Current Status of the Δ-full Chiral Nuclear Forces

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in collaboration with

H. Krebs and E. Epelbaum

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Outline

- Introduction&Motivation
- 2-N forces with explicit Δ
- 3-N forces with explicit Δ (2-π–exchange topology)
- Summary and Outlook

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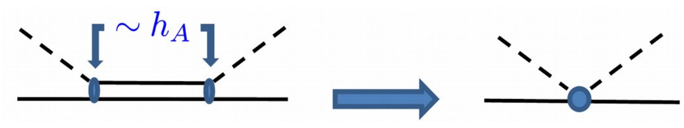
  Hemmert, Holstein, Kambor '98
- Δ gives a large contribution to LECs (c_i) via resonance saturation
  Bernard, Kaiser, Meißner '97



$h_A \approx 1.34$ large!

Δ–resonance saturation of the πN LECs

Krebs, AG, Epelbaum, to appear soon

$$c_1(\Delta) \;=\; 0, \quad c_2(\Delta) \;=\; \frac{4\,h_A^2}{9\,\Delta}, \quad c_3(\Delta) \;=\; -\frac{4\,h_A^2}{9\,\Delta}, \quad c_4(\Delta) \;=\; \frac{2\,h_A^2}{9\,\Delta}$$

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Fits to KH PWA, Koch' 86

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Q^4, KH PWA	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26
$\epsilon^3 + Q^4$, KH PWA	-0.85	0.44	-1.91	1.49	2.07	-2.45	0.66	-3.86	-0.12	-7.05	3.39	-0.38	2.85
Δ-resonance saturation contribution	0	2.81	-2.81	1.40	2.39	-2.39	0	-4.77	1.87	-4.15	4.15	-0.17	1.32
level of Δ-resonance saturation, %	0	81	59	42	39	35	0	40	123	40	68	46	41

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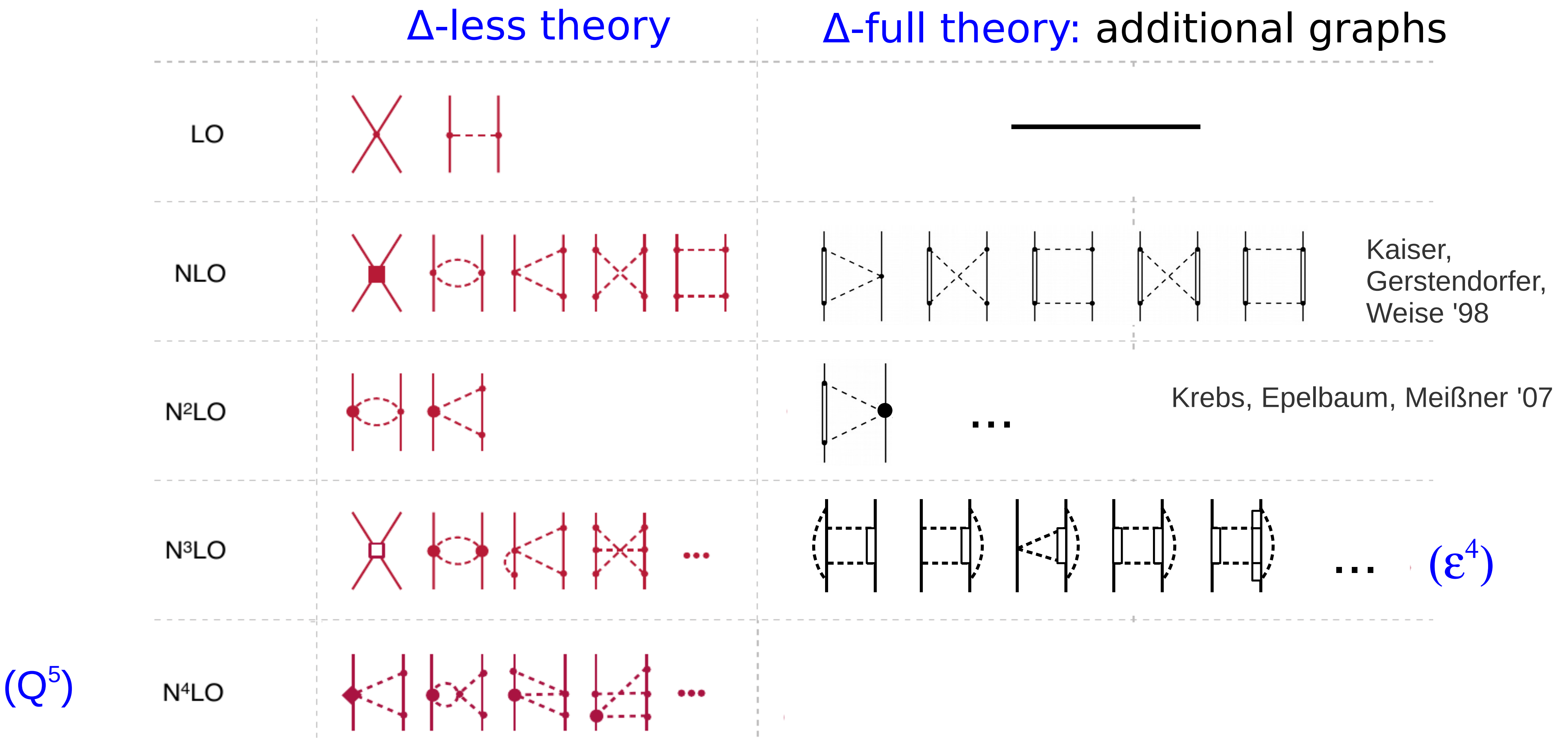
Small scale expansion of 2NF



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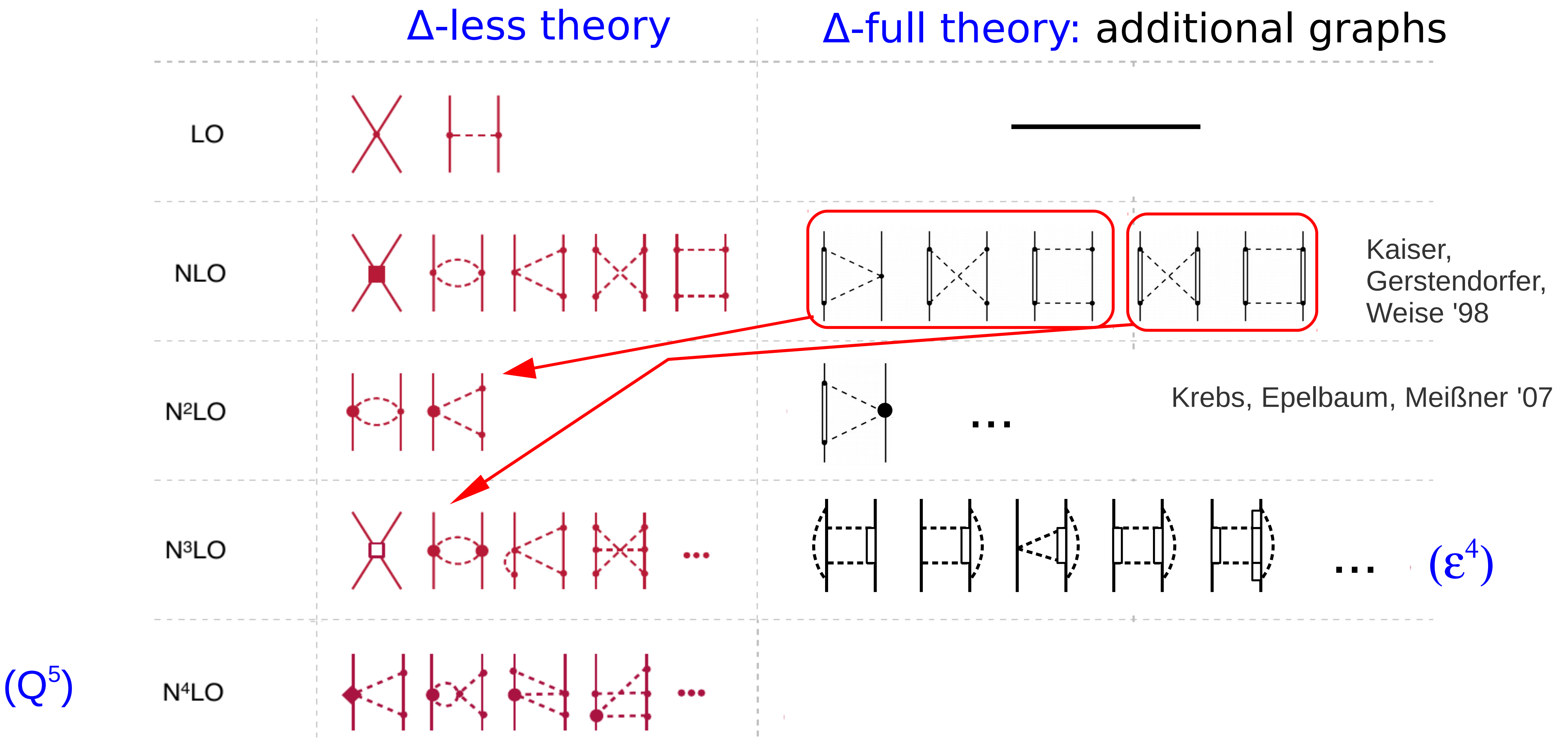

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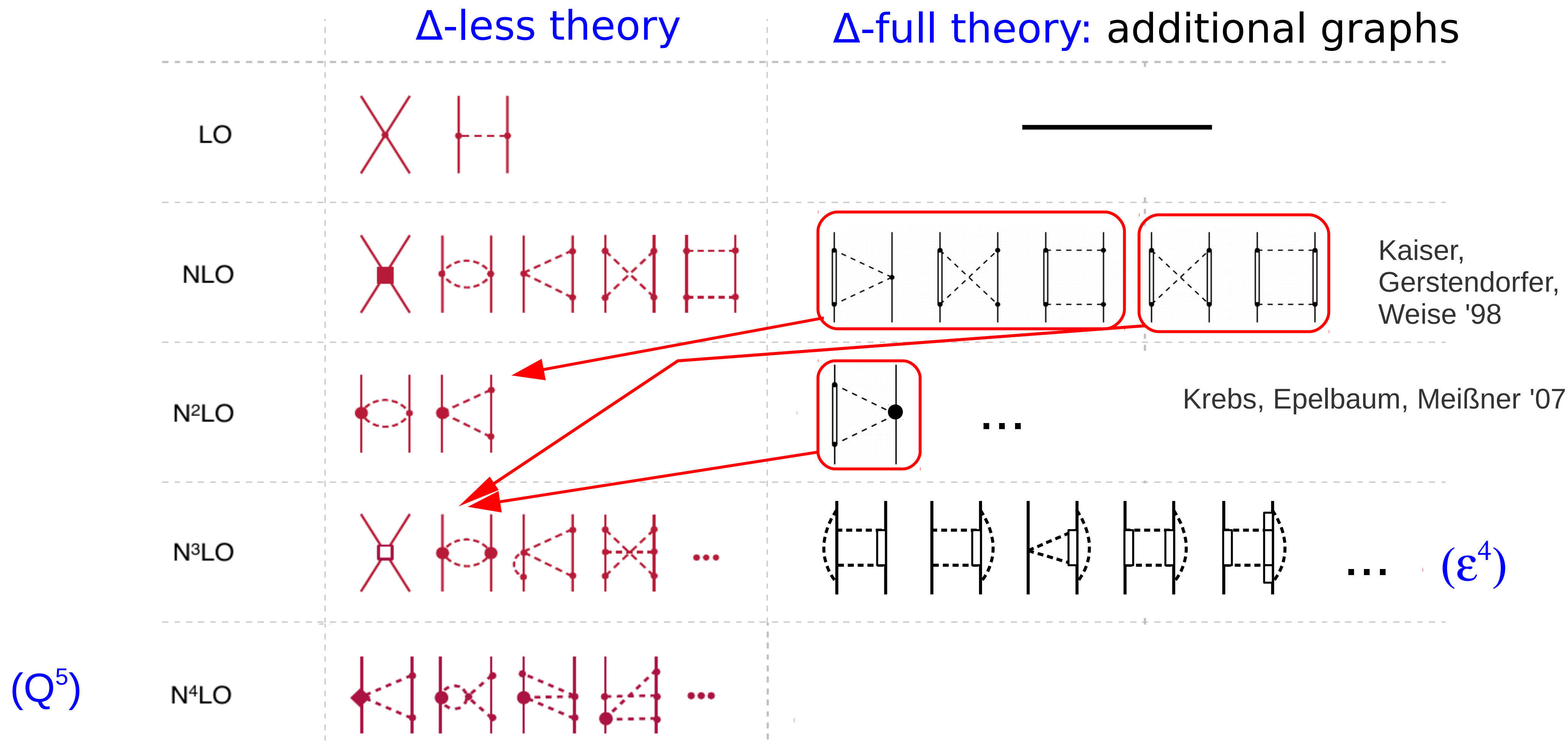

