Nuclear physics around the unitarity limit

Sebastian König

Nuclear Theory Workshop

TRIUMF, Vancouver, BC

February 28, 2017

SK, H.W. Grießhammer, H.-W. Hammer, U. van Kolck, arXiv:1607.04623 [nucl-th]
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Perturbative perspectives for nuclear effective field theories

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Prelude

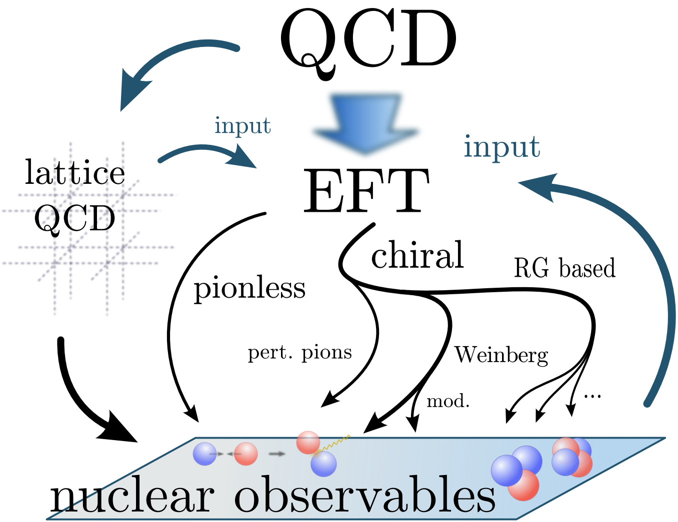
hierarchy of forces (natural in EFT)

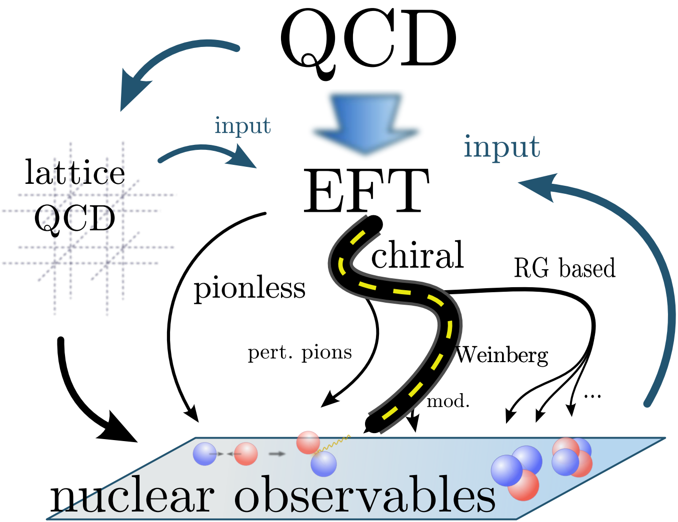
many-body forces $\leftrightarrow$ two-body off-shell tuning

Various approaches depart from focusing on two-body input. . .

- **JISP16** Shirokov *et al.*, PLB **644** 33 (2007)
  $\hookrightarrow$ two-body only, but input from nuclei up to ^{16}O
- **N2LO$_{\text{opt}}$, N2LO$_{\text{sat}}$** Ekstöm *et al.*, PRL **110** 192502 (2013), PRC **91** 051301 (2015)
  simultaneous fit to NN + light nuclei, saturation properties
- **SRG-evolved 2N + N2LO 3N** Simonis *et al.*, PRC **93** (2016)
  $\hookrightarrow$ predict realistic saturation properties
- **nuclear lattice calculations** Elhatisari *et al.*. PRL **117** 132501 (2016)
  $\hookrightarrow$ use input from α-α scattering
- . . .

