Improved nuclear structure corrections for spectroscopy of muonic atoms

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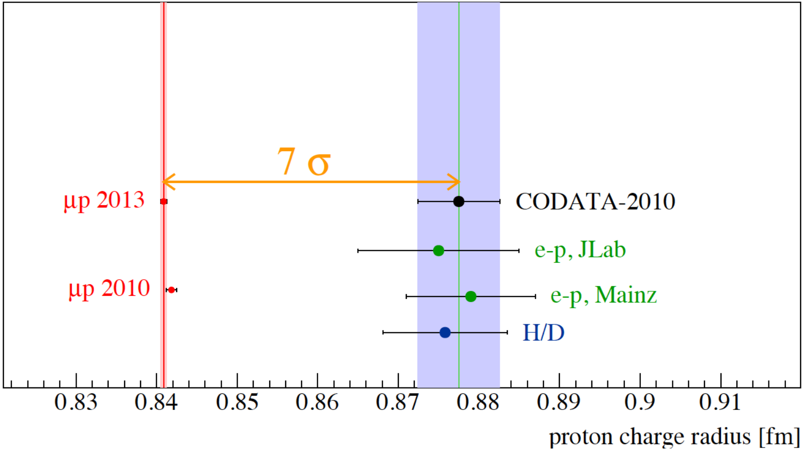
[5]The Hebrew University of Jerusalem

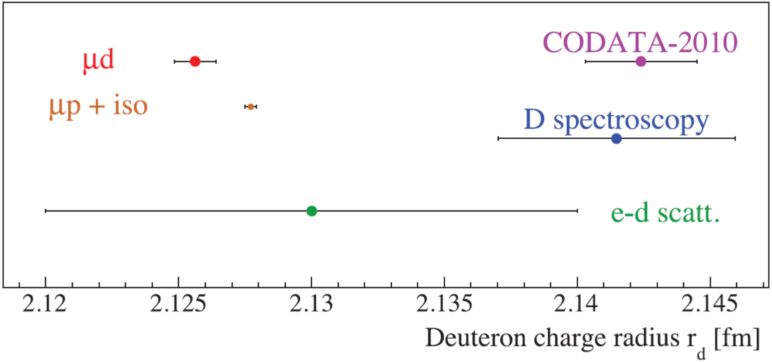
Progress in Ab Initio Techniques in Nuclear Physics — Feb. 28 2017

How big is the proton?



R. Pohl & J. Krauth @ CREMA

How big is the deuteron?



R. Pohl *et al.*, **Science 2016**

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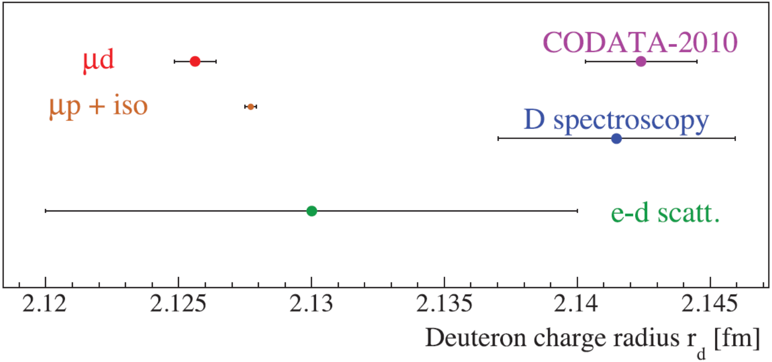
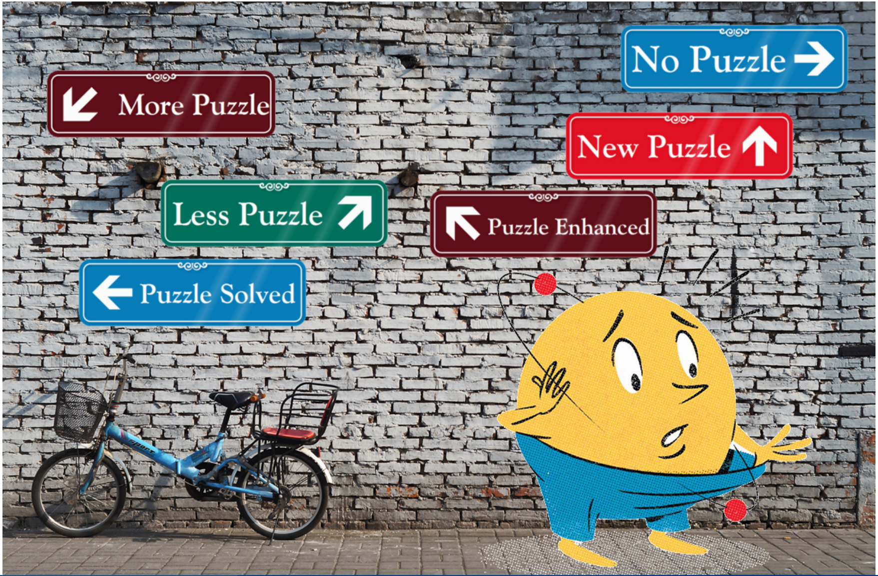


