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# Microscopic optical potentials derived from nucleon-nucleon chiral potentials

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Progress in Ab Initio Techniques in Nuclear Physics

March 3, 2017

# Outline

1. Introduction
2. Statement of the problem & approximations
3. Comparison with experimental data
4. Summary & outlook

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# Introduction

## Purpose

Study the domain of applicability of microscopic two-body chiral potentials in the construction of an optical potential

### Optical potentials

- **Phenomenological**

  Many adjustable parameters set up fitting a large amount of experimental data

- **Microscopic**

  Built in terms of the underlying NN scattering amplitudes

  → Based on an effective NN interaction

### Applications

- Nucleon-nucleus elastic scattering
- Inelastic scattering
- Other nuclear reactions

# Why a microscopic approach?

- Microscopic optical potentials do not contain adjustable parameters
- We expect a **greater predictive power** when applied to situations where experimental data are not yet available

→ Study of unstable nuclei

The Scattering of Fast Nucleons from Nuclei

A. K. Kerman

*Massachusetts Institute of Technology, Cambridge, Massachusetts*

H. McManus

*Chalk River Laboratory, Chalk River, Ontario, Canada*

and

R. M. Thaler

*Los Alamos Scientific Laboratory, Los Alamos, New Mexico*

Received May 27, 1959

PHYSICAL REVIEW C VOLUME 30, NUMBER 6 DECEMBER 1984

Momentum space approach to microscopic effects in elastic proton scattering

A. Picklesimer

*Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742*
*and Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

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(Received 6 August 1984)

# Statement of the problem

Lippmann-Schwinger (LS) equation for nucleon-nucleus scattering

$$T = V + VG_0(E)T$$

Separation of the LS equation

$$T = U + UG_0(E)PT$$

$$U = V + VG_0(E)QU$$

Transition operator for the elastic scattering

$$T_{\rm el} \equiv PTP = PUP + PUPG_0(E)T_{\rm el}$$

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Free propagator

$$G_0(E) = (E - H_0 + i\epsilon)^{-1}$$

Free Hamiltonian

$$H_0 = h_0 + H_A$$

External interaction

$$V = \sum_{i=1}^{A} v_{0i}$$

# The spectator expansion

PHYSICAL REVIEW C VOLUME 52, NUMBER 4 OCTOBER 1995

**Propagator modifications in elastic nucleon-nucleus scattering within the spectator expansion**

C.R. Chinn,[1,2] Ch. Elster,[3] R.M. Thaler,[1,4] and S.P. Weppner[3]
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[2]*Center for Computationally Intensive Physics, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831*
[3]*Institute of Nuclear and Particle Physics, and Department of Physics, Ohio University, Athens, Ohio 45701*
[4]*Physics Department, Case Western Reserve University, Cleveland, Ohio 44106*

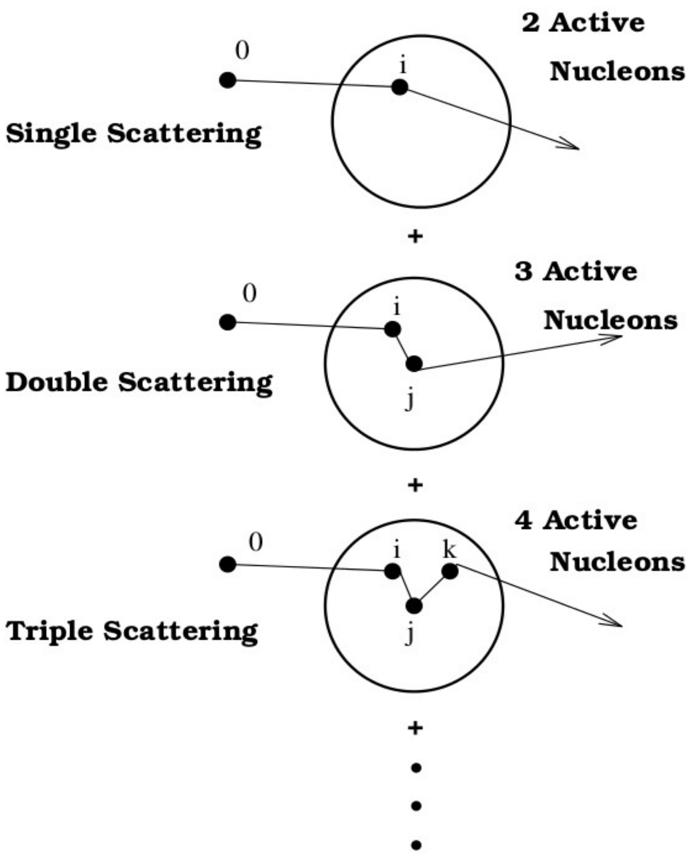
(Received 27 March 1995)

The theory of the elastic scattering of a nucleon from a nucleus is presented in the form of a spectator expansion of the optical potential. Particular attention is paid to the treatment of the free projectile-nucleus propagator when the coupling of the struck target nucleon to the residual target must be taken into consideration. First order calculations within this framework are shown for neutron total cross sections and for proton scattering for a number of target nuclides at a variety of energies. The calculated values of these observables are in very good agreement with measurement.

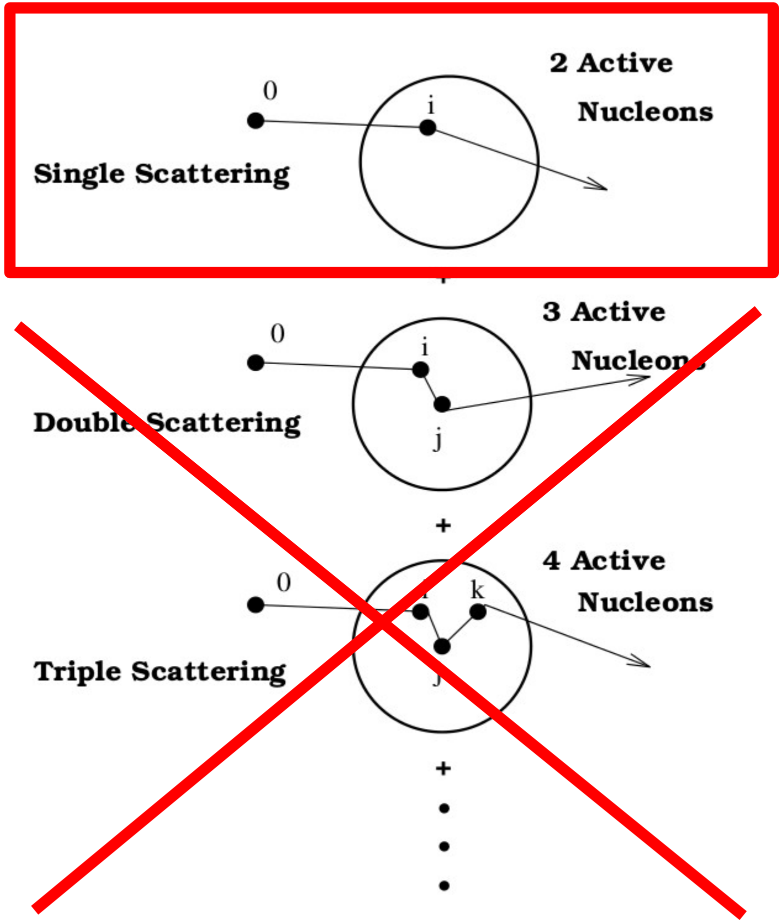
## The spectator expansion for the optical potential operator

$$U = \sum_{i=1}^{A} \tau_i + \sum_{i,j\neq i}^{A} \tau_{ij} + \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{ijk} + \cdots$$