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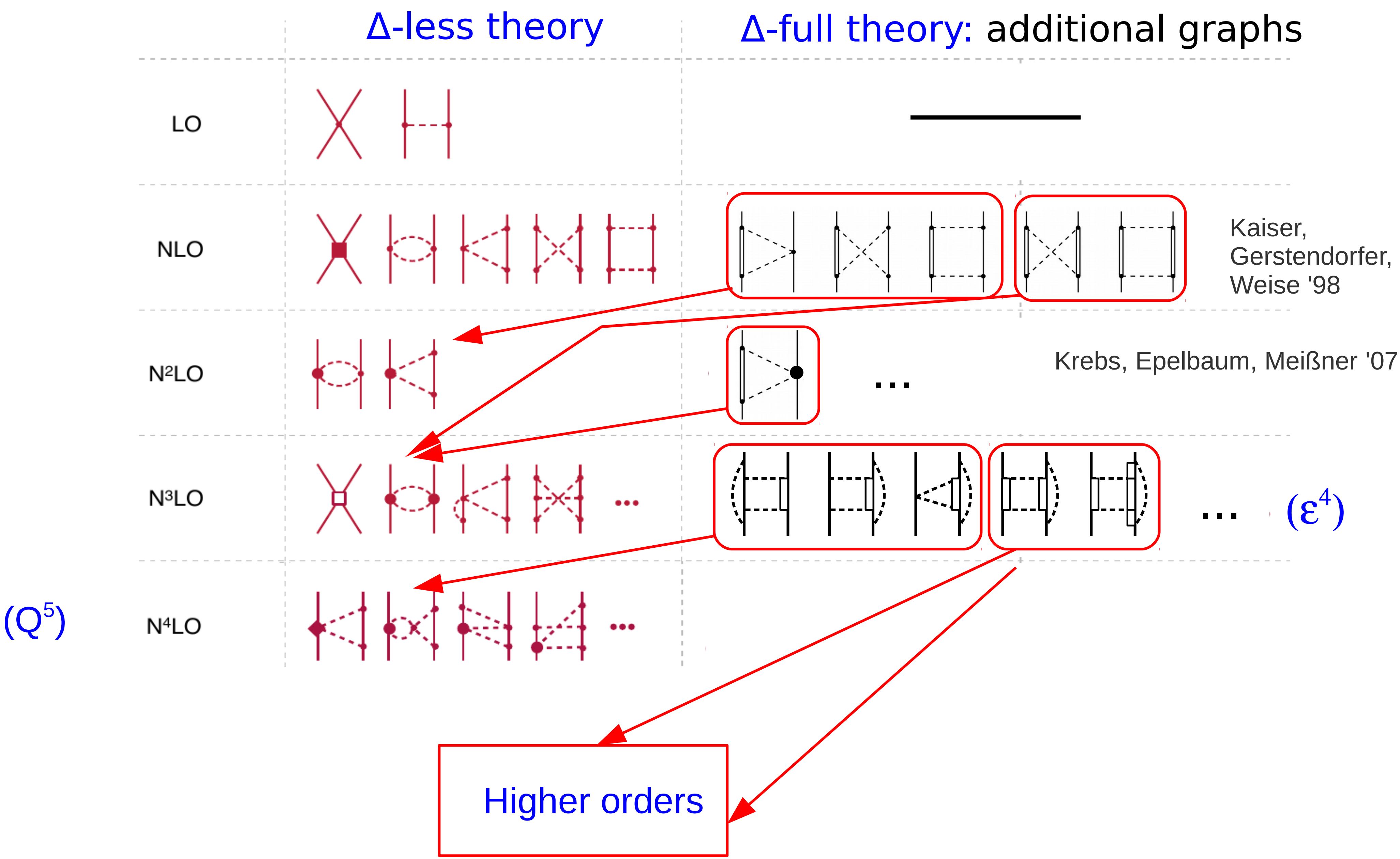


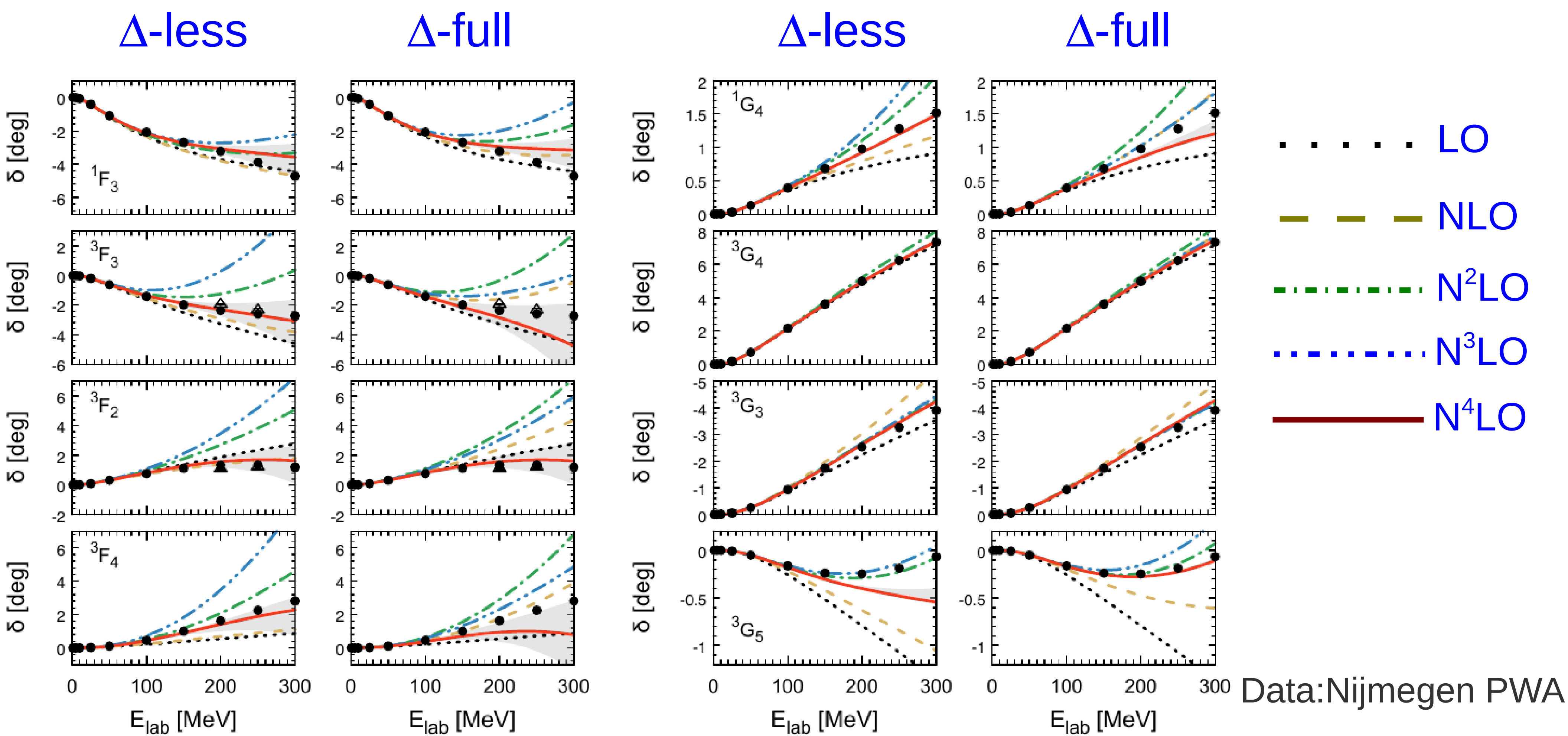
N^3LO 2N-forces with explicit Δ

- Only 2-pion-exchange contribution are considered (the long range part)
- $1/m_N$ corrections to NLO diagrams are included
- Results for peripheral phases, no refitting of LEC's
- No additional parameters, h_A and g_1 ($\pi N\Delta$ and $\pi\Delta\Delta$) are extracted from the fit to πN scattering
- Sensitivity to the choice of c_i is studied

F and G waves

LECs from fit to KH PWA

Bands:0.5 GeV<cut off<1.5 GeV



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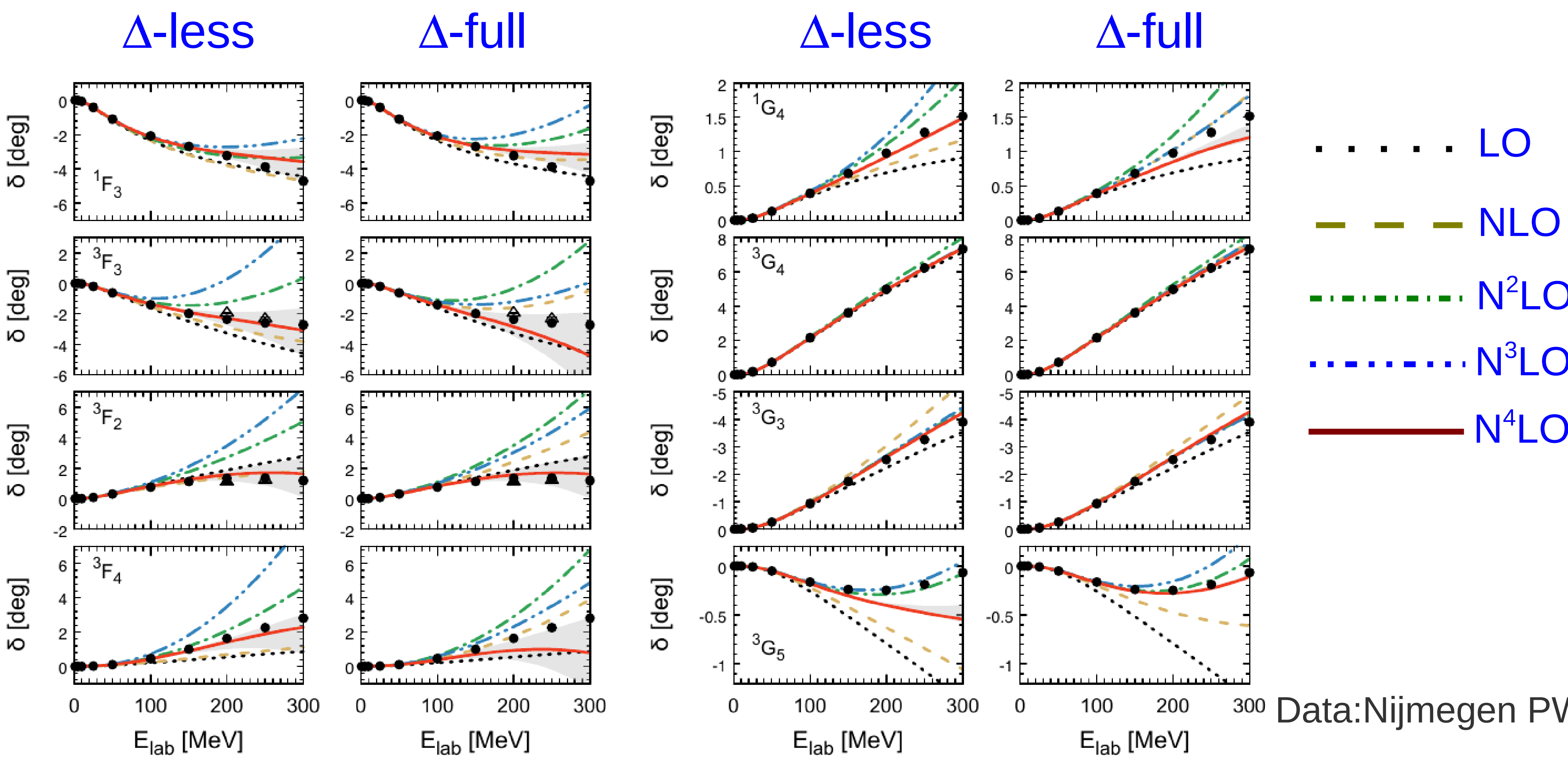
Comparable discription of the data in Δ-less and Δ-full case at N⁴LO



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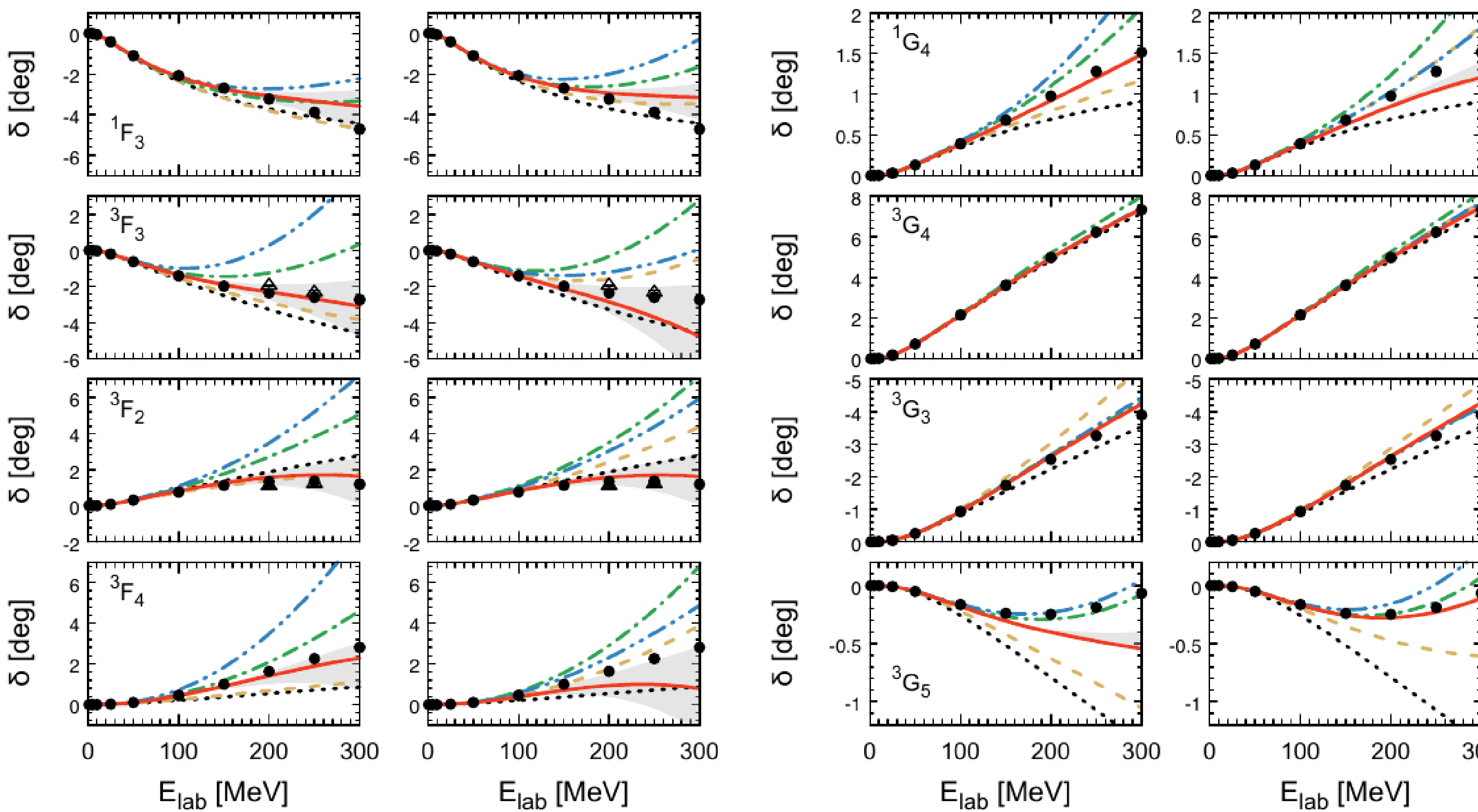
F-waves are sensitive to the short-range physics

