Recent developments and applications of three-nucleon interactions

Kai Hebeler

Vancouver, February 27, 2018

Progress in Ab Initio Techniques in Nuclear Physics

TRIUMF

SFB 1245

TECHNISCHE UNIVERSITÄT DARMSTADT

Outline

Calculation of semilocal momentum space **3N matrix elements** (SMS)

★strategy

★challenges

★status

- Few-body results A=3 and A=4
- Many-body results up to ^{16}O

→ *Thomas Hüther's talk+poster*

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Calculation of semilocal momentum space **3N matrix elements** (SMS)

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- Few-body results A=3 and A=4
- Many-body results up to ^{16}O

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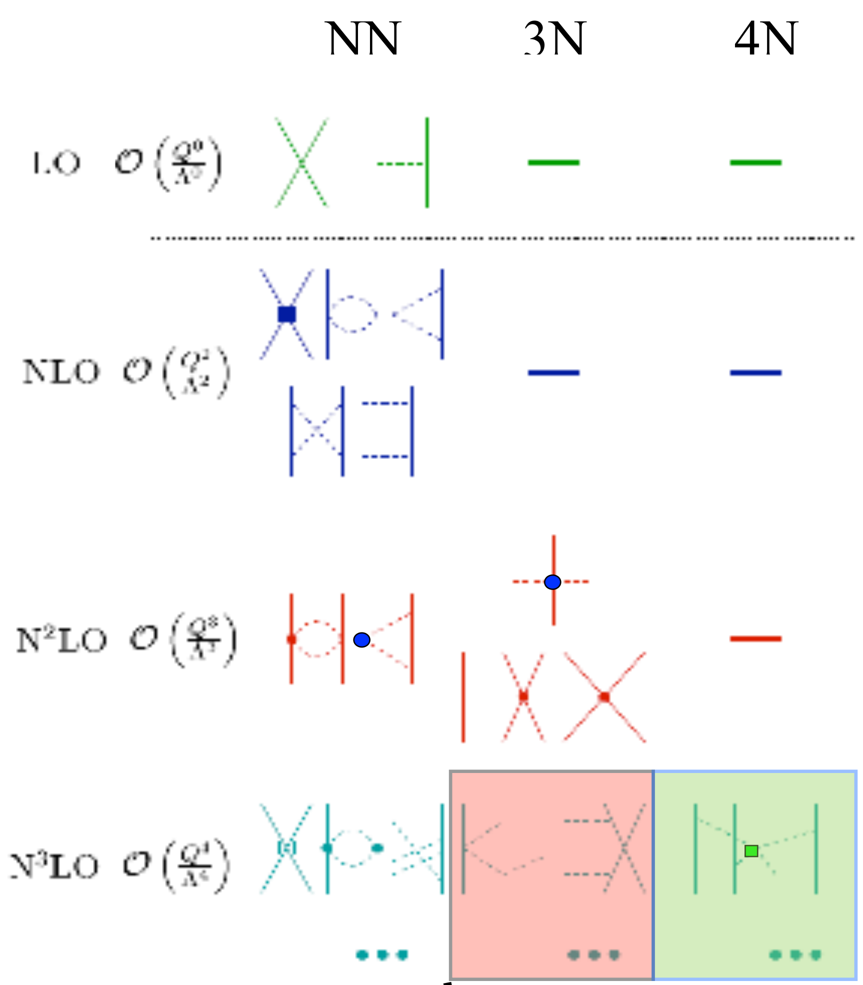
Novel efficient interface for **NN and 3N interactions** in a **non-partial-wave** basis

- First application to nuclear matter plus first fits of 3N couplings to nuclear matter

- applications to other systems and frameworks?

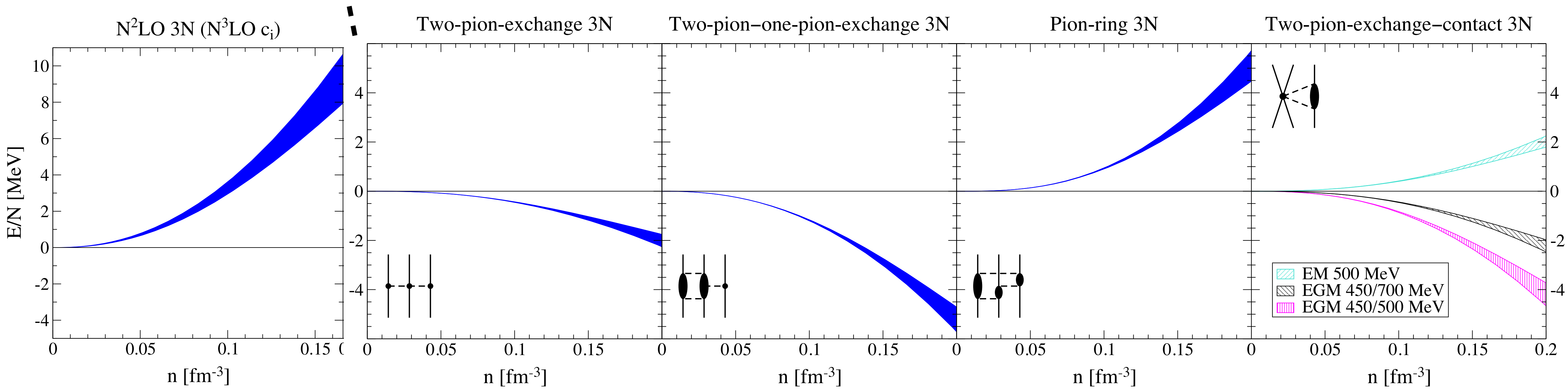
Power counting in chiral 3N sector: Contributions of many-body forces at N³LO in neutron matter



- first calculations of N³LO 3NF and 4NF contributions to EOS of neutron matter
- found large contributions in Hartree Fock appr., comparable to size of N²LO contributions

Krüger, Tews, KH, Schwenk, PRC 88, 025802 (2013)

Tews, Krüger, KH, Schwenk, PRL 110, 032504 (2013)



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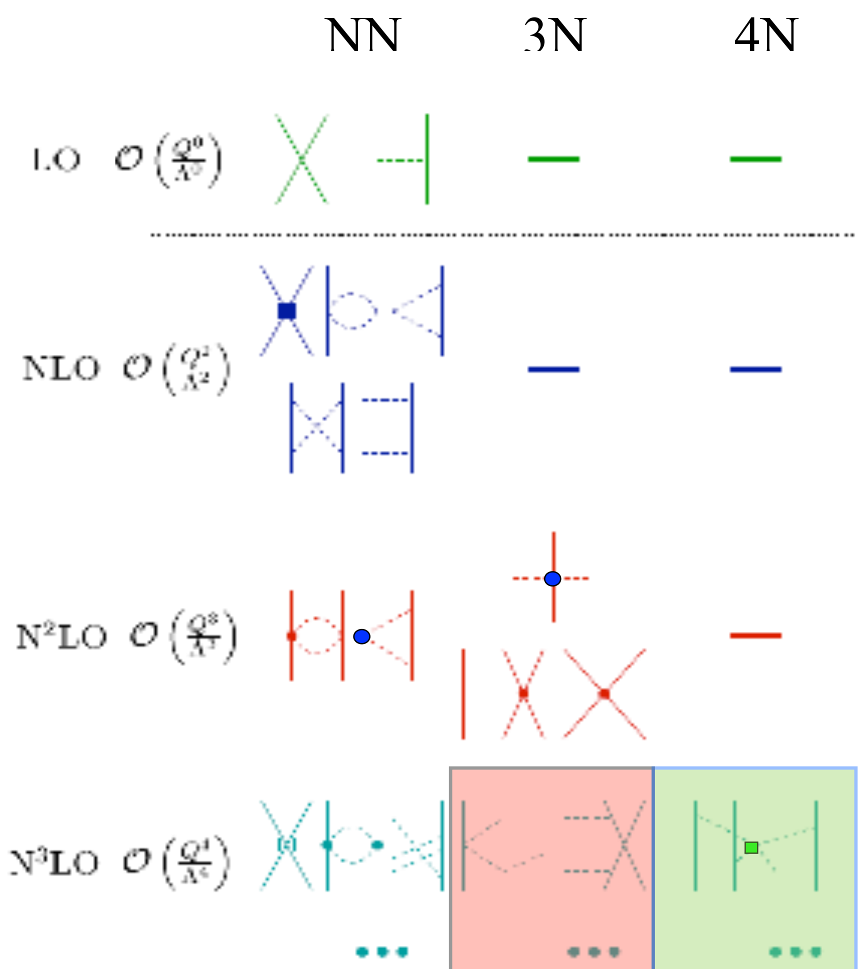

- first calculations of N³LO 3NF and 4NF contributions to EOS of neutron matter
- found large contributions in Hartree Fock appr., comparable to size of N²LO contributions
- 4NF contributions small

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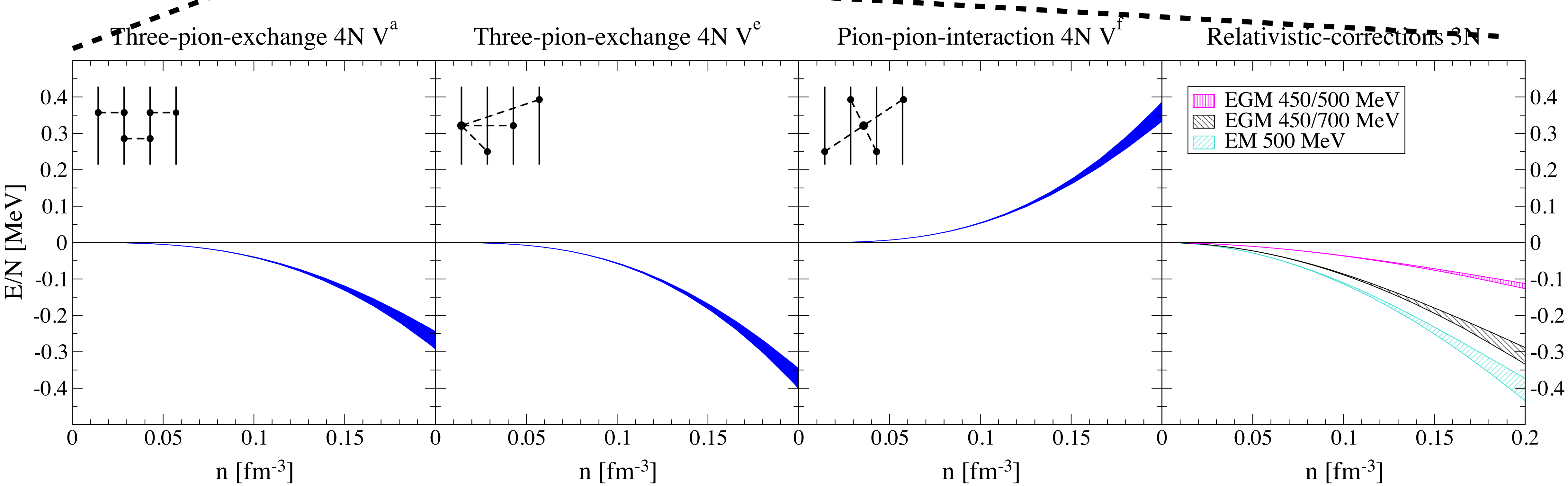
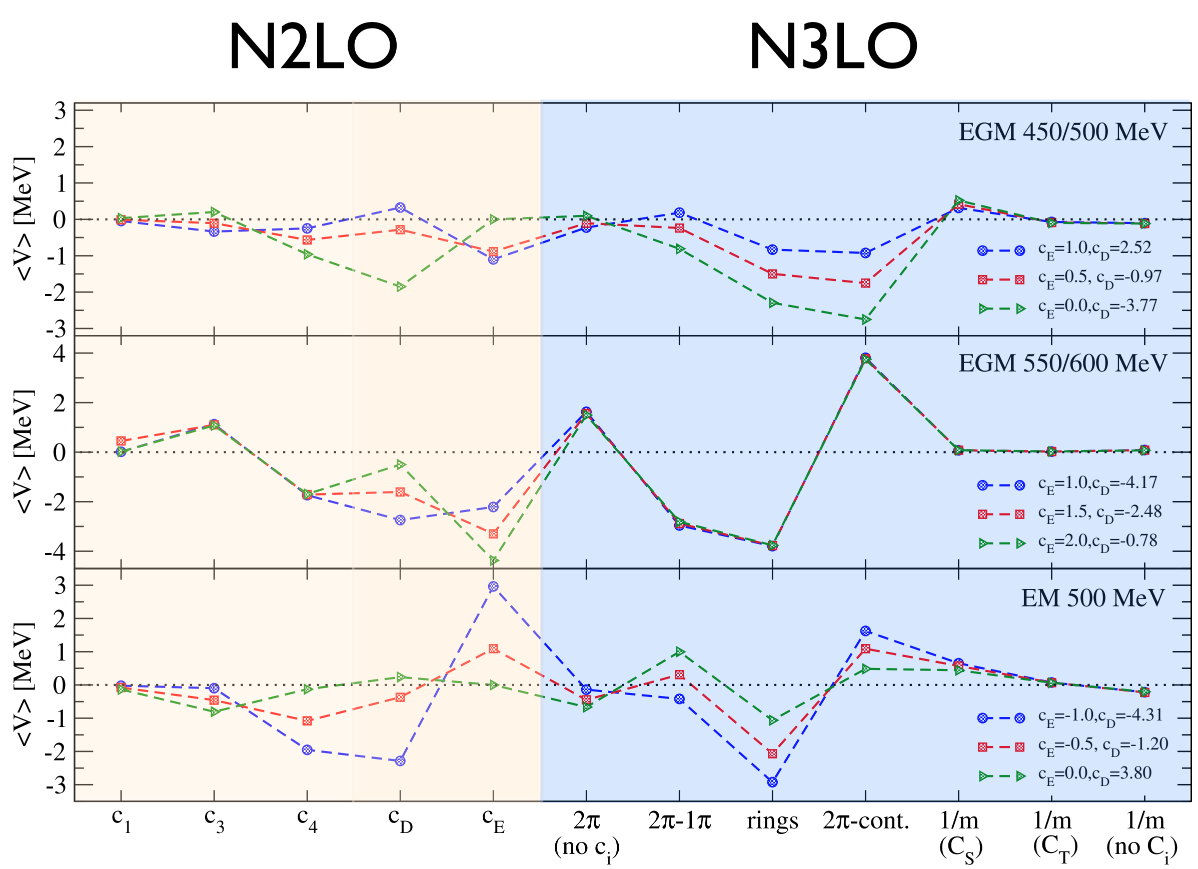