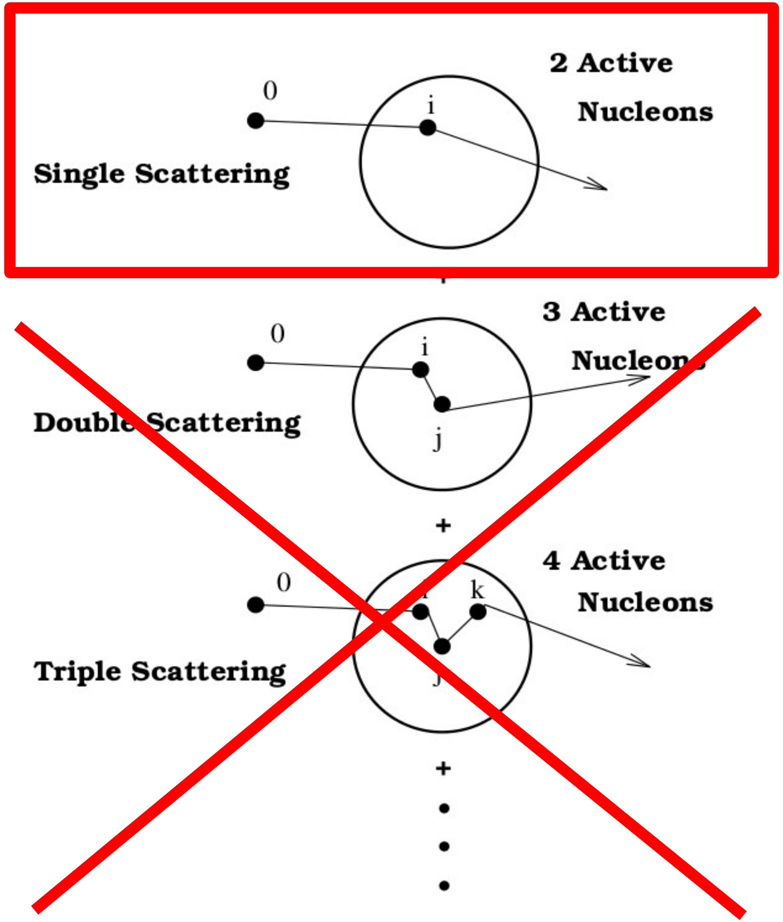
# The single-scattering approximation



# The single-scattering approximation



# The single-scattering approximation




Optical potential operator

$$U = \sum_{i=1}^{A} \tau_i$$

The first-order term

$$\tau_i = v_{0i} + v_{0i} G_0(E) Q \tau_i$$

# The impulse approximation

We neglect the coupling of the struck target nucleon with the residual nucleus

The interaction between the two nucleons is considered as free: $\tau_i \approx t_{0i}$

The free NN t matrix

$$t_{0i} = v_{0i} + v_{0i} g_i t_{0i}$$

The free two-body propagator

$$g_i = \frac{1}{E - h_0 - h_i + i\epsilon}$$

Optical potential operator

$$U = \sum_{i=1}^{A} t_{0i}$$

Useful approximation for the intermediate-energy regime

↓

Only two-particle integral equations

# The first-order optical potential

## Elastic scattering amplitude

$$T_{\rm el}(\boldsymbol{k}', \boldsymbol{k}; E) = U(\boldsymbol{k}', \boldsymbol{k}; \omega) + \int d^3p \frac{U(\boldsymbol{k}', \boldsymbol{p}; \omega)\, T_{\rm el}(\boldsymbol{p}, \boldsymbol{k}; E)}{E(k_0) - E(p) + i\epsilon}$$

## The first-order optical potential

$$U(\boldsymbol{q}, \boldsymbol{K}; \omega) = \frac{A-1}{A} \sum_{N=n,p} \int d^3P\, \eta(\boldsymbol{P}, \boldsymbol{q}, \boldsymbol{K})\, t_{pN}\left[\boldsymbol{q}, \frac{A+1}{A}\boldsymbol{K} - \boldsymbol{P}; \omega\right]$$
$$\times\, \rho_N \left[\boldsymbol{P} - \frac{A-1}{A}\frac{\boldsymbol{q}}{2}, \boldsymbol{P} + \frac{A-1}{A}\frac{\boldsymbol{q}}{2}\right]$$

---

Momentum transfer

$$\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$$

Total momentum

$$\boldsymbol{K} = \frac{1}{2}(\boldsymbol{k}' - \boldsymbol{k})$$

# Optimum factorization approximation

Expansion of the t matrix in a Taylor series in ***P***

$$\eta(\boldsymbol{P})\,t_{pN}(\boldsymbol{P}) = \eta(\boldsymbol{P}_0)\,t_{pN}(\boldsymbol{P}_0) + (\boldsymbol{P}-\boldsymbol{P}_0)\,\partial_{\boldsymbol{P}_0}\Big[\eta(\boldsymbol{P}_0)\,t_{pN}(\boldsymbol{P}_0)\Big] + \cdots$$

Time-reversal invariance of the ground state density matrix

$$\int d^3P\,\boldsymbol{P}\,\rho_N\left[\boldsymbol{P}-\frac{A-1}{A}\frac{\boldsymbol{q}}{2},\boldsymbol{P}+\frac{A-1}{A}\frac{\boldsymbol{q}}{2}\right] = 0$$

Neutron and proton density profiles

$$\rho_N(q) = \int d^3P\,\rho_N\left[\boldsymbol{P}-\frac{A-1}{A}\frac{\boldsymbol{q}}{2},\boldsymbol{P}+\frac{A-1}{A}\frac{\boldsymbol{q}}{2}\right]$$

