Deuteron electrodisintegration with low-resolution potentials

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TRIUMF ab initio workshop 2017

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Supported in part by NSF, DOE, SciDAC-3 NUCLEI project
Basic Problem

- **Goal**: Extract nuclear properties from experiments and predict them with theory

\[ \frac{d\sigma}{d\Omega} \propto \left| \langle \psi_{\text{final}} | \hat{O}(q) | \psi_{\text{initial}} \rangle \right|^2 \]

Use factorization to isolate individual components and extract process-independent nuclear properties

Nucleon knockout reaction

\[ d(e,e')p \text{ with low-resolution potentials} \]
Basic Problem

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  \]

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\[ d(e,e')p \text{ with low-resolution potentials} \]
Separation between long- and short-distance physics is not unique, but defined by the scale $\mu_f$

Form factor $F_2$ is independent of $\mu_f$, but pieces are not

$f_a(x, \mu_f^2 = Q^2)$ runs with $Q^2$ but is process independent
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When does factorization hold?

Which process-independent nuclear properties can we extract?

What is the scale/scheme dependence of the extracted properties?
SRG makes scale dependence obvious

- Unitary transformations widely used to soften nuclear Hamiltonians leading to accelerated convergence
- SRG scale $\lambda$ sets the scale for decoupling high- and low-momentum and separating structure and reaction

\[ \psi_d(k) \quad [\text{fm}^{3/2}] \]

$\lambda = \infty$  
$\lambda = 2.0 \quad \text{fm}^{-1}$  
$\lambda = 1.5 \quad \text{fm}^{-1}$  
AV18  
$L = 2$

\[ \sigma \sim \left| \left\langle \psi_f | \hat{O} | \psi_i \right\rangle \right|^2 \Rightarrow \hat{O} \text{ must change to keep observables invariant} \]

UV physics absorbed in operator (cf. Chiral EFTs)

\[ p < \lambda \quad p > \lambda \]
SRG makes scale dependence obvious

- Unitary transformations widely used to soften nuclear Hamiltonians leading to accelerated convergence
- SRG scale $\lambda$ sets the scale for decoupling high- and low-momentum and separating structure and reaction
- Transformed wave function $\rightarrow$ no high momentum components
- $\sigma \sim |\langle \psi_f | \hat{O} | \psi_i \rangle|^2 \Rightarrow \hat{O}$ must change to keep observables invariant
- UV physics absorbed in operator (cf. Chiral EFTs)
Use deuteron electrodisintegration to investigate scale/scheme dependence of factorization between nuclear structure and nuclear reaction

\[
\frac{d\sigma}{d\Omega} \propto (v_L f_L + v_T f_T + v_{TT} f_{TT} \cos 2\phi_p + v_{LT} f_{LT} \cos \phi_p)
\]

\(v_L, v_T, \ldots\) - electron kinematic factors. \(f_L, f_T, \ldots\) - deuteron structure functions
Test ground: $^2\text{H}(e, e' \ p)n$

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- \( v_L, v_T, \ldots \) - electron kinematic factors. \( f_L, f_T, \ldots \) - deuteron structure functions

- Components depend on the scale \( \lambda \). Cross section does not!
Evolutionary effects

$^2\text{H}(e, e' \ p) n$ calculations done using AV18 potential with $\lambda = \infty$ and $\lambda = 1.5 \ \text{fm}^{-1}$

$$f_L \sim \sum_{m_s, m_J} |\langle \psi_f | J_0 | \psi_i \rangle|^2$$

Effects due to evolution of one or more components of $\langle \psi_f | J_0 | \psi_i \rangle$ as a function of kinematics $\rightarrow$ scale dependence of factorization

Proof of principle calculation using simplified $J_0$. Comparison to experiment not warranted
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Quasi-free ridge (QFR): \(\omega_{\text{photon}} = 0\).
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Quasi-free ridge (QFR): \(\omega_{\text{photon}} = 0\)

Weak scale dependence at QFR which gets progressively stronger away from it

SNM et al., PRC 92, 064002 (2015)
Results at QFR

- At the quasi-free ridge
  \[ E'(\text{in MeV}) \approx 10 q^2 (\text{in fm}^{-2}) \]

- \[ f_L \sim \sum_{m_s, m_J} |\langle \psi_f | J_0 | \psi_i \rangle|^2 \]