3NF power counting in 3H for different regulators

nonlocal



KH et al.,
PRC 91, 044001 (2015)

→ *Hermann's talk*

3NF power counting in 3H for different regulators

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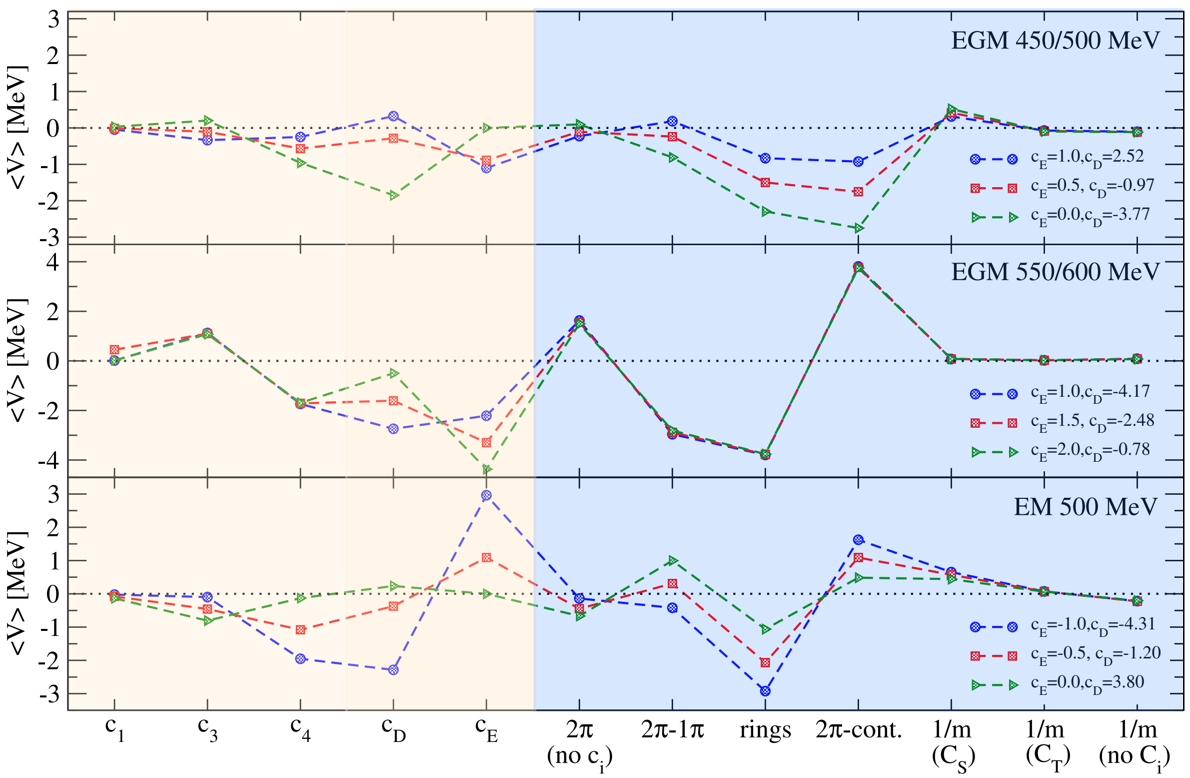
N2LO N3LO



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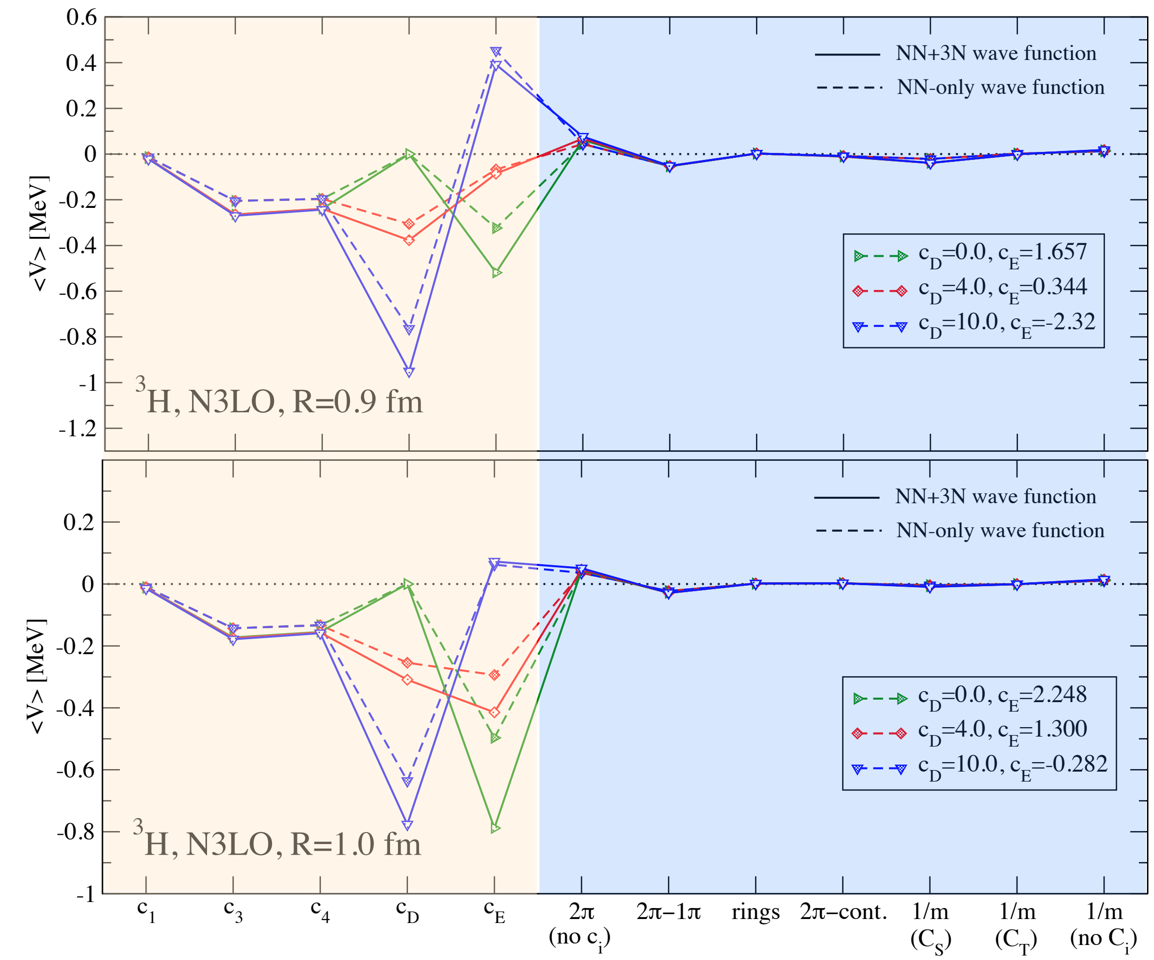
semilocal (coordinate space)

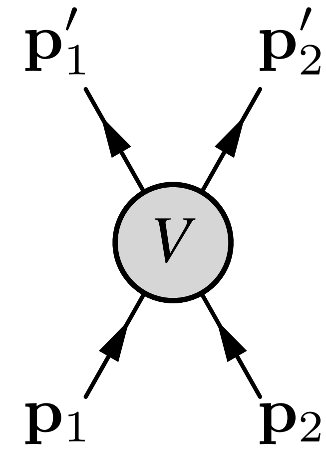
N2LO N3LO



- size of N3LO contribution not suppressed for shown nonlocal interactions
- N3LO contributions suppressed for semilocal interactions
- technical challenges for semilocal interactions:
  - ★ forces non-perturbative, large basis spaces/RG evolution needed
  - ★ implementation of 3N forces hard, stability problems for scattering calculations
  - ★ Derivation and implementation of nuclear currents hard → *Hermann's talk*

Regularization schemes for nuclear interactions (here: NN)

Separation of long- and short-range physics



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$$\mathbf{p}' = (\mathbf{p}_1' - \mathbf{p}_2')/2$$
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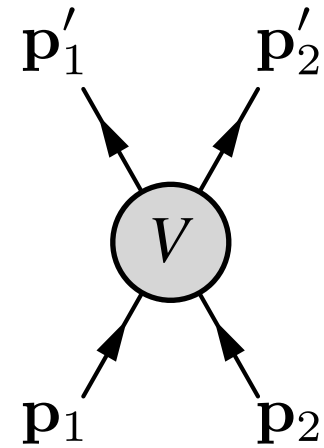

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$$V_{\mathrm{NN}}(\mathbf{p}, \mathbf{p}') \to \exp\left[-\left((p^2 + p'^2)/\Lambda^2\right)^n\right] V_{\mathrm{NN}}(\mathbf{p}, \mathbf{p}')$$

Epelbaum, Glöckle, Meissner, NPA 747, 362 (2005)
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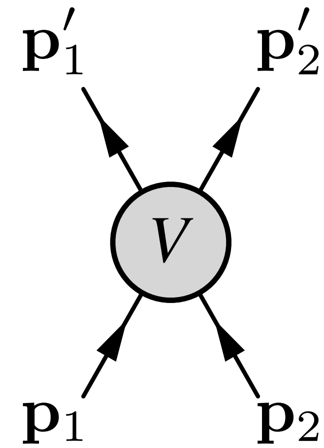

