Shell-model calculations with chiral three-body force

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Chiral EFT and 3NF

Chiral effective field theory (EFT)

S. Weinberg, Phys. A **96**, 327 (1979).
R. Machleidt and D. Entem, Physics Reports **503**, 1 (2011).

Degrees of freedom and symmetry

Nucleons and pions
Chiral symmetry

→

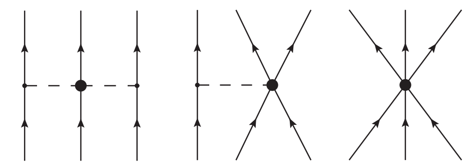
Soft scale Q
Hard scale Λ_χ

Perturbative expansion of Lagrangian

$(Q/\Lambda_\chi)^n$ Power counting
Theoretical error

Many-body forces on an equal footing

At N^2LO ($n = 3$),
three-nucleon force (3NF) appears.



5 low-energy constants (LECs)
(2 of them appear for the first time)

S. Weinberg, Phys. Lett. B **295**, 114 (1992).
U. van Kolck, Phys. Rev. C **49**, 2932 (1994).

Regularization

Theory valid in the scale $Q \ll \Lambda_\chi$,

$$V_{3N} \longmapsto u_\nu(q,\Lambda)\, V_{3N}\, u_\nu(q',\Lambda),$$

with the regulator u_ν of the cutoff Λ.

→ **Discussed later**.

Out of our scope

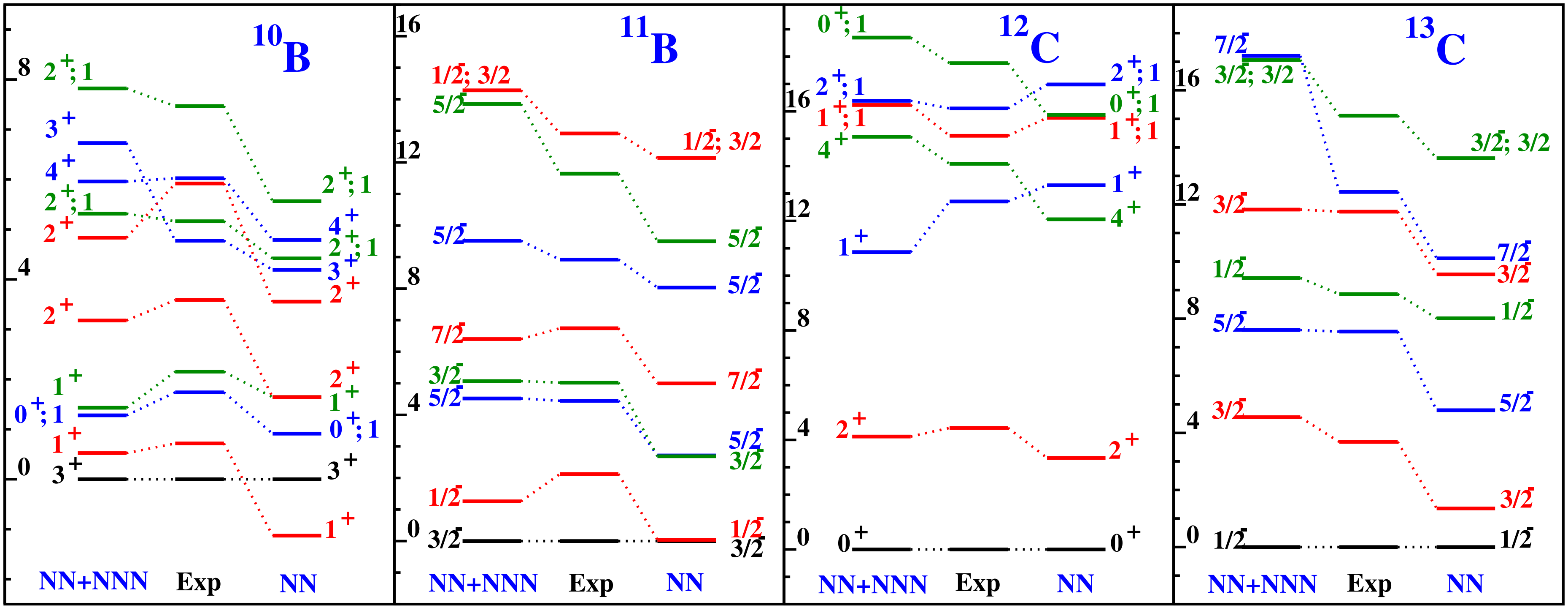
Fujita-Miyazawa, Tucson-Melbourne, Ulbana, etc.

J. Fujita and H. Miyazawa, Prog. Theor. Phys. **17**, 360 (1957).
S. A. Coon *et al.*, Nucl. Phys. **A317**, 242 (1979).
J. Carlson *et al.*, Nucl. Phys. **A401**, 59 (1983).

p-shell nuclei

ab initio no-core shell model (NCSM)

P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007).

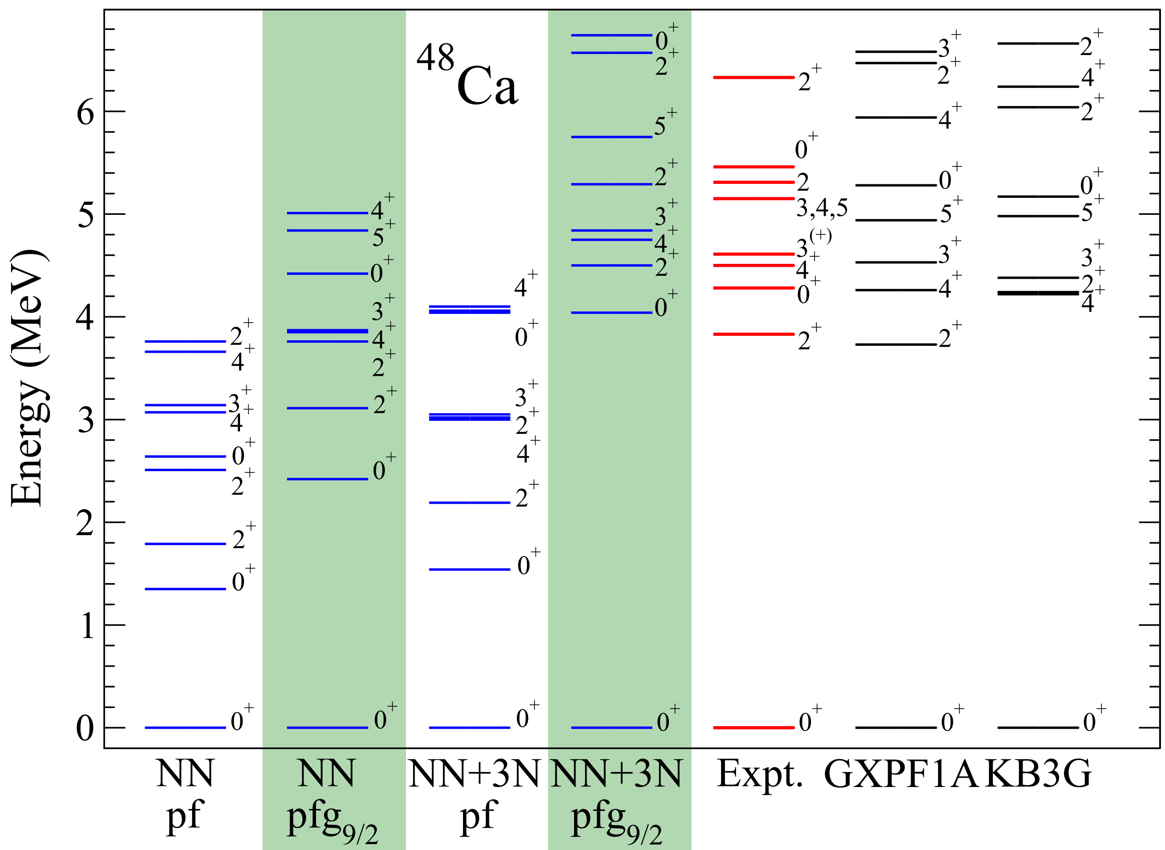


- Inclusion of 3NF improves drastically the **order** (**qualitative**) of excited states, **absolute value** (**quantitative**) as well, compared to the experimental data.