Nuclear EFT overview




QCD
input
lattice
QCD
input
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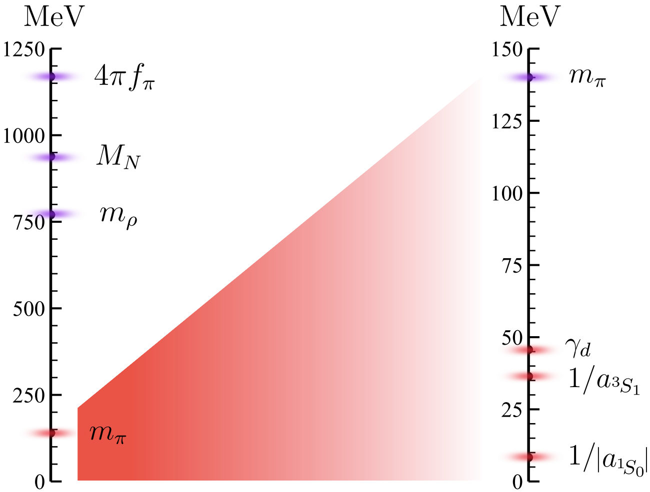
...
nuclear observables

Nuclear scales revisited



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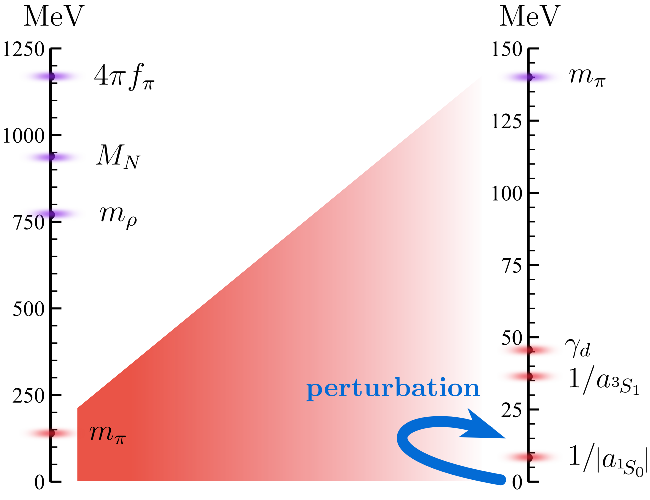

unitarity limit: $1/a = 0$

SK *et al.* JPG **43** 055106 (2016)

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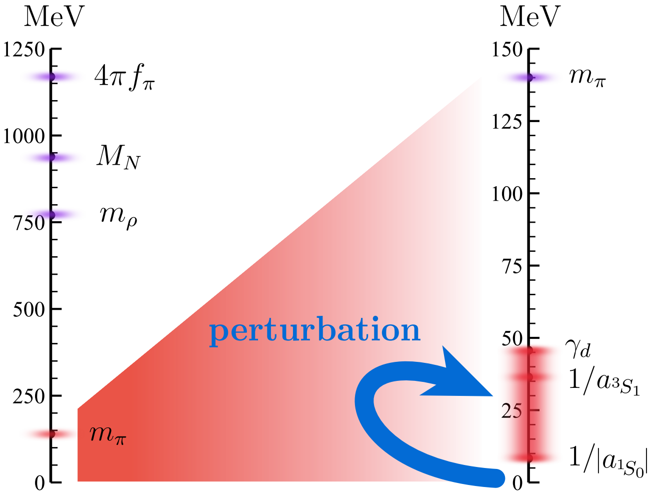

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SK *et al.* 1607.04623

The unitarity expansion

SK, H.W. Grießhammer, H.-W. Hammer, U. van Kolck, arXiv:1607.04623 [nucl-th]

Basic setup

- two-body physics (LECs) $\leftrightarrow$ effective range expansion
- assume $a_{s={}^1S_0,t={}^3S_1} = \infty \iff 1/a_{s,t} = 0$ at leading order
- **still need LO pionless three-body force!**
  $\hookrightarrow$ reproduce triton energy exactly
- finite a, Coulomb, ranges $\rightarrow$ perturbative corrections!

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Capture gross features at leading order, build up the rest as perturbative "fine structure!"