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(momentum space)

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cf. Navratil, Few-body Systems 41, 117 (2007)

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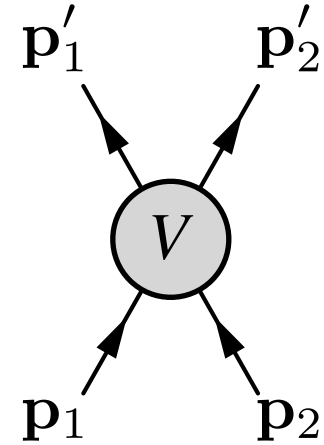

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$$V_{\rm NN}^{\pi}(\mathbf{r}) \rightarrow \left(1 - \exp\left[-\left(r^2/R^2\right)^n\right]\right) V_{\rm NN}^{\pi}(\mathbf{r})$$
$$\delta(\mathbf{r}) \rightarrow \alpha_n \exp\left[-(r^2/R^2)^n\right]$$

Gezerlis et. al, PRL, 111, 032501 (2013)

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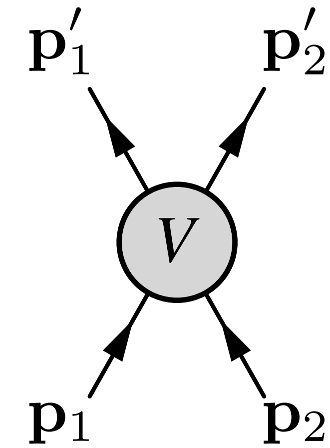

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semi-local

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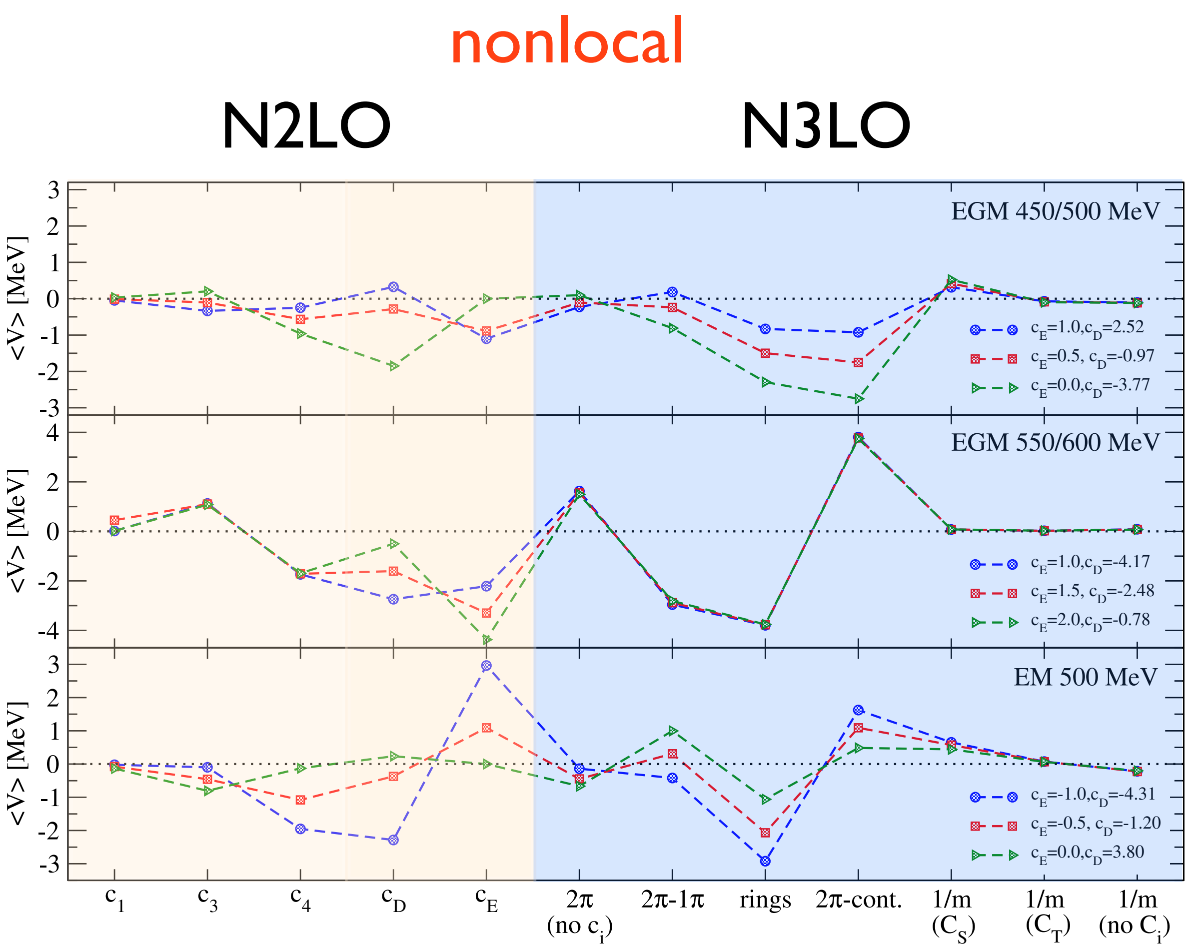
Epelbaum et. al, PRL, 115, 122301 (2015)

3NF power counting for different regulators



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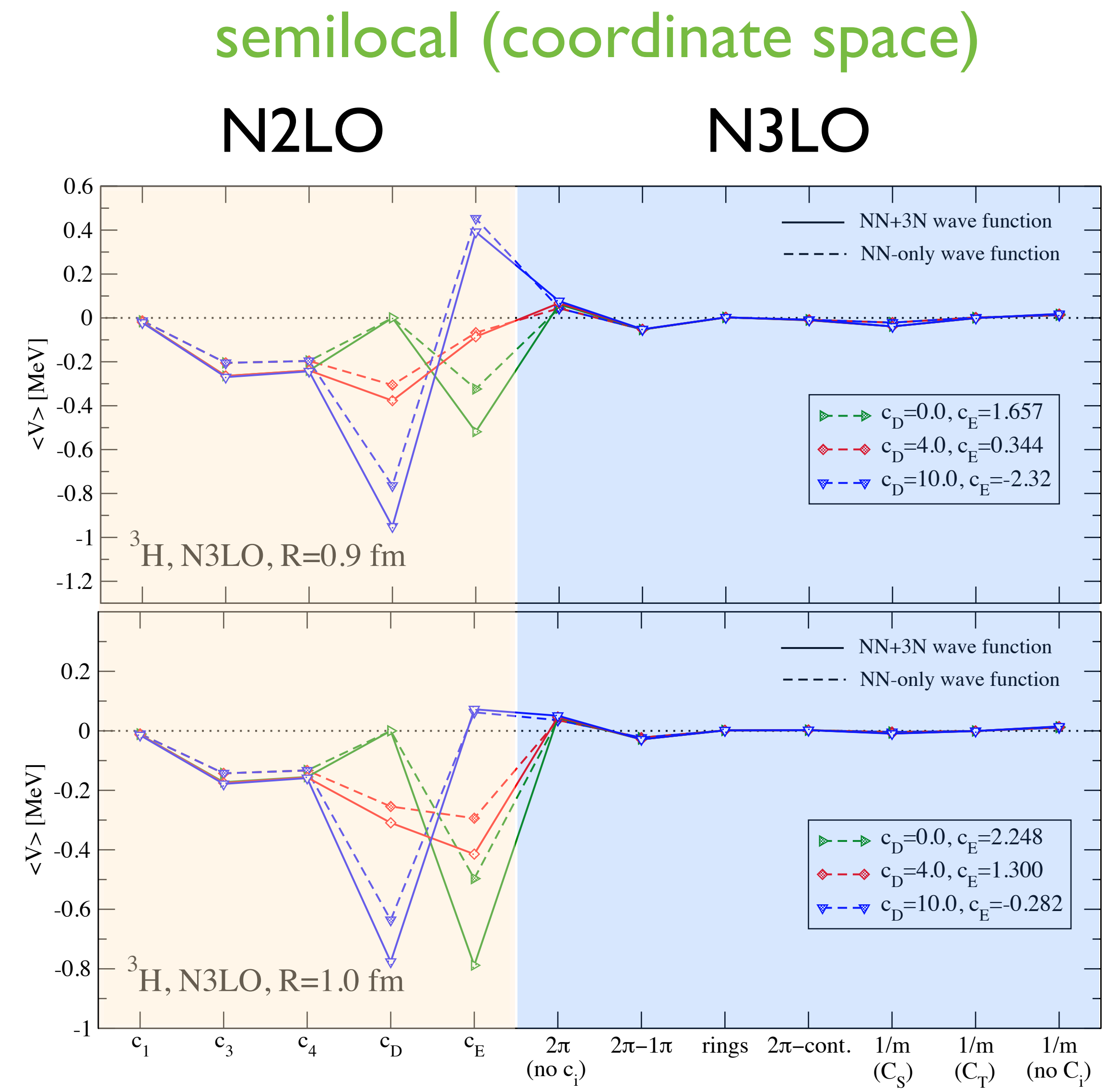


Development of improved novel semilocal NN+3N interactions regularised in momentum space.

$$V_\pi(\mathbf{p},\mathbf{p}') \to V_\pi(\mathbf{p},\mathbf{p}')e^{-(\mathbf{q}^2+m_\pi^2)/\Lambda^2}$$

Reinert et al., arXiv:1711.08821

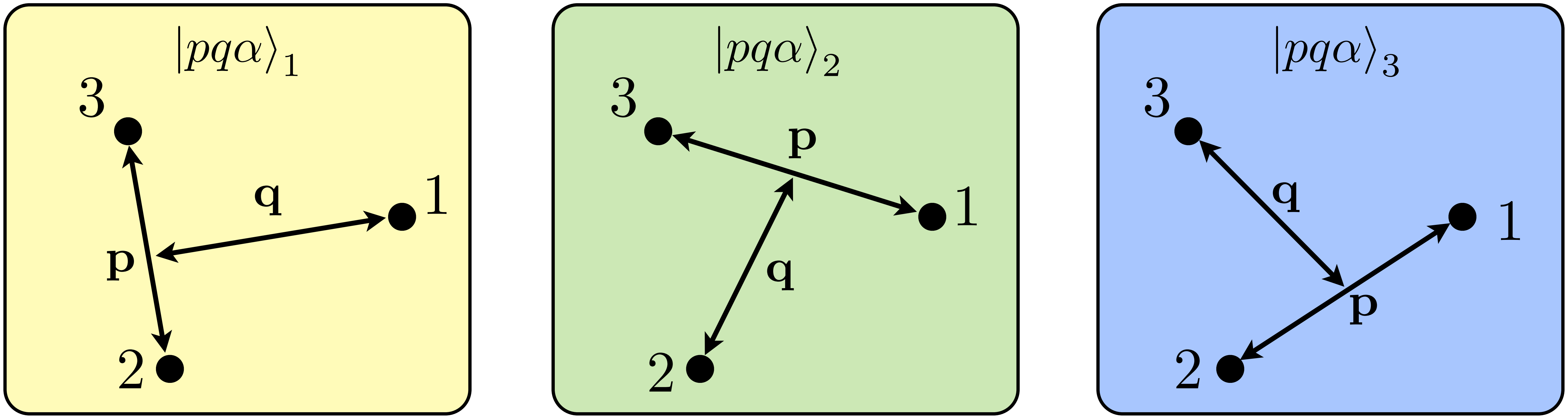
→ *Hermann's talk*

Calculation of N2LO 3NFs completed. Benchmarks and fits in progress!