fp-shell nuclei

Shell model with ^{40}Ca core

J. D. Holt *et al.*, Phys. Rev. C **90**, 024312 (2014).

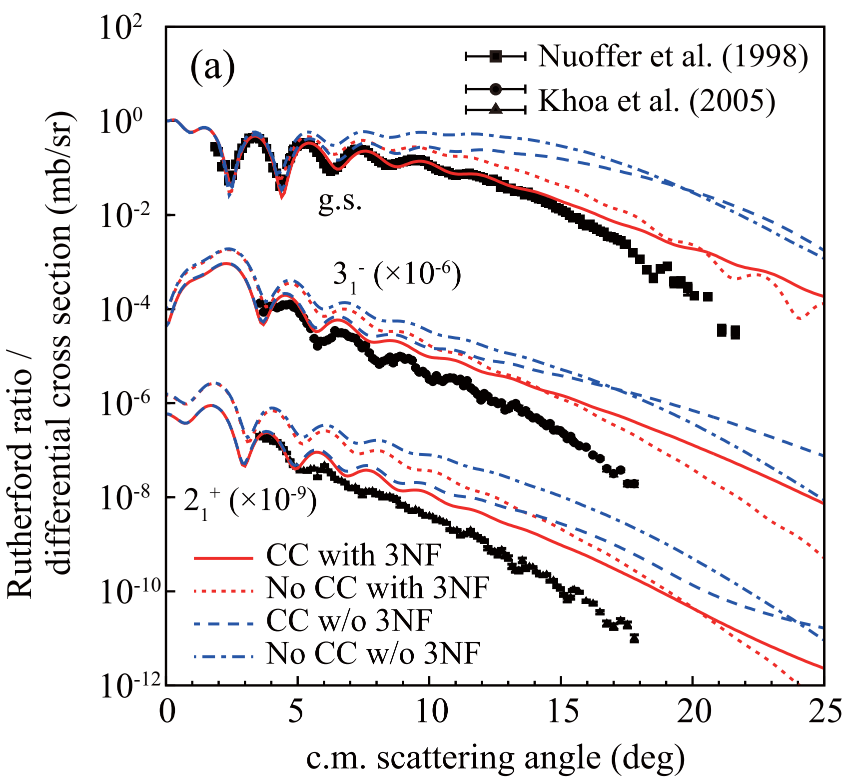


- The 3NF effect with $g_{9/2}$ is significant.

Nucleus-nucleus scattering

Elastic and inelastic ^{16}O-^{16}O scattering

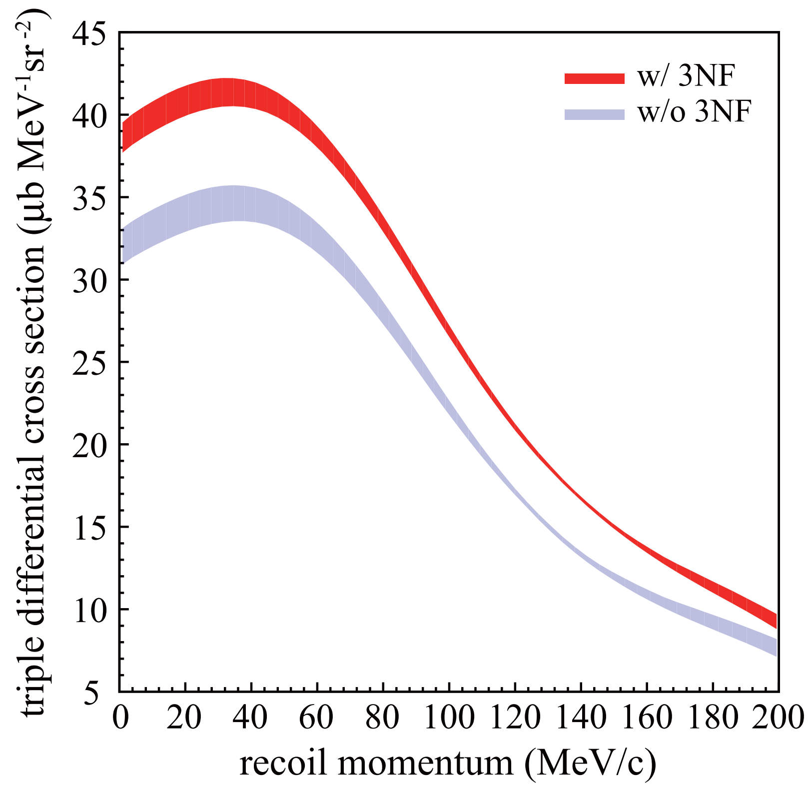
K. Minomo *et al.*, Phys. Rev. C **93**, 014607 (2016).



- Backward angle → high density
- Similar effect on ^{12}C-^{12}C scattering.

Knock-out reaction $^{40}Ca(p, 2p)^{39}K$

K. Minomo *et al.*, Phys. Rev. C **96**, 024609 (2017).

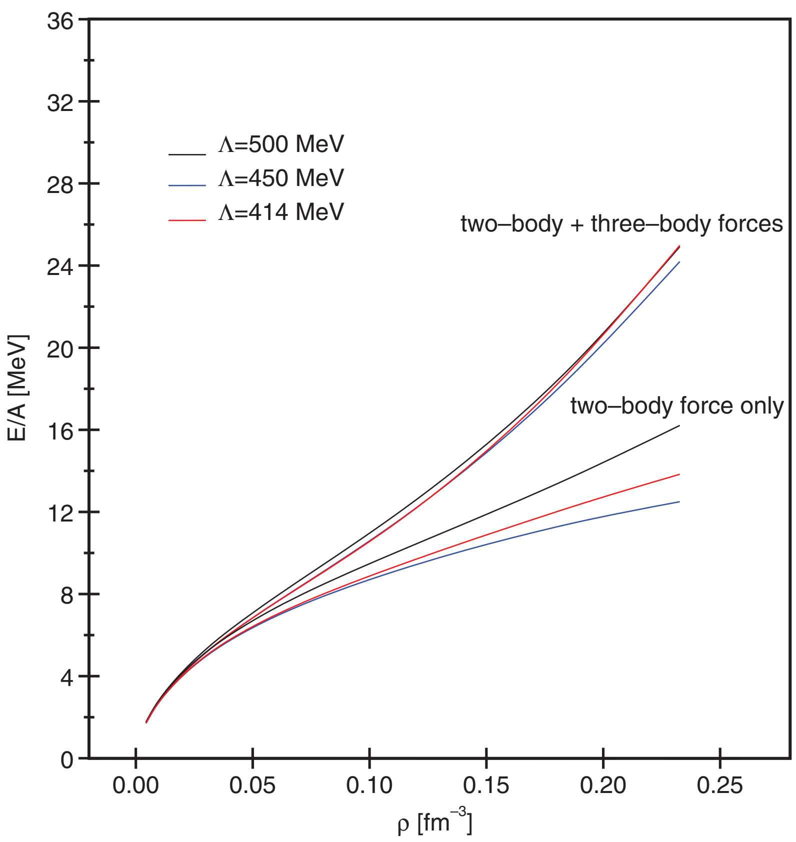


- Specific kinematical-condition.

Significance of 3NF | Nuclear matter

Pure neutron matter

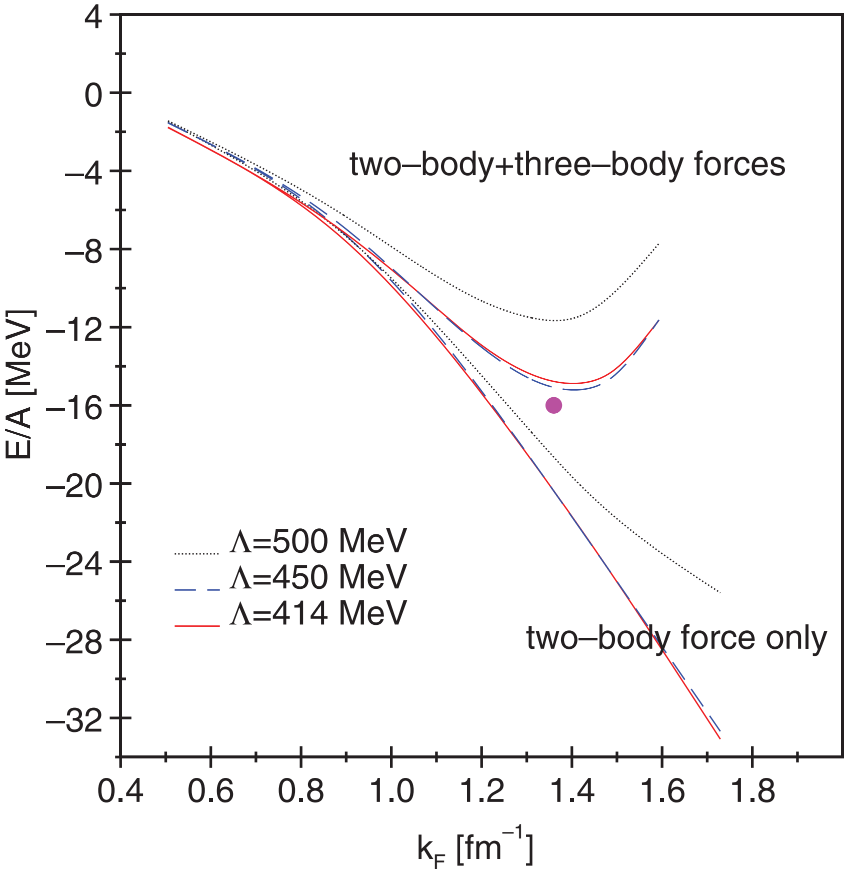
L. Coraggio *et al*., Phys. Rev. C **87**, 014322 (2013).



- Only the 2-pion exchange term contributes.

Symmetric nuclear matter

L. Coraggio *et al*., Phys. Rev. C **89**, 044321 (2014).



- Crucial 3NF effect for saturation.