A factorized form of the potential is obtained choosing $\boldsymbol{P}_0 = 0$

# Optimum factorization approximation

## The optimum factorized optical potential

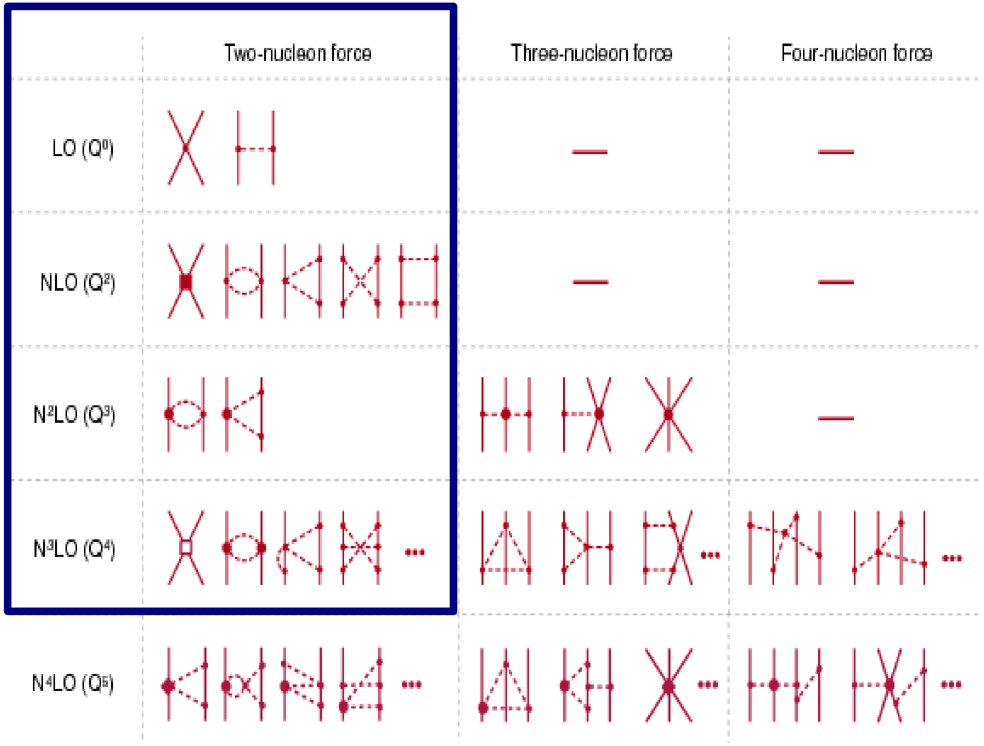
$$U(\boldsymbol{q},\boldsymbol{K};\omega) = \frac{A-1}{A}\eta(\boldsymbol{q},\boldsymbol{K}) \sum_{N=n,p} t_{pN}\left[\boldsymbol{q},\frac{A+1}{A}\boldsymbol{K};\omega\right]\rho_N(q)$$

## Basic ingredients

1. Nucleon-nucleon interaction

2. Neutron and proton densities

# Chiral potential up to N$^3$LO




- QCD symmetries are consistently respected
- Systematic expansion (order by order we know exactly the terms to be included)
- Theoretical errors. Order by order in a power expansion, the uncertainties are of order of $\mathcal{O}(Q/\Lambda_\chi)^\nu$
- Two- and many-body forces belong to the same framework

Chiral potential up to the fourth order
Only the two-body part

# Chiral potentials up to N$^3$LO

## Machleidt *et al.* (EM)

- Three possible choices for the LS cut-off:
  $\Lambda$ = 450, 500, 600 MeV
- Dimensional regularization of the two-pion exchange term in the potential

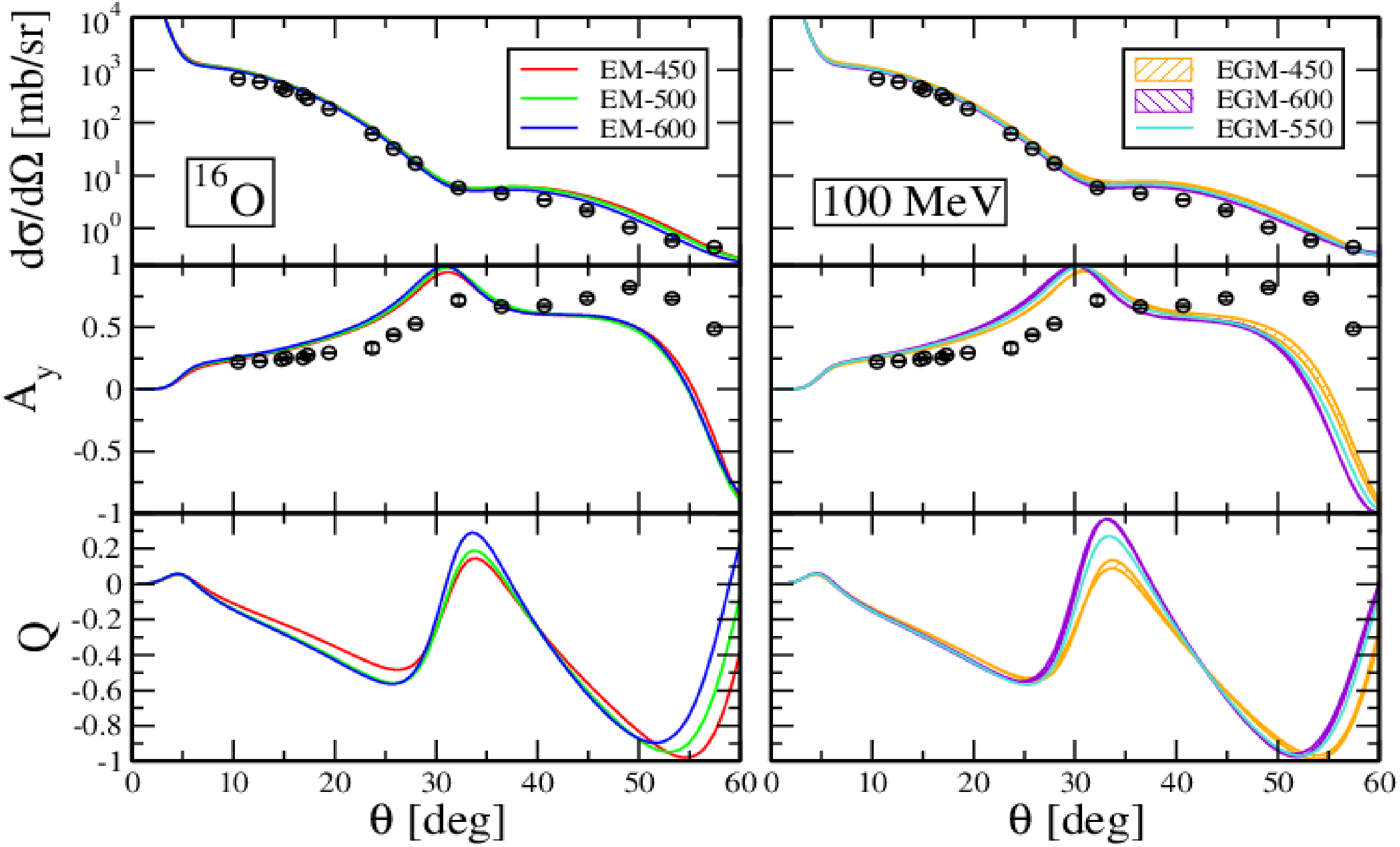
Phys. Rev. C **68**, 041001 (2003)
Phys. Rev. C **75**, 024311 (2007)
Phys. Rev. C **87**, 014322 (2013)
Phys. Rev. C **88**, 054002 (2013)