- Long-range part of the wave function probed at QFR \( \rightarrow \) invariant under SRG evolution

\[ E' = 100 \text{ MeV} \quad q^2 = 10 \text{ fm}^{-2} \]

\[ \langle \psi_f | J_0 | \psi_i \rangle \]
Results at QFR

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- Long-range part of the wave function probed at QFR → invariant under SRG evolution

\[ E' = 10 \text{ MeV} \quad q^2 = 4 \text{ fm}^{-2} \]

\[ E' = 100 \text{ MeV} \quad q^2 = 10 \text{ fm}^{-2} \]

\[ \langle \psi_f | J_0 | \psi_i \rangle \]

\[ \langle \psi_f | J_0 | \psi_\lambda \rangle \]

\[ \langle \psi_\lambda | J_0 | \psi_i \rangle \]

\[ \langle \psi_f | J_\lambda | \psi_i \rangle \]
Results below QFR

\[ \langle \psi_f | J_0 | \psi_i \rangle = \langle \phi | J_0 | \psi_i \rangle + \langle \phi | t^\dagger G_0^\dagger J_0 | \psi_i \rangle \]

IA

FSI

Below QFR two terms add constructively

Wave function in IA probed between 1.7 and 3.4 fm\(^{-1}\) \(\Rightarrow\) \(|\langle \psi_f | J_0 | \psi^\lambda_i \rangle| < |\langle \psi_f | J_0 | \psi_i \rangle|\)

\[ E' = 30 \text{ MeV} \quad q^2 = 25 \text{ fm}^{-2} \]

\[ f_L [\text{fm}] \]

\[ \theta [\text{deg}] \]

\[ \lambda = \infty \quad \lambda = 1.5 \text{ fm} \]

AV18

\[ L = 2 \]

\[ \psi_d(k) [\text{fm}^{3/2}] \]

\[ k [\text{fm}^{-1}] \]

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d(e,e')p with low-resolution potentials
Results below QFR

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\[ \langle \psi_f^\lambda | J_0 | \psi_i \rangle = \langle \phi | J_0 | \psi_i \rangle + \langle \phi | t^\dagger \lambda G_0^\dagger J_0 | \psi_i \rangle \]

\[ |\langle \phi | t^\dagger \lambda G_0^\dagger J_0 | \psi_i \rangle| < |\langle \phi | t^\dagger G_0^\dagger J_0 | \psi_i \rangle| \]

\[ E' = 30 \text{ MeV} \quad q^2 = 25 \text{ fm}^{-2} \]

\[ \langle \psi_f | J_0 | \psi_i \rangle \quad \langle \psi_f | J_0 | \psi_i^\lambda \rangle \quad \langle \psi_f^\lambda | J_0 | \psi_i \rangle \]

\[ \lambda = \infty \quad \lambda = 1.5 \quad \text{AV18} \]

\[ L = 2 \]

\[ \Psi_d(k) \quad k \left[ \text{fm}^{-1} \right] \]

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d(e, e')p with low-resolution potentials
Results below QFR

\[
\langle \psi_f | J_0 | \psi_i \rangle = \langle \phi | J_0 | \psi_i \rangle + \langle \phi | t^{\dagger} G_0^{\dagger} J_0 | \psi_i \rangle
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\[
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\]

Effect of evolution of current opposite to the evolution of initial and final states

\[E' = 30 \text{ MeV} \quad q^2 = 25 \text{ fm}^{-2}\]

\(f_L \text{ [fm]}\)

\(\theta [\text{deg}]\)

\(\lambda = \infty\)

\(\lambda = 1.5\)

AV18

L = 2

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\(d(e,e')p\) with low-resolution potentials
Results below QFR

\[ \langle \psi_f | J_0 | \psi_i \rangle = \langle \phi | J_0 | \psi_i \rangle + \langle \phi | t^\dagger G_0^\dagger J_0 | \psi_i \rangle \]

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Wave function in IA probed between 1.7 and 3.4 fm\(^{-1} \Rightarrow |\langle \psi_f | J_0 | \psi_i^\lambda \rangle| < |\langle \psi_f | J_0 | \psi_i \rangle|\]

\[ \langle \psi_f^\lambda | J_0 | \psi_i \rangle = \langle \phi | J_0 | \psi_i \rangle + \langle \phi | t^\dagger \lambda G_0^\dagger J_0 | \psi_i \rangle \]

