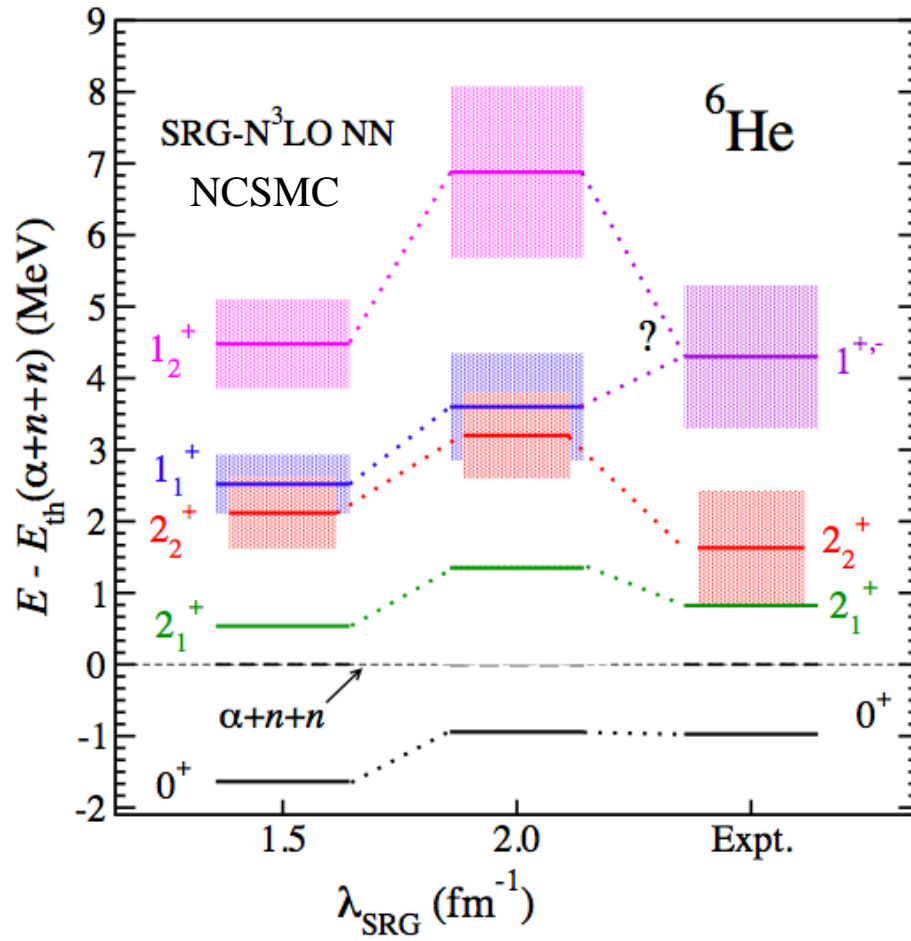
# The continuum spectrum can be extracted from the phase shifts



We can compare the spectrum given by the different formalisms

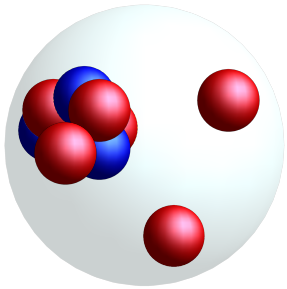


We can also study the dependence in the evolution parameter  $\lambda_{\text{SRG}}$

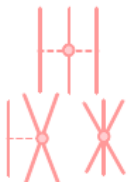
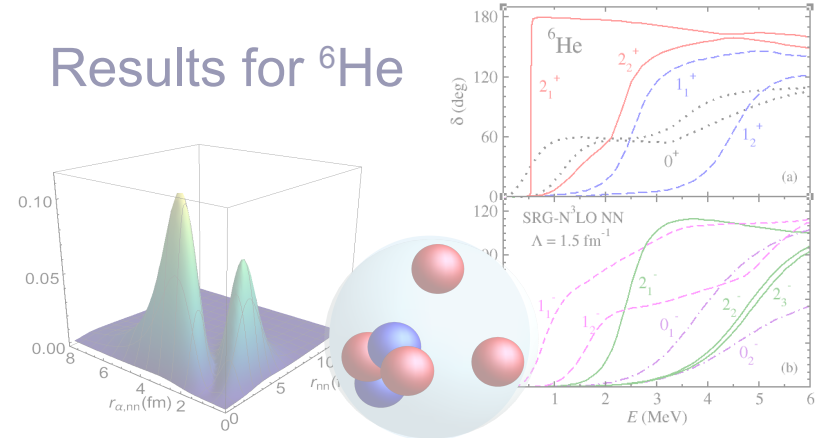