## Epelbaum *et al.* (EGM)

- Three possible choices for the LS cut-off:
  $\Lambda$ = 450, 550, and 600 MeV
- Spectral function representation
  $\Lambda'$ = 500, 600, and 700 MeV
- Available choices
  $(\Lambda, \Lambda')$ = (450, 500), (450, 700), (550, 600), (600, 600), (600, 700)

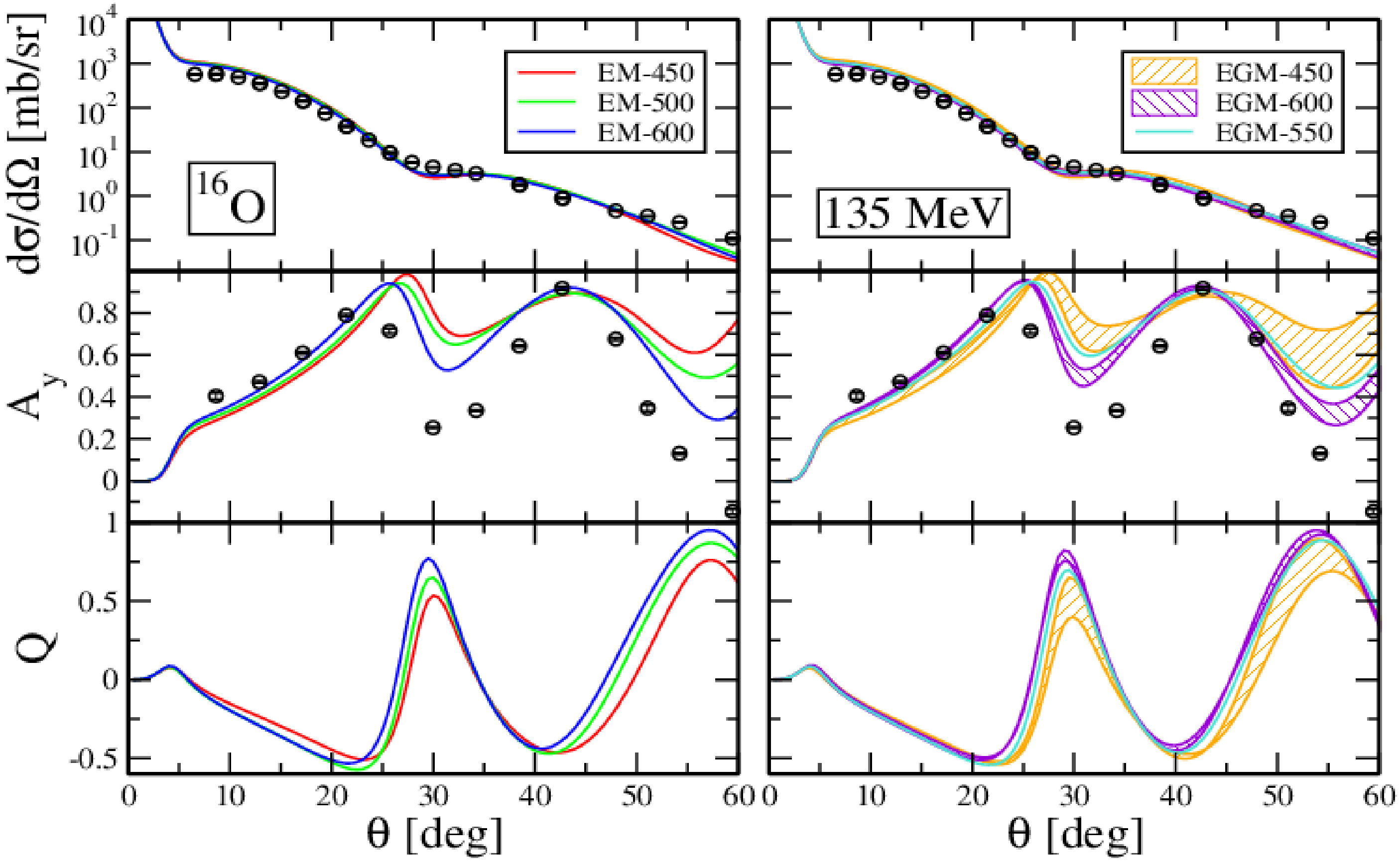
Nucl. Phys. A **747**, 362 (2005)

# Comparison with experimental data




M. Vorabbi, P. Finelli, and C. Giusti, Phys. Rev. C **93**, 034619 (2016)

# Comparison with experimental data




M. Vorabbi, P. Finelli, and C. Giusti, Phys. Rev. C **93**, 034619 (2016)

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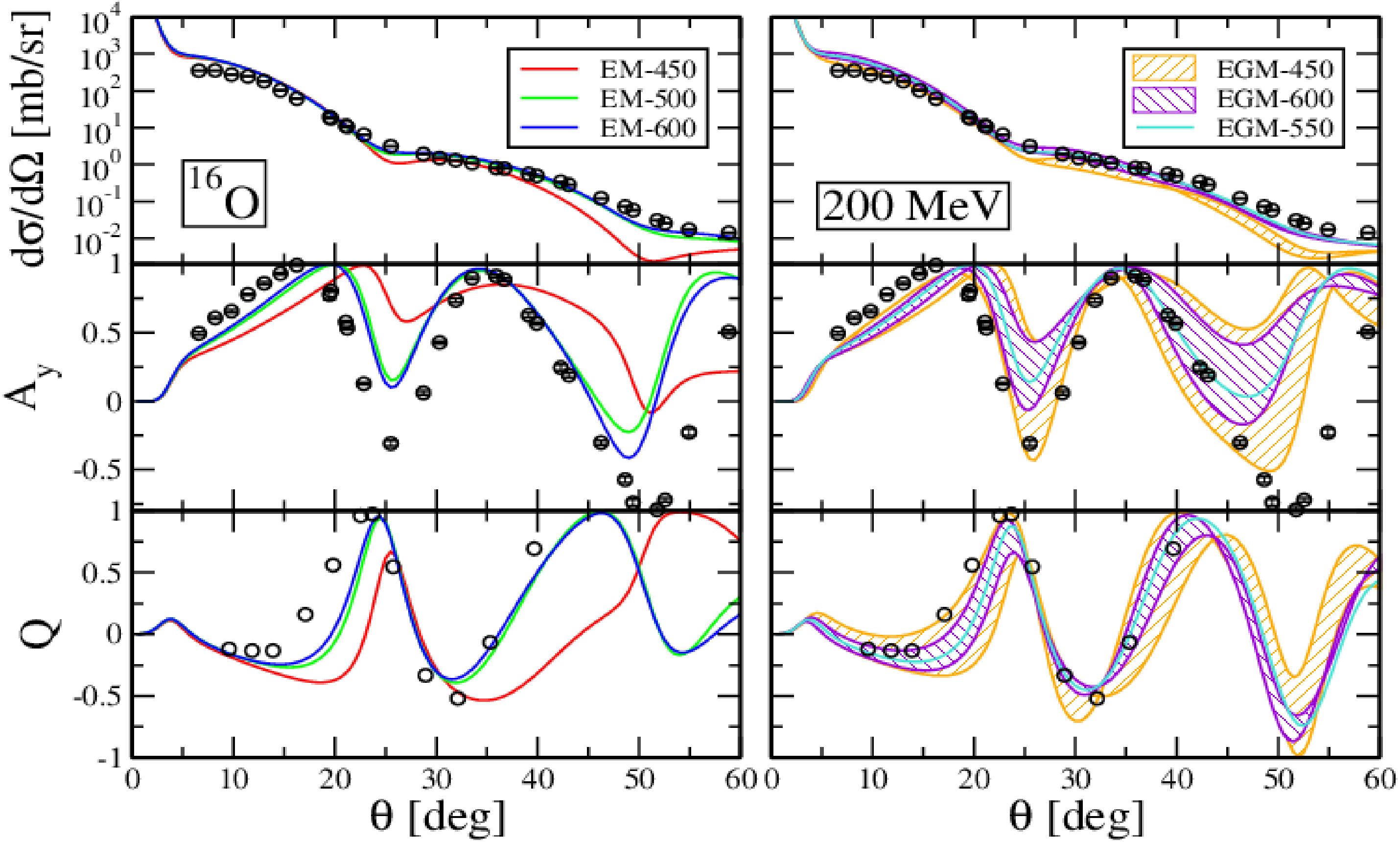