Motivation

- Including the **3NF based on the chiral EFT** in **realistic shell-model (RSM)** calculations.
- Investigating 3NF effect with elucidating **cutoff dependence**, **LEC dependence**, **nuclides dependence**, etc.

  → It is necessary to develop **our own code** for the 3-body matrix elements (MEs).

This presentation

- Formulation of the 3-body MEs is given in detail.
- Preliminary results are shown.

Purpose

Motivation

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釈迦に説法 Shaka ni seppo.	=	Preaching to Buddha. ~ Teaching something to someone who knows more than you.

Antisymmetrized 3-body ME

Separation of CM motion

$$
\left| [[\bullet\bullet]\bullet]_{JT} \right\rangle = \sum (\text{coeff.}) \left\{ 9j \right\} \left\{ 9j \right\}
$$

***jj* coupling → *LS* coupling**

$$
\times \left\{ 6j \right\} \langle\langle \text{ HOB } \rangle\rangle \langle\langle \text{ HOB } \rangle\rangle
$$

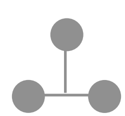
Talmi transformations

I. Talmi, Helv. Phys. Acta **25**, 185 (1952).

$$
\times \left\{ 6j \right\} \left\{ 6j \right\} \left\{ 9j \right\}
$$

Recoupling for antisymmetrization

$$
\times \left| \left[\left| \quad \right\rangle \left| \text{CM} \right\rangle \right]_{JT} \right\rangle
$$

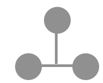
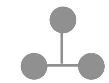


※ Harmonic-oscillator bracket (HOB)

Numerical way (diagonalization of antisymmetrizer)

P. Navrátil *et al.*, Phys. Rev. C **61**, 044001 (2000).

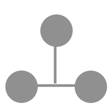
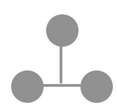
JT-coupled state ⟶ Jacobi-HO state

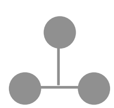
$\left| \left[[\bullet\bullet]\bullet \right]_{JT} \right\rangle_A = \sqrt{6}\hat{\mathcal{A}}_3 \left| \left[[\bullet\bullet]\bullet \right]_{JT} \right\rangle = (\text{coeff.}) \left| \text{CM} \right\rangle \sqrt{6}\hat{\mathcal{A}}_3 \Big|$  $\Big\rangle = (\text{coeff.}) \left| \text{CM} \right\rangle \Big|$  $\Big\rangle_A$

$$\hat{\mathcal{A}}_3 = \frac{1}{3!}\left[\mathbb{1} - \hat{\mathcal{P}}_{ab} - \hat{\mathcal{P}}_{bc} - \hat{\mathcal{P}}_{ca} + \hat{\mathcal{P}}_{ab}\hat{\mathcal{P}}_{bc} + \hat{\mathcal{P}}_{ab}\hat{\mathcal{P}}_{ca} \right]$$

Spectral decomposition

$$\hat{\mathcal{A}}_3 = \sum_{\nu} \epsilon_{\nu} \left| \nu \right\rangle \left\langle \nu \right|$$

$\hat{\mathcal{A}}_3 \Big| i;$  $\Big\rangle = \sum_{j\nu} C^i_{\nu} C^j_{\nu} \Big| j;$  $\Big\rangle$

$C^i_{\nu} = \Big\langle \nu \Big| i;$  $\Big\rangle$

$\hat{\mathcal{A}}_3$: idempotent

$$\epsilon_{\nu} = \begin{cases} 1 & \text{(physical states)} \\ 0 & \text{(spurious states)} \end{cases}$$

P. Navrátil *et al.*, Phys. Rev. C **59**, 611 (1999).

Coefficients obtained numerically.

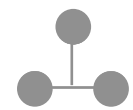
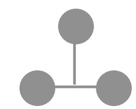
Numerical way (diagonalization of antisymmetrizer)

P. Navrátil *et al.*, Phys. Rev. C **61**, 044001 (2000).

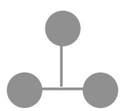
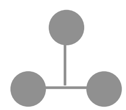
Eigenvalue equation

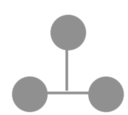
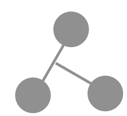
$$\left(\mathcal{A}_{ij} \right) \left(C^j_\nu \right) = \epsilon_\nu \left(C^i_\nu \right)$$

Constrain

$\hat{\mathcal{P}}_{ab} \left| i; \right.$  $\rangle = - \left| i; \right.$  $\rangle$

Partially antisymmetrized states

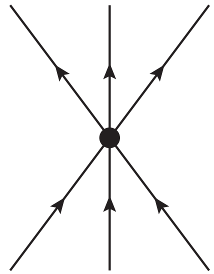
$\mathcal{A}_{ij} = \langle i;$  $\left| \hat{\mathcal{A}}_3 \right| j;$  $\rangle$

$= \frac{1}{3} \left[\delta_{ij} - (\text{coeff.}) \langle i; \right.$  $\left| j; \right.$  $\left. \rangle \right]$

$$= \frac{1}{3} \left[\delta_{ij} - (\text{coeff.}) \langle\langle \text{ HOB } \rangle\rangle \right]$$

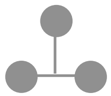
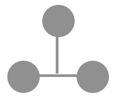
- This approach is general to perform the antisymmetrization for *A*-body system.

Contact term



E. Epelbaum *et al.*, Phys. Rev. C **66**, 064001 (2002).
P. Navrátil, Few-Body Syst. **41**, 117 (2007).

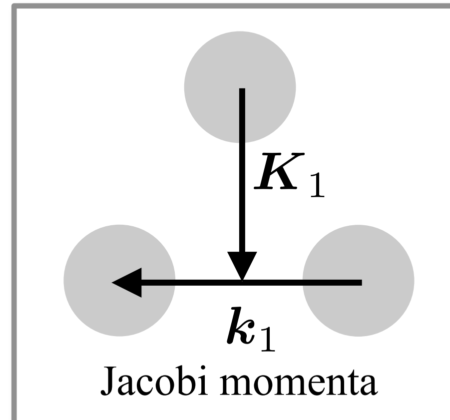
Final form

$\langle$  $\left| W_{3N}^{(\mathrm{ct})} \right|$  $\rangle$

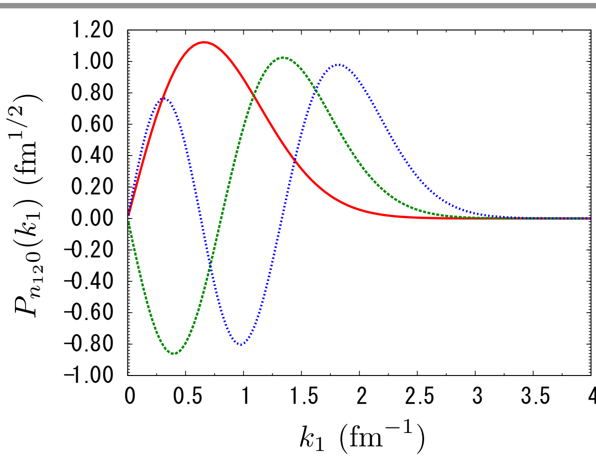
$$= (\mathrm{coeff.}) \ni \textbf{LEC } c_E$$

$$\times \iint dk_1 dK_1 k_1 K_1 P_{n_{12}0}(k_1) P_{n0}(K_1) u_\nu(k_1, K_1, \Lambda)$$

$$\times \iint dk_1' dK_1' k_1' K_1' P_{n_{12}'0}(k_1') P_{n'0}(K_1') u_\nu(k_1', K_1', \Lambda)$$



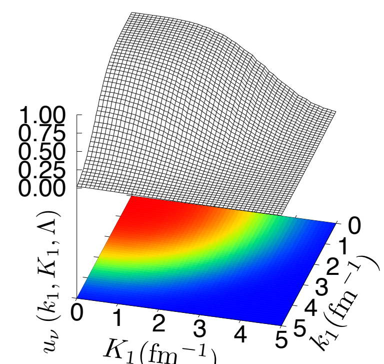
Momentum-space HO



Only *s*-wave due to our regulator.

Regulator (non-local form)

$$u_\nu(k_1, K_1, \Lambda) = \exp\left[-\left(\frac{k_1^2 + K_1^2}{2\Lambda^2}\right)^\nu\right]$$



One-pion exchange + contact term

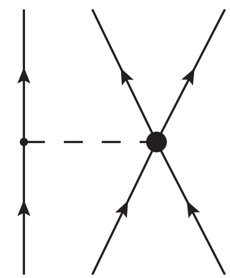
E. Epelbaum *et al.*, Phys. Rev. C **66**, 064001 (2002).
P. Navrátil, Few-Body Syst. **41**, 117 (2007).