Δ-less Δ-full Δ-less Δ-full



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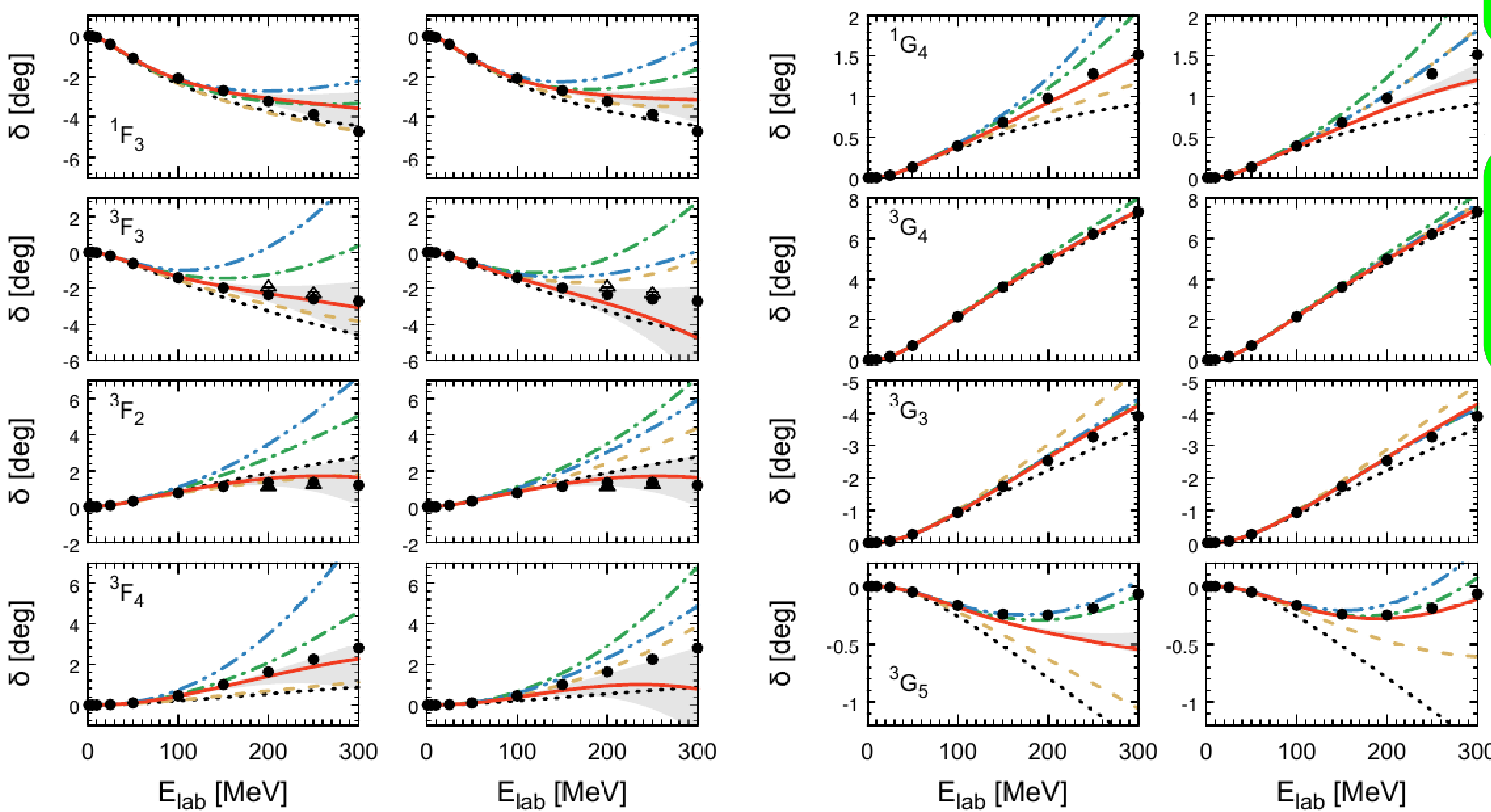


. . . . . . LO

Convergence of chiral expansion is slightly better in Δ-full case

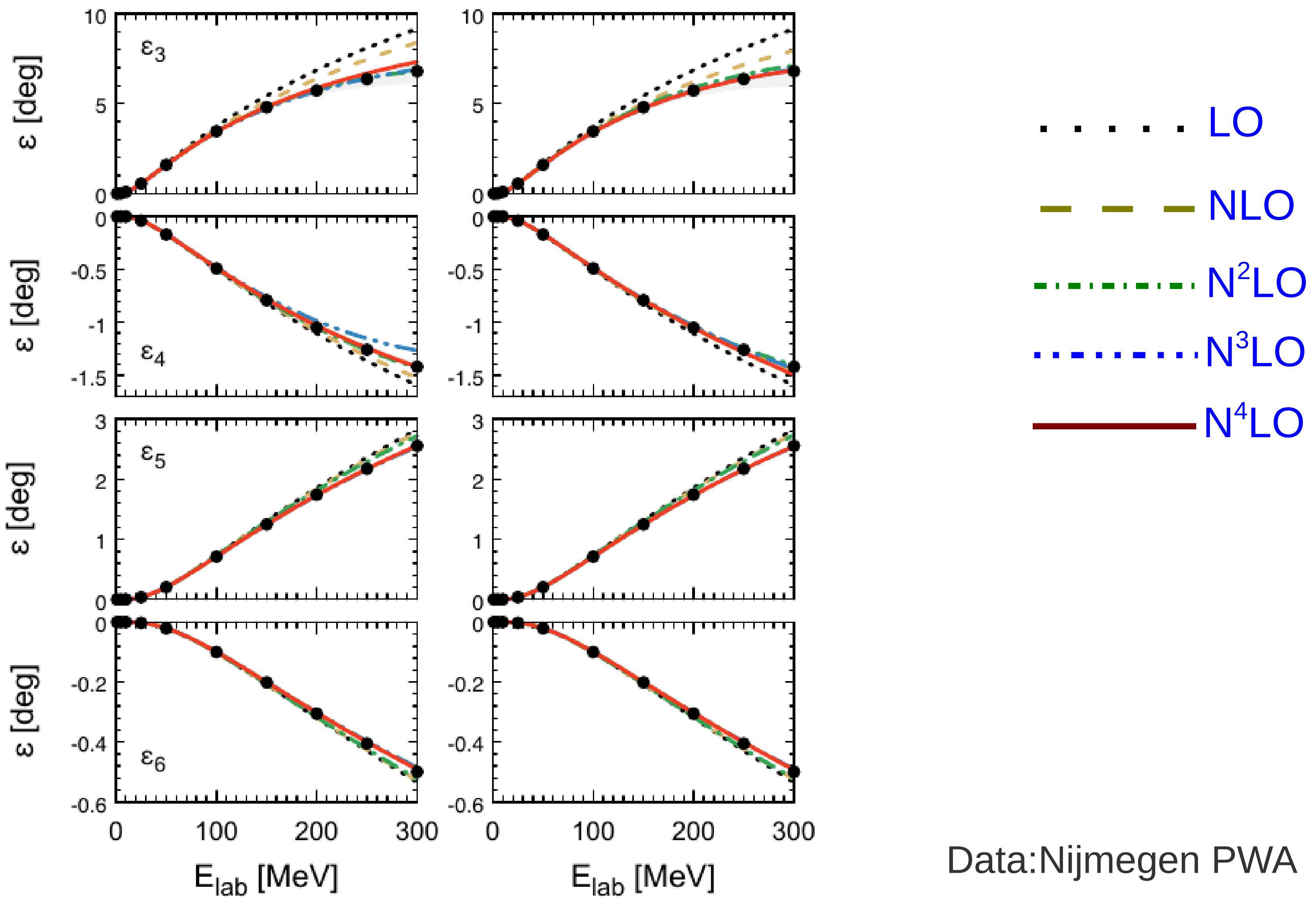
N^{4}LO

Data:Nijmegen PWA

Mixing angles ε_3, ε_4, ε_5, ε_6

LECs from fit to KH PWA

Δ-less Δ-full



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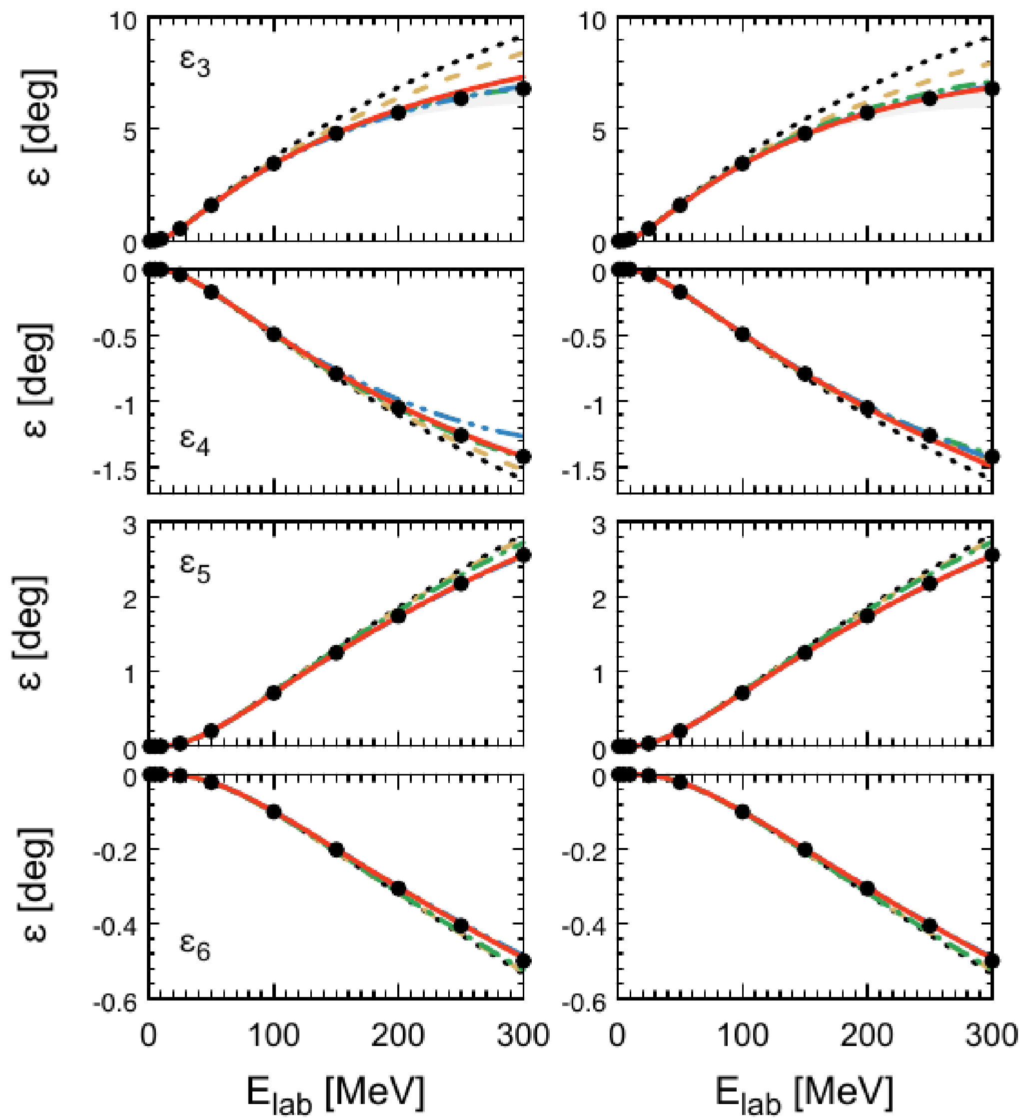
Δ-less   Δ-full