→ *Thomas Hüther's talk*

Representation of 3N interactions in momentum space

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$$



Due to the large number of matrix elements, the traditional way of computing matrix elements requires extreme amounts of computer resources.

$$N_p \simeq N_q \simeq 15$$
$$N_\alpha \simeq 30 - 180 \quad \longrightarrow \quad \dim[\langle pq\alpha|V_{123}|p'q'\alpha'\rangle] \simeq 10^7 - 10^{10}$$

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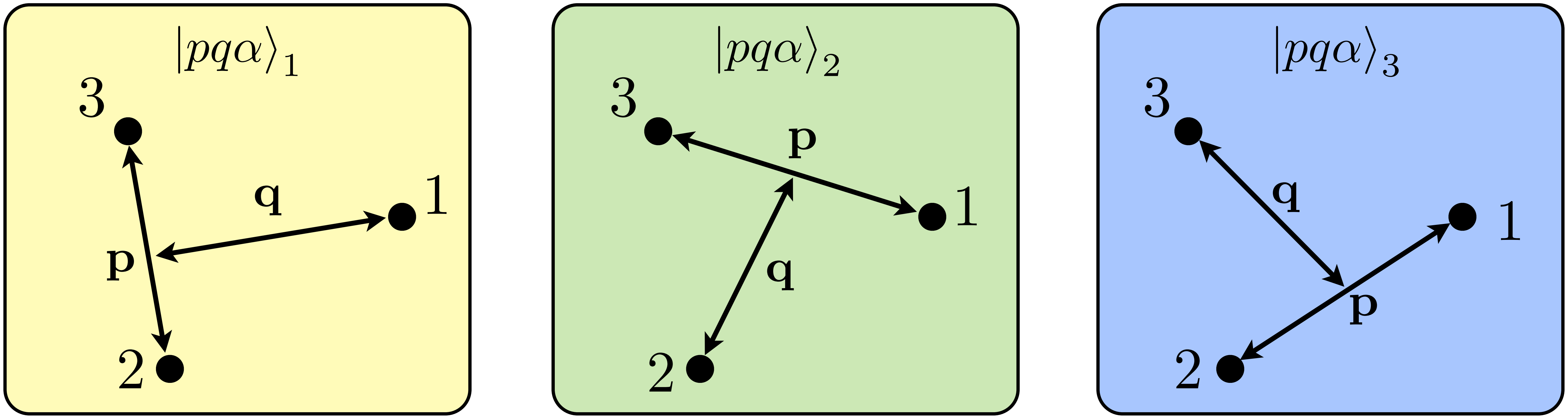

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A 'new' algorithm allows efficient calculation.

KH, Krebs, Epelbaum, Golak, Skibinski, PRC 91, 044001(2015)

Calculation of 3N forces in momentum partial-wave representation

$$\langle pq\alpha|V_{123}|p'q'\alpha'\rangle \sim \sum_{m_i} \int d\hat{\mathbf{p}}\, d\hat{\mathbf{q}}\, d\hat{\mathbf{p}}'\, d\hat{\mathbf{q}}' Y_l^m(\hat{\mathbf{p}}) Y_{\bar{l}}^{\bar{m}}(\hat{\mathbf{q}})\, \langle \mathbf{p}\mathbf{q}ST|V_{123}|\mathbf{p}'\mathbf{q}'S'T'\rangle\, Y_{l'}^{m'}(\hat{\mathbf{p}}') Y_{\bar{l}'}^{\bar{m}'}(\hat{\mathbf{q}}')$$

traditional method:

- reduce dimension of angular integrals from 8 to 5 by using symmetry
- discretize angular integrals and perform all sums numerically

much more efficient method:

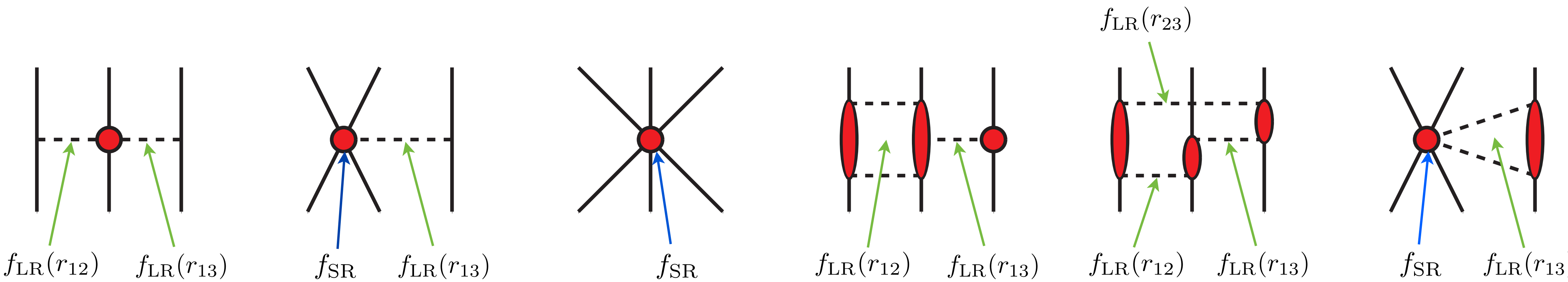
- use that all interaction contributions (except rel. corr.) are local:

$$\begin{aligned} \langle \mathbf{p}\mathbf{q}|V_{123}|\mathbf{p}'\mathbf{q}'\rangle &= V_{123}(\mathbf{p}-\mathbf{p}', \mathbf{q}-\mathbf{q}') \\ &= V_{123}(p-p', q-q', \cos\theta) \end{aligned}$$

$\longrightarrow$ allows to perform all except for 3 integrals analytically

- only a few small discrete internal sums need to be performed for each external momentum and angular momentum

Semi-local regularization of 3NF (coordinate space)



$1/m$

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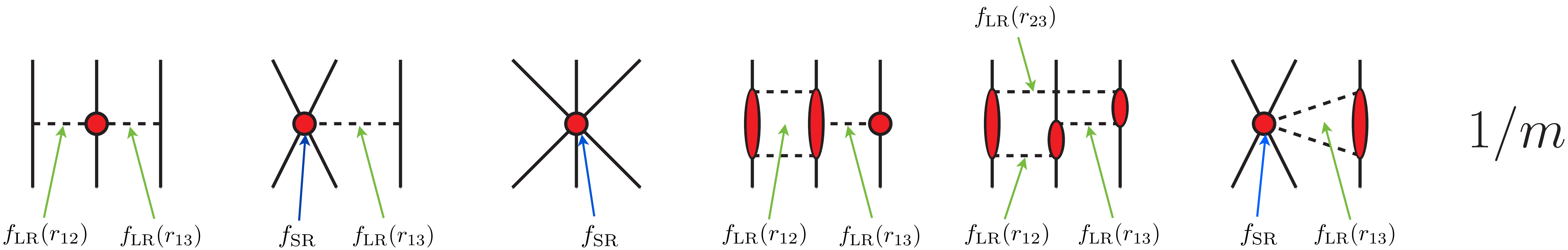

Computational strategy:

(1) calculate unregularized 3NF in sufficiently large partial-wave basis

(2) fourier transform coordinate space regulator to momentum space

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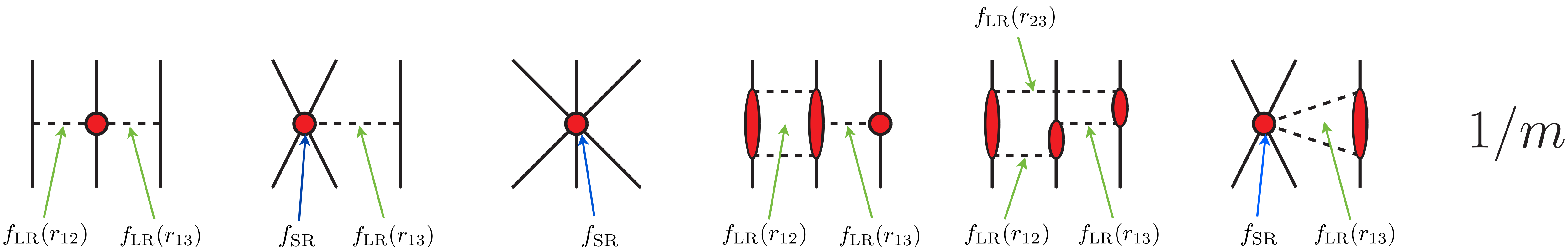

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(3) decompose regulator f_{LR} in partial wave momentum basis

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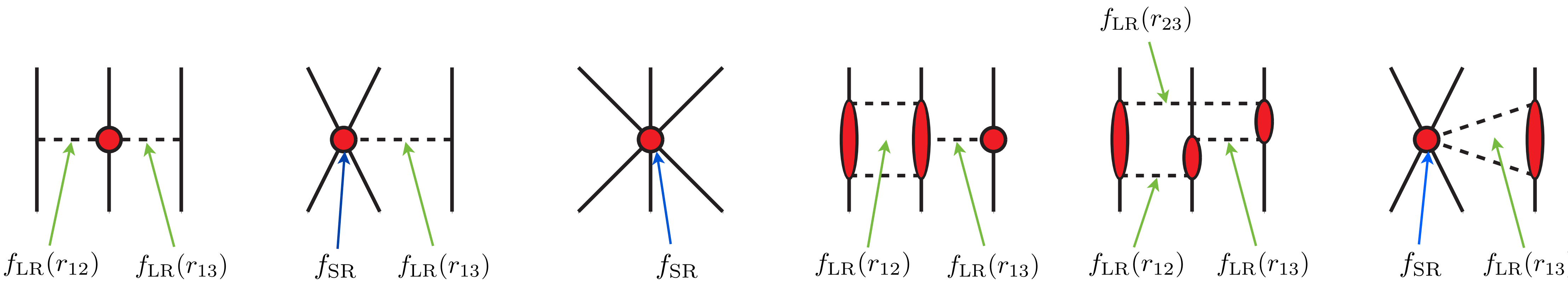

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(4) perform convolution integrals:

$$\langle pq\alpha | V_{123}^{\rm reg} | p'q'\alpha' \rangle = \int d\tilde{q}\, \tilde{q}^2 \int d\tilde{p}\, \tilde{p}^2 \sum_{\tilde{\alpha}} \langle pq\alpha | V_{123} | \tilde{p}\tilde{q}\tilde{\alpha} \rangle \, \langle \tilde{p}\tilde{q}\tilde{\alpha} | f_{LR} | p'q'\alpha' \rangle$$

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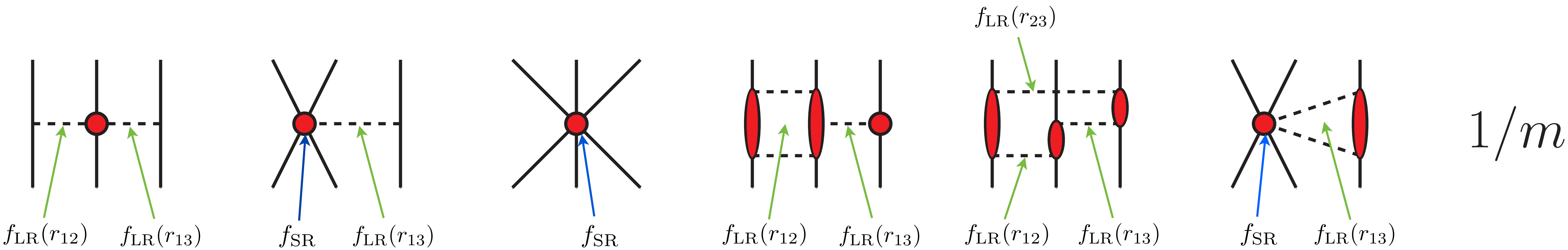