Irreducible-tensor expression

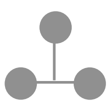
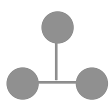
$$\frac{\boldsymbol{\sigma}_c \cdot \boldsymbol{q}_c}{q_c^2 + m_\pi^2} \boldsymbol{\sigma}_b \cdot \boldsymbol{q}_c = \sum_{\lambda_0 \lambda_1 \lambda_2} \sum_{\mathcal{K}_1 \mathcal{K}_2} (\text{coeff.}) f_{\lambda_2}^{(\lambda_0)}(K_1, K_1')$$

Multipole-expansion function

$$\times \left[[\sigma_1(c) \otimes \sigma_1(b)]_{\lambda_0} \otimes \left[Y_{\mathcal{K}_1}\left(\hat{\boldsymbol{K}}_1\right) \otimes Y_{\mathcal{K}_2}\left(\hat{\boldsymbol{K}}_1'\right) \right]_{\lambda_0} \right]_{00}$$



Final form

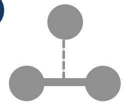
$\Big\langle$  $\Big| W_{3N}^{(1\pi)} \Big|$  $\Big\rangle$

$$= \sum_{\lambda_0 \lambda_1 \lambda_2} (\text{coeff.}) \iiiint dk_1 dk_1' dK_1 dK_1' k_1 k_1' K_1^{\lambda_0 - \lambda_1 + 1} K_1'^{\lambda_1 + 1} f_{\lambda_2}^{(\lambda_0)}(K_1, K_1')$$

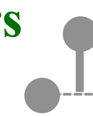
$\ni$ LEC c_D

$$\times P_{n_{12}0}(k_1) P_{n_{12}'0}(k_1') P_{nl}(K_1) P_{n'l'}(K_1') u_\nu(k_1, K_1, \Lambda) u_\nu(k_1', K_1', \Lambda)$$

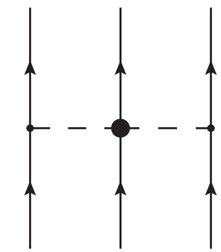
Only *s*-wave HO in *a*-*b* motion



Non *s*-wave appears in (*ab*)-*c* motion



Two-pion exchange term



$$w_{3N}^{(2\pi)} = w_{3N}^{(2\pi;c_1)} + w_{3N}^{(2\pi;c_3)} + w_{3N}^{(2\pi;c_4)}$$

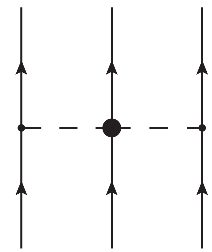
LEC c_1 **LEC c_3** **LEC c_4**

Irreducible-tensor expression (c_1 term)

- The operator $\dfrac{(\boldsymbol{\sigma}_b \cdot \boldsymbol{q}_b)(\boldsymbol{\sigma}_c \cdot \boldsymbol{q}_c)}{(q_b^2 + m_\pi^2)(q_c^2 + m_\pi^2)}$ contains two transferred momenta, $\boldsymbol{q}_b$ and $\boldsymbol{q}_c$, which make calculations cumbersome.

3-body ME with HO | Jacobi-HO ME

Two-pion exchange term



$$w_{3N}^{(2\pi)} = w_{3N}^{(2\pi;c_1)} + w_{3N}^{(2\pi;c_3)} + w_{3N}^{(2\pi;c_4)}$$

LEC c_1 **LEC c_3** **LEC c_4**

Irreducible-tensor expression (c_1 term)

- The operator $\dfrac{(\boldsymbol{\sigma}_b \cdot \boldsymbol{q}_b)\,(\boldsymbol{\sigma}_c \cdot \boldsymbol{q}_c)}{(q_b^2 + m_\pi^2)\,(q_c^2 + m_\pi^2)}$ contains two transferred momenta,

  $\boldsymbol{q}_b$ and $\boldsymbol{q}_c$, which make calculations cumbersome.

  → A technique to tackle this operator has been suggested previously.

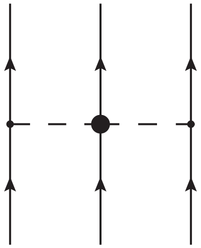
(1) Dividing two propagators using complete set

D. Hüber *et al.*, Few-Body Syst. **22**, 107 (1997).

$$\left\langle i; \middle| \frac{(\boldsymbol{\sigma}_b \cdot \boldsymbol{q}_b)\,(\boldsymbol{\sigma}_c \cdot \boldsymbol{q}_c)}{(q_b^2 + m_\pi^2)\,(q_c^2 + m_\pi^2)} \middle| j; \right\rangle$$

$$= \sum_h \left\langle i; \middle| \frac{\boldsymbol{\sigma}_c \cdot \boldsymbol{q}_c}{q_c^2 + m_\pi^2} \middle| h; \right\rangle \left\langle h; \middle| \frac{\boldsymbol{\sigma}_b \cdot \boldsymbol{q}_b}{q_b^2 + m_\pi^2} \middle| j; \right\rangle$$

Two-pion exchange term



$$w_{3N}^{(2\pi)} = w_{3N}^{(2\pi;c_1)} + w_{3N}^{(2\pi;c_3)} + w_{3N}^{(2\pi;c_4)}$$

LEC c_1 **LEC c_3** **LEC c_4**

Irreducible-tensor expression (c_1 term)

- The operator $\dfrac{(\boldsymbol{\sigma}_b \cdot \boldsymbol{q}_b)(\boldsymbol{\sigma}_c \cdot \boldsymbol{q}_c)}{(q_b^2 + m_\pi^2)(q_c^2 + m_\pi^2)}$ contains two transferred momenta, $\boldsymbol{q}_b$ and $\boldsymbol{q}_c$, which make calculations cumbersome.

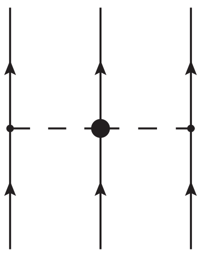
→ A technique to tackle this operator has been suggested previously.

(2) Local interaction in coordinate space with the regulator depending on transferred momenta

P. Navrátil, Few-Body Syst. **41**, 117 (2007).

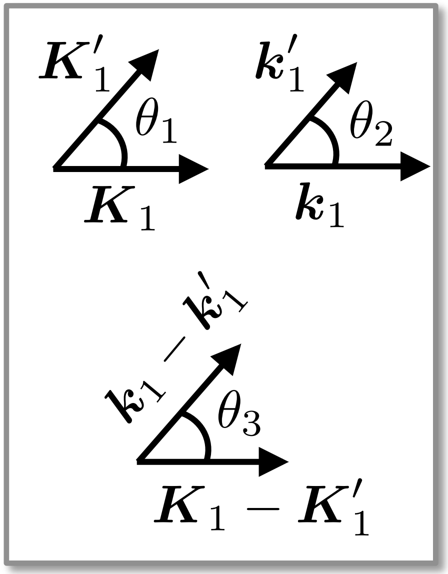
$$U_\nu(q_i, \Lambda_0) = \exp\left[-\left(\frac{q_i}{\Lambda_0}\right)^{2\nu}\right]$$

Two-pion exchange term



Irreducible-tensor expression (c_1 term)

Our approach → **straight forward** and a **brute-force method**



- The operator can be expressed as a function F of $k_1, k'_1, K_1, K'_1, \cos\theta_1, \cos\theta_2, \text{ and } \cos\theta_3$.

Triple-fold multipole expansion

$$\frac{(\boldsymbol{\sigma}_b \cdot \boldsymbol{q}_b)(\boldsymbol{\sigma}_c \cdot \boldsymbol{q}_c)}{(q_b^2 + m_\pi^2)(q_c^2 + m_\pi^2)}$$

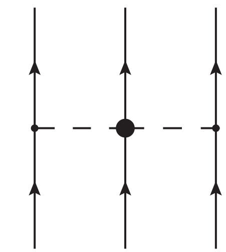
$$= \sum\sum\sum F(k_1, k'_1, K_1, K'_1, \cos\theta_1, \cos\theta_2, \cos\theta_3)$$

$$= \sum\sum\sum (\text{coeff.}) f_{\lambda_1\lambda_2\lambda_3}(k_1, k'_1, K_1, K'_1)$$ **Multipole-expansion function**

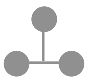
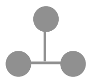
23 summations

$$\times \left[\left[\sigma_1(b) \otimes \left[Y_{\mathcal{L}_1}\left(\hat{\boldsymbol{k}}_1\right) \otimes Y_{\mathcal{L}_2}\left(\hat{\boldsymbol{k}}'_1\right)\right]_{L_2}\right]_{L_1} \otimes \left[\sigma_1(c) \otimes \left[Y_{\mathcal{L}_3}\left(\hat{\boldsymbol{K}}_1\right) \otimes Y_{\mathcal{L}_4}\left(\hat{\boldsymbol{K}}'_1\right)\right]_{L_3}\right]_{L_1}\right]_{00}$$

Two-pion exchange term



c_1 term

$\langle$  $\left| W_{3N}^{(2\pi;c_1)} \right|$  $\rangle$

$$= \sum\sum\sum \,(\text{coeff.}) \quad \ni \; c_1\text{, ten } 3j\text{, eight } 6j\text{, five } 9j \text{ symbols, etc.}$$