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Effect of evolution of current opposite to the evolution of initial and final states
Results above QFR

- Scale dependence qualitatively different above the quasi-free ridge

\[ \langle \psi_f | J_0 | \psi_i \rangle = \langle \phi | J_0 | \psi_i \rangle + \langle \phi | t^\dagger G^\dagger J_0 | \psi_i \rangle \]  

- Above QFR two terms add destructively

\[ E' = 100 \text{ MeV} \quad q^2 = 0.5 \text{ fm}^{-2} \]
Results above QFR

- Scale dependence qualitatively different above the quasi-free ridge
  \[ \langle \psi_f | J_0 | \psi_i \rangle = \langle \phi | J_0 | \psi_i \rangle + \langle \phi | t^\dagger G_0^\dagger J_0 | \psi_i \rangle \]
  
  1. \[ \text{IA} \]
  2. \[ \text{FSI} \]

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- Can be explained by looking at the effect of evolution on the overlap matrix elements
  [SNM et al., PRC 92, 064002 (2015)]
Results above QFR

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- Above QFR two terms add destructively

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- Scale dependence depends on the kinematics, but in a systematic way
Evolution effects on individual components

\[ f_L \propto \sum_{m_s,m_J} \langle \psi_f | J_0 | \psi_i \rangle = \sum_{m_s,m_J} \langle \psi_f^\lambda | J_0^\lambda | \psi_i^\lambda \rangle \]

- Looked at effects of evolution on the observable \( f_L \)
- Look at changes due to evolution for individual components and their implications
- Evolution of \( \psi_{\text{deut}} \): suppression of high-momentum components \( \rightarrow \) accelerated convergence of nuclear structure calculations
Evolving the final state

\[ |\Delta \psi(k)| \]

\[ 3S_1 \]

\[ p' = 0.85 \text{ fm}^{-1} \]
\[ E' \approx 30 \text{ MeV} \]

\[ \psi_f^\lambda(p'; k) = \phi_{p'} + \Delta \psi(\lambda(p'; k)) \]

\[ \text{IA} \quad \text{FSI} \]

\[ \text{For } p' \gtrsim \lambda, \psi_f^\lambda(p'; k) \text{ localized around the outgoing momentum } p' \]
Evolution minimizes FSI contribution

Local decoupling $\Rightarrow$ increased validity of IA
Evolution minimizes FSI contribution

- Local decoupling $\Rightarrow$ increased validity of IA

$$f_L(\langle \psi_f | J_0 | \psi_i \rangle) \approx f_L(\langle \phi | J_0^\lambda | \psi_i^\lambda \rangle)$$

$E' = 100$ MeV $q^2 = 36$ fm$^{-2}$

$E' = 120$ MeV $q^2 = 49$ fm$^{-2}$
Varying $\lambda$ shuffles the physics between short- and long-distance parts

$\lambda$ decreases $\rightarrow$ blob size increases.

One-body current operator develops two and higher body components

\[ \langle k_1 T_1 | J_0(q) | k_2 T = 0 \rangle = \]
\[ \frac{1}{2} \left( G_E^p + (-1)^T G_E^n \right) \delta(k_1 - k_2 - q/2) + \frac{1}{2} \left( (-1)^T G_E^p + G_E^n \right) \delta(k_1 - k_2 + q/2) \]

Naive expectation: RG changes to $J_0(q)$ complicates reaction calculations

Sushant More  d(e,e')p with low-resolution potentials
EFT for the current

- $\langle \psi_f | J_0(q) | \psi_i \rangle = \langle \psi^\lambda_f | J^\lambda_0(q) | \psi^\lambda_i \rangle$

- Low-momentum component of $J^\lambda_0(q)$ most relevant
EFT for the current

- $\langle \psi_f | J_0(q) | \psi_i \rangle = \langle \psi_f^\lambda | J_0^\lambda(q) | \psi_i^\lambda \rangle$

- Low-momentum component of $J_0^\lambda(q)$ most relevant.