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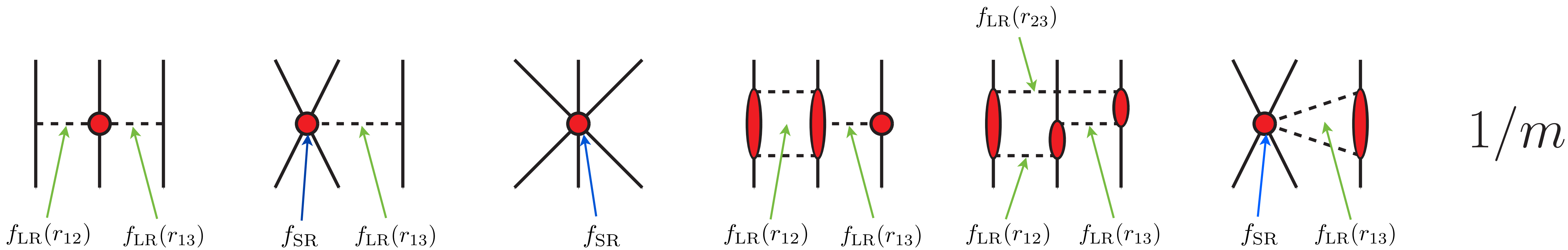

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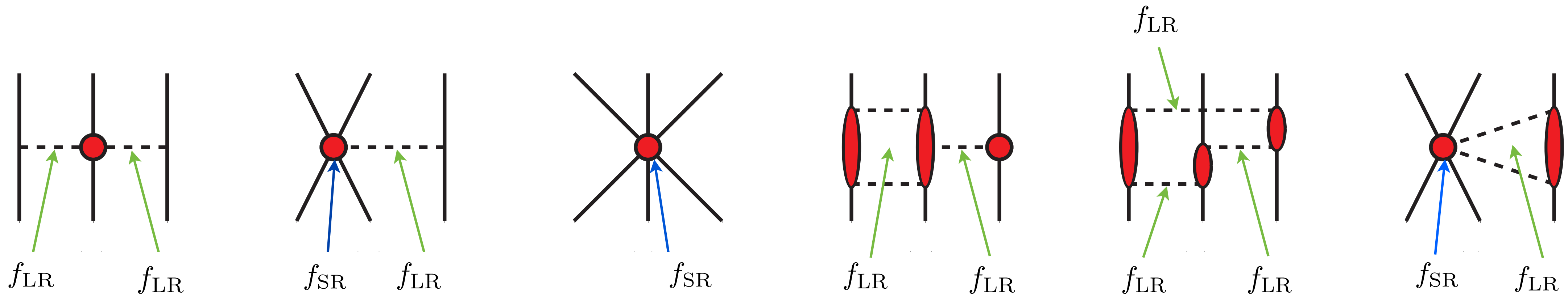
(6) antisymmetrize interactions (optional)

Semi-local regularization of 3NF (momentum space)



$$f_{\rm LR} = f_{\rm LR}(\mathbf{q}) = \exp\left[-(\mathbf{q}^2 + m_\pi^2)/\Lambda^2\right]$$ → *Hermann Krebs' talk*

$$V_{123} = V_{123}(p - p', q - q', \cos\theta) = V_{123}(\tilde{p}, \tilde{q}, \cos\theta)$$

Example:

N2LO 2pi topology: $$V_{123}^{2\pi} \sim \frac{1}{(\mathbf{q}_2^2 + m_\pi^2)(\mathbf{q}_2^3 + m_\pi^2)} = \frac{1}{((\mathbf{p} - \mathbf{q}/2)^2 + m_\pi^2)((\mathbf{p} + \mathbf{q}/2)^2 + m_\pi^2)}$$

$$\longrightarrow \boxed{V_{123}^{2\pi, reg} \sim \frac{f_{\rm LR}(\mathbf{q}_2) f_{\rm LR}(\mathbf{q}_3)}{(\mathbf{q}_2^2 + m_\pi^2)(\mathbf{q}_2^3 + m_\pi^2)}}$$

Status and storage of 3NF matrix elements

- Calculation of matrix elements at N2LO completed

- all 3N topologies are calculated and stored separately, allows to easily adjust values of LECs $c_1, c_3, c_4, c_D, c_E, C_S$ and C_T

- calculated matrix elements of Faddeev components

$$\langle pq\alpha | V_{123}^{i} | p'q'\alpha' \rangle$$

as well as fully and partially antisymmetrized matrix elements

$$\langle pq\alpha | (1 + P_{123} + P_{132}) V_{123}^{i} (1 + P_{123} + P_{132}) | p'q'\alpha' \rangle$$

$$\langle pq\alpha | V_{123}^{i} (1 + P_{123} + P_{132}) | p'q'\alpha' \rangle$$

http://www.hdfgroup.org

- HDF5 file format for efficient I/O

First results:
→ *Thomas Hüther's talk*

Novel efficient many-body framework for nuclear matter (and other problems?)

Main code developer:
Christian Drischler

Problem:

Evaluation of MBPT diagrams beyond second order in perturbation theory becomes complicated and tedious in partial wave representation.

Present frameworks too inefficient for including matter properties in force fits.

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Strategy:

Implementation of NN and 3N forces without partial wave decomposition.

Calculate MBPT diagrams in vector basis

$$|12...n\rangle = |\mathbf{k}_1 m_{s_1} m_{t_1}\rangle \otimes |\mathbf{k}_2 m_{s_2} m_{t_2}\rangle \otimes ... \otimes |\mathbf{k}_n m_{s_n} m_{t_n}\rangle$$

using Monte-Carlo techniques. Implementation efficient and very transparent.

Drischler et al. arXiv:1710.08220 (2017)

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Status:

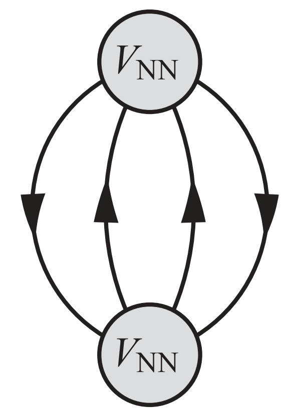
Implementation of nonlocal NN plus 3N forces up to N3LO complete.

Implemented MBPT diagrams up to 4th order for state-of-the-art interactions.

Entem et al. PRC 96, 024004 (2017)

Example: Second order diagram in MBPT

$$E^{(2)}_{\mathrm{NN+3N,eff}} = \frac{1}{4}\left[\prod_{i=1}^{4}\mathrm{Tr}_{\sigma_i}\int\frac{d\mathbf{k}_i}{(2\pi)^3}\right]\left|\langle 12|V^{(2)}_{\mathrm{as}}|34\rangle\right|^2$$
$$\times\frac{n_{\mathbf{k}_1}n_{\mathbf{k}_2}(1-n_{\mathbf{k}_3})(1-n_{\mathbf{k}_4})}{\varepsilon_{\mathbf{k}_1}+\varepsilon_{\mathbf{k}_2}-\varepsilon_{\mathbf{k}_3}-\varepsilon_{\mathbf{k}_4}}(2\pi)^3$$
$$\times\,\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4).$$



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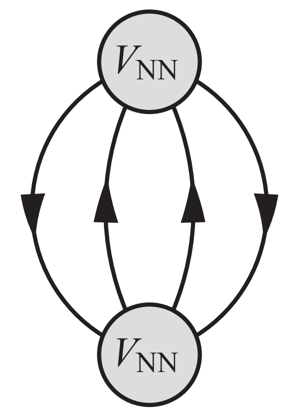


Partial wave representation:

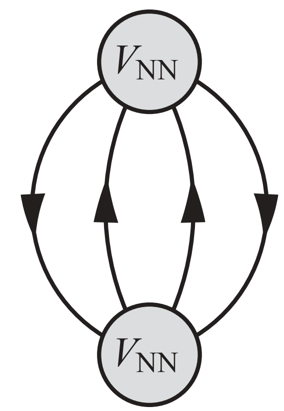
$$\sum_{S,M_S,M_S'}\left|\langle\mathbf{k}SM_S|V^{(2)}_{\mathrm{as}}|\mathbf{k}'SM_S'\rangle\right|^2$$
$$=\sum_L P_L(\cos\theta_{\mathbf{k},\mathbf{k}'})\sum_{J,l,l',S}\sum_{\tilde{J},\tilde{l},\tilde{l}'}(4\pi)^2 i^{(l-l'+\tilde{l}-\tilde{l}')}(-1)^{\tilde{l}+l'+L}$$
$$\times\,\mathcal{C}^{L0}_{l0\tilde{l}'0}\mathcal{C}^{L0}_{l'0\tilde{l}0}\sqrt{(2l+1)(2l'+1)(2\tilde{l}+1)(2\tilde{l}'+1)}$$
$$\times\,(2J+1)(2\tilde{J}+1)\begin{Bmatrix} l & S & J\\ \tilde{J} & L & \tilde{l}'\end{Bmatrix}\begin{Bmatrix} J & S & l'\\ \tilde{l} & L & \tilde{J}\end{Bmatrix}$$
$$\times\,\langle k|V^{(2)}_{Sl'lJ}|k'\rangle\langle k'|V^{(2)}_{S\tilde{l}'\tilde{l}\tilde{J}}|k\rangle[1-(-1)^{l+S+1}]$$
$$\times\,[1-(-1)^{\tilde{l}+S+1}],$$

e.g., KH, Schwenk
PRC 82, 014314 (2013)

- hard to automatize and generalize to higher order diagrams
- prone to mistakes

Example: Second order diagram in MBPT

$$E^{(2)}_{\mathrm{NN+3N,eff}} = \frac{1}{4}\left[\prod_{i=1}^{4} \mathrm{Tr}_{\sigma_i}\int \frac{d\mathbf{k}_i}{(2\pi)^3}\right] \left|\langle 12|V^{(2)}_{\mathrm{as}}|34\rangle\right|^2$$
$$\times \frac{n_{\mathbf{k}_1} n_{\mathbf{k}_2}(1-n_{\mathbf{k}_3})(1-n_{\mathbf{k}_4})}{\varepsilon_{\mathbf{k}_1}+\varepsilon_{\mathbf{k}_2}-\varepsilon_{\mathbf{k}_3}-\varepsilon_{\mathbf{k}_4}}(2\pi)^3$$
$$\times \delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4).$$



Partial wave representation:

$$\sum_{S,M_S,M_S'} \left|\langle \mathbf{k} S M_S|V^{(2)}_{\mathrm{as}}|\mathbf{k}' S M_S'\rangle\right|^2$$
$$= \sum_L P_L(\cos\theta_{\mathbf{k},\mathbf{k}'}) \sum_{J,l,l',S} \sum_{\tilde{J},\tilde{l},\tilde{l}'} (4\pi)^2 i^{(l-l'+\tilde{l}-\tilde{l}')}(-1)^{\tilde{l}+l'+L}$$
$$\times \mathcal{C}^{L0}_{l0\tilde{l}'0}\mathcal{C}^{L0}_{l'0\tilde{l}0}\sqrt{(2l+1)(2l'+1)(2\tilde{l}+1)(2\tilde{l}'+1)}$$
$$\times (2J+1)(2\tilde{J}+1)\begin{Bmatrix} l & S & J \\ \tilde{J} & L & \tilde{l}' \end{Bmatrix}\begin{Bmatrix} J & S & l' \\ \tilde{l} & L & \tilde{J} \end{Bmatrix}$$
$$\times \langle k|V^{(2)}_{Sl'lJ}|k'\rangle\langle k'|V^{(2)}_{S\tilde{l}'\tilde{l}\tilde{J}}|k\rangle[1-(-1)^{l+S+1}]$$
$$\times [1-(-1)^{\tilde{l}+S+1}],$$

e.g., KH, Schwenk PRC 82, 014314 (2013)

- hard to automatize and generalize to higher order diagrams
- prone to mistakes

Single-particle vector representation:

$$\frac{E^{(2)}_{\mathrm{NN}}}{V} = +\frac{1}{4}\sum_{\substack{ij\\ab}} \frac{\langle ij|\mathcal{A}_{12}V_{\mathrm{NN}}|ab\rangle\,\langle ab|\mathcal{A}_{12}V_{\mathrm{NN}}|ij\rangle}{D_{ijab}}$$

- each diagram a compact single line of code
- straightforward to automatize code generation
- adaptive evaluation of integrals using Monte-Carlo techniques