R. Pohl *et al.*, **Science 2016**

See Javier Hernandez's talk at the next session


No Puzzle
More Puzzle
New Puzzle
Less Puzzle
Puzzle Enhanced
Puzzle Solved

CREMA @ PSI

Extract precise **charge radii** $\boldsymbol{R_c}$ from Lamb shift (LS) in:

- μH (published 2010,2013: **proton radius puzzle)**
- μD (published 2016: **deuteron radius puzzle)**
- $\mu\,^4$He$^+$ (measured 2014, finalizing: **agreement with** $e{-}^4$**He ?!**)
- $\mu\,^3$He$^+$ (measured 2014, analyzing: **???)**

  $\Longrightarrow$ radius puzzle(s), QED tests, He isotope shift, nuclear *ab initio*, ...
- $\mu\,^3$H, $\mu\,^6$He$^+$, $\mu\,^{6,7}$Li^{+2} ... (possible?)

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Extract **magnetic radii** R_m from Hyper-fine splitting (HFS) in:

- μH & $\mu\,^3$He$^+$ (approved)

CREMA @ PSI

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- μH & $\mu\,^3$He$^+$ (approved)

FAMU @ RIKEN-RAL / J-PARC

- HFS in μH in two new methods (planned)

Precise R_c/R_m from μA LS/HFS

Require accurate theoretical inputs from QED, hadron and nuclear physics

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}\left(R_c\right) + \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}$$

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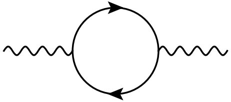
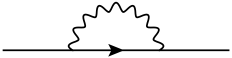
- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects
- Theory of μ-p, D, ^{3,4}He$^+$ reexamined

  Martynenko *et al.* **'07**, Borie **'12**, Krutov *et al.* **'15**

  Karshenboim *et al.* **'15**, Krauth *et al.* **'15** ...





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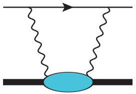
$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}(R_c) + \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}$$

- Nuclear finite-size corrections (elastic):
  - leading term (OPE): $\delta_{size} = \frac{m_r^3}{12}(Z\alpha)^4 \times R_c^2$
  - Zemach/Friar term (TPE): $\delta_{\mathrm{Zem}} = -\frac{m_r^4}{24}(Z\alpha)^5 \times \langle r^3 \rangle_{(2)} \quad \propto R_c^3$
    - can be calculated from g.s. charge distribution,
      Friar **'79**, Borie **'12('14)**, Krutov *et al.* **'15**
    - extracted from experimental form factors,
      Sick **'14**
    - or avoided due to cancellations with δ_{pol}
      Pachucki '11 & Friar '13 (μD)

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}\,(R_c) + \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}$$

- Nuclear polarization corrections (inelastic TPE):
  - least well-known
  - related to nuclear response functions:
    $S_O(\omega) = \displaystyle\sum\!\!\!\!\!\!\int_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$
  - can be calculated (continuum few-body problem)
  - or extracted from data (very imprecise)



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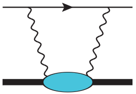
  - can be calculated (continuum few-body problem)
  - or extracted from data (very imprecise)
  - sometimes rewritten as:
    $\boldsymbol{\delta_{\mathrm{TPE}} \equiv \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}}$