# Summary and outlook

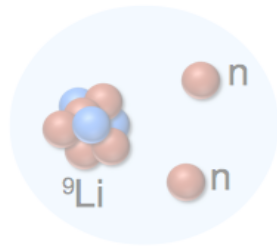
NCSMC for  
3-cluster problems



Results for  ${}^6\text{He}$



3-nucleon force



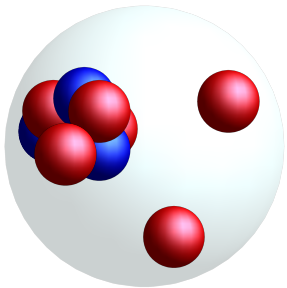
Heavier halo



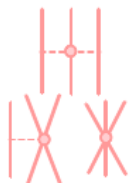
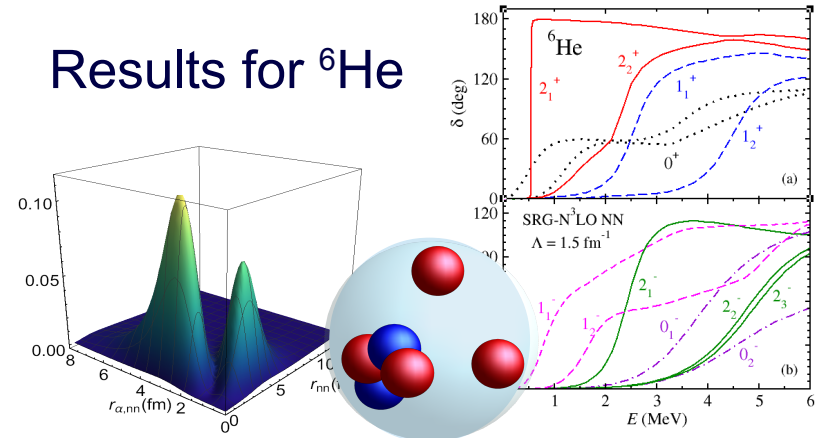
Reactions with 3-body channels

# Summary and outlook

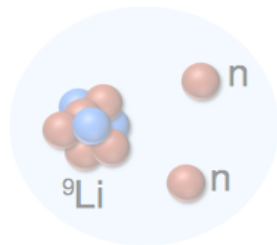
NCSMC for  
3-cluster problems



Results for  ${}^6\text{He}$



3-nucleon force



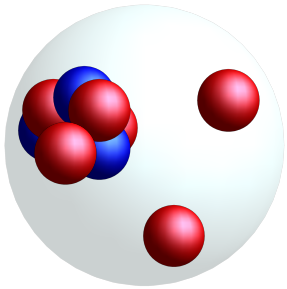
Heavier halo



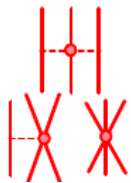
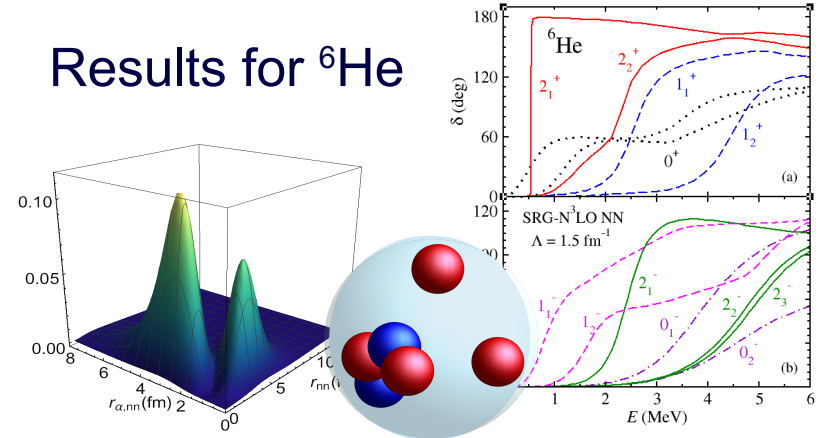
Reactions with 3-body channels

# Summary and outlook

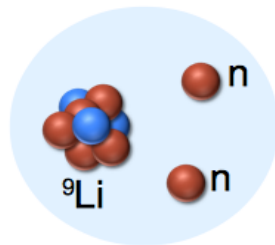
NCSMC for  
3-cluster problems



Results for  ${}^6\text{He}$



3-nucleon force



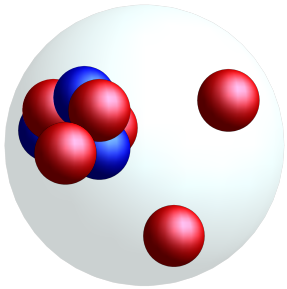
Heavier halo



Reactions with 3-body channels

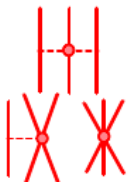
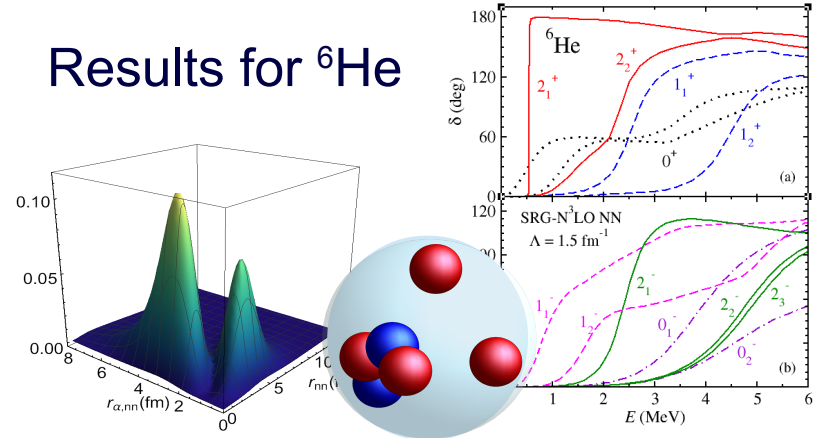
# Summary and outlook

NCSMC for  
3-cluster problems

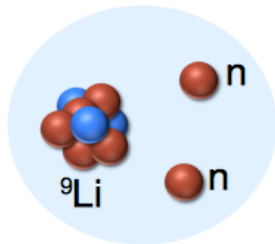


Thank you!!!

Results for  ${}^6\text{He}$



3-nucleon force



Heavier halo



Reactions with 3-body channels



**Thank you!**