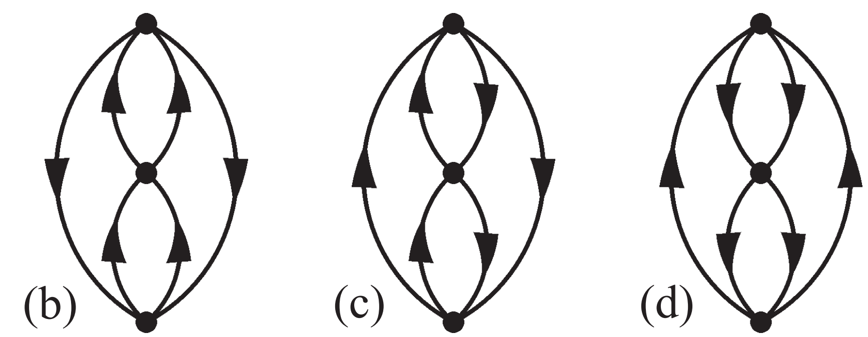
Higher-order contributions

example: third order (particle-particle, hole-hole, particle-hole)

$$\frac{E_1^{(3)}}{V} = +\frac{1}{8}\sum_{\substack{ijkl\\ab}} \frac{\langle ij|\mathscr{A}_{12}V_{\mathrm{NN}}|ab\rangle \langle kl|\mathscr{A}_{12}V_{\mathrm{NN}}|ij\rangle \langle ab|\mathscr{A}_{12}V_{\mathrm{NN}}|kl\rangle}{D_{ijab}D_{klab}},$$

$$\frac{E_2^{(3)}}{V} = +\sum_{\substack{ijk\\abc}} \frac{\langle ij|\mathscr{A}_{12}V_{\mathrm{NN}}|ab\rangle \langle ak|\mathscr{A}_{12}V_{\mathrm{NN}}|ic\rangle \langle bc|\mathscr{A}_{12}V_{\mathrm{NN}}|jk\rangle}{D_{ijab}D_{jkbc}},$$

$$\frac{E_3^{(3)}}{V} = +\frac{1}{8}\sum_{\substack{ij\\abcd}} \frac{\langle ij|\mathscr{A}_{12}V_{\mathrm{NN}}|ab\rangle \langle ab|\mathscr{A}_{12}V_{\mathrm{NN}}|cd\rangle \langle cd|\mathscr{A}_{12}V_{\mathrm{NN}}|ij\rangle}{D_{ijab}D_{ijcd}}.$$



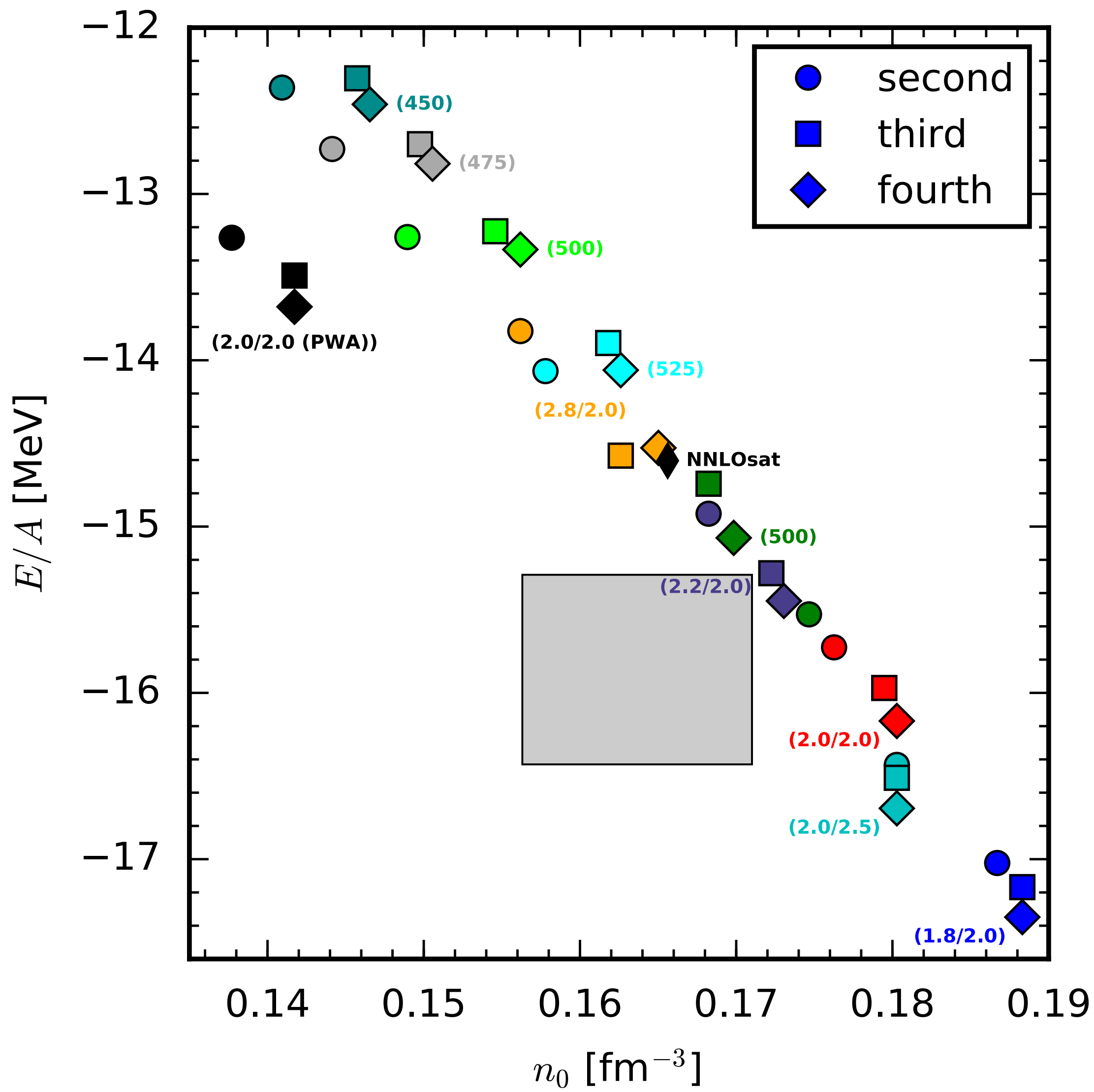
Status:

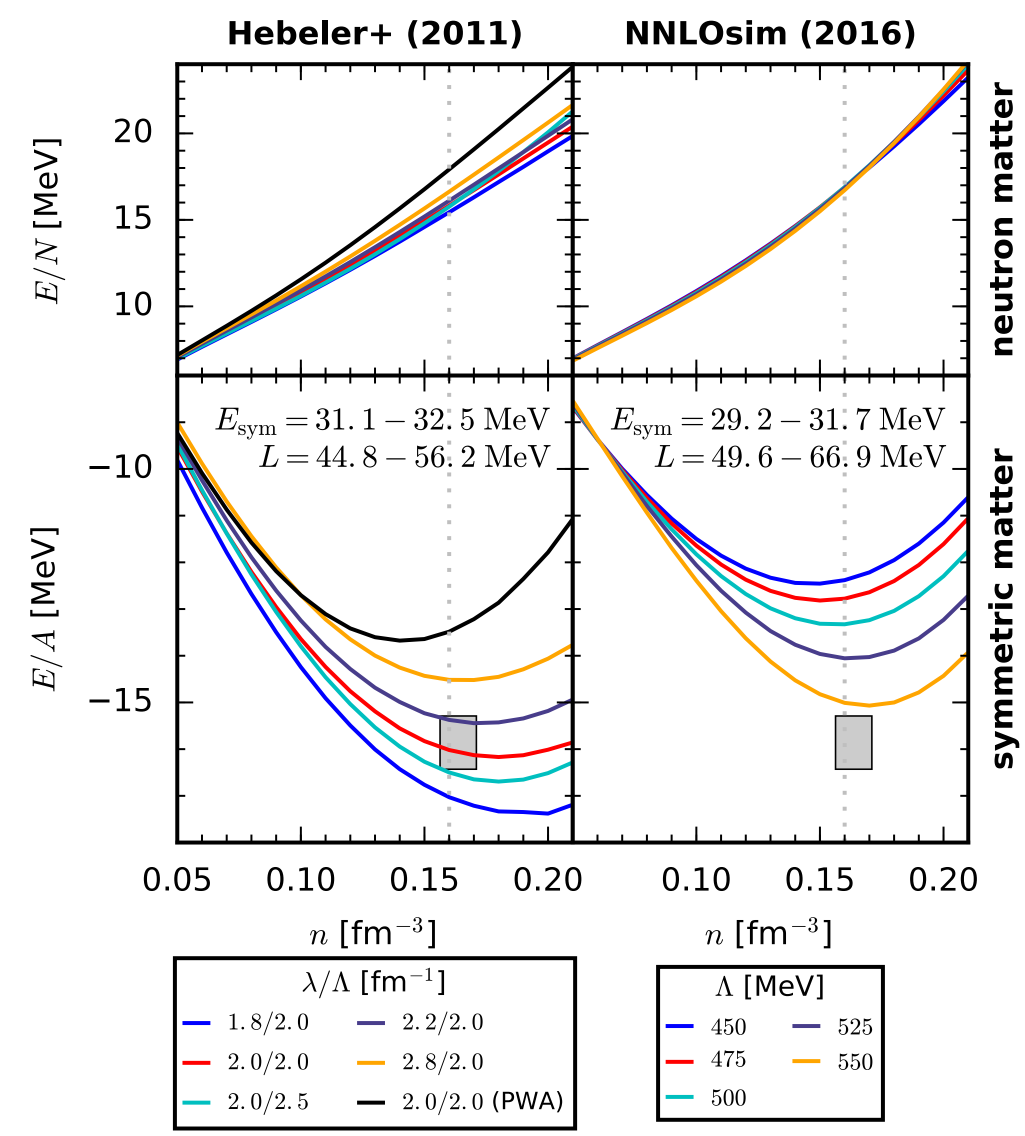
- implemented all NN diagrams up to fourth order in MBPT, 3N interactions up to third order
- implemented all NN and 3N interactions (nonlocal) up to N3LO
- possible to also use NN matrix elements stored in partial wave basis by partial wave resummation
- interaction interface suitable for all many-body frameworks that require matrix elements in a momentum vector single-particle basis

Proof of principle:
Fits of 3N interactions to saturation properties of nuclear matter

- incorporation of saturation properties in fits was not possible so far due to insufficient efficiency of many-body calculations

- performed calculations up to 4th order for set of presently used NN interactions, natural convergence pattern   Drischler et al., arXiv:1710.08220 (2017)

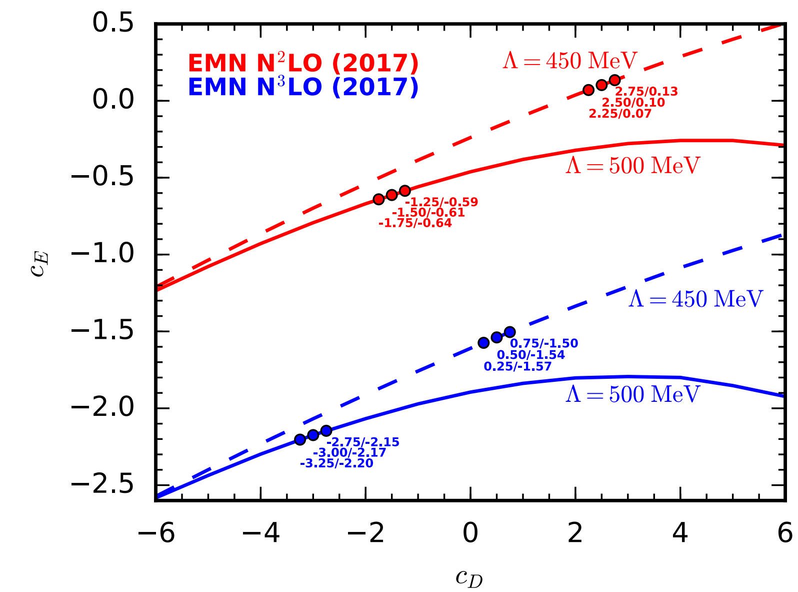




Proof of principle:
Fits of 3N interactions to saturation properties of nuclear matter