19 summations

$$\times \iiiint dk_1 dK_1 dk_1' dK_1' \, k_1^{2-\lambda_b-\lambda_b'+\lambda_3-\lambda_3'} \, k_1'^{\lambda_b'+\lambda_3'+1} \, K_1^{2-\lambda_c+\lambda_b-\lambda_b''+\lambda_3-\lambda_3''} \, K_1'^{\lambda_c+\lambda_b''+\lambda_3''+1}$$

$$\times f_{\lambda_1\lambda_2\lambda_3}(k_1, k_1', K_1, K_1') P_{n_{12}l_{12}}(k_1) \, P_{n_{12}'l_{12}'}(k_1') \, P_{nl}(K_1) \, P_{n'l'}(K_1') \, u_\nu\,(k_1, K_1, \Lambda) \, u_\nu\,(k_1', K_1', \Lambda)$$

c_3 and c_4 terms

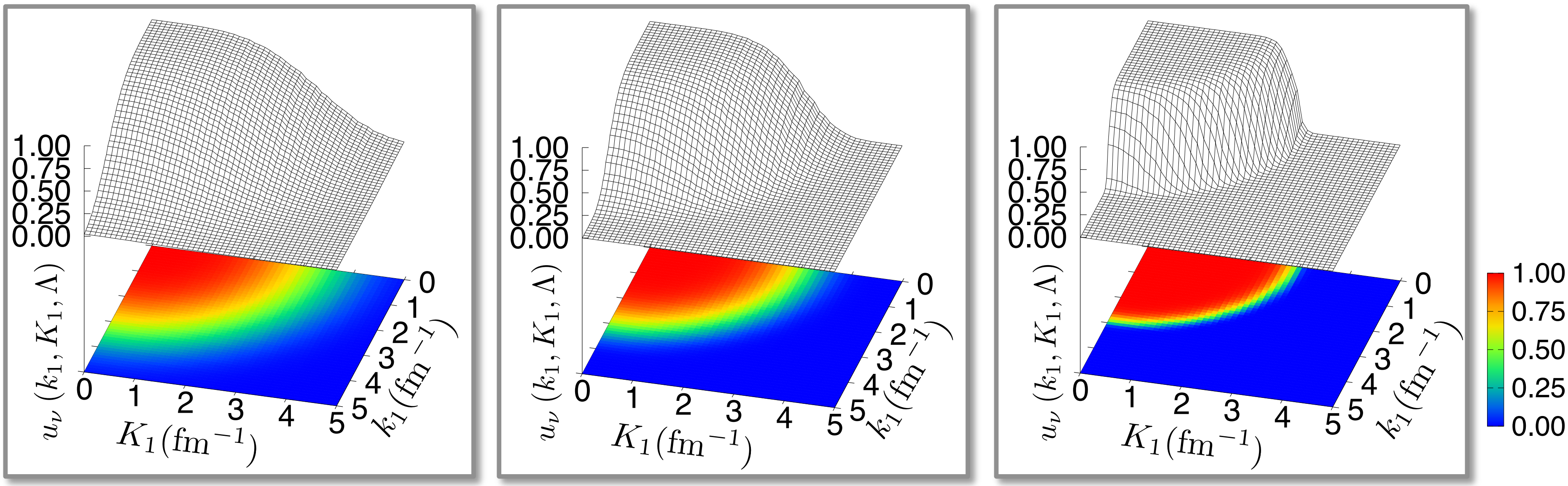
- c_3 term: an additional summation
- c_4 term: two additional summations

Three sets of the regulator and LECs

L. Coraggio *et al.*, Phys. Rev. C **89**, 044321 (2014).

$$u_\nu\left(k_1, K_1, \Lambda\right) = \exp\left[-\left(\frac{k_1^2 + K_1^2}{2\Lambda^2}\right)^\nu\right]$$

$\Lambda = 500\ \mathrm{MeV}, \nu = 2$ | $\Lambda = 450\ \mathrm{MeV}, \nu = 3$ | $\Lambda = 414\ \mathrm{MeV}, \nu = 10$



R. Machleidt and D.R. Entem, Phys. Rep. **503** 1 (2011).

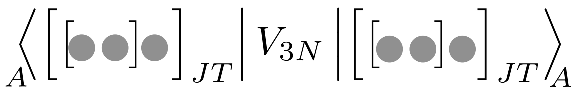
L. Coraggio *et al.*, Phys. Rev. C **87**, 014322 (2013).

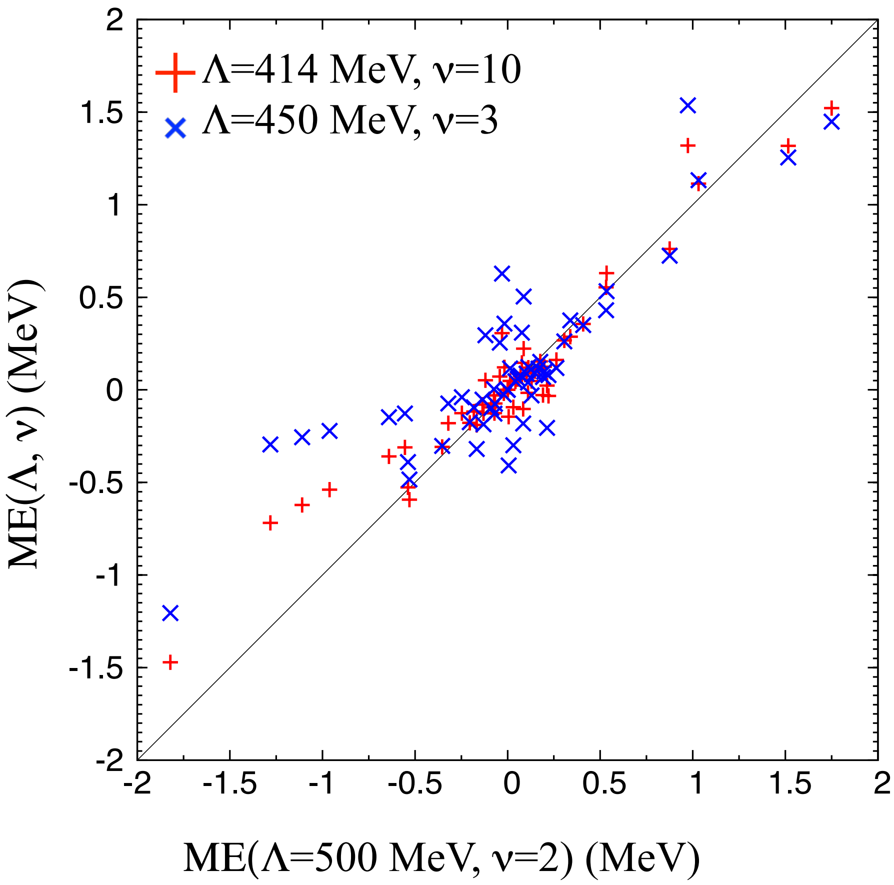
L. Coraggio *et al.*, Phys. Rev. C **75**, 024311 (2007).

- The value of the LECs c_D and c_E are determined from the **^{3}H and ^{3}He binding energy** and their **Gamow-Teller MEs**.