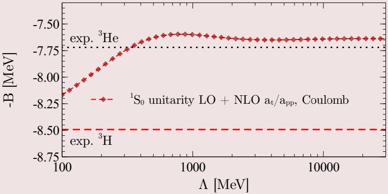
- **shift focus away from two-body details**
- nuclear sweet spot $1/a_{s,t} < Q_A < 1/R$?
- **note:** zero-energy deuteron at LO and NLO
- exact $SU(4)_W$ symmetry at LO!
  cf. Vanasse+Phillips, FB Syst. **58** 26 (2017)

original cupcake by Slashme (via Wiki Commons)

Helium results

^{3}He at $^1S_0$ and full unitarity

- good NLO established for $^1S_0$ unitarity



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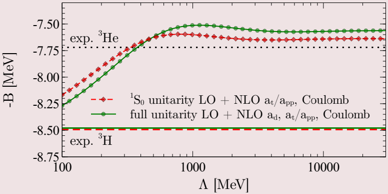

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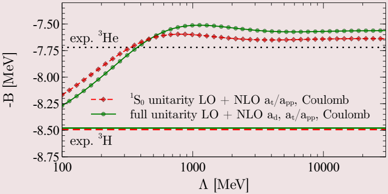
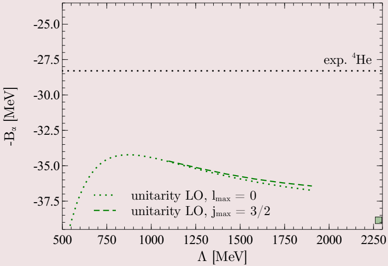


^{4}He (zero-range, no Coulomb)



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cf. also Platter (2004)

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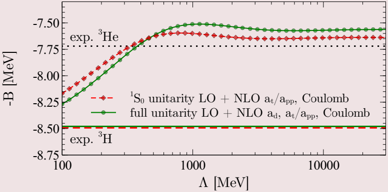

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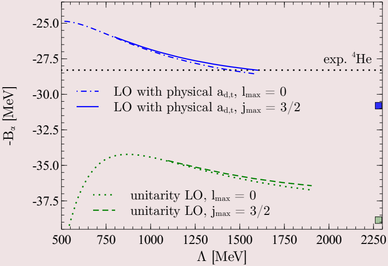

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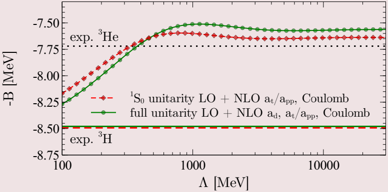

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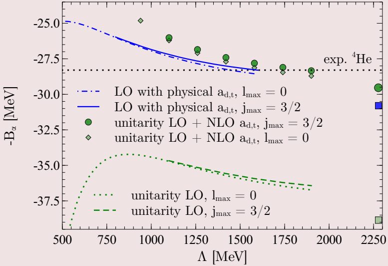

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Some details

- binding energies at LO: find zeros of $\det(\mathbf{1} - K(E))$,
  $K(E)$ = Faddeev(-Yakubowsky) kernel
- NLO energy shift: $\Delta E = \langle\Psi|V^{(1)}|\Psi\rangle$, $|\Psi\rangle$ = LO wavefunction

$$|\Psi\rangle = (\mathbf{1} - P_{34} - PP_{34})(1+P)\,|\psi_A\rangle + (\mathbf{1}+P)(\mathbf{1}+\tilde{P})\,|\psi_B\rangle$$

wavefunction convergence slower than eigenvalue convergence!

$\hookrightarrow$ need more mesh points and partial-wave components...

Energy balance

- sample calculation with physical scattering lengths at LO:

Λ / MeV	800	1000	1200	1400
E_{kin} / MeV	113.67	140.58	168.44	197.09
E_{pot} / MeV	-139.77	-167.41	-195.76	-224.62

- E_{kin} and E_{pot} not observable
- sum converges as cutoff is increased, individual values do not!