The accuracy of R_c is limited by δ_{TPE}

Example — μD:

$$\begin{aligned}
\Delta E^{\mathrm{LS}}_{\mathrm{QED}} &= 228.77356(75)\,\mathrm{meV} \\
\Delta E^{\mathrm{LS}}_{\mathrm{rad.-dep.}} &= -6.11025(28)\, r_{\mathrm{d}}^2\,\mathrm{meV/fm}^2 + 0.00300(60)\,\mathrm{meV} \\
\Delta E^{\mathrm{LS}}_{\mathrm{TPE}} &= 1.70910(2000)\,\mathrm{meV}
\end{aligned}$$

J. Krauth *et al.* (CREMA), **Ann. Phys. (2016)**; R. Pohl *et al.* (CREMA), **Science 2016**

Status — prior to $\mu^{3,4}$He$^+$ measurements:

- Uncertainty in δ_{pol}: $\sim 20\%$
- Required: $\sim 5\%$
  (to determine R_c with $\sim 10^{-4}$ accuracy)

We have performed the first *ab-initio* calculation of δ_{Zem} and δ_{pol} for $A = 3, 4$

we used state-of-the-art nuclear forces

- **AV18+UIX**
- **χEFT: N3LO (Entem & Machleidt) + N2LO (Navrátil)**

$\Longrightarrow$ estimate nuclear physics uncertainty

we employ established few-body methods

- **EIHH**: Effective interaction Hyperspherical Harmonics **(bound method)**
- **LIT**: Lorentz Integral Transform **(continuum method)**
- **LSR**: **A new method** based on the Lanczos algorithm
  NND *et al.*, **Phys. Rev. C (2014)**

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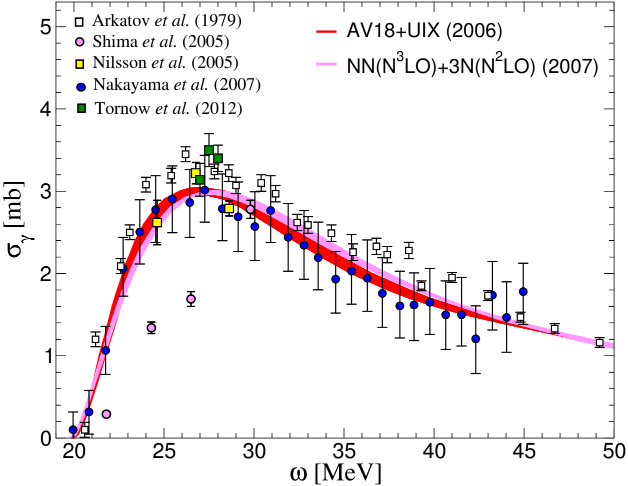
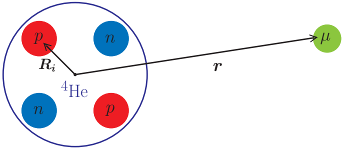
electric dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{D1}(\omega)$



- Hamiltonian for muonic atoms

$$H = H_{nucl} + H_\mu + \Delta H$$
$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



- Corrections to the point Coulomb from protons

$$\Delta H = \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$

- Evaluate inelastic effects of ΔH on muonic spectrum in 2^{nd}-order perturbation theory

$$\delta_{\rm pol} = \sum_{N \neq N_0, \mu} \langle N_0 \mu_0 | \Delta H | N \mu \rangle \frac{1}{E_{N_0} - E_N + \epsilon_{\mu 0} - \epsilon_\mu} \langle N \mu | \Delta H | \mu_0 N_0 \rangle$$

$|\mu_0\rangle$: muon wave function for $2S/2P$ state

Systematic contributions to nuclear polarization

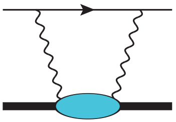
δ_{NR} **Non-Relativistic** limit

$\delta_L + \delta_T$ **L**ongitudinal and **T**ransverse **relativistic** corrections