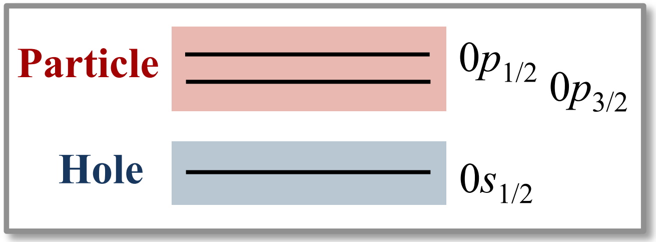
p-shell

JT-coupled ME

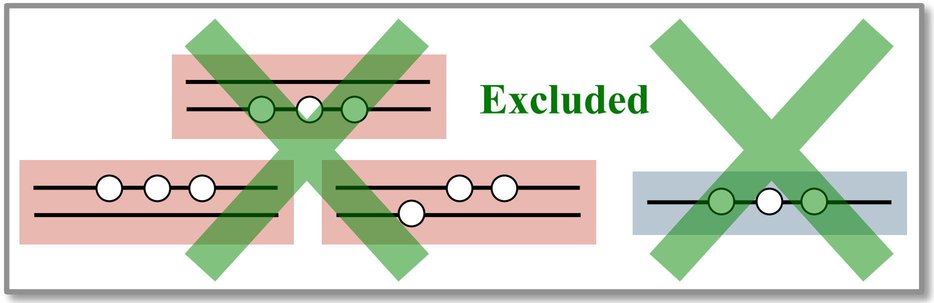




Model space $\hbar\omega = 19$ MeV



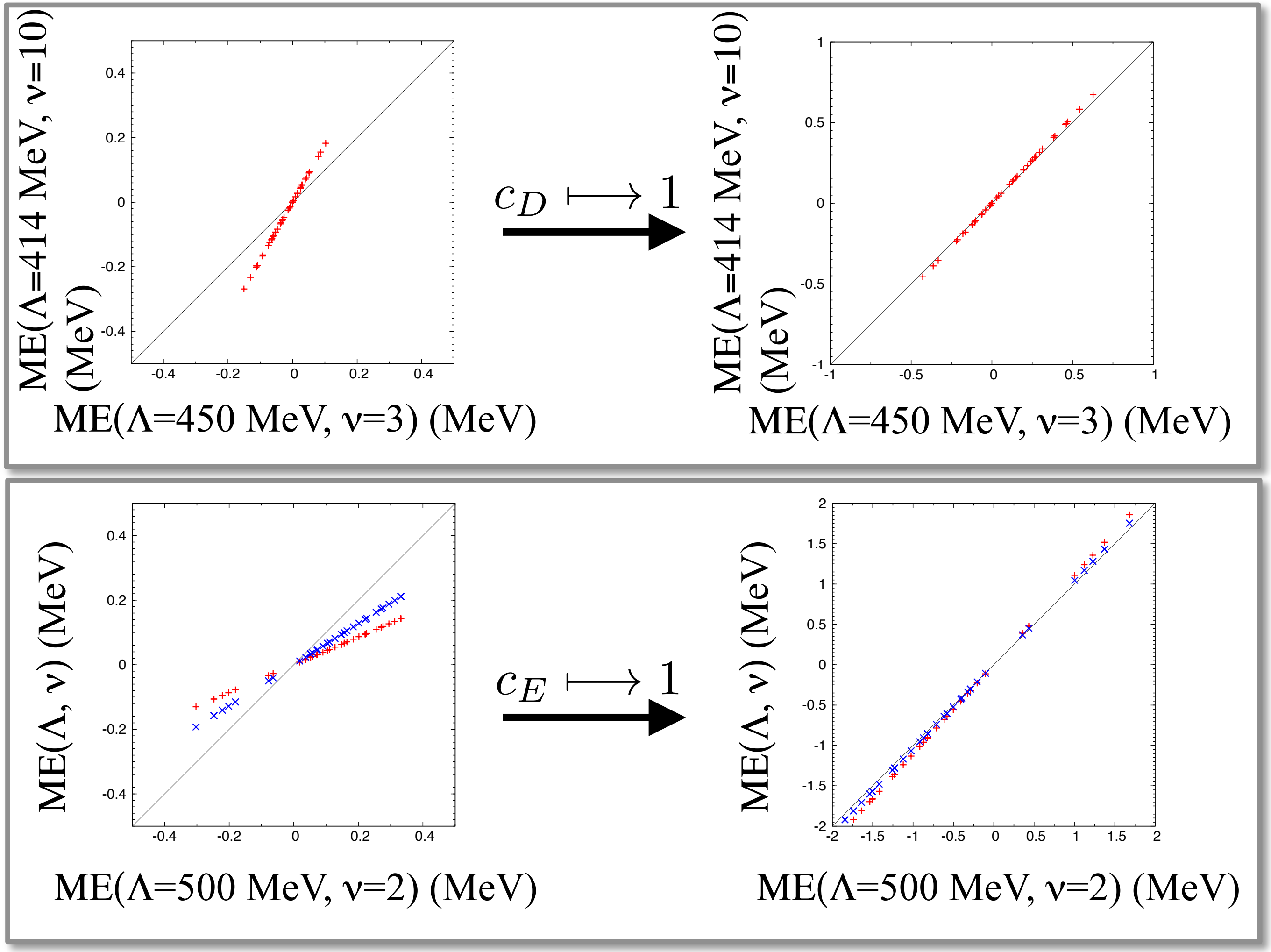
First-order approximation



Result | Correlation plot of 3-body MEs

1-pion + contact

Contact

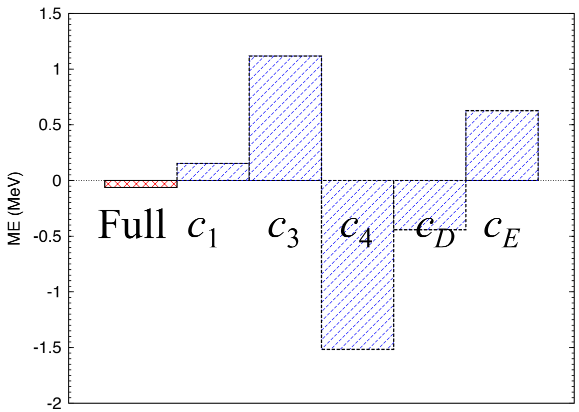


- The difference originates from LECs rather than the regulator itself for the **short range terms**.
- High sensitivity on the regulator for the **two-pion exchange term**.

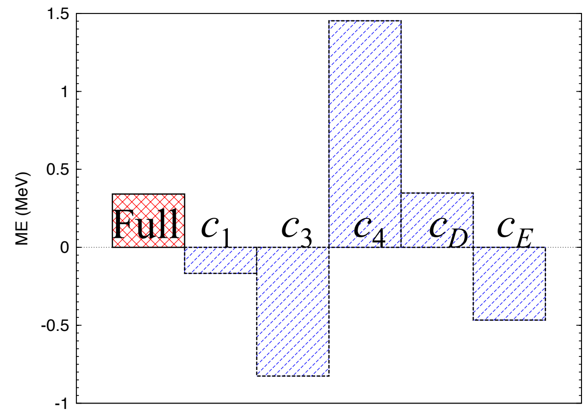
A few examples of MEs $_A\left\langle \left[[a'b']_{J'_{12}T'_{12}} c'\right]_{JT} \right| V_{3N} \left| \left[[ab]_{J_{12}T_{12}} c\right]_{JT} \right\rangle_A$

- The c_4 MEs play largest contribution, almost universally.

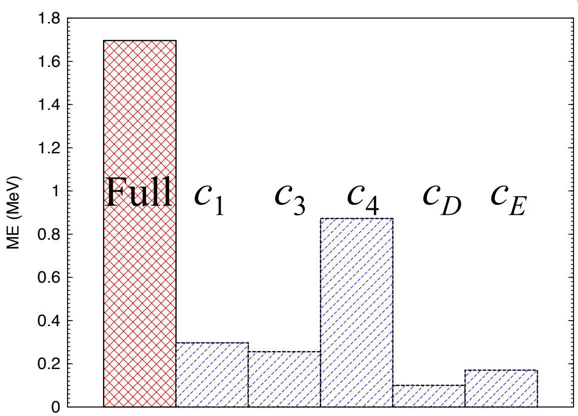
$$_A\left\langle \left[[s_{\frac{1}{2}}s_{\frac{1}{2}}]_{01}p_{\frac{3}{2}}\right]_{\frac{3}{2}\frac{3}{2}} \right| V_{3N} \left| \left[[s_{\frac{1}{2}}s_{\frac{1}{2}}]_{01}p_{\frac{3}{2}}\right]_{\frac{3}{2}\frac{3}{2}} \right\rangle_A$$



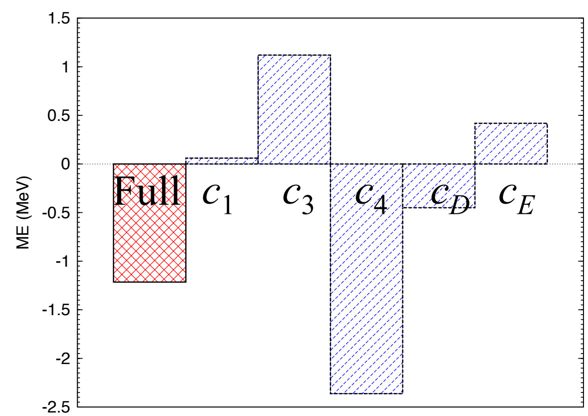
$$_A\left\langle \left[[s_{\frac{1}{2}}s_{\frac{1}{2}}]_{01}p_{\frac{3}{2}}\right]_{\frac{3}{2}\frac{3}{2}} \right| V_{3N} \left| \left[[s_{\frac{1}{2}}s_{\frac{1}{2}}]_{10}p_{\frac{3}{2}}\right]_{\frac{3}{2}\frac{3}{2}} \right\rangle_A$$



$$_A\left\langle \left[[s_{\frac{1}{2}}p_{\frac{3}{2}}]_{11}s_{\frac{1}{2}}\right]_{\frac{1}{2}\frac{1}{2}} \right| V_{3N} \left| \left[[s_{\frac{1}{2}}s_{\frac{1}{2}}]_{10}p_{\frac{3}{2}}\right]_{\frac{1}{2}\frac{1}{2}} \right\rangle_A$$



$$_A\left\langle \left[[s_{\frac{1}{2}}p_{\frac{3}{2}}]_{11}s_{\frac{1}{2}}\right]_{\frac{1}{2}\frac{1}{2}} \right| V_{3N} \left| \left[[s_{\frac{1}{2}}p_{\frac{3}{2}}]_{11}s_{\frac{1}{2}}\right]_{\frac{1}{2}\frac{1}{2}} \right\rangle_A$$



A few examples of MEs $\left\langle \left[\left[a'b' \right]_{J'_{12}T'_{12}} c' \right]_{JT} \middle| V_{3N} \middle| \left[\left[ab \right]_{J_{12}T_{12}} c \right]_{JT} \right\rangle_A$