M. Vorabbi, P. Finelli, and C. Giusti, Phys. Rev. C **93**, 034619 (2016)

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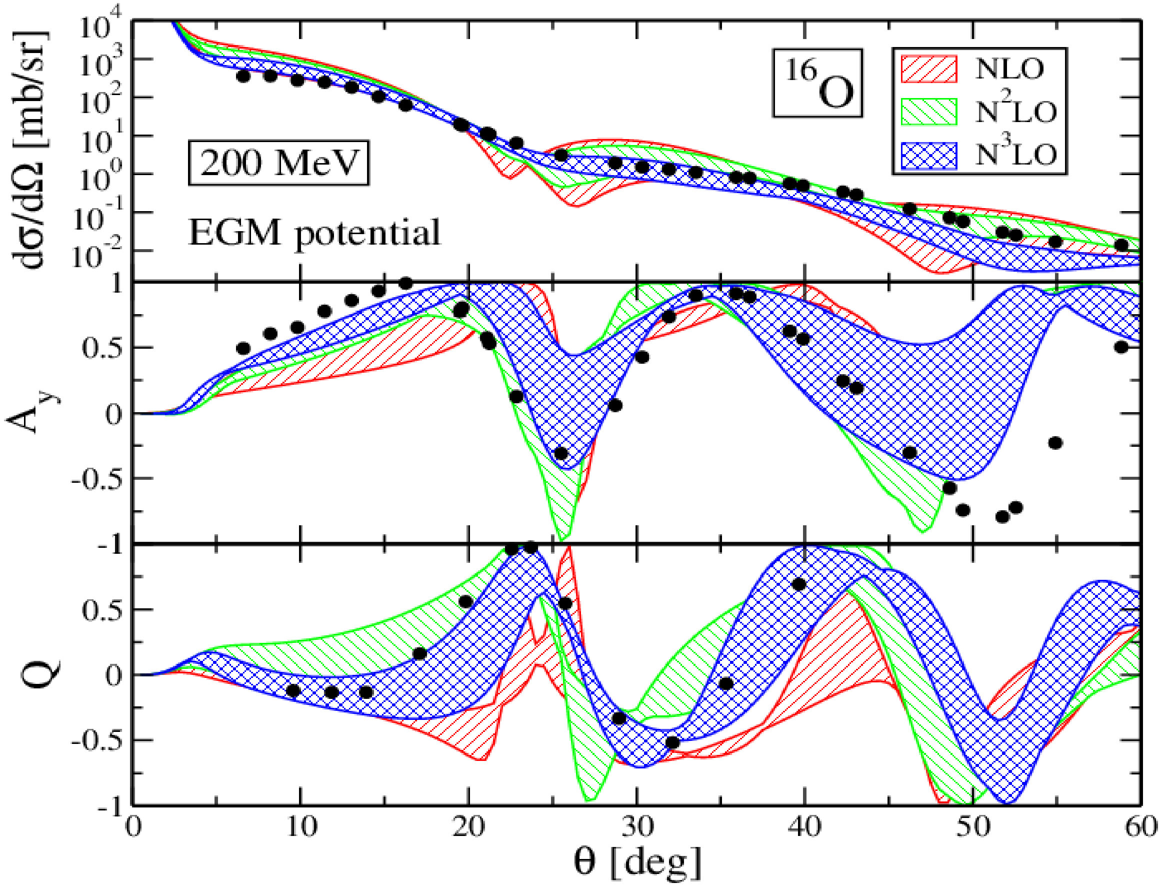