δ_C **Coulomb** distortions

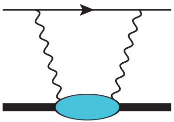
δ_{NS} Corrections from **finite Nucleon Size**

- **Neglect Coulomb interactions in the intermediate state**



Non-relativistic limit

- **Neglect Coulomb interactions in the intermediate state**
- **Expand muon matrix element in powers of**
  $\eta \equiv \sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$



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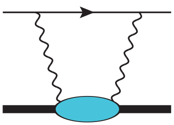
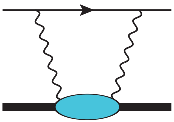


- **Neglect Coulomb interactions in the intermediate state**
- **Expand muon matrix element in powers of** $\eta \equiv \sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$
  - $|\boldsymbol{R}-\boldsymbol{R}'| \Longrightarrow$ "virtual" distance the proton travels in 2γ exchange
  - uncertainty principal $|\boldsymbol{R}-\boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
  - $\eta \sim \sqrt{\frac{m_r}{m_N}} \approx 0.3$

Non-relativistic limit



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$$\boxed{P_{NR}(\omega,\boldsymbol{R},\boldsymbol{R}') \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]}$$

$$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)} \quad \sim \quad \eta^2 \quad + \quad \eta^3 \quad + \quad \eta^4$$

NR limit at LO: $\boldsymbol{\delta_{NR}^{(0)}}$

$$\delta_{NR} = \boldsymbol{\delta_{NR}^{(0)}} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$$

- $\delta_{NR}^{(0)} \propto \eta^2$

$$\delta_{D1}^{(0)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

- $S_{D_1}(\omega) \Longrightarrow$ electric dipole response function $[\ \hat{D}_1 = RY_1(\hat{R})\ ]$
- $\delta_{D1}^{(0)}$ is the dominant contribution to $\delta_{\rm pol}$
- $\Longrightarrow$ Rel. and Coulomb corrections added at this order

NR limit at NLO: $\boldsymbol{\delta}_{NR}^{(1)}$

$$\delta_{NR} = \delta_{NR}^{(0)} + \boldsymbol{\delta_{NR}^{(1)}} + \delta_{NR}^{(2)}$$

- $\delta_{NR}^{(1)} \propto \eta^3$

$$\boxed{\delta_{NR}^{(1)} = \delta_{R3pp}^{(1)} + \delta_{Z3}^{(1)}}$$

$$\delta_{R3pp}^{(1)} = -\frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R} d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \langle N_0 | \hat{\rho}^\dagger(\boldsymbol{R}) \hat{\rho}(\boldsymbol{R}') | N_0 \rangle$$

$$\delta_{Z3}^{(1)} = \frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R} d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0(\boldsymbol{R}) \rho_0(\boldsymbol{R}')$$

- $\delta_{R3pp}^{(1)} \Rightarrow$ 3rd-order proton-proton correlation
- $\delta_{Z3}^{(1)} \Longrightarrow$ 3rd Zemach moment

NR limit at NLO: $\boldsymbol{\delta}_{NR}^{(1)}$

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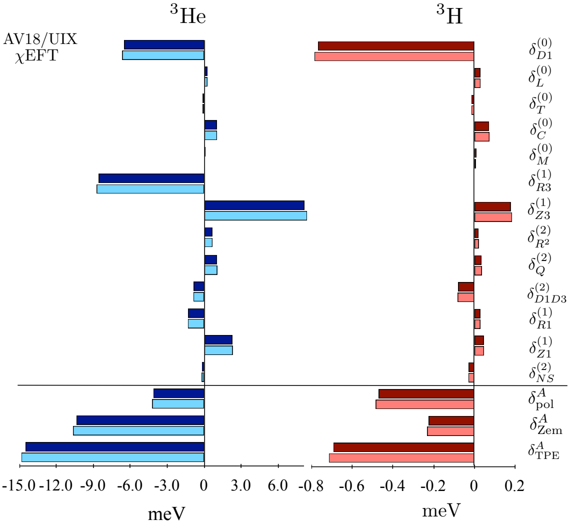
- $\delta_{R3pp}^{(1)} \Rightarrow$ 3rd-order proton-proton correlation
- $\delta_{Z3}^{(1)} \Longrightarrow$ 3rd Zemach moment

  cancels *elastic* Zemach moment of finite-size corrections

  c.f. Pachucki '11 & Friar '13 (μD) $\Longrightarrow \boldsymbol{\delta_{\mathrm{TPE}}} \equiv |\delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}|$