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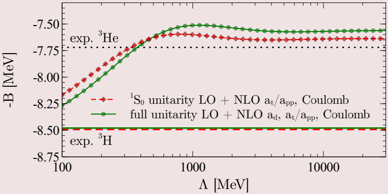

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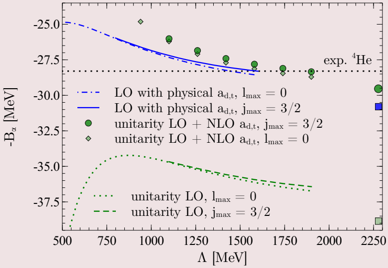

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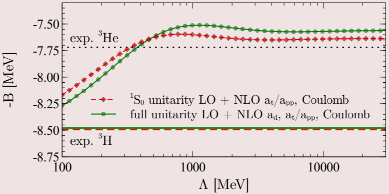

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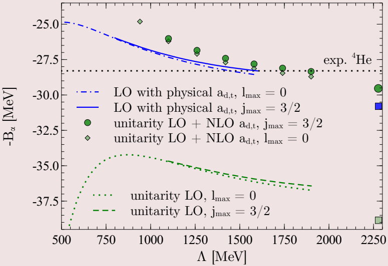
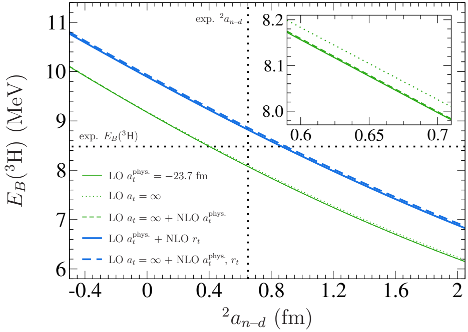


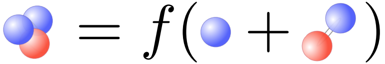
- ^{4}He resonance state ~ 0.3 MeV above ^{3}He + p threshold
- just below threshold at unitarity LO
- boson calculations with nuclear scales
  $\rightsquigarrow$ shift by about $0.2 - 0.5$ MeV

SK, Hammer, Grießhammer, van Kolck (2016/17)
cf. also Platter (2004)

Few-nucleon correlations



Phillips line



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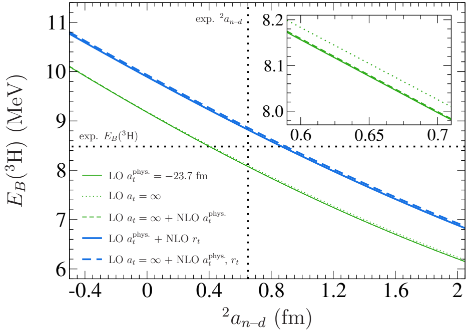

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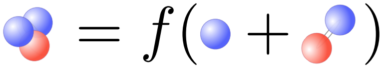

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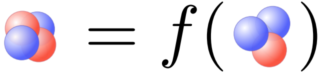

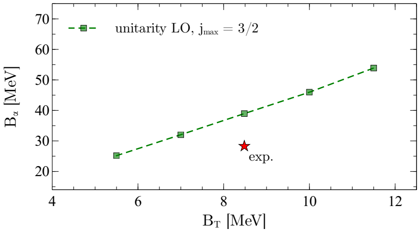

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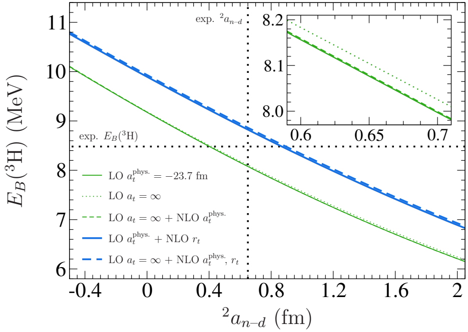