Comparable description of the data in Δ-less and Δ-full case at N^4LO

N^2LO

N^3LO

N^4LO

Data:Nijmegen PWA

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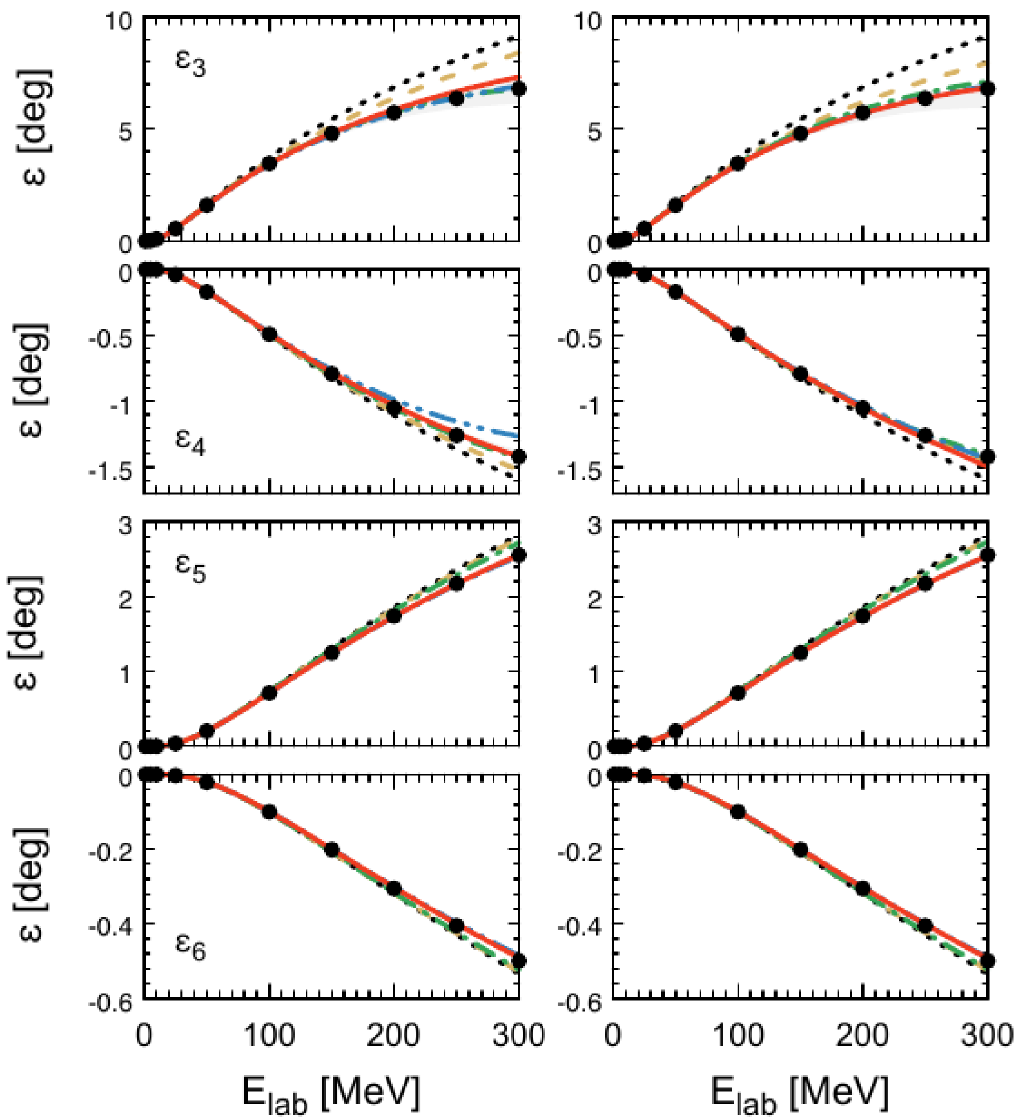
LECs from fit to KH PWA

Δ-less　　Δ-full



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H and I waves

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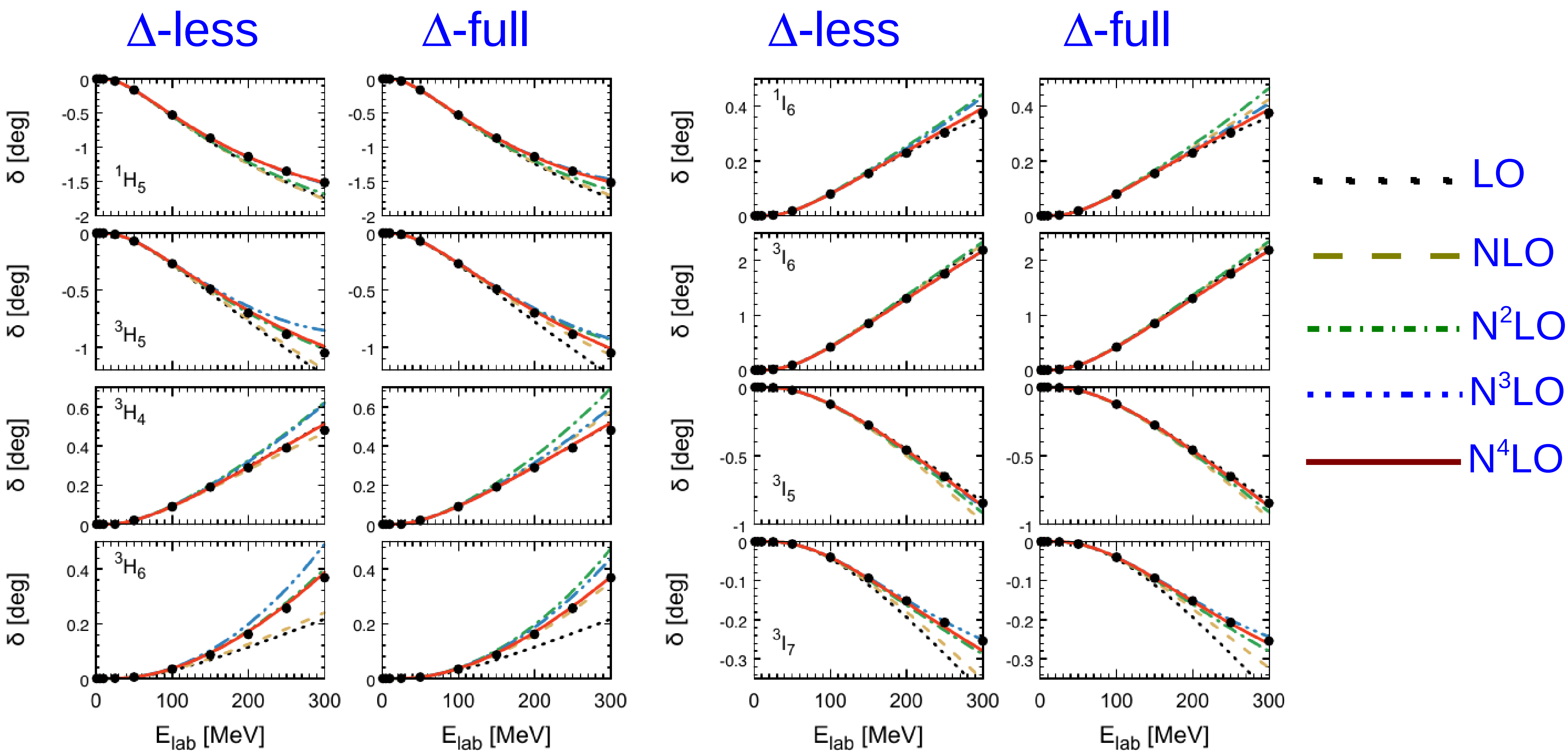

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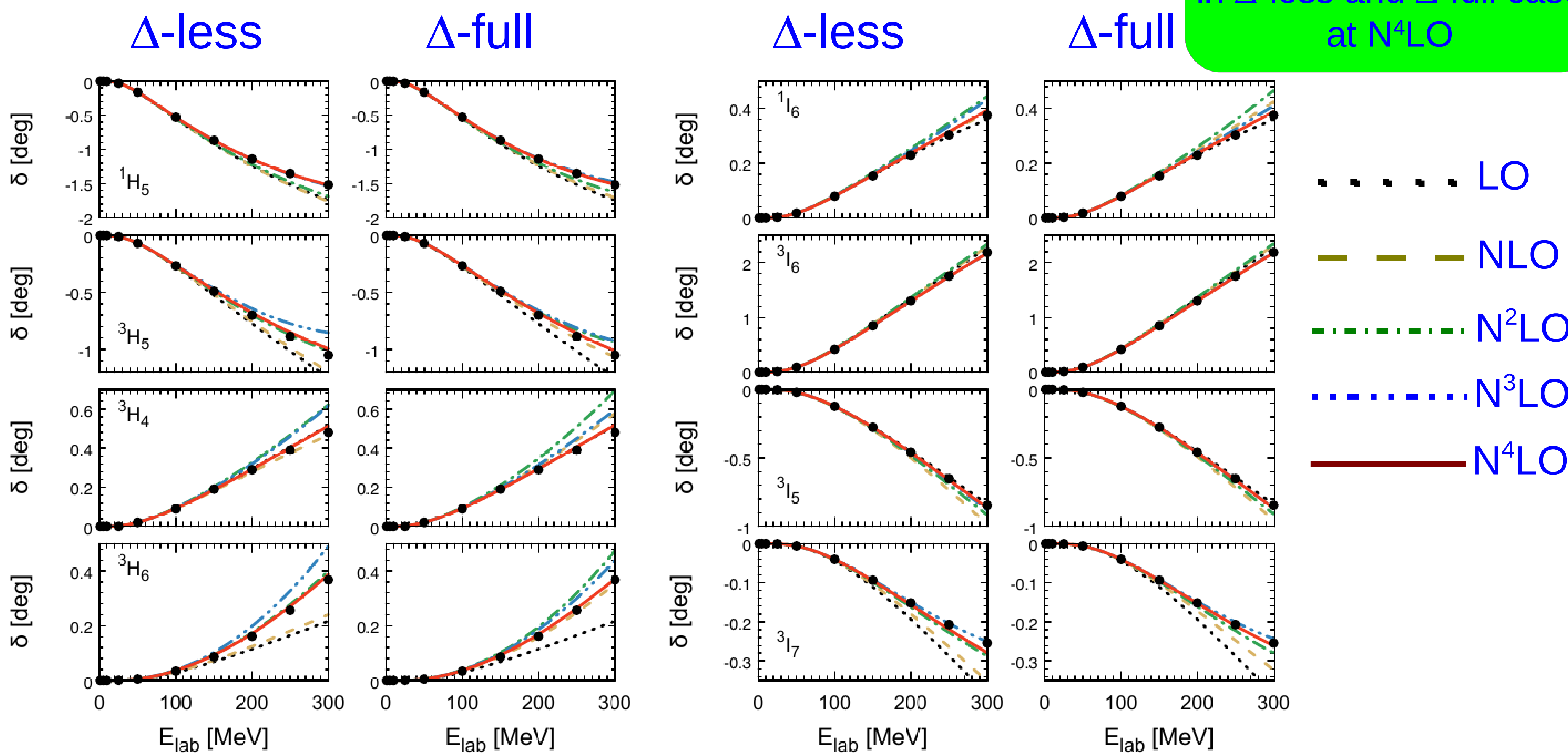

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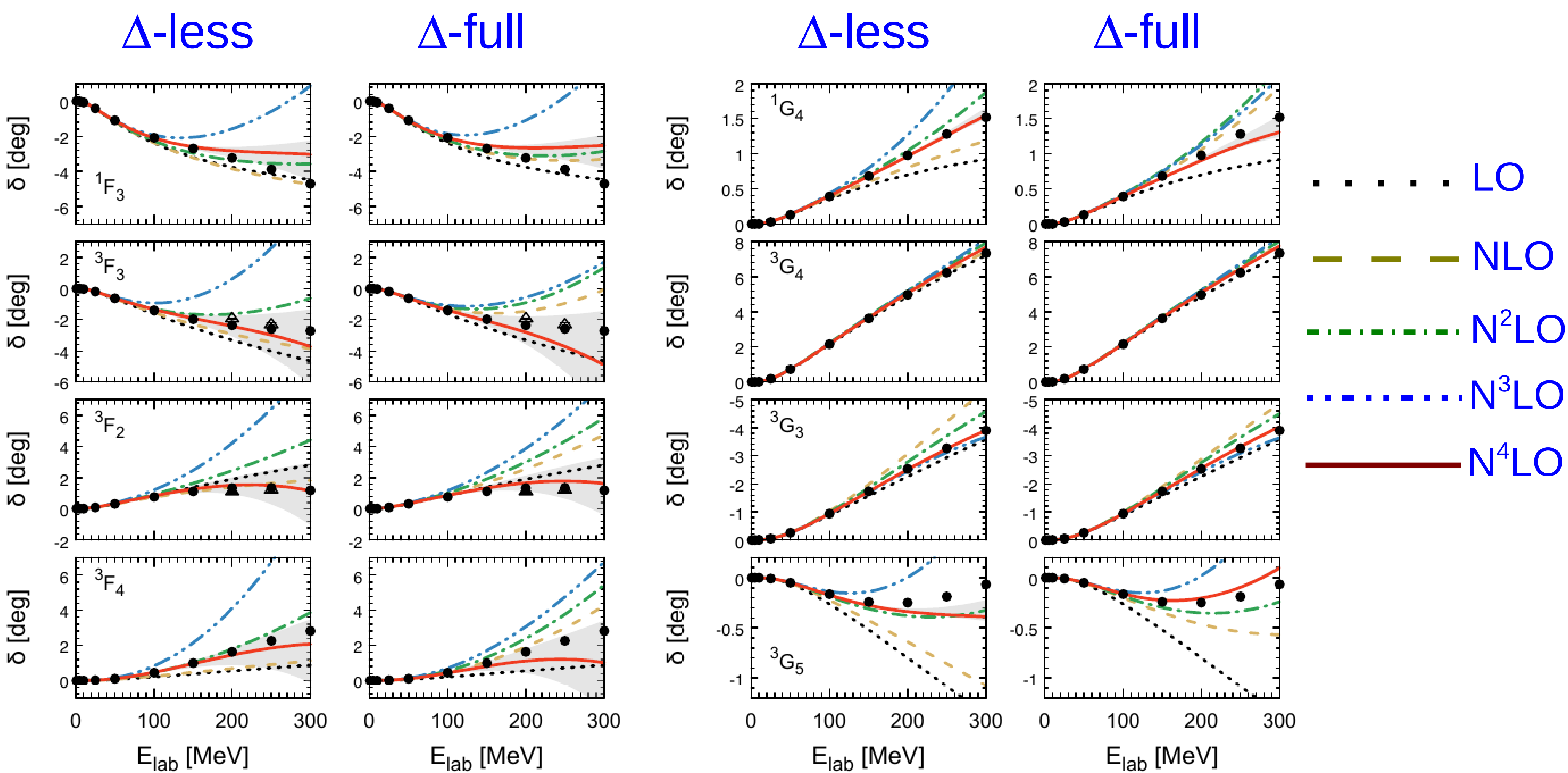
F and G waves