Results: $A = 3$



NND *et al.*, **Phys. Lett. B (2016)**

Error type	$\mu\,^3\mathrm{He}^+$ $\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\mu\,^3\mathrm{H}$ $\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.4	0.1	0.1	0.1	0.0	0.1
Nuclear model	1.5	1.8	1.7	2.2	2.3	2.2
ISB	2.0	0.2	0.5	0.9	0.2	0.6
Nucleon size	1.6	1.5	0.6	0.6	1.3	0.0
Relativistic	0.6	-	1.5	1.4	-	0.3
Coulomb	1.2	-	0.3	0.3	-	0.2
Multipole expansion	2.0	-	0.6	2.0	-	1.4
Higher $Z\alpha$	1.5	-	0.4	0.7	-	0.5
Magnetic MEC	0.4	-	0.1	0.3	-	0.2
Total	4.1%	2.3%	2.5%	3.6%	2.7%	2.7%

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Two-photon exchange correction to $2S$-$2P$ splitting in muonic ^{3}He ions

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(Received 2 December 2016; published 27 January 2017)

We calculate the two-photon exchange correction to the Lamb shift in muonic ^{3}He ions within the dispersion relations framework. Part of the effort entailed making analytic fits to the electron-^{3}He quasielastic scattering data set, for purposes of doing the dispersion integrals. Our result is that the energy of the $2S$ state is shifted downwards by two-photon exchange effects by 15.14(49) meV, in good accord with the result obtained from a potential model and effective field theory calculation.

TABLE I. Individual contributions to ΔE_{2S} from two-photon exchange in μ-^{3}He, in units of meV.

Contribution	This work	Refs. [21,22]
Elastic	−10.93(27)	−10.49(24)
δ^N_{Zem}		−0.52(3)
Inelastic	−5.81(40)	−4.45(21)
Nuclear	−5.50(40)	−4.17(17)
Nucleon	−0.31(2)	−0.28(12)
Subtraction	1.60(12)	
Nuclear	1.39(12)	
Nucleon	0.21(3)	
Total TPE	−15.14(49)	−15.46(39)

Summary of results: $2 \leqslant A \leqslant 4$

System	Our Ref.	Unc.	Experimental Status
$\mu\,{}^{2}\mathrm{H}$	Phys. Lett. B '14	$1\% \to 1.3\%$	published *Science* '16
$\mu\,{}^{4}\mathrm{He}^{+}$	Phys. Rev. Lett. '13	$20\% \to 6\%$	measured, unpublished
$\mu\,{}^{3}\mathrm{He}^{+}$	Phys. Lett. B '16	$20\% \to 4\%$	measured, unpublished
$\mu\,{}^{3}\mathrm{H}$	Phys. Lett. B '16	4%	measurable?

- Our results agree with other values and are more accurate

$\Rightarrow$ Unc. comparable with $\sim 5\%$ experimental needs
$\Rightarrow$ Will improve precision of R_c from Lamb shifts
$\Rightarrow$ May help shed light on the "proton (deuteron) radius puzzle"

The work is not completed yet ...

Error type	$\mu\,^3\mathrm{He}^+$ δ^A_{pol}	δ^A_{Zem}	δ^A_{TPE}	$\mu\,^3\mathrm{H}$ δ^A_{pol}	δ^A_{Zem}	δ^A_{TPE}
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Finite nucleon-size corrections in coordinate-space

1. Replace point-nucleon limit

$$\Delta H = \frac{Z\alpha}{r} - \alpha \sum_{i}^{Z} \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|}$$

Finite nucleon-size corrections in coordinate-space

1. Replace point-nucleon limit with a convolution of nucleon charge densities

$$\Delta H = \frac{Z\alpha}{r} - \alpha \sum_i^Z \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \Longrightarrow \frac{Z\alpha}{r} - \alpha \sum_i^A \int d\boldsymbol{R}' \, \frac{n_i(\boldsymbol{R}' - \boldsymbol{R}_i)}{|\boldsymbol{r} - \boldsymbol{R}'|}$$