- incorporation of saturation properties in fits was not possible so far due to insufficient efficiency of many-body calculations

- performed fits for 3NF at N2LO and N3LO to 3H and matter for new family of NN forces by Entem, Machleidt and Nosyk   Entem et al. PRC 96, 024004 (2017)



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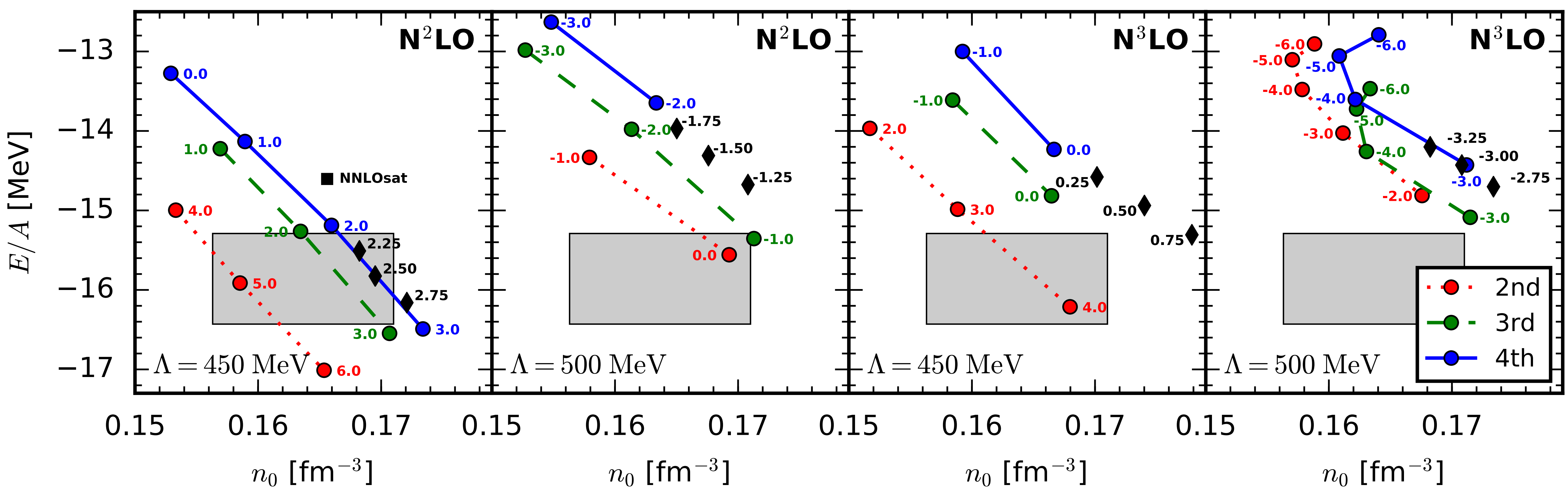

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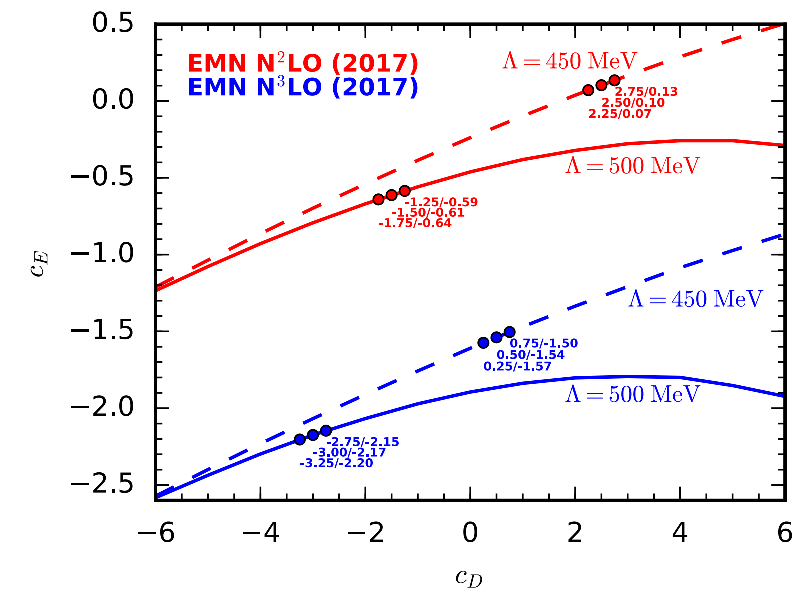

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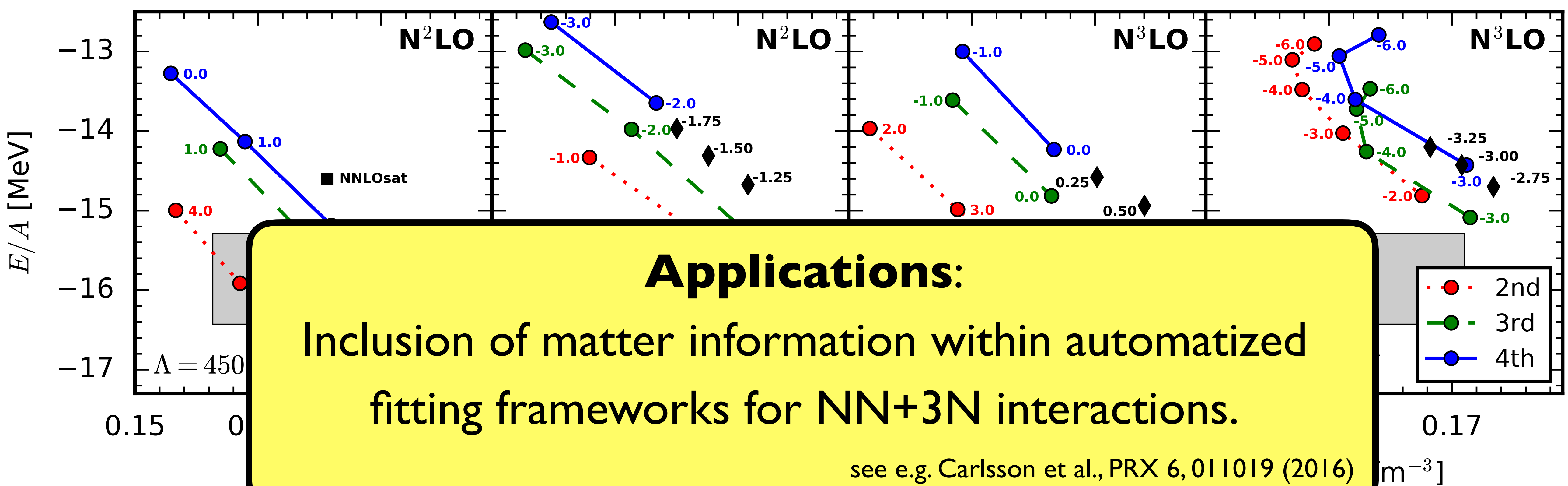


Applications:

Inclusion of matter information within automatized fitting frameworks for NN+3N interactions.

see e.g. Carlsson et al., PRX 6, 011019 (2016)

Thank you!

Backup slides

Status of 3NF matrix element calculation

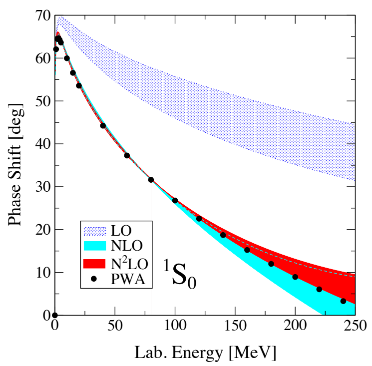
$\mathcal{J}$	$J_{12}^{\max}$	c_1, c_3, c_4, c_D, c_E	$1/m$ OPE (fixed β_8, β_9)
$1/2$	8	✓	✓
$3/2$	8	✓	✓
$5/2$	7	✓	✓
$7/2$	7	✓	✓
$9/2$	6	✓	✓

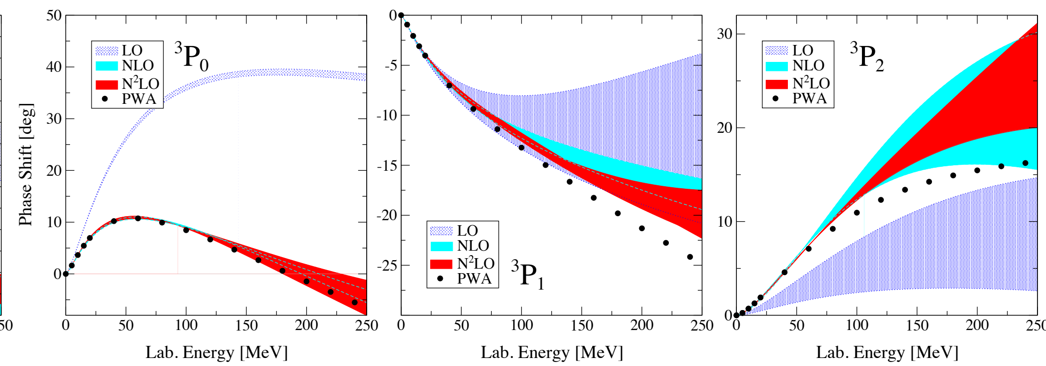
- $N_p = 25, p_{\max} = 10 \text{ fm}^{-1}$ $N_q = 25, q_{\max} = 10 \text{ fm}^{-1}$
- Computed for cutoffs $\Lambda = 400, 450, 500, 550 \text{ MeV}$
- Calculation of 1/m TPE expensive! need to fix values of β_8, β_9

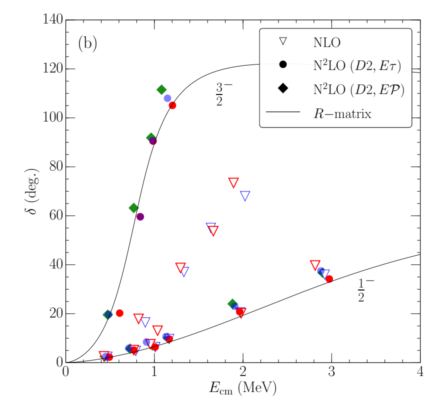
Recent and current developments of novel nuclear interactions

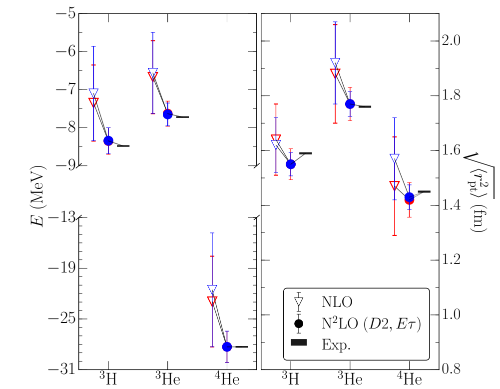
1. local EFT interactions, suitable for Quantum Monte Carlo calculations

status: NN plus 3N up to N2LO, calculations of few-body systems and neutron matter









Gezerlis et al.,
PRL 111, 032501 (2013)

Gezerlis et al.,
PRC 90, 054323 (2014)

Lynn et al.,
PRL 116, 062501 (2016)