LECs from Roy-Steiner Equation (subthreshold expansion)

Siemens et al. '17



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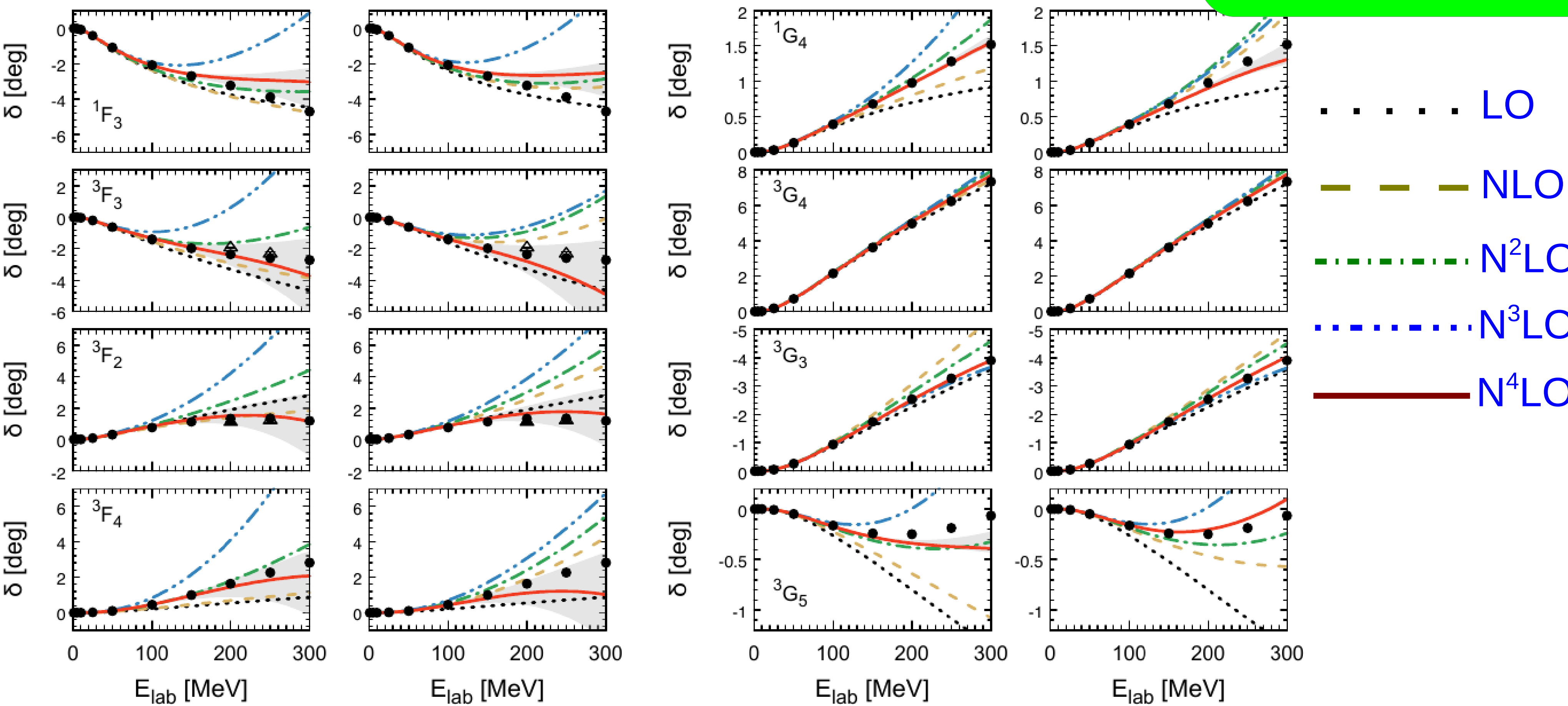
Description
of the data
similar to the one with
KH LECs

Δ-less Δ-full Δ-less Δ-ful



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Mixing angles ε_3, ε_4, ε_5, ε_6

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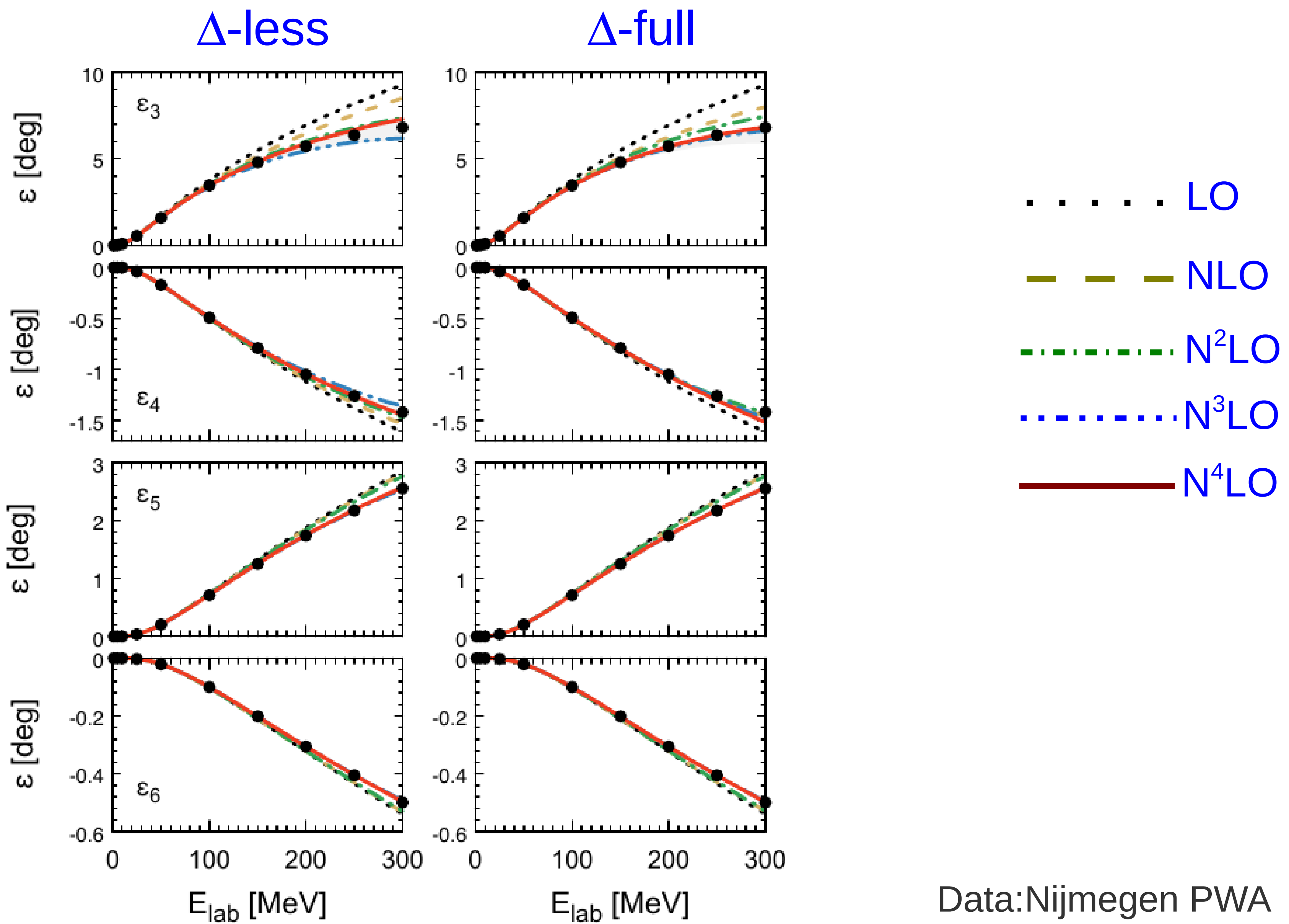

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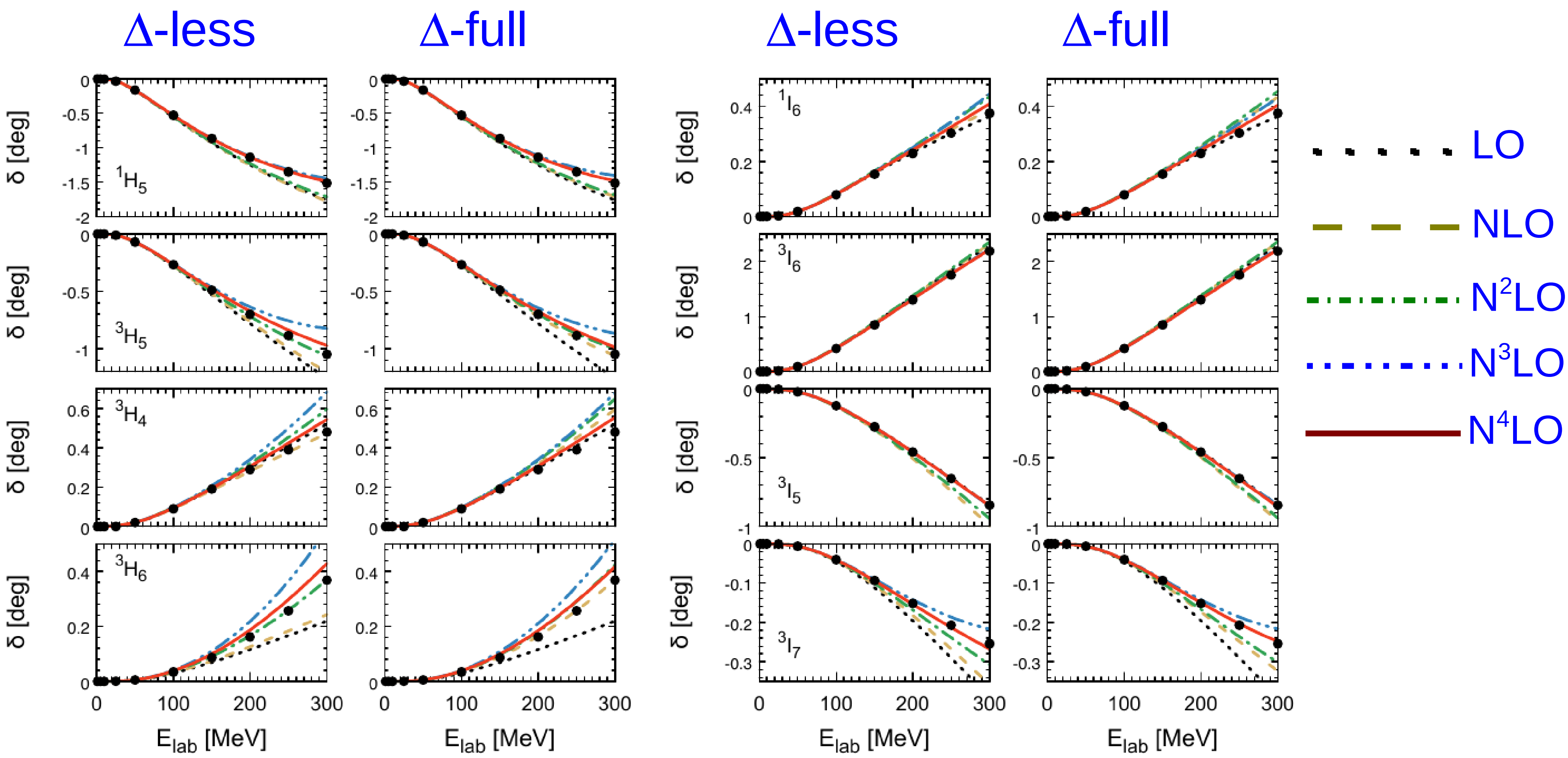
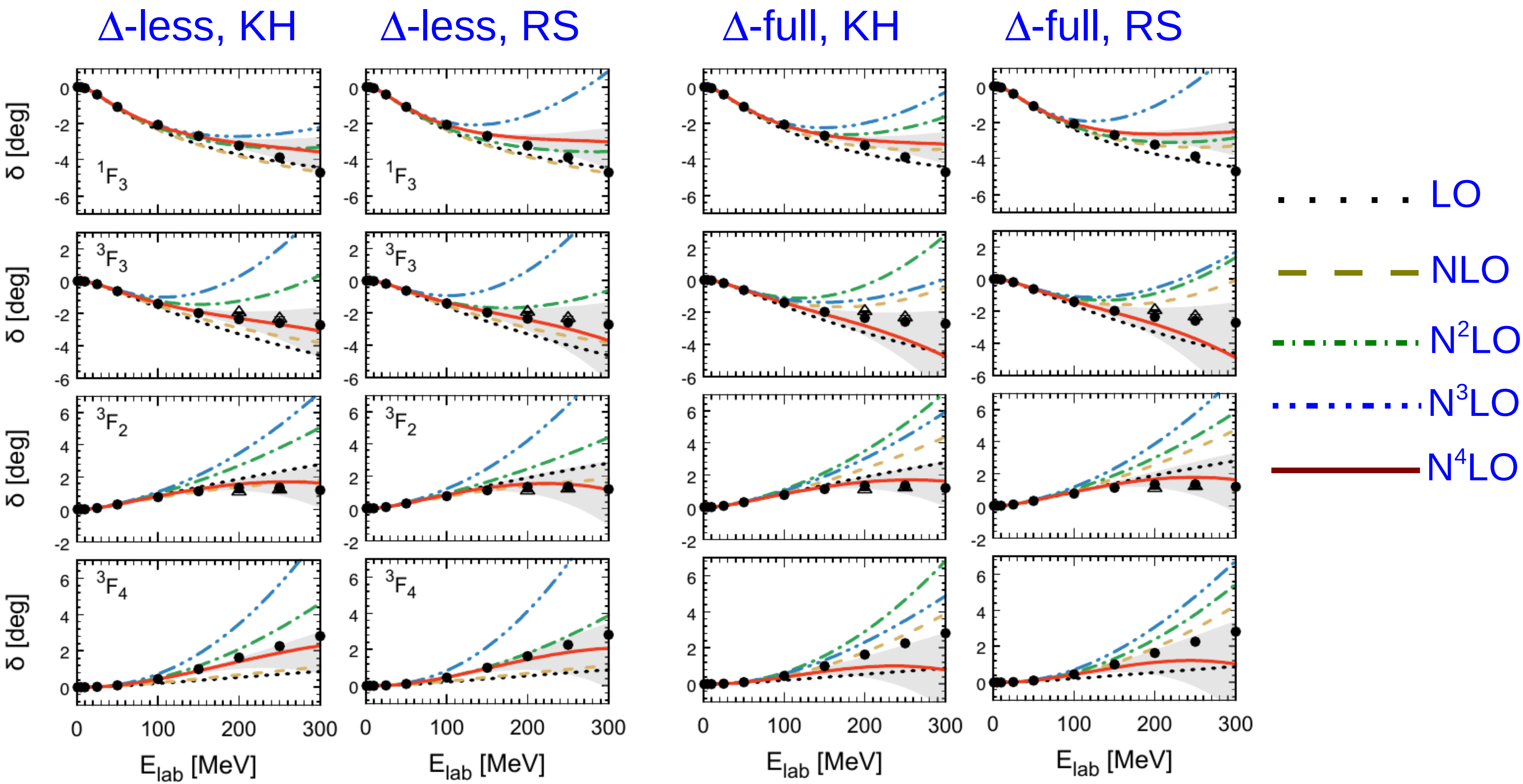