2. Fourier transform to their electric form-factors in **the low-q^2 approximation**

$$G^E_{p/n}(q^2) \simeq G^E_{p/n}(0) - \frac{\langle r^2_{\mathrm{ch}} \rangle_{p/n}}{6} q^2$$

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$$G^E_{p/n}(q^2) \simeq G^E_{p/n}(0) - \frac{\langle r^2_{\rm ch}\rangle_{p/n}}{6} q^2$$

3. Then substitute

$$q^2 \to \nabla^2_R$$

- In momentum-space

$$\langle r^3 \rangle_{(2)} = \frac{48}{\pi} \int_0^\infty \frac{dq}{q^4} \left[F_E^2(q^2) - 1 + \frac{(qR)^2}{3} \right]$$

- In coordinate-space

$$\langle r^3 \rangle_{(2)} = \iint d\boldsymbol{R} d\boldsymbol{R}' \rho_0(\boldsymbol{R}) \rho_0(\boldsymbol{R}') |\boldsymbol{R} - \boldsymbol{R}'|^3$$

Calculating Zemach moments

- In momentum-space

$$\langle r^3 \rangle_{(2)} = \frac{48}{\pi} \int_0^\infty \frac{dq}{q^4} [F_E^2(q^2) - 1 + \frac{(qR)^2}{3}]$$

- In coordinate-space (Hybrid method)

$$\langle r^3 \rangle_{(2)} = \iint d\boldsymbol{R} d\boldsymbol{R}' \rho_0(\boldsymbol{R}) \rho_0(\boldsymbol{R}') |\boldsymbol{R} - \boldsymbol{R}'|^3 = \frac{2}{\pi} \int dq\ q\ F_E^2(q^2) \int dR sin(qR) |R|^4$$

where in practice

$$\rho_0(\boldsymbol{R}) = \rho_{\text{ch}}^{\text{point}}(\boldsymbol{R}) = \int \frac{d\boldsymbol{q}}{2\pi^3} e^{i\boldsymbol{q}\cdot\boldsymbol{R}} F_E^{\text{point}}(q^2)$$

- In momentum-space

$$\langle r^3\rangle_{(2)} = \frac{48}{\pi}\int_0^\infty \frac{dq}{q^4}\left[F_E^2(q^2) - 1 + \frac{(qR)^2}{3}\right]$$

- In coordinate-space (Hybrid method)

$$\langle r^3\rangle_{(2)} = \iint d\boldsymbol{R}d\boldsymbol{R}'\rho_0(\boldsymbol{R})\rho_0(\boldsymbol{R}')|\boldsymbol{R}-\boldsymbol{R}'|^3 = \frac{2}{\pi}\int dq\ q\ F_E^2(q^2)\int dRsin(qR)|R|^4$$

where in practice

$$\rho_0(\boldsymbol{R}) = \rho_{\rm ch}^{\rm point}(\boldsymbol{R}) = \int \frac{d\boldsymbol{q}}{2\pi^3}e^{i\boldsymbol{q}\cdot\boldsymbol{R}}F_E^{\rm point}(q^2)\ \text{but}\ F_E^{\rm full}(q^2) = \frac{1}{Z}\sum_{i=p,n}F_i^{point}(q^2)\cdot G_i^E(q^2)$$

Calculating Zemach moments - Results

Using AV18+UIX Nuclear interaction and Kelly parameterization for the nucleon form-factors:
(brackets show only uncertainties due to the method and K_{max} convergence)

Nucleons	Method	^{3}He	^{3}H
Low-q^2	momentum	-	-
Low-q^2	coordinate	27.18(5)	18.86(0)
Full	momentum	27.58(15)	19.27(9)
Full	coordinate	27.65(5)	19.30(0)
Exp. [Sick '14]	SOG	28.15(70)	-

Calculating Zemach moments - ^{3}He Benchmark

Using AV18+UIX Nuclear interaction and Kelly parameterization for the nucleon form-factors:
(Calculations in non-rel. impulse approximation, i.e., w/o MECs, Darwin-Foldy, etc.
Values in fm^n)