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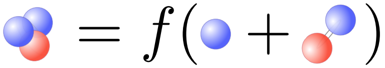

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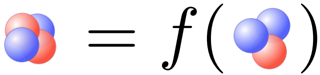

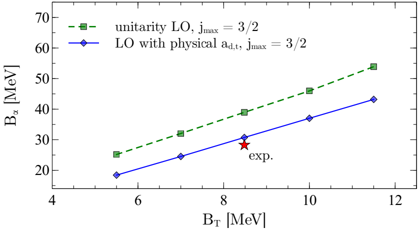

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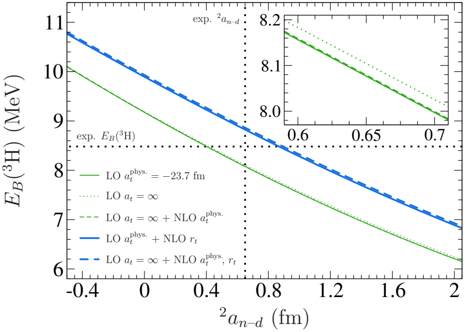

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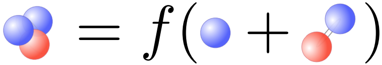

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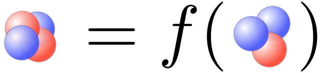
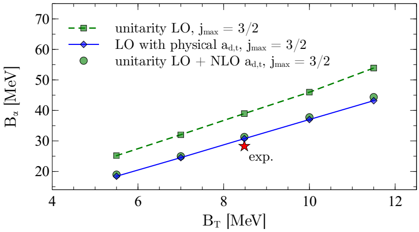


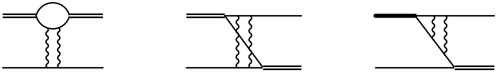


Unitarity expansion(s) at second order

Various contributions at N²LO. . .

SK, 1609.03163 [nucl-th]

1. **quadratic scattering-length corrections**
2. two-photon exchange



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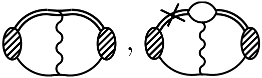
4. isospin-breaking effective ranges: $r_{pp} \neq r_{np}$
5. mixed Coulomb and range corrections! e.g. 

   $\rightsquigarrow$ **log. divergence, new pd counterterm!**

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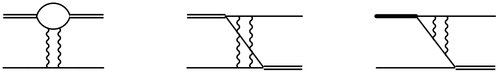

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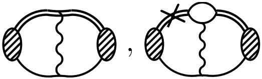

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Energy shift from T-matrix

$$B_2 = \lim_{E \to -B_0} \frac{(E + B_0)^2 \, \mathcal{T}^{(2)}(E; k, p) + B_1 (E + B_0) \mathcal{T}^{(1)}(E; k, p)}{\mathcal{B}^{(0)}(k) \, \mathcal{B}^{(0)}(p)}$$

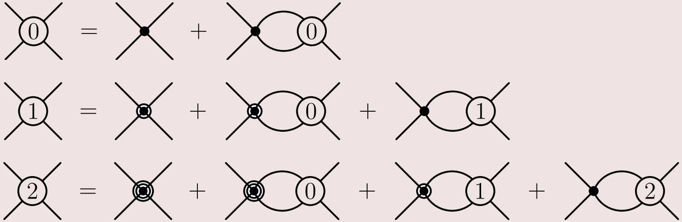
cf. Ji+Phillips (2013), Vanasse (2013)

The perturbative deuteron

Use efficient method to calculate T-matrix in perturbation theory!