\[ \langle \bar{3}S_1; k_1 | J_0^{\lambda=\infty} | 3S_1; k_2 \rangle \ q^2 = 36 \text{ fm}^{-2} \]

\[ \langle \bar{3}S_1; k_1 | J_0^{\lambda=3.0} | 3S_1; k_2 \rangle \ q^2 = 36 \text{ fm}^{-2} \]
EFT for the current

- \[ \langle \psi_f | J_0(q) | \psi_i \rangle = \langle \psi_f^{\lambda} | J_0^{\lambda}(q) | \psi_i^{\lambda} \rangle \]

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- $\langle \psi_f | J_0(q) | \psi_i \rangle = \langle \psi_f^\lambda | J_0^\lambda(q) | \psi_i^\lambda \rangle$
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Low-momentum component of \( J_0^\lambda(q) \) most relevant

\[ \langle {}^3S_1; k_1 | J_0^\lambda(q) | {}^3S_1; k_2 \rangle = g_0^q + g_2^q(k_1^2 + k_2^2) + \cdots \]

\[ \langle {}^3S_1; k_1 | J_0^{1.5}(q) | {}^3S_1; k_2 \rangle q^2 = 36 \text{ fm}^{-2} \]
EFT for the current

\[ \langle \tilde{P}^3_1; k_1 | J^\lambda_0 (q) | \tilde{S}^3_1; k_2 \rangle = g^q_1 k_1 + \cdots \]
\[ \langle \tilde{D}^3_2; k_1 | J^\lambda_0 (q) | \tilde{S}^3_1; k_2 \rangle = g^{q,D}_2 k_1^2 + \cdots \]
\[ \langle \psi^\lambda_f | J^\lambda_0 (q) | \psi^\lambda_i \rangle \approx \langle \psi^\lambda_f | J^\lambda_0 (q) | \psi^\lambda_i \rangle \tilde{S}^3_1 \]

\[ \langle \psi^\lambda_f | J^\lambda_0 | \psi^\lambda_i \rangle \tilde{S}^3_1 \]

\[ \langle \psi^\lambda_f | J^\lambda_0 | \psi^\lambda_i \rangle \tilde{S}^3_1 = \langle \psi^\lambda_f | \tilde{S}^3_1 \rangle \langle \tilde{S}^3_1 | J^\lambda_0 \rangle \langle \tilde{S}^3_1 | \psi^\lambda_i \rangle + \langle \psi^\lambda_f | \tilde{P}^3_1 \rangle \langle \tilde{P}^3_1 | J^\lambda_0 \rangle \langle \tilde{S}^3_1 | \psi^\lambda_i \rangle + \cdots \]

\[ \langle \tilde{P}^3_1; k_1 | J^\lambda_0=1.5 | \tilde{S}^3_1; k_2 \rangle q^2 = 36 \text{fm}^{-2} \]
Results from low-momentum potential

\[
\langle \psi_f^\lambda | J_0^\lambda (q) | \psi_{\text{deut}}^\lambda \rangle \\
= g_0^q \psi_f^\lambda(0) \psi_{\text{deut}}^\lambda(0) + \cdots
\]
\[
\langle \psi_f^\lambda | J_0^\lambda (q) | \psi_{\text{deut}}^\lambda \rangle = g_0^q \psi_f^{\lambda \ast} (r) \psi_{\text{deut}}^{\lambda} (r) \bigg|_{r=0} + \cdots
\]

Results from low-momentum potential

\[ E' = 20 \text{ MeV} \quad q^2 = 36 \text{ fm}^{-2} \]

- \( \langle \psi_f | J_0 | \psi_i \rangle \)
- EFT: S (up to \( k^4 \))
- EFT: S + P

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d(e,e')p with low-resolution potentials
Results from low-momentum potential

\[ \langle \psi_f^\lambda | J_0^\lambda (q) | \psi_{\text{deut}}^\lambda \rangle = g^q_0 \psi_f^\lambda (r)^* \psi_{\text{deut}}^\lambda (r) \bigg|_{r=0} + \cdots \]

- \( f_L \) from EFT \( \approx f_L \) exact
- Agreement made better by going to higher order terms in EFT expansion

\[ E' = 20 \text{ MeV} \quad q^2 = 36 \text{ fm}^{-2} \]
Convergence in partial wave channels

\[ E' = 20 \text{ MeV} \quad q^2 = 36 \text{ fm}^{-2} \]

\[ \langle \psi_f^\lambda | J_0^\lambda | \psi_i^\lambda \rangle_{l_{\text{max}}=0} \]