Data:Nijmegen PWA

F-waves: Impact of using different LECs



Data:Nijmegen PWA

F-waves: Impact of using different LECs

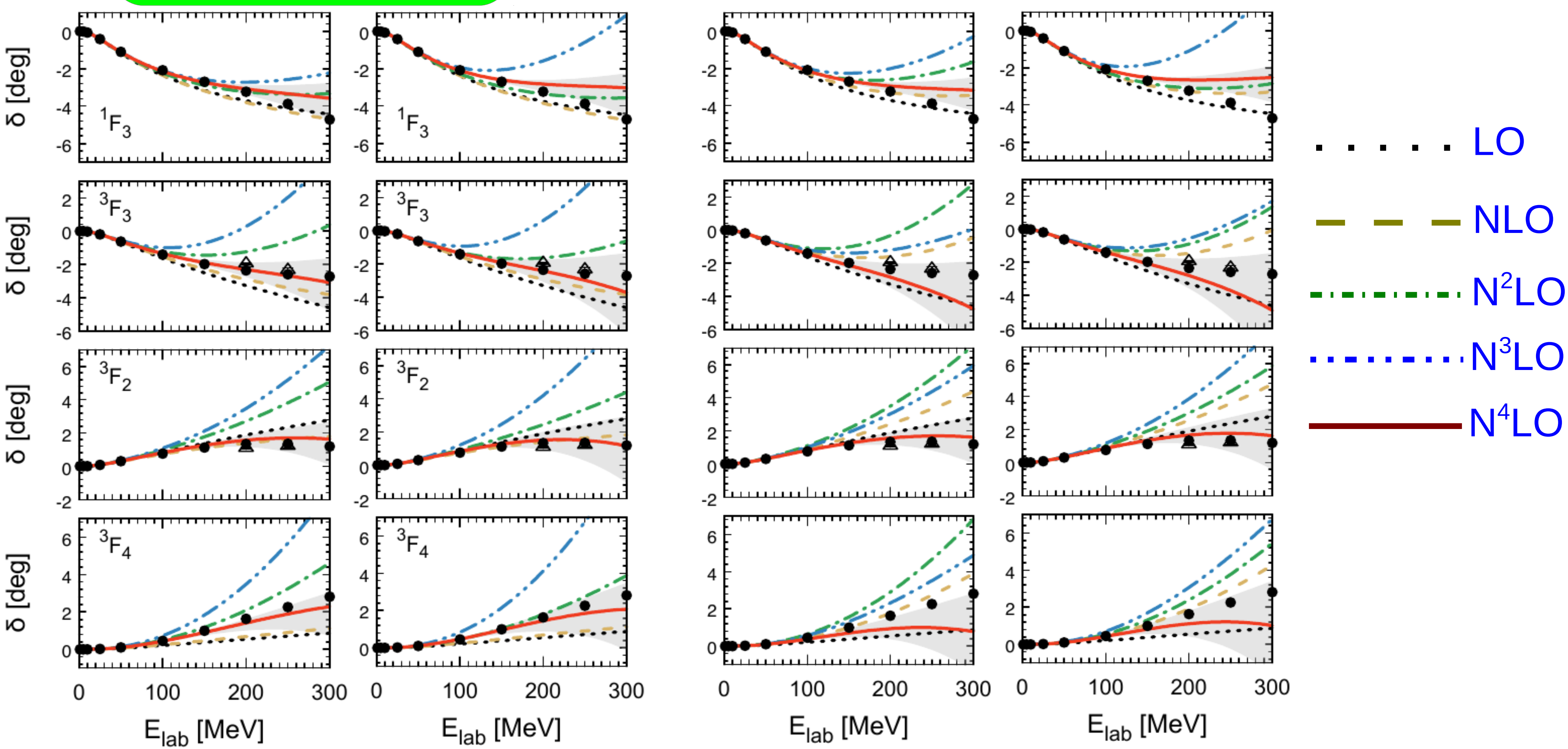
Little difference between KH and RS results

Δ

, RS

Δ-full, KH

Δ-full, RS



Data:Nijmegen PWA

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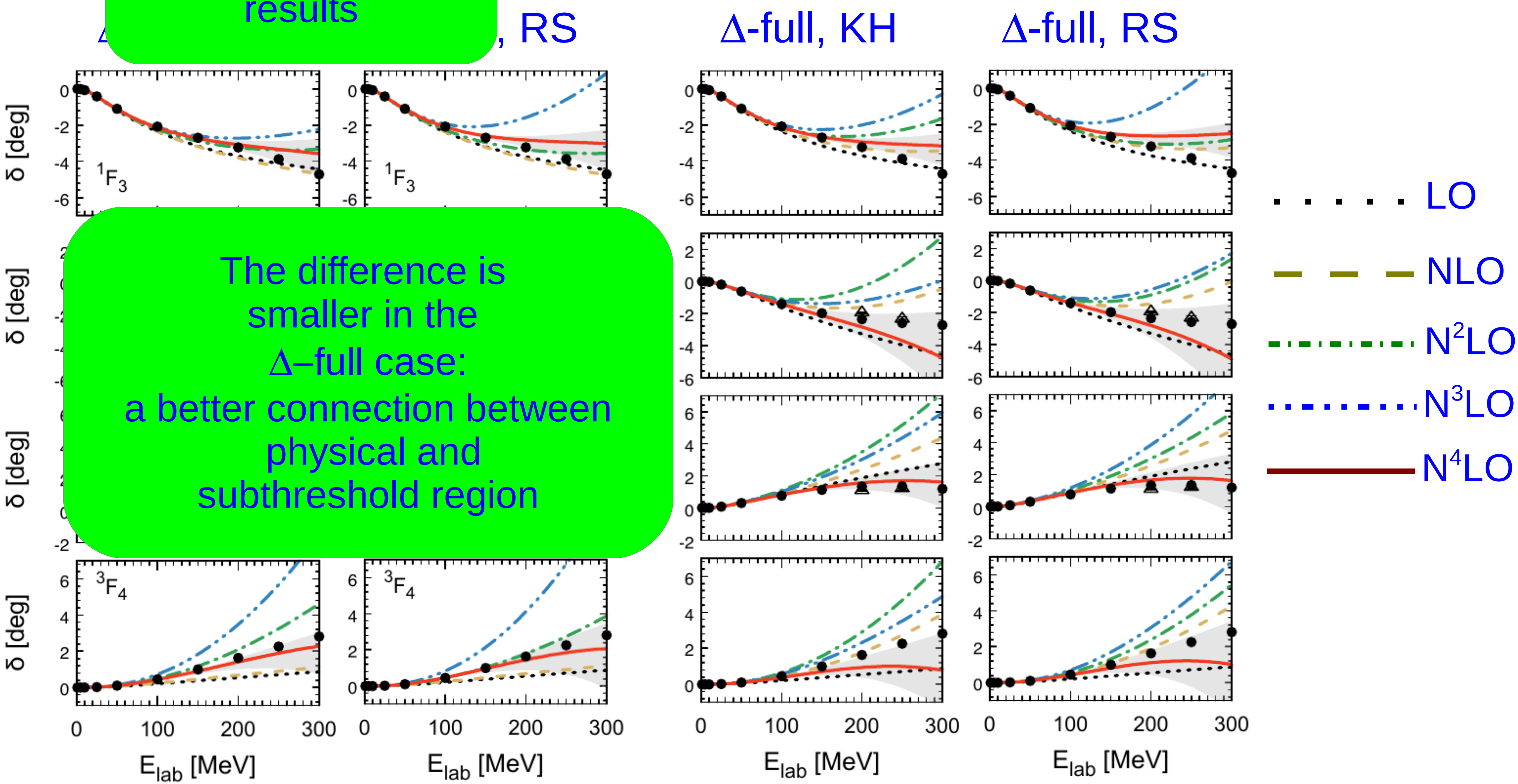
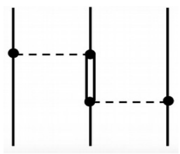
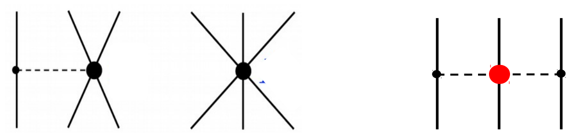
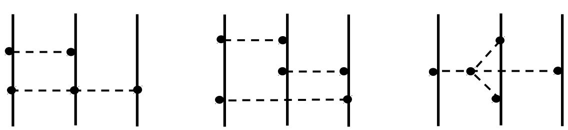
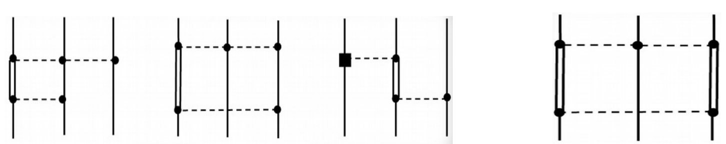
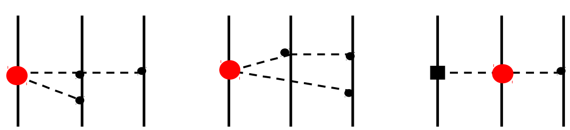


The difference is smaller in the Δ–full case: a better connection between physical and subthreshold region

Data:Nijmegen PWA

Small scale expansion of 3NF

	Δ-less theory	Δ-full theory: additional graphs	
NLO	—		
N^2LO		—	
N^3LO	 …		(ε^4)
(Q^5) N^4LO	 …	…	