Ab-initio Method	Algorithm	$R_{\rm ch}$	$\langle R^3 \rangle_{(2)}$	$\langle R^4 \rangle$
GFMC	mom.	1.954(3)	27.90(20)	35.1(4)
HH-momentum	mom.	1.953(1)	27.56(20)	32.5(1.3)
EIHH (coor.)	mom.	1.953(7)	27.58(15)	33.8(5)
	coor.	1.953(3)	27.65(05)	34.1(2)
Exp. [Sick '14]	SOG	1.973(14)	28.15(70)	32.9(1.60)

In collaboration with Saori Pastore, Maria Piarulli, and Bob Wiringa

Questions Remain

1. Can we improve the multipole (η) expansion of $\delta_{\rm pol}$?
2. Can we improve the nucleon-size treatment of $\delta_{\rm pol}$?

Questions Remain

1. Can we improve the multipole (η) expansion of δ_{pol}?
2. Can we improve the nucleon-size treatment of δ_{pol}?

- Answer(s):

  Yes!

1. There is an alternative (η-less) derivation which leads to a new multipole expansion.
2. This derivation allows including the full form-factors.

Non-Rel. point-nucleon case

$$\delta_{\text{pol}} = c_0 \sum_\ell \int dq I_\ell(q) \qquad \text{(integrated numerically)}$$

$$I_\ell(q) = \int d\omega \frac{R_\ell(q,\omega)}{q^2(q^2+2m_r\omega)}$$

$$R_\ell(q,\omega) = \sum_{N\neq N_0} \left|\langle N_0||\hat{J}_\ell(q\boldsymbol{R})||N\rangle\right| \delta(\omega-\omega_N)$$

$$\hat{J}_\ell(q\boldsymbol{R}) = \sum_i^Z j_\ell(qR) Y_\ell(\hat{R})$$

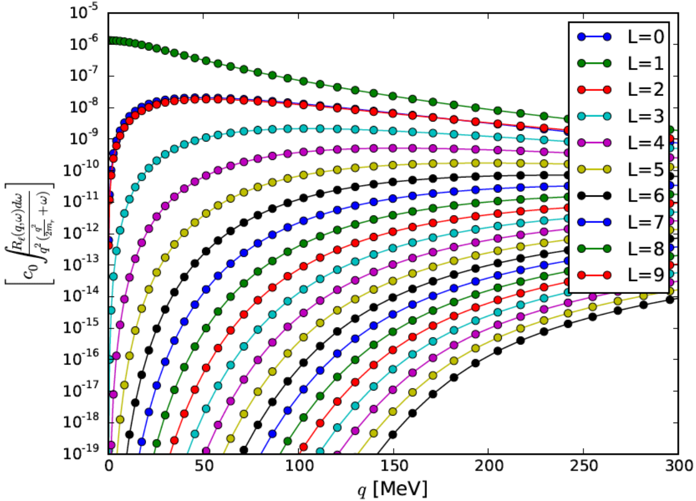
Including nucleon form-factors

$$R_\ell \Longrightarrow \sum_i R_\ell^i \cdot G_i^E$$

$$\hat{J}_\ell \Longrightarrow \sum_i \hat{J}_\ell^i$$

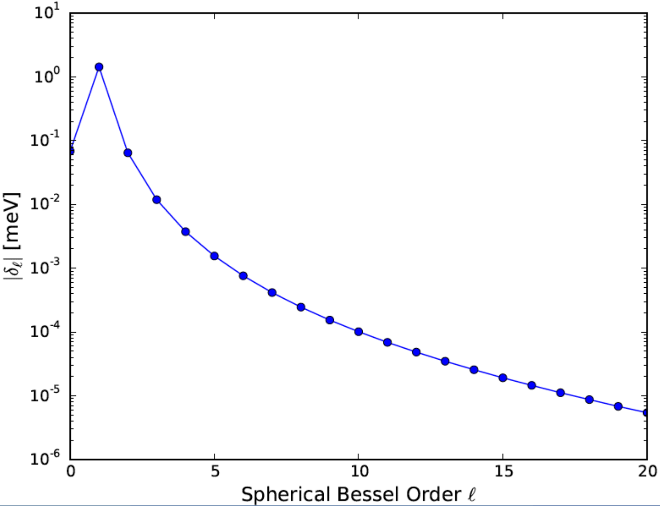
$$i = p/n$$

$I_\ell(q)$ (μD)