EFT: S (LO contact)

\[ \langle \psi_f^\lambda | J_0^\lambda | \psi_i^\lambda \rangle_{l_{\text{max}}=1} \equiv \langle \psi_f^\lambda ; 3S_1 | J_0^\lambda \text{ exact} | \psi_i^\lambda ; 3S_1 \rangle \]
Convergence in partial wave channels

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EFT: S (up to \( k^2 \))

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- EFT: S (up to \( k^4 \))

\[ \langle \psi^\lambda_j | J^\lambda_0 | \psi^\lambda_i \rangle_{l_{\text{max}}=0} \equiv \langle \psi^\lambda_j ; {}^3 S_1 | J_0^\lambda_{\text{exact}} | \psi^\lambda_i ; {}^3 S_1 \rangle \]

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Convergence in partial wave channels

\[ E' = 20 \text{ MeV} \quad q^2 = 36 \text{ fm}^{-2} \]

\[ \langle \psi^\lambda_f | J_0^\lambda | \psi_i^\lambda \rangle_{l_{\text{max}}=0} \]

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EFT: S (up to \( k^4 \))

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\[ \langle 3 P_i ; k_1 | J_0^\lambda_{\text{EFT}} | 3 S_1 ; k_2 \rangle_{\text{LO}} \equiv g_{P_i} k_1 \]
\(q\)-factorization of \(f_L\)

- \(f_L \equiv f_L(p', \theta; q)\)
  - \(p'\) and \(\theta\): outgoing nucleon
  - \(q\): momentum transfer

- For \(p' \ll q\), \(f_L\) scales with \(q\)
  \(f_L(p', \theta; q) \rightarrow g(p', \theta) B(q)\)

- Note that \(f_L\) is a strong function of \(q\)

\[ \text{ratio of } f_L \text{ at } p_i' \text{ to } p_0' \]

\[ q \text{ [fm}^{-1}] \]

\[ 10^{-1} \quad 10^{-2} \quad 10^{-3} \quad 10^{-4} \quad 10^{-5} \]

\[ \text{G}_E^p = 1, \text{G}_E^n = 0 \]

\[ p_0' = 0.1 \quad \theta = 15^0 \]

\[ \text{Sushant More} \quad d(e,e')p \text{ with low-resolution potentials} \]
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  - \( p' \) and \( \theta \): outgoing nucleon
  - \( q \): momentum transfer

- For \( p' \ll q, f_L \) scales with \( q \)
  - \( f_L(p', \theta; q) \to g(p', \theta)B(q) \)

- Note that \( f_L \) is a strong function of \( q \)

- Follows from the LO term in EFT expansion:
  \[
  \langle \psi_f^\lambda | J_0^\lambda (q) | \psi_{\text{deut}}^\lambda \rangle \approx \frac{g_0^q \left. \psi_f^\lambda(p'; r) \psi_{\text{deut}}^\lambda(r) \right|_{r=0}}{f_L(p'_i, \theta, q)} \]

\( p'_i = 0.2 \)
\( p'_i = 0.4 \)
\( p'_i = 0.5 \)
\( p'_i = 0.8 \)
\( p'_0 = 0.1 \)
\( \theta = 15^0 \)

\( G_E^p = 1, \ G_E^n = 0 \)
Summary and Moving Forward

- Scale dependence abounds... in a systematic way which can be accounted for
- Conventional wisdom: low-resolution potentials ill-suited for (high-\(q\)) reactions calculations
  \[ \rightarrow \text{RG changes to } \hat{O}_q \text{ tractable} \]
- Local decoupling of \(\psi_f^\lambda\) → increased validity of impulse approximation
- Explanation of factorization straightforward in low-momentum picture
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To do:

- Make the EFT picture more quantitative
- Extend to \(A > 2\). Basis for consistent construction of operators
- Consistently extract process-independent quantities from experiments
  \(\rightarrow\) What is the best scale to use?
  \(\rightarrow\) What are the controlled approximations that we can make?
  \(\rightarrow\) Model dependence of SRC, spectroscopic factors, ...
Back up

Sushant More  d(e,e')p with low-resolution potentials