Small scale expansion of 3NF

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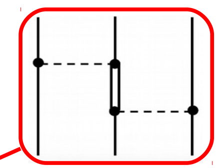
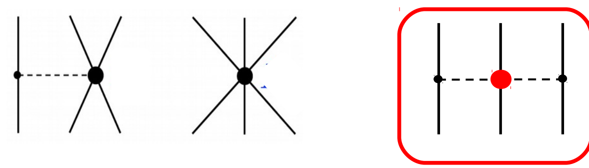
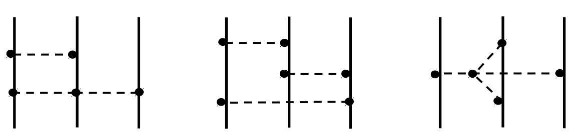
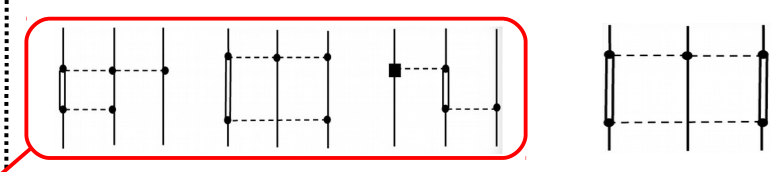
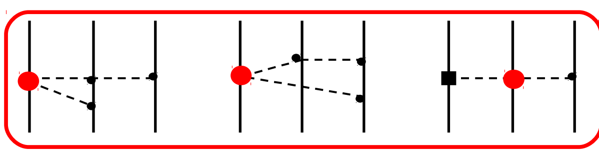
	NLO	—		
	N^2LO		—	
	N^3LO	 …		(ε^4)
(Q^5)	N^4LO	 …	…	

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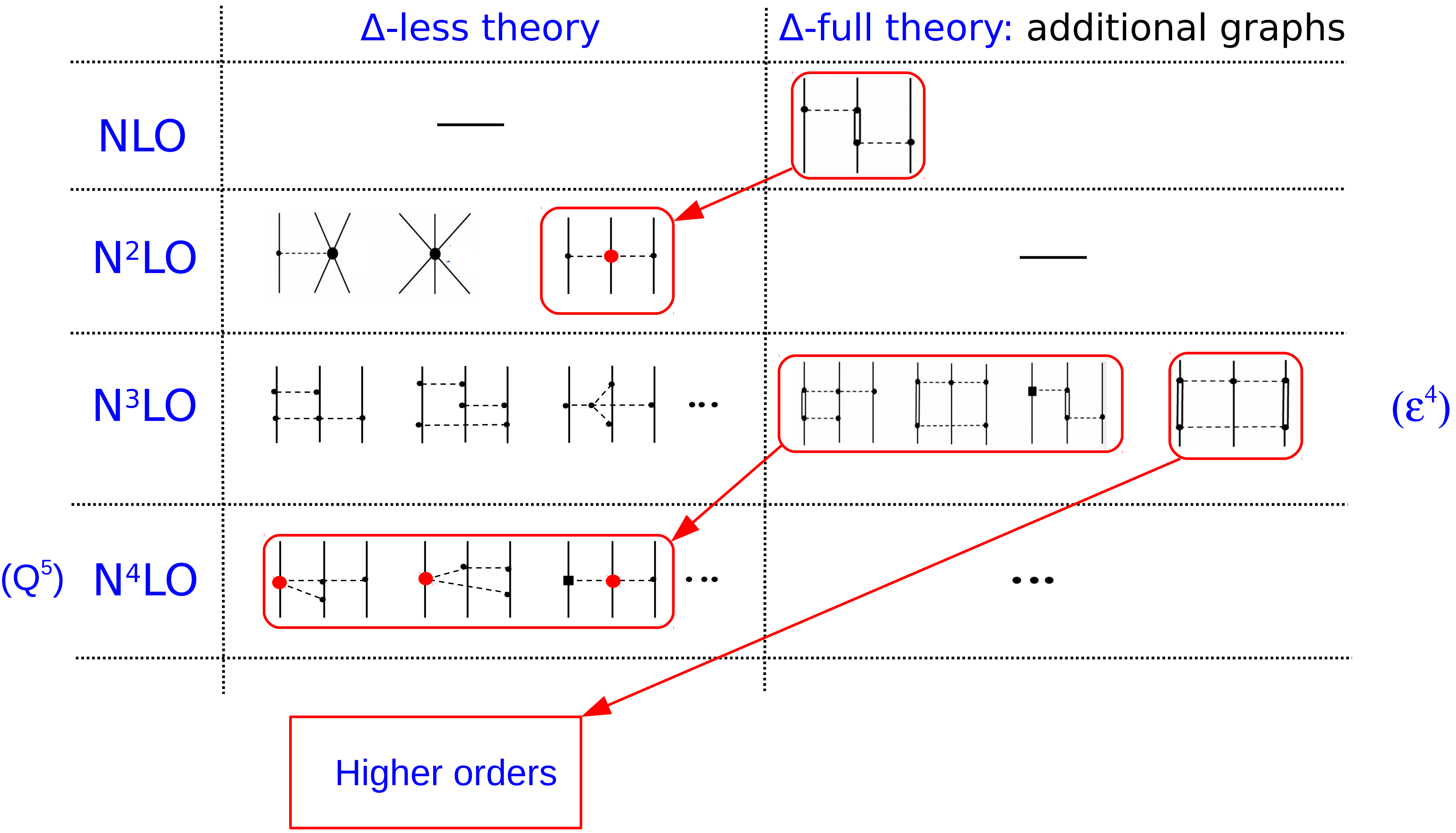
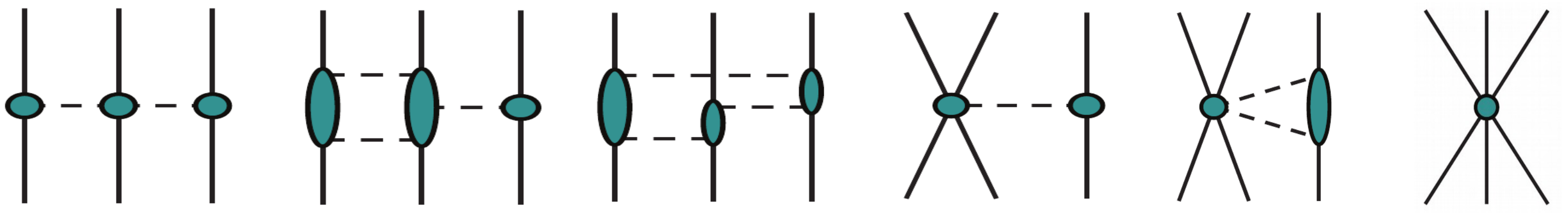


3NF topologies

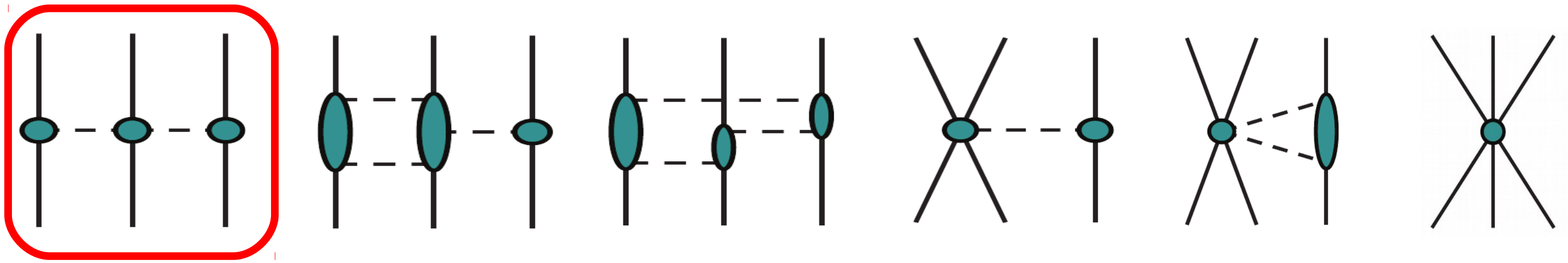


2π– exchange

2π–1π– exchange

ring

3NF topologies



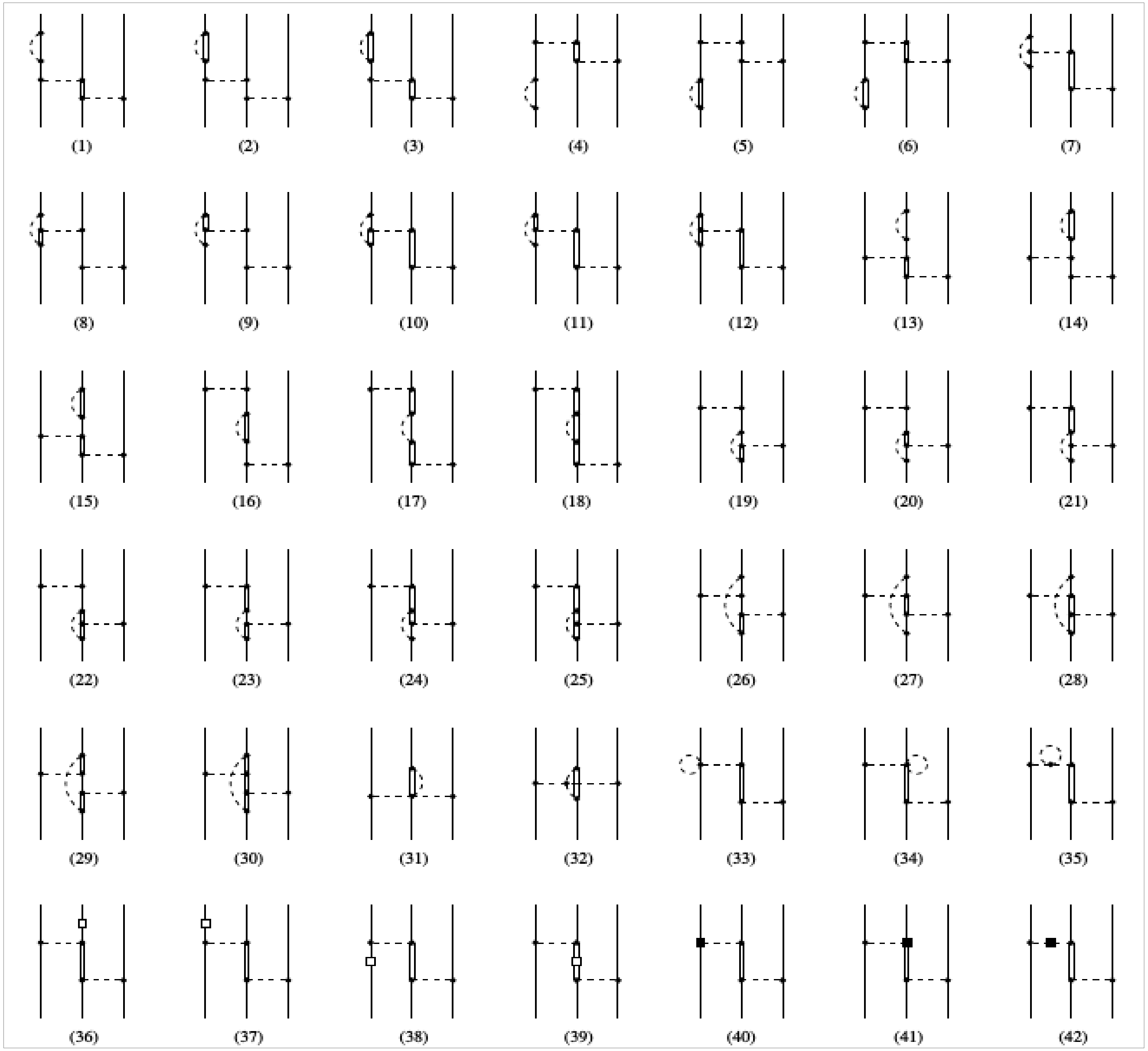
2π–exchange

2π–1π–exchange

ring

- ➔ Only the longest-range part considered (coordinate space)
- ➔ Scheme independent
- ➔ No unknown parameters

2–π–exchange diagrams for 3NF at N3LO (ε^3) (Δ-contributions)

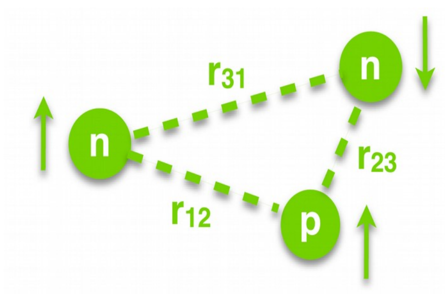


Most general structure of a local 3NF

Epelbaum, AG, Krebs, Schat '15

Up to N^{4}LO all considered contributions are local

Constraints:
- Locality
- Isospin symmetry
- Parity and time-reversal invariance



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$$\tilde{\mathcal{G}}_1 = 1$$
$$\tilde{\mathcal{G}}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$$
$$\tilde{\mathcal{G}}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$$
$$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_3$$
$$\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$\tilde{\mathcal{G}}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$$
$$\tilde{\mathcal{G}}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$$
$$\tilde{\mathcal{G}}_8 = \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_3$$
$$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1$$
$$\tilde{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_3$$
$$\tilde{\mathcal{G}}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_2$$
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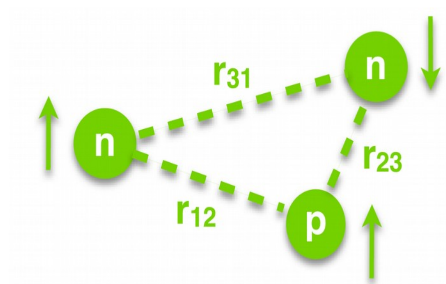
$$\tilde{\mathcal{G}}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_3$$
$$\tilde{\mathcal{G}}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$$
$$\tilde{\mathcal{G}}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_3 \cdot \hat{r}_{23} \, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$$
$$\tilde{\mathcal{G}}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \hat{r}_{23} \, \vec{\sigma}_3 \cdot \hat{r}_{12} \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$$