Vanasse, PRC **88** 044001 (2013)

T-matrix perturbation theory for $C_0 = C_0^{(0)} + C_0^{(1)} + C_0^{(2)} + \cdots$



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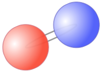
- . . . but it moves to $\kappa^{(1)} = 1/a_t$ at N^{2}LO



$\leftrightarrow$ expansion in momentum, not energy

$$B_0 = \frac{(\kappa^{(0)})^2}{M_N} \;,\; B_1 = \frac{2\kappa^{(0)}\kappa^{(1)}}{M_N} \;,\; B_2 = \frac{(\kappa^{(1)})^2}{M_N} \;,\; \kappa^{(0)} \to 0$$

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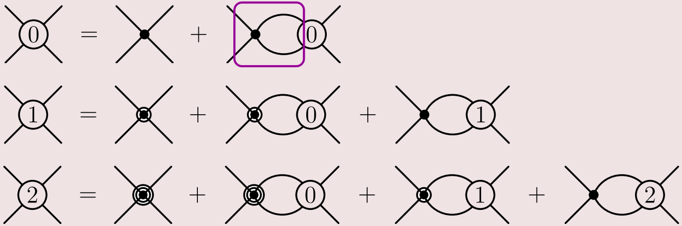

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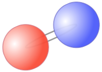

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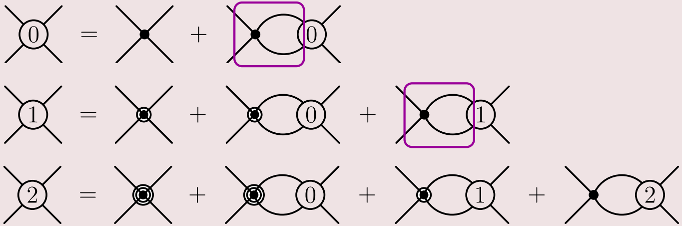

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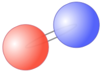

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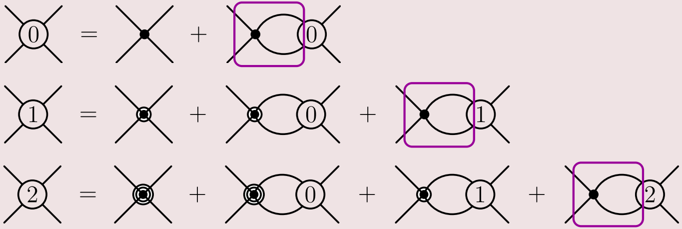

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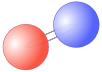

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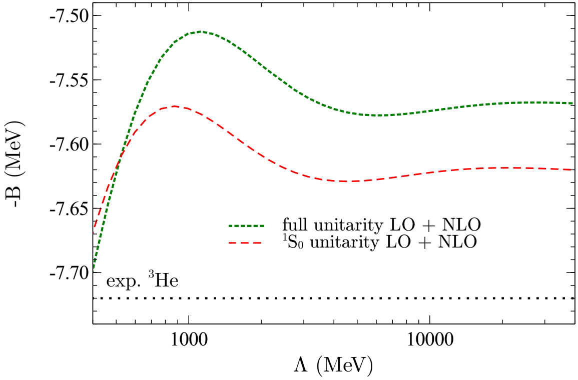

More ^{3}He results

SK, arXiv:1609.03163 [nucl-th]

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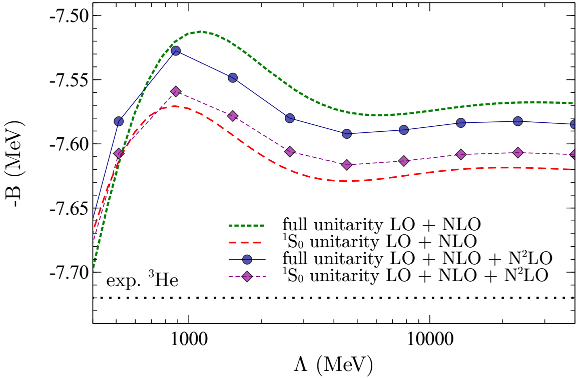
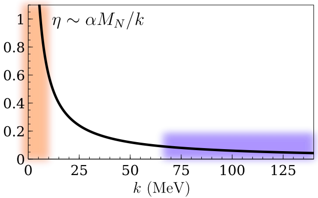