M. Vorabbi, P. Finelli, and C. Giusti, Phys. Rev. C **93**, 034619 (2016)

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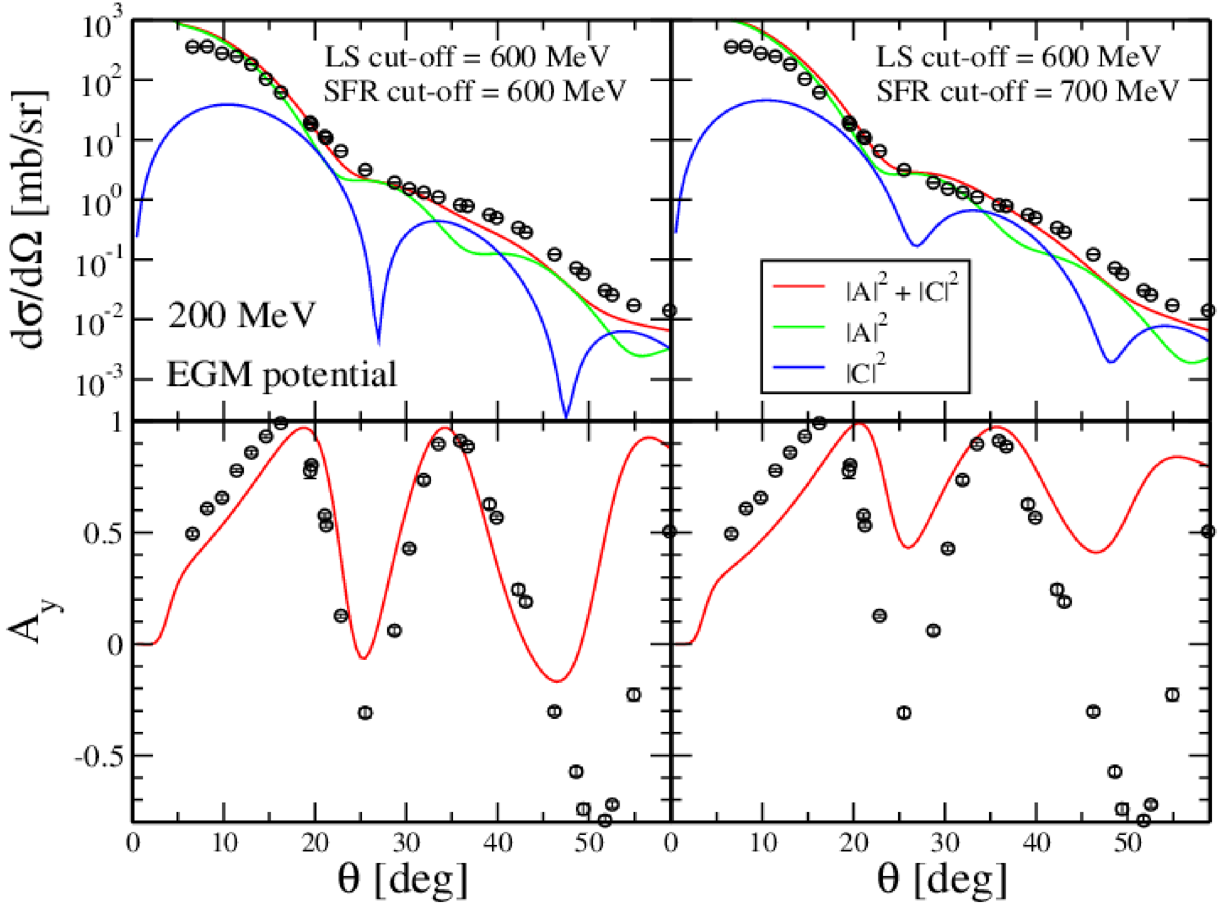


M. Vorabbi, P. Finelli, and C. Giusti, Phys. Rev. C **93**, 034619 (2016)

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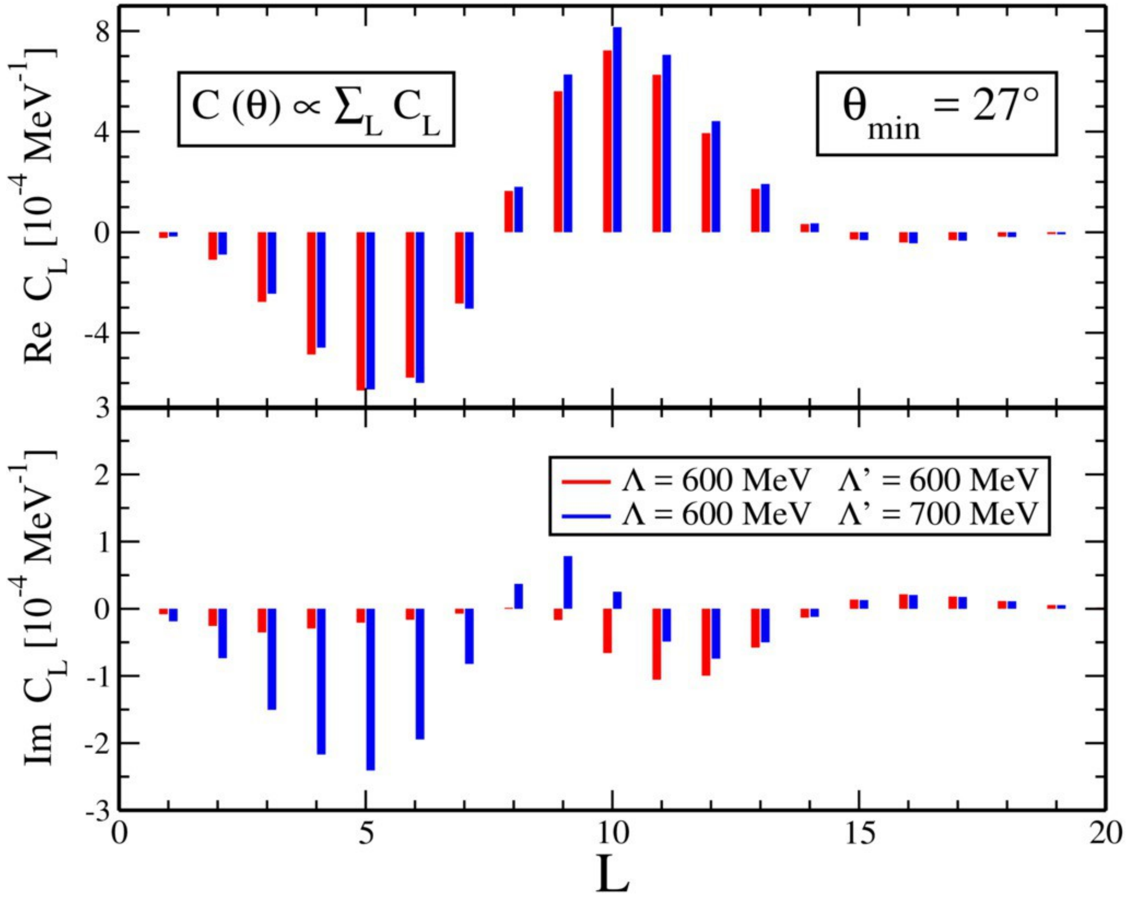