Constraints:
- Locality
- Isospin symmetry
- Parity and time-reversal invariance

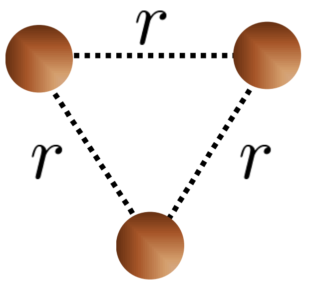


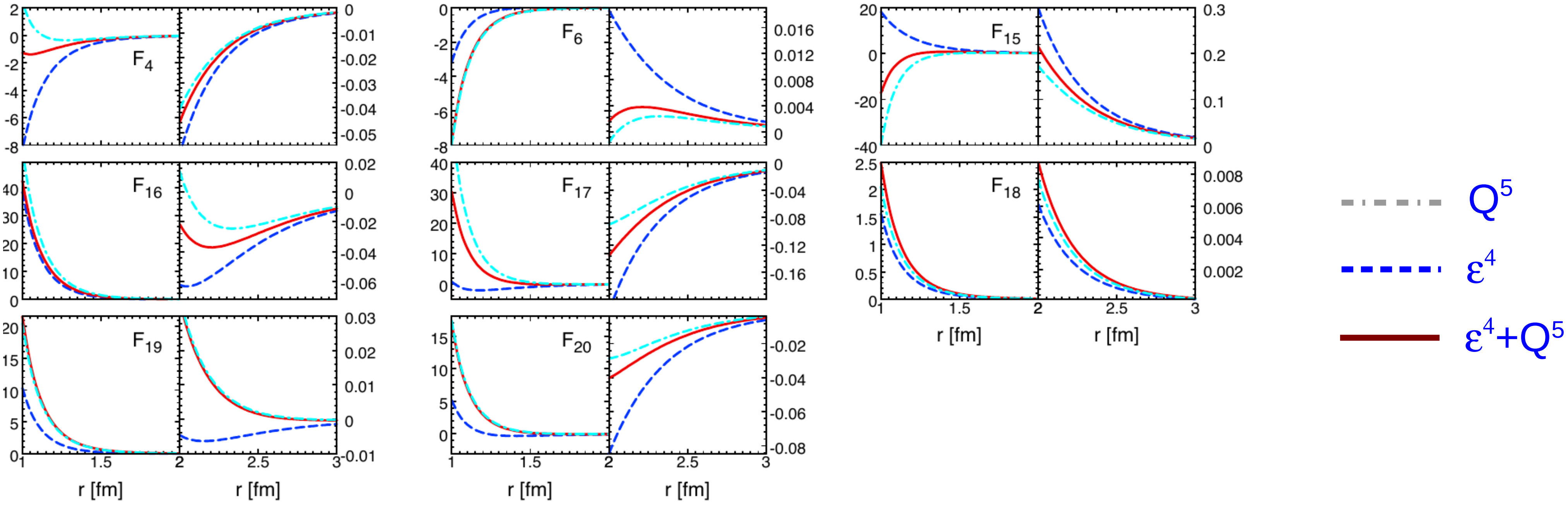
$$V_{\rm 3N}^{\rm full} = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i \mathcal{F}_i(r_{12}, r_{23}, r_{31}) + 5\,\text{permutations}$$

Two-pion-exhcange 3NF in Δ–full and Δ–less approach

Krebs, AG, Epelbaum, to appear soon

TPE “structure functions” F_i in MeV”
in equilateral-triangle configuration





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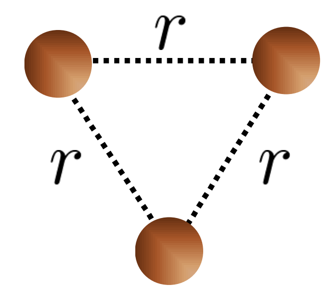

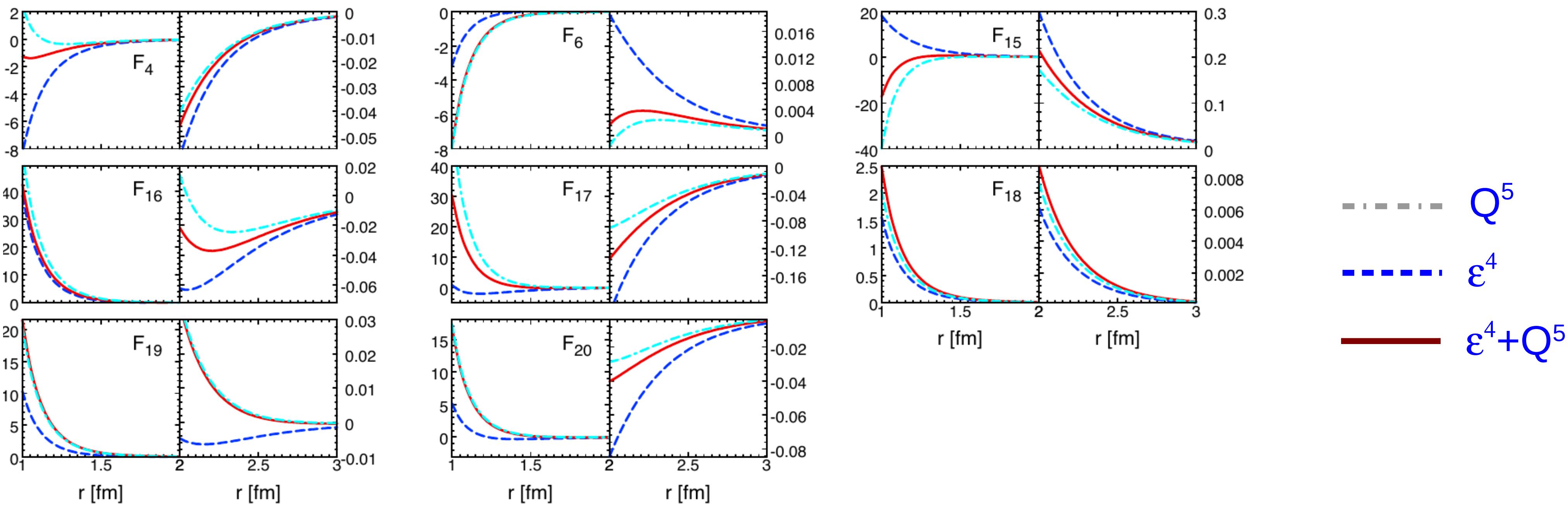

➔ Δ–saturation at “short” distances (1-1.5 fm): F_6, F_{16}, F_{18}, F_{19}, F_{20}

Two-pion-exhcange 3NF in Δ–full and Δ–less approach

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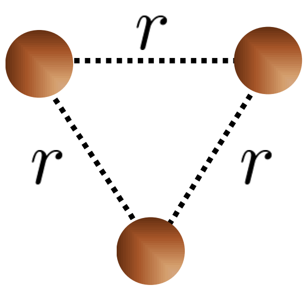
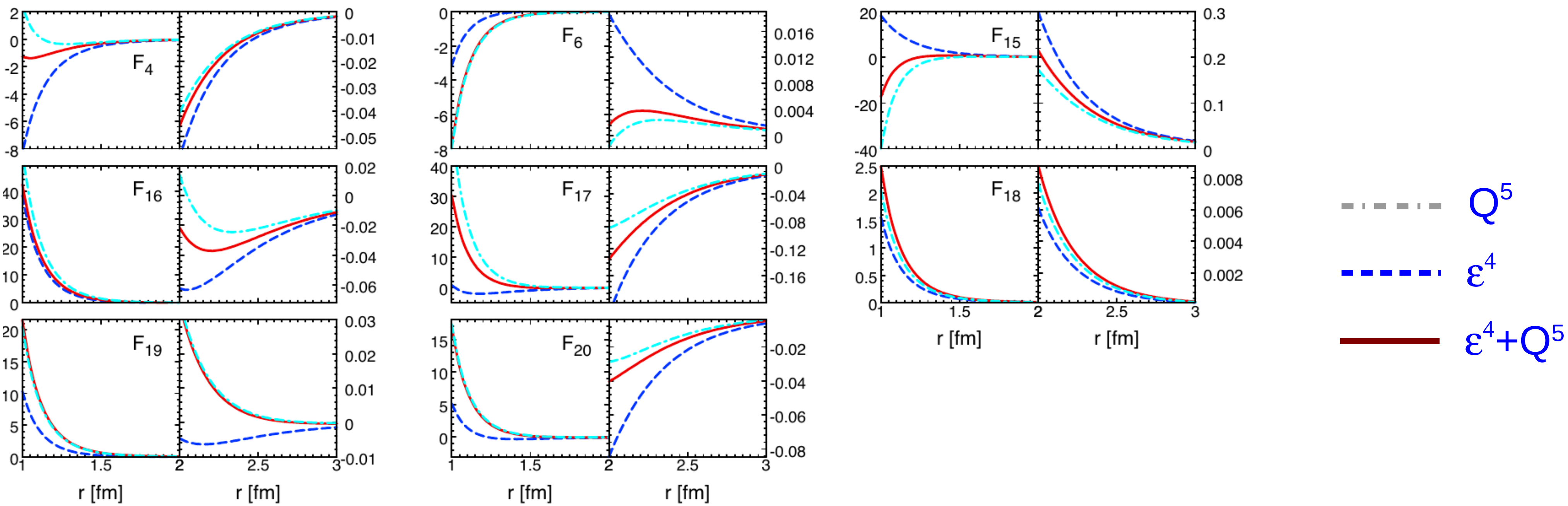




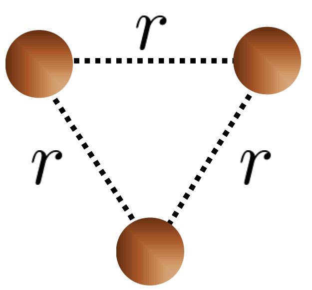
➔ Δ–saturation at "short" distances (1-1.5 fm): F_6, F_{16}, F_{18}, F_{19}, F_{20}

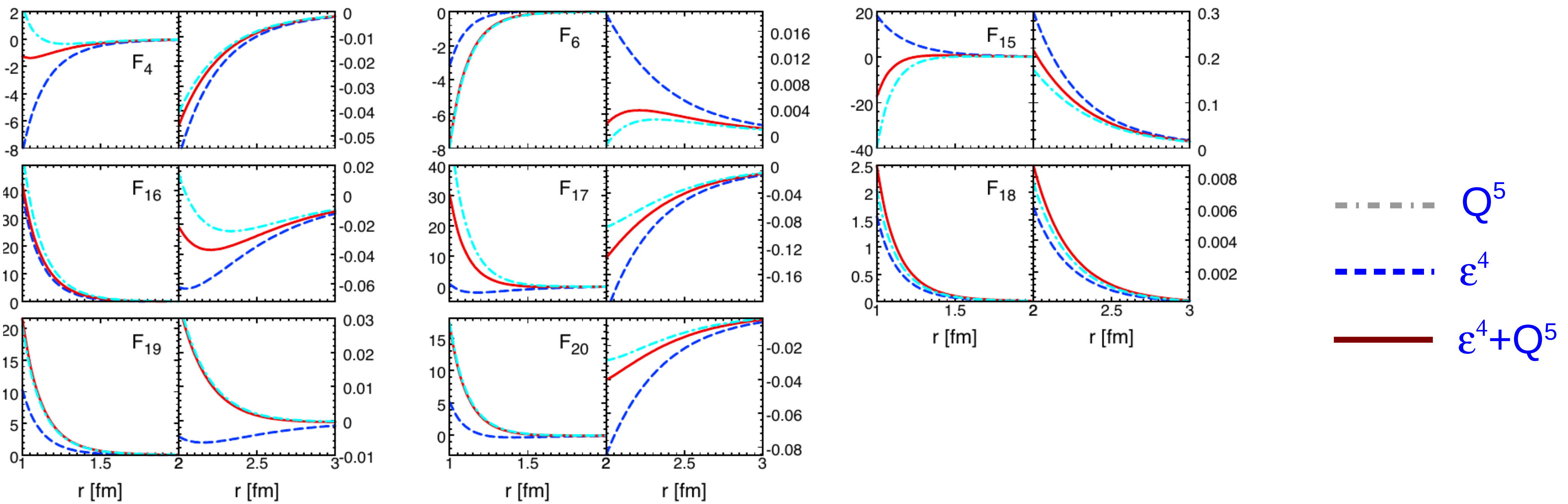
➔ Δ–saturation at large distances (2.5-3.0 fm): all F_i

Two-pion-exhcange 3NF in Δ–full and Δ–less approach

Krebs, AG, Epelbaum, to appear soon

TPE "structure functions" F_i in MeV"
in equilateral-triangle configuration





- ➔ Δ–saturation at "short" distances (1-1.5 fm): F_6, F_{16}, F_{18}, F_{19}, F_{20}
- ➔ Δ–saturation at large distances (2.5-3.0 fm): all F_i
- ➔ Convergence of chiral expansion at large distances

Summary

- ➔ Current results for Δ-full chiral 2-nucleon and 3-nucleon forces at N^3LO (N^4LO) are presented
- ➔ 2-nucleon forces (peripheral phases): description of the data is comparable to the Δ-less case, convergence pattern is better, dependence on the choice of LECs is weaker
- ➔ 3-nucleon forces: at large distances resonance Δ–saturation works, chiral expansion seems to converge

Outlook

- ➔ Fitting short-range part of Δ-full chiral 2N forces to data.
- ➔ Calculating shorter-range Δ-full 3N forces.