- The c MEs play largest contribution, almost universally

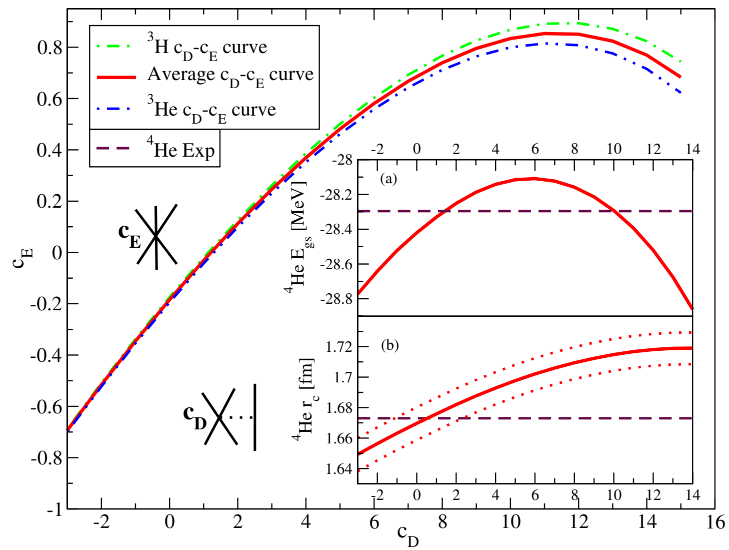
- LECs

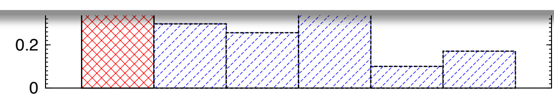
$$c_1 = -0.81 \text{ GeV}^{-1}, \; c_3 = -3.20 \text{ GeV}^{-1}, \; c_4 = 5.40 \text{ GeV}^{-1}$$

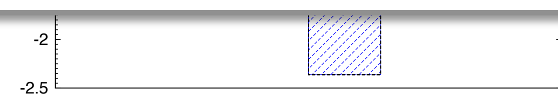
D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003).

$$c_D = -1.0, \; c_E = -0.34$$

P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007).







Result | Framework of RSM calculations

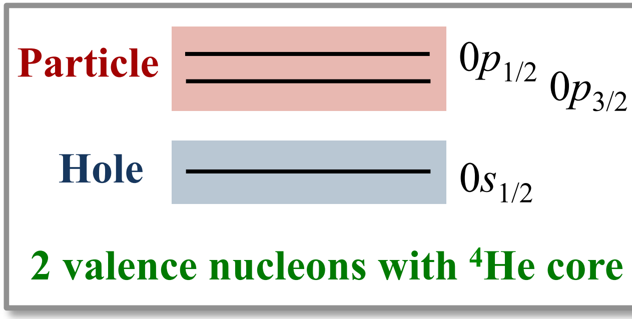
No empirical input

Interactions and LECs

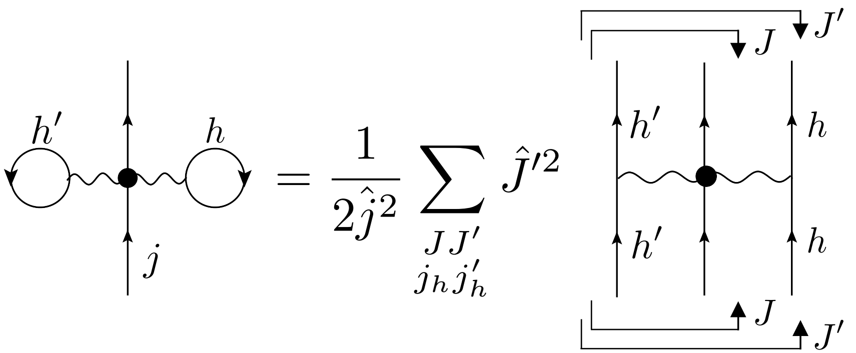
2NF: Chiral EFT N^{3}LO, **3NF**: Chiral EFT N^{2}LO

D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003).
P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007).

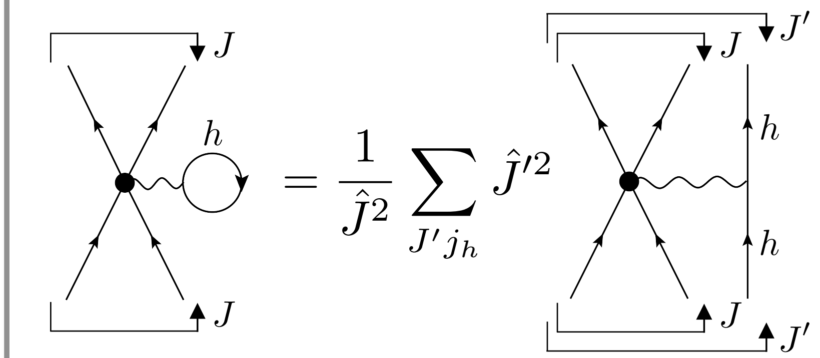
Model space $\hbar\omega = 19$ MeV



Normal-ordered SPE (1st order)



Normal-ordered 2BME (1st order)



Many-body perturbation theory

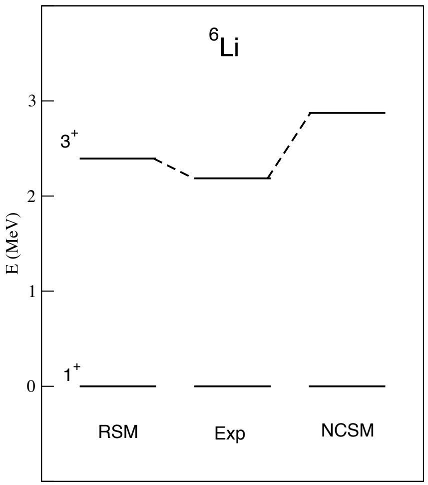
2NF: Up to the 3rd-order of the folded-diagram expansion
3NF: Up to the 1st-order, at this moment

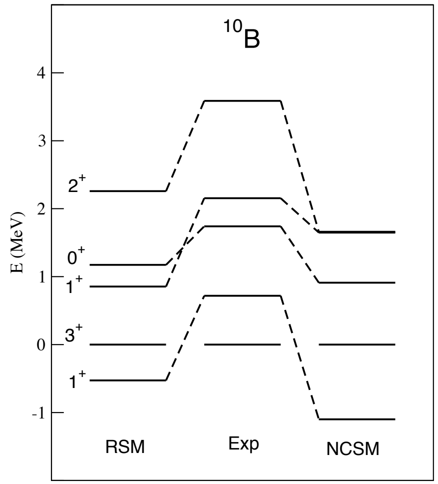
Renormalization

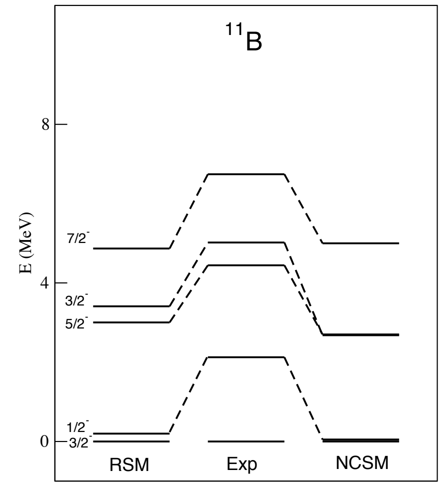
Our realistic forces are **NOT** renormalized.

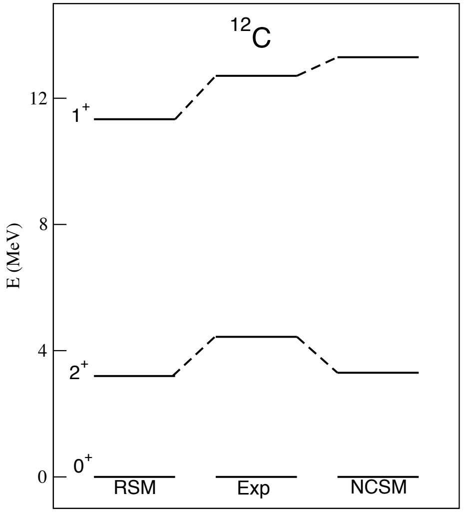
2NF only (very preliminary)

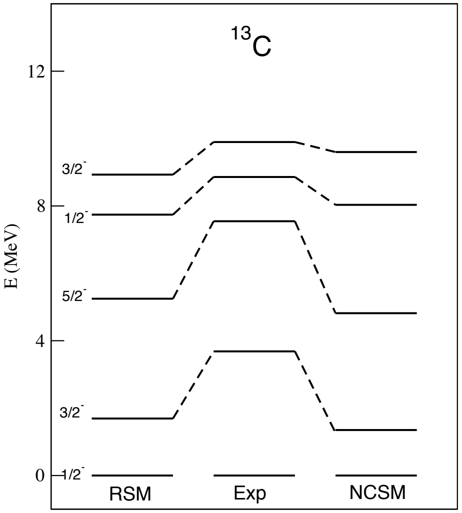
- Comparison with no-core shell model (NCSM) P. Navrátil *et al*., Phys. Rev. Lett. **99**, 042501 (2007).