M. Vorabbi, P. Finelli, and C. Giusti, Phys. Rev. C **93**, 034619 (2016)

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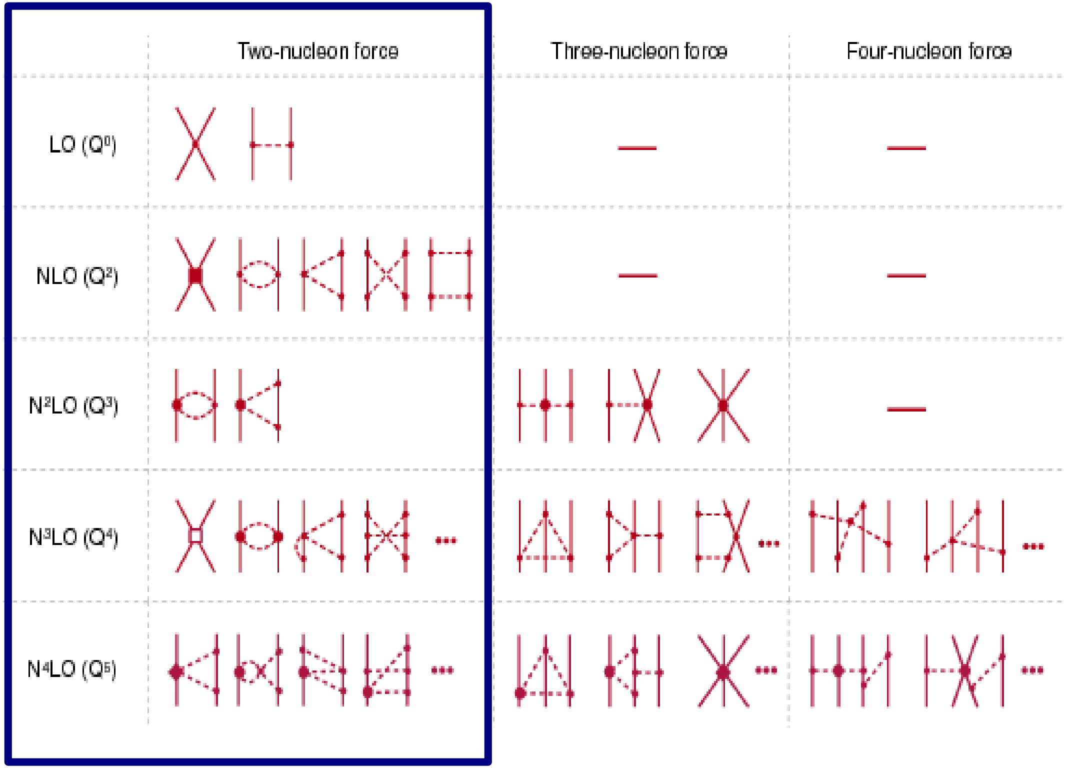
# Chiral potentials up to N$^4$LO




## Machleidt *et al.* (EKMN)

Phys. Rev. C **91**, 014002 (2015)

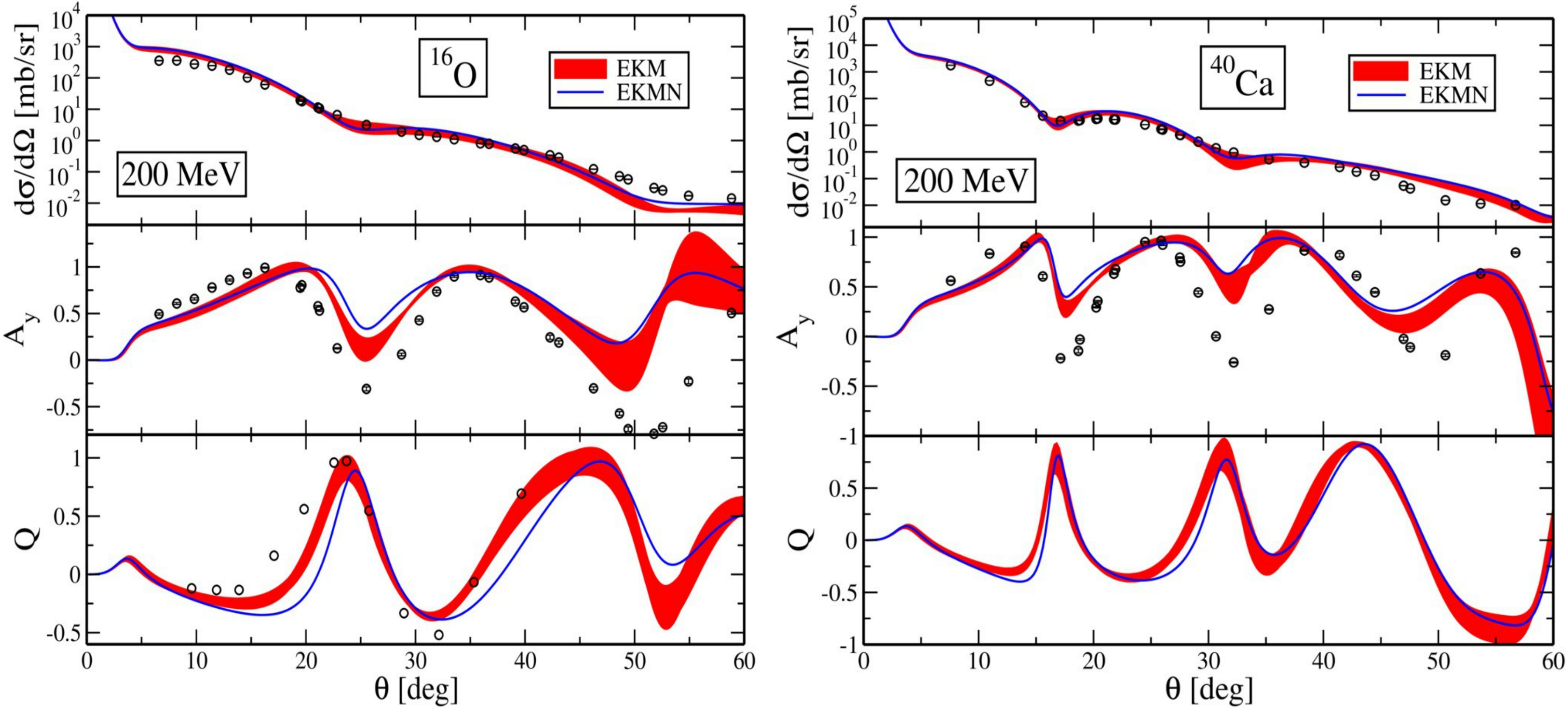
- LS cut-off: $\Lambda$ = 500 MeV
- Dimensional regularization of the two-pion exchange term

