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Canada's national laboratory
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SRG Evolution of One-, Two- and Three-body Operators

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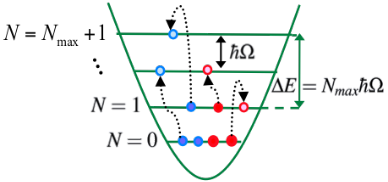
The No-Core Shell Model

Want to solve the eigenvalue problem:

$$H\left|\Psi_k\right\rangle = E_k\left|\Psi_k\right\rangle, \text{ where } H = \sum_i^A T_i + \sum_{i<j} V_{ij} + \sum_{i<j<f} V_{ijf} + \cdots$$

Expand in anti-symmetrized products of harmonic oscillator single-particle states



$$\left|\Psi_k\right\rangle = \sum_{N=0}^{N_{max}} \sum_j c_{Nj}^k \left|\Phi_{Nj}\right\rangle$$

Calculations should converge to the exact value as $N_{max} \to \infty$

Problem: the size of the model space increases rapidly with particle number

The SRG method uses a unitary transformation to decouple high and low momentum physics allowing faster convergence of calculations

$$H_\alpha = U_\alpha H U_\alpha^\dagger$$

$$\frac{\mathrm{d}H_\alpha}{\mathrm{d}\alpha} = [[T, H_\alpha], H_\alpha] \text{ where } H_{\alpha=0} = H$$

Note: SRG transformations introduce higher-body terms in the Hamiltonian

$$U_\alpha H U_\alpha^\dagger = H_\alpha^{(1)} + H_\alpha^{(2)} + H_\alpha^{(3)} + \dots$$

$$H\left|\Psi_k\right\rangle = E_k\left|\Psi_k\right\rangle \;\rightarrow\; H_\alpha\left|\Psi_{k,\alpha}\right\rangle = E_k\left|\Psi_{k,\alpha}\right\rangle$$

General operators must also be transformed:

$$\left\langle\Psi_f\right|\hat{O}\left|\Psi_i\right\rangle = \left\langle\Psi_{f,\alpha}\right|\hat{O}_\alpha\left|\Psi_{i,\alpha}\right\rangle \text{ where } \hat{O}_\alpha = U_\alpha\hat{O}U_\alpha^\dagger$$

$$U_\alpha = \sum_k \left|\Psi_{k,\alpha}\right\rangle\left\langle\Psi_k\right|$$

Implementation in two-body relative coordinates:

For $|\Psi_k\rangle = |kJ^\pi TT_z\rangle$, U_α is constructed in blocks: $U_\alpha^{J^\pi TT_z}$
Non-scalar operators may connect states with different quantum numbers:

$$\langle k'J'^{\pi'}T'T'_z||\hat{O}^{(K)}||kJ^\pi TT_z\rangle = \langle k'J'^{\pi'}T'T'_z,\alpha||U_\alpha^{J'^{\pi'}T'T'_z}\hat{O}^{(K)}U_\alpha^{\dagger J^\pi TT_z}||kJ^\pi TT_z,\alpha\rangle$$

H_α, O_α: 2-body part determined in A=2 system

PHYSICAL REVIEW C **67**, 055206 (2003)

Parameter-free effective field theory calculation for the solar proton-fusion and hep processes

T.-S. Park,[1,2,3] L. E. Marcucci,[4,5] R. Schiavilla,[6,7] M. Viviani,[5,4] A. Kievsky,[5,4] S. Rosati,[5,4] K. Kubodera,[1,2]
D.-P. Min,[8] and M. Rho[1,9]

- 1-body: Gamow-Teller (GT)

$$A_l = -g_A \tau_l^- e^{-i q \cdot r_l} \left[\boldsymbol{\sigma}_l + \frac{2(\bar{\boldsymbol{p}}_l \boldsymbol{\sigma}_l \cdot \bar{\boldsymbol{p}}_l - \boldsymbol{\sigma}_l \bar{\boldsymbol{p}}_l^2) + i \boldsymbol{q} \times \bar{\boldsymbol{p}}_l}{4 m_N^2} \right]$$

- 2-body: Axial Meson Exchange Current (MEC)

$$\begin{aligned} A_{12} = \frac{g_A}{2 m_N f_\pi^2} \frac{1}{m_\pi^2 + \boldsymbol{k}^2} \Big[& -\frac{i}{2} \tau_\times^- \boldsymbol{p}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \boldsymbol{k} \\ & + 4 \hat{c}_3 \boldsymbol{k}\boldsymbol{k} \cdot (\tau_1^- \boldsymbol{\sigma}_1 + \tau_2^- \boldsymbol{\sigma}_2) + \left(\hat{c}_4 + \frac{1}{4} \right) \tau_\times^- \boldsymbol{k} \times [\boldsymbol{\sigma}_\times \times \boldsymbol{k}] \Big] \\ & + \frac{g_A}{m_N f_\pi^2} [2 \hat{d}_1 (\tau_1^- \boldsymbol{\sigma}_1 + \tau_2^- \boldsymbol{\sigma}_2) + \hat{d}_2 \tau_\times^a \boldsymbol{\sigma}_\times], \qquad (19) \end{aligned}$$

$$\hat{O} = GT^{(1)} \to \hat{O}_\alpha = GT^{(1)} + GT^{(2)}_\alpha + \dots$$

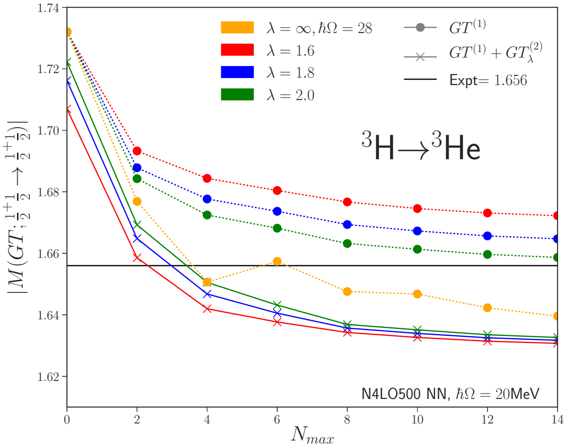
Operator:

Gamow-Teller (1-body)

$$\langle GT^{(2)}_\alpha \rangle_{A=2} = \langle (GT^{(1)})_\alpha \rangle_{A=2} - \langle GT^{(1)} \rangle_{A=2}$$

Potential: "N^{4}LO NN"

- chiral NN @ N^{4}LO, Machleidt PRC96 (2017), 500MeV cutoff



Results: β-decay ${}^3\text{H} \rightarrow {}^3\text{He}$

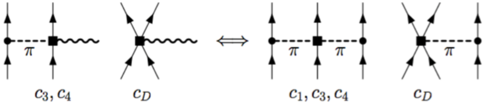
$$\hat{O} = GT^{(1)} + MEC^{(2)} \rightarrow \hat{O}_\alpha = GT^{(1)} + GT_\alpha^{(2)} + MEC_\alpha^{(2)} + \ldots$$

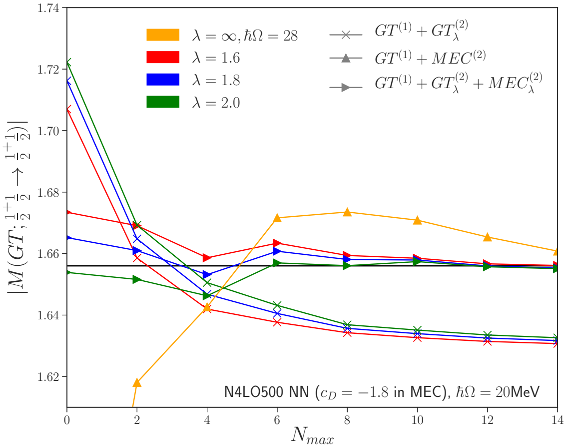
Operator:

Gamow-Teller (1-body) + chiral meson exchange current (2-body) Park (2003)

Potential: "N^{4}LO NN"

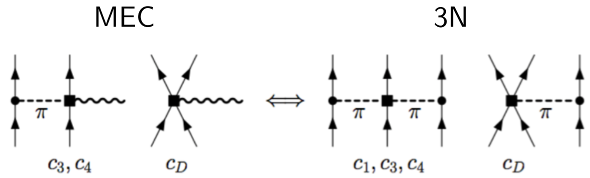
- chiral NN @ N^{4}LO, Machleidt PRC96 (2017), 500MeV cutoff
- LEC $c_D = -1.8$ determined

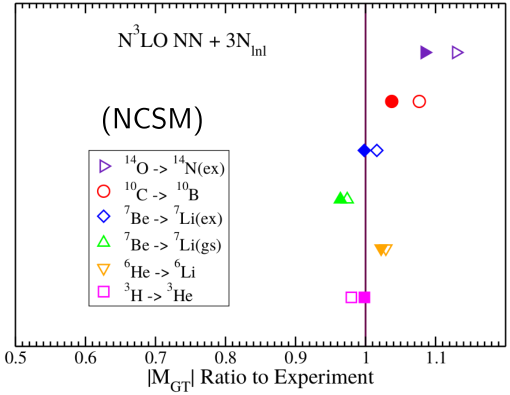




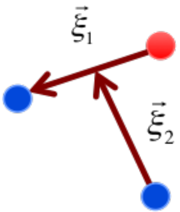
Application to Heavier Nuclei

- Does inclusion of the MEC explain g_A quenching?
- The effect of the inclusion is greater in heavier nuclei
- SRG evolved matrix elements used in coupled-cluster and IM-SRG methods (up to Sn^{100})





3-Body Operators and Double-Beta Decay



Summer 2017 at Lawrence Livermore National Laboratory - with S. Quaglioni

- Added functionality for evolving operators in 3-body space
- 2-body operators must be embedded into antisymmetric 3-body basis
- 2-body $0\nu\beta\beta$ matrix elements & 3-body GT$\times$MEC product terms
  - produced by J. Engel (UNC)

H_α, O_α:

- 2-body part determined in A=2 system
- 3-body part determined in A=3 system
- ...

- Operators must be SRG evolved to converge to the correct result
- A method was developed for arbitrary operators
- So far implemented: GT, axial MEC, $0\nu\beta\beta$, radius, E2
- Results for β-decay strengths: ^{3}H$\rightarrow$^{3}He, ^{6}He$\rightarrow$^{6}Li and other nuclei
- ^{3}H$\rightarrow$^{3}He used to determine the low-energy constant c_D in chiral MEC
- To do:
  - Benchmark $0\nu\beta\beta$ matrix elements
  - Implement electromagnetic operators (M1, E1)
  - Analyze 3-body vs. 2-body evolution
  - so far: SRG induced 3-body contributions are not significant in GT+MEC