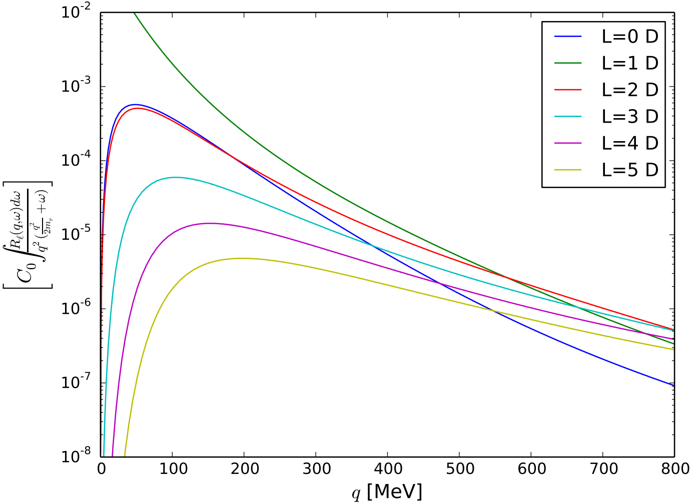


L=0
L=1
L=2
L=3
L=4
L=5
L=6
L=7
L=8
L=9
$\left[c_0 \int \frac{R_\ell(q,\omega)d\omega}{q^2(\frac{q^2}{2m_r}+\omega)}\right]$
q [MeV]
0
50
100
150
200
250
300
10-5
10-6
10-7
10-8
10-9
10-10
10-11
10-12
10-13
10-14
10-15
10-16
10-17
10-18
10-19

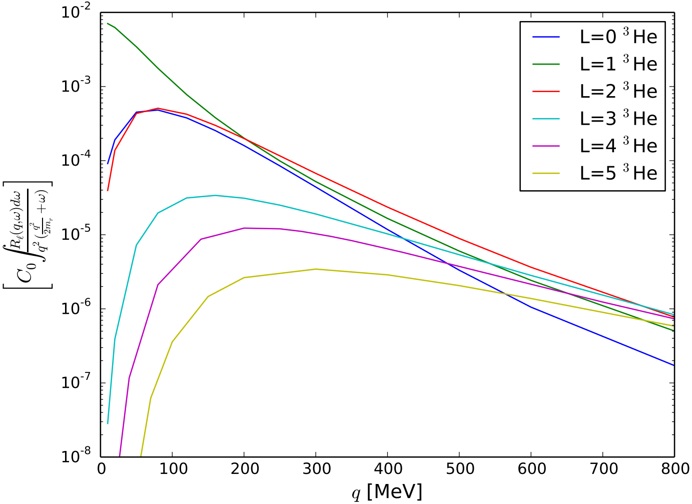
δ_ℓ (μD)



$I_\ell(q)$ (μD)


L=0 D
L=1 D
L=2 D
L=3 D
L=4 D
L=5 D
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
100
200
300
400
500
600
700
800
q [MeV]
$\left[C_0 \int \frac{R_\ell(q,\omega)d\omega}{q^2(\frac{q^2}{2m_r}+\omega)}\right]$

$I_\ell(q)$ $(\mu^3\text{He}^+)$



- We will continue to improve the nuclear corrections, which are the bottleneck for LS measured in μA, $2 \leqslant A \leqslant 4$ by
  - Improving the treatment of nucleon-sizes in $\delta_{\rm Zem}$ and $\delta_{\rm pol}$
  - Using the η-less multipole expansion
  - Including ISB, MECs, etc.

- and in the future
  - Quantify & reduce nuclear uncertainty
    See Javier Hernandez's talk at the next session for $A = 2$
  - Use novel nuclear potentials
  - Extend to LS in $A \geqslant 6$, HFS in $A = 3$, ...

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Thank you!
Merci!