## Epelbaum *et al.* (EKM)

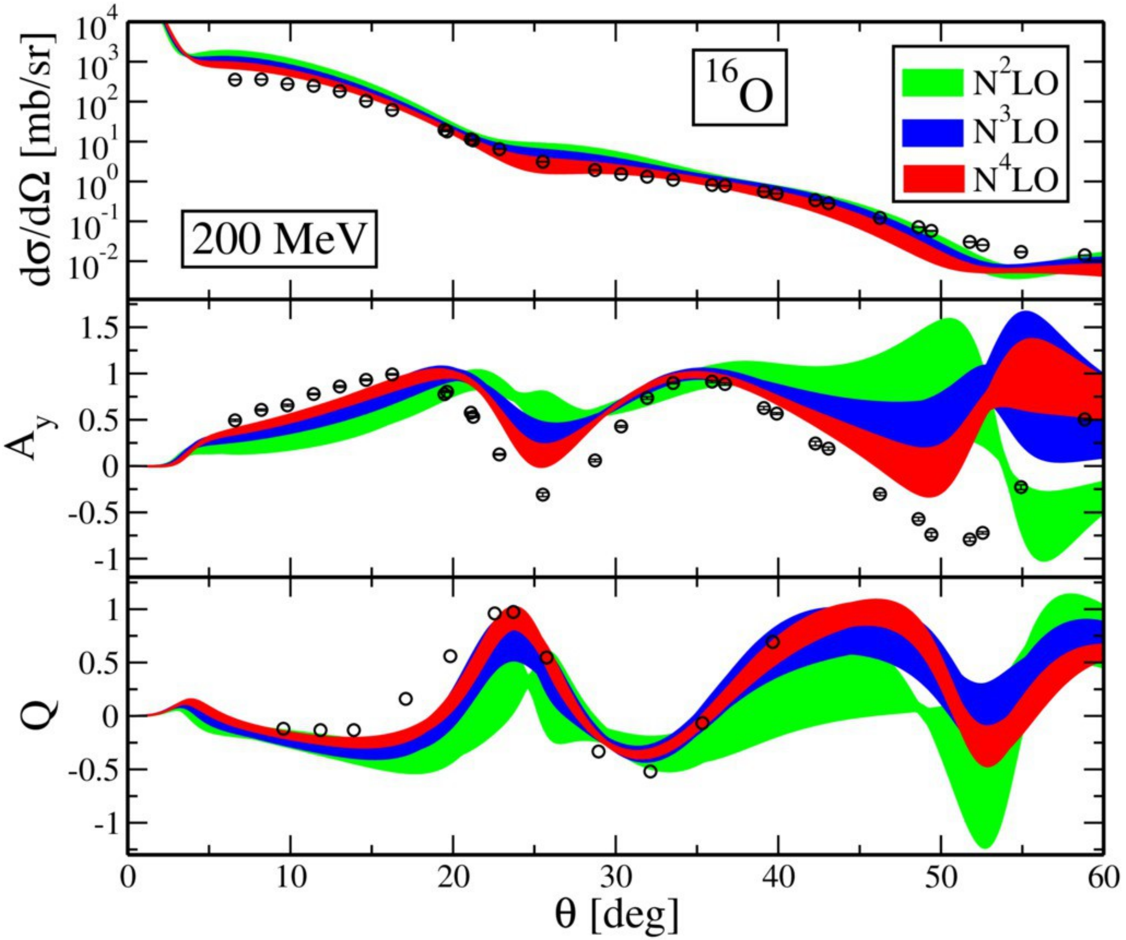
Phys. Rev. Lett. **115**, 122301 (2015)

- Semi-local regularization
  R = 0.8, **0.9**, 1.0, 1.1, 1.2 fm

# Comparison with experimental data



# Comparison with experimental data



# Density-dependent NN chiral potentials

## Inclusion of density-dependent corrections to the bare NN force

J. W. Holt, N. Kaiser, W. Weise, Phys. Rev. C **81**, 024002 (2010)

Equation for the optical potential

$$U = V + VG_0(E)QU$$

External interaction

$$V = \sum_{i=1}^{A} \tilde{v}_{0i}$$

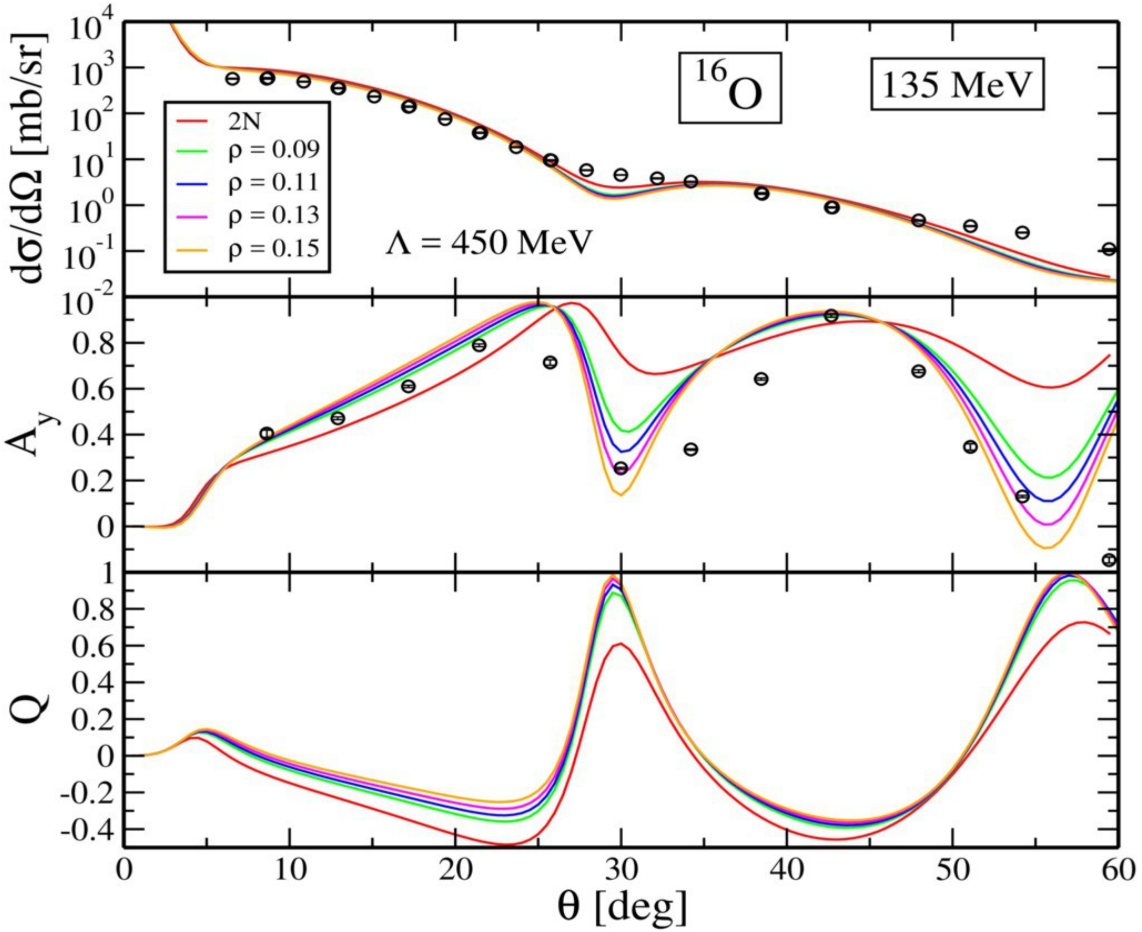
DDNN chiral potential operator

$$\tilde{v}_{0i}(\rho) = v_{0i} + v_{0i}^{\mathrm{med}}(\rho)$$

Effective density-dependent in-medium NN interaction derived from the LO chiral 3N force

Bare NN interaction

# Density-dependent NN chiral potentials



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# Summary and outlook

## Conclusions

- Close results and a good description of the experimental cross sections are obtained for proton energies up to about 135 MeV
- A better agreement with empirical data is obtained at 200 MeV with higher values of the LS cut-off
- EGM-600 potential provides a better description of experimental data with the SFR cut-off = 600 MeV

## Future improvements

- Improve the inclusion of density-dependent corrections
- Computation of the folding integral
- Inclusion of medium effects

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# Merci!

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