$\hookrightarrow$ **good convergence of half- and full-unitarity expansions!**

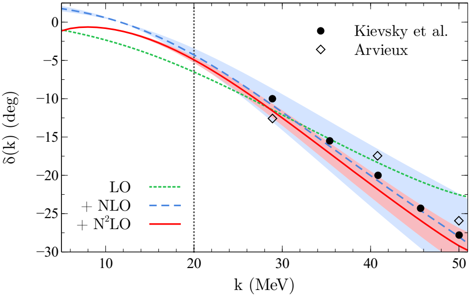
Perturbative p-d phase shifts

At intermediate energies, Coulomb is perturbative for pp/pd scattering!

SK *et al.* (2015); SK (2016/17)



$\eta \leq 1/3$ for $k \geq 20$ MeV



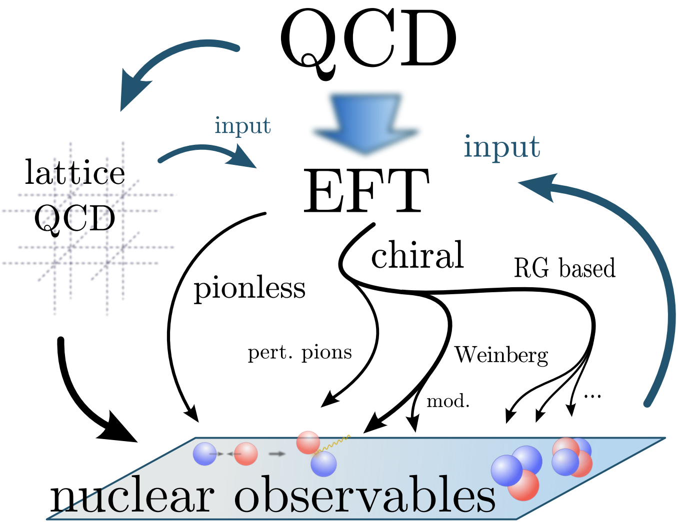
Perturbative subtracted phase shifts

$$\delta(k) \equiv \delta_{\text{full}}(k) - \delta_{\text{c}}(k)$$

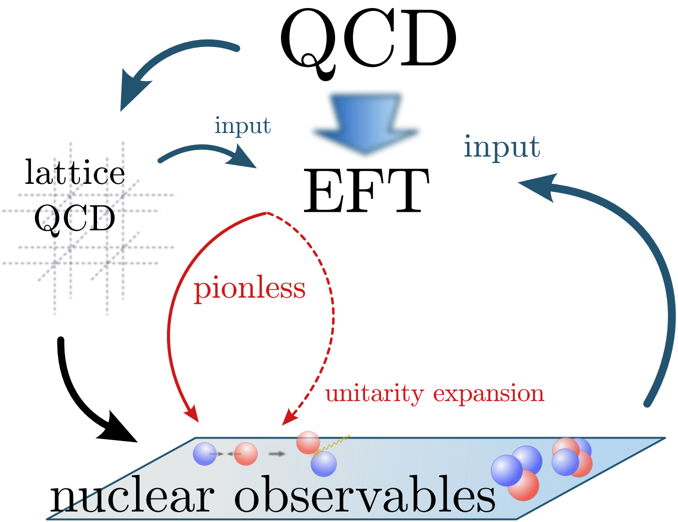
$$= \delta^{(0)}_{\text{full}}(k) - \cancel{\delta^{(0)}_{\text{c}}(k)} + \delta^{(1)}_{\text{full}}(k) - \delta^{(1)}_{\text{c}}(k) + \delta^{(2)}_{\text{full}}(k) - \delta^{(2)}_{\text{c}}(k) + \cdots$$

cf. also SK, Hammer (2014)

Summary and outlook



Summary and outlook



Summary and outlook