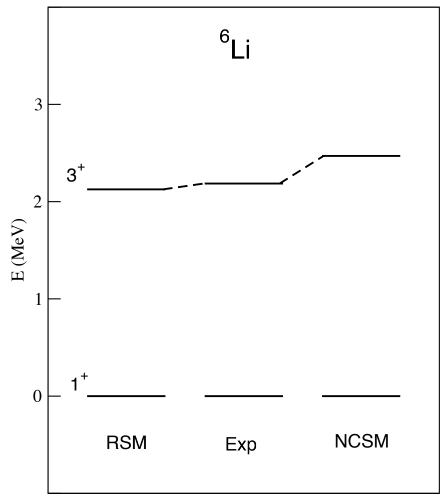


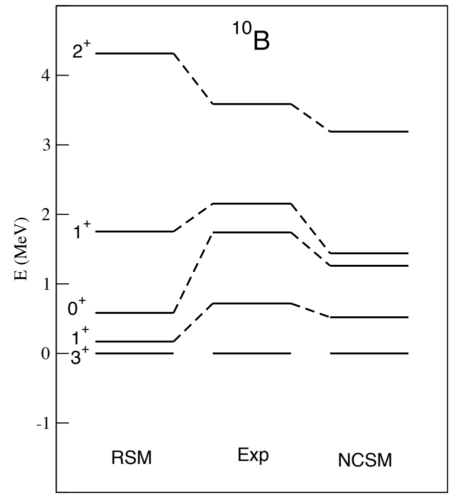


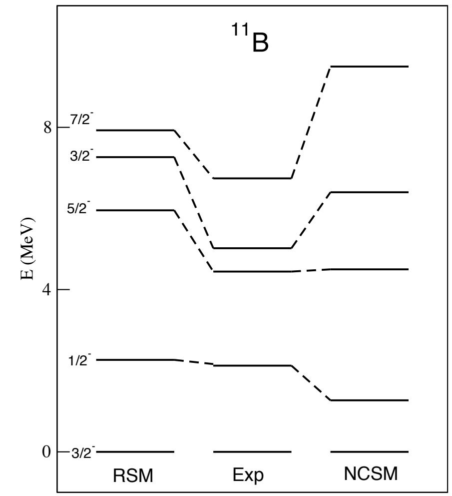


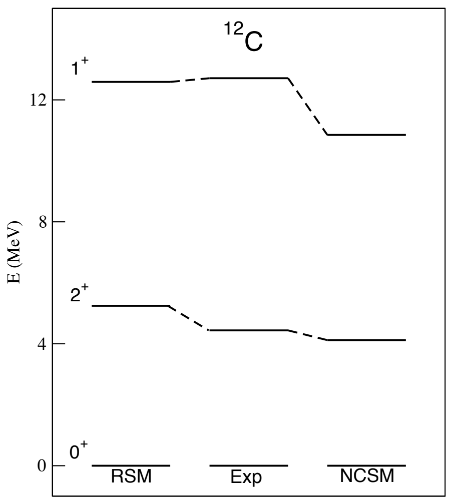
2NF + 3NF (very preliminary)

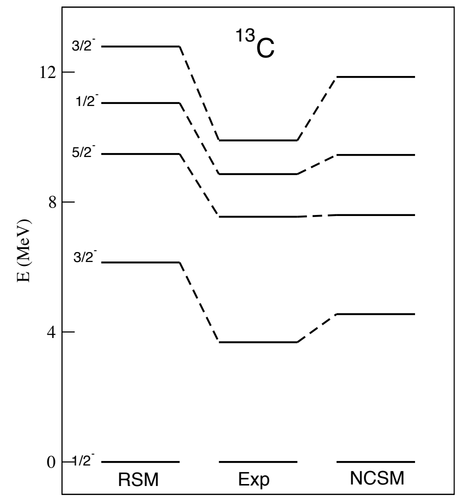
- Comparison with no-core shell model (NCSM) P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007).





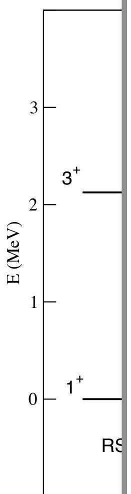


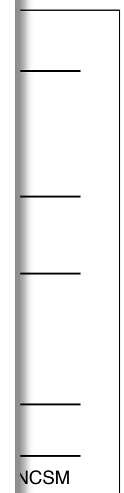




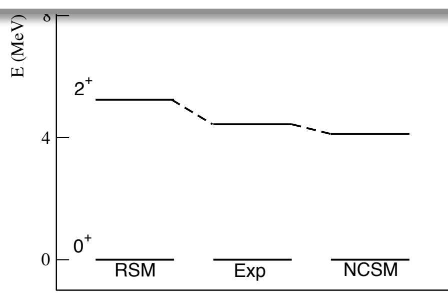
2NF + 3NF (very preliminary)

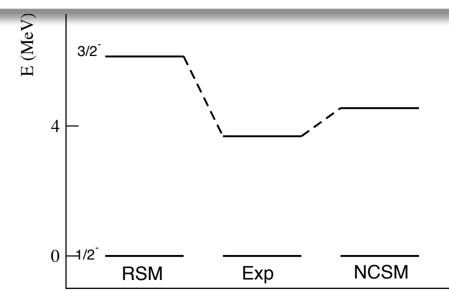
- Comparison with no-core shell model (NCSM) P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007).





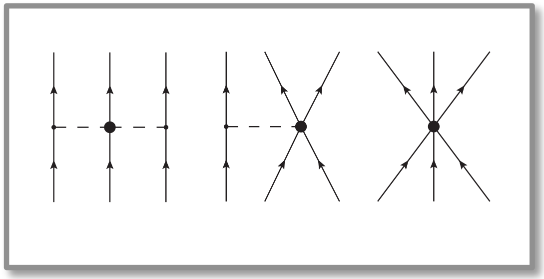
- Comparison between RSM and NCSM are satisfactory for the **2NF** only.
- Comparison between SM and NCSM are satisfactory for **low-lying spectra** when including **3NF**, less satisfactory for **higher-energy states**.
- RSM calculations with three-body forces are **very preliminary**, there are more correlations to be considered:
  → Include contributions beyond the first order.
  → Include the many-body correlations induced by the 3NF.



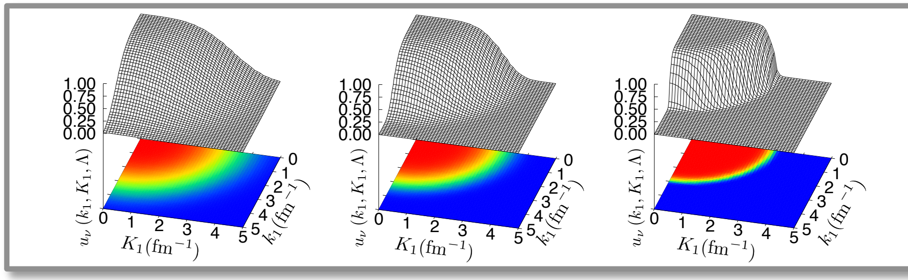


Summary

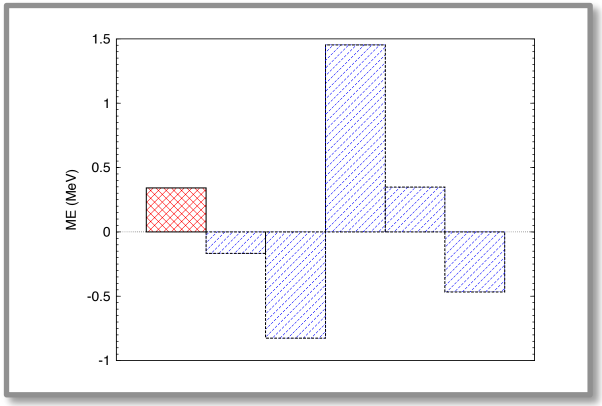
Chiral 3NF at N^2LO



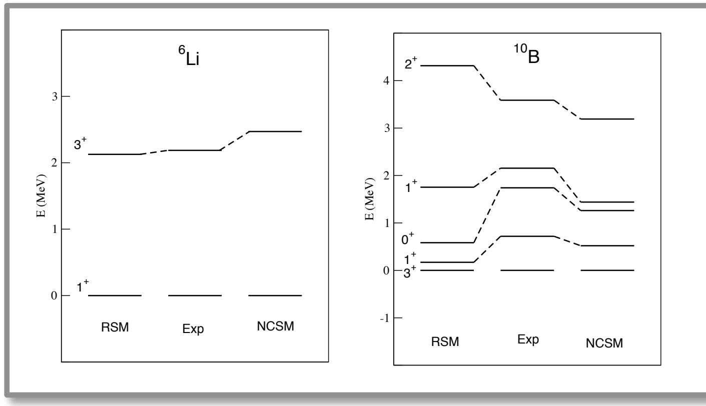
Cutoff and LEC dependence



Contribution of each term



Preliminary spectra



Future plan

- Include **beyond the first-order** contribution of 3NF.
- Include the **many-body correlations** induced by the 3NF.
- Tailor our numerical code for hybrid (MPI + OpenMP) calculations.