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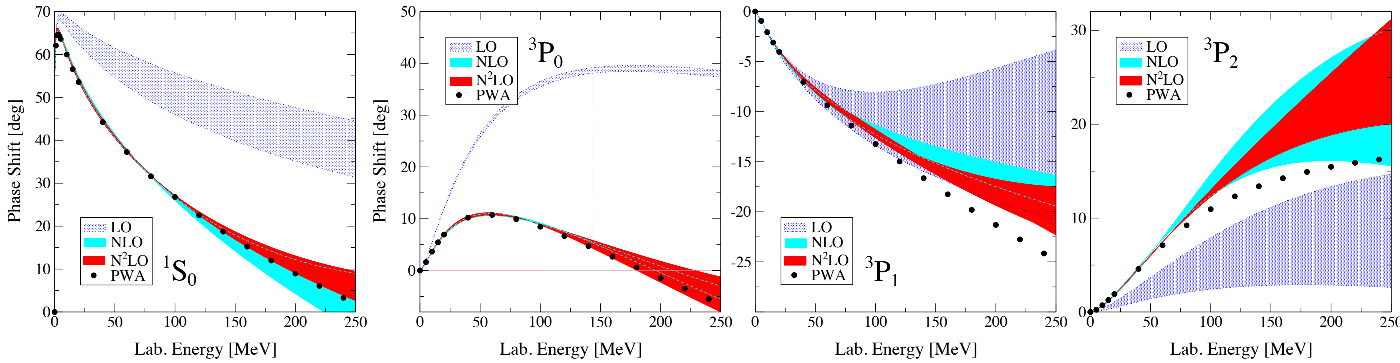

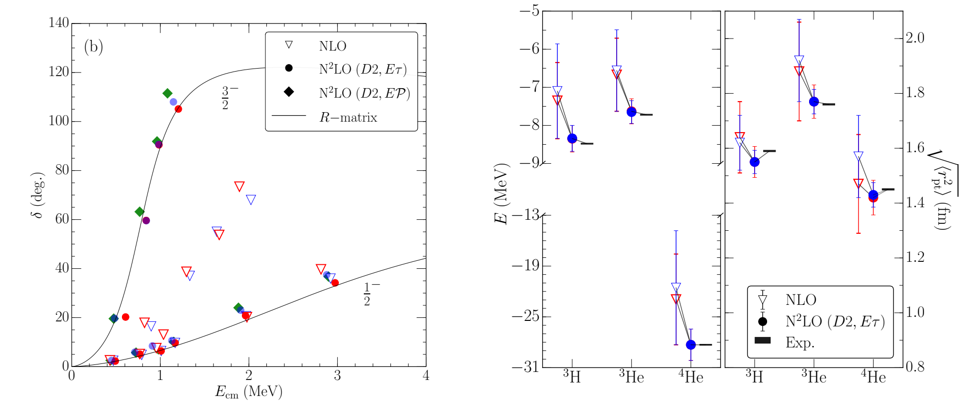

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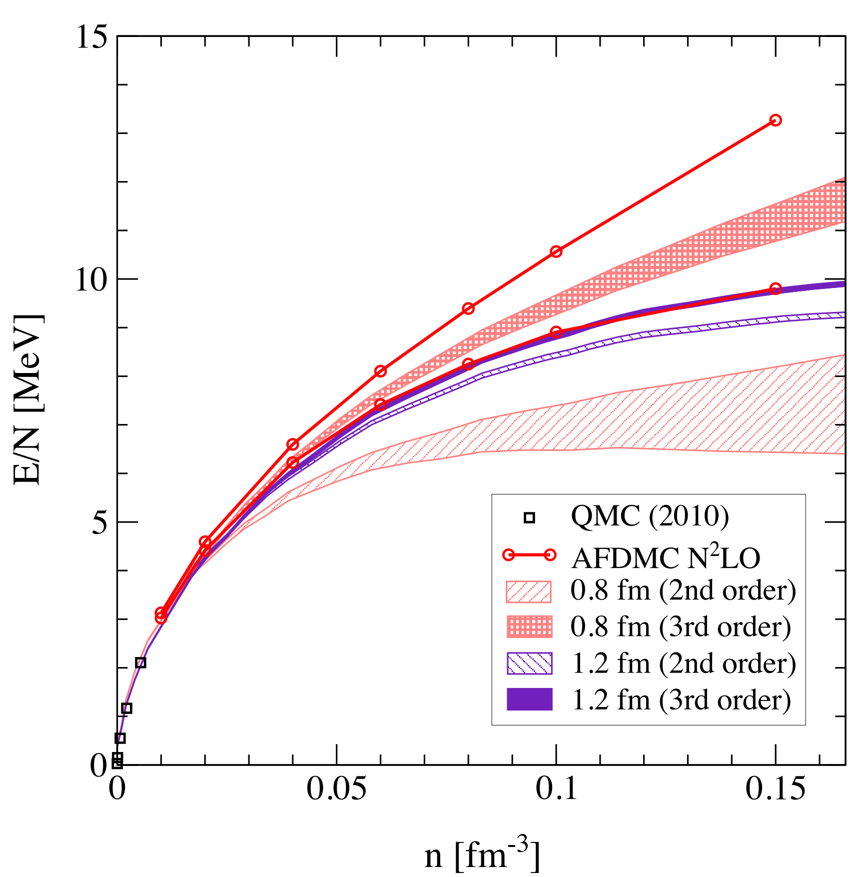

first Quantum Monte Carlo of neutron matter based on chiral EFT interactions

perfect agreement for soft interactions, first direct validation of calculations within many-body perturbation theory

Gezerlis et. al,
PRL 111, 032501 (2013)

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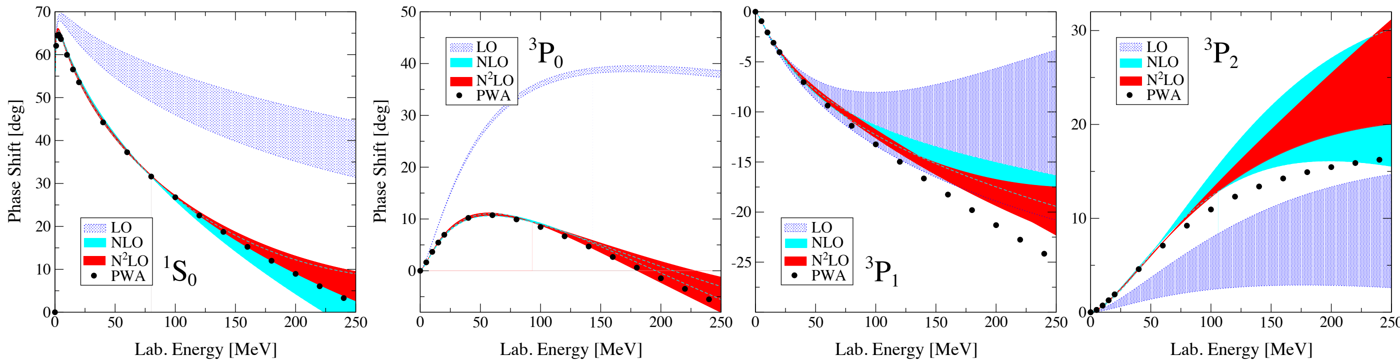

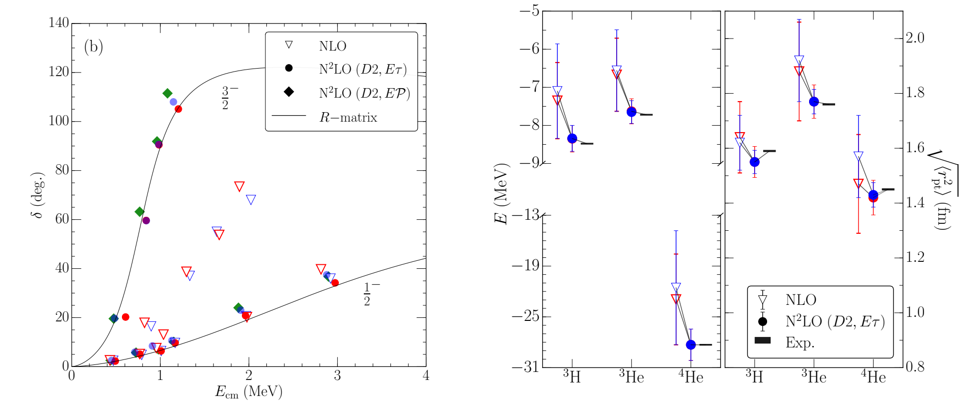

Gezerlis et al.,
PRL 111, 032501 (2013)

Gezerlis et al.,
PRC 90, 054323 (2014)

Lynn et al.,
PRL 116, 062501 (2016)

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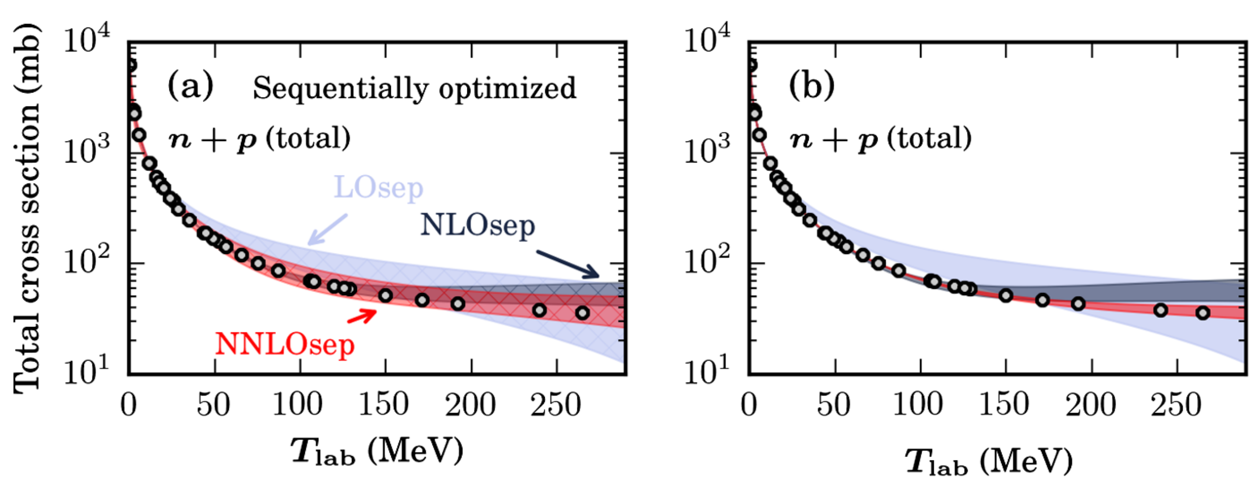
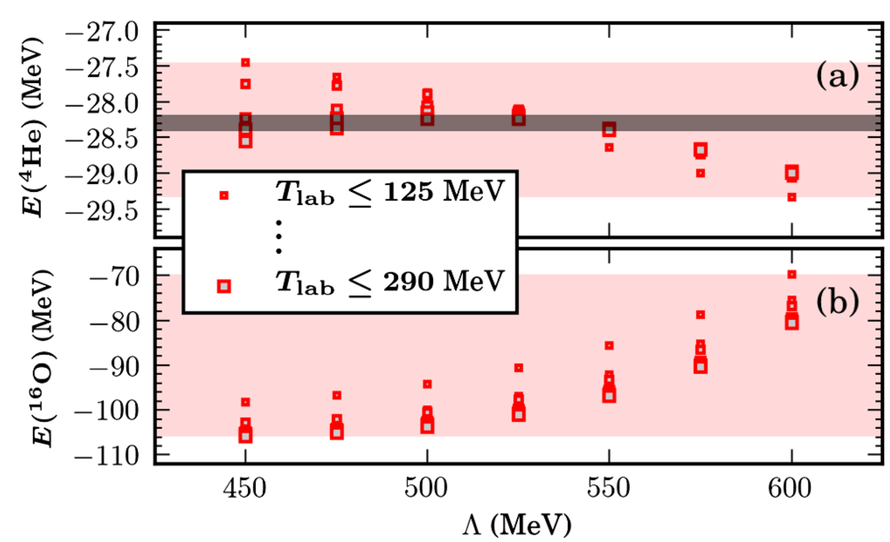
2. simultaneous fit of NN and 3N forces to two- and few-body observables

   status: NN plus 3N up to N2LO, N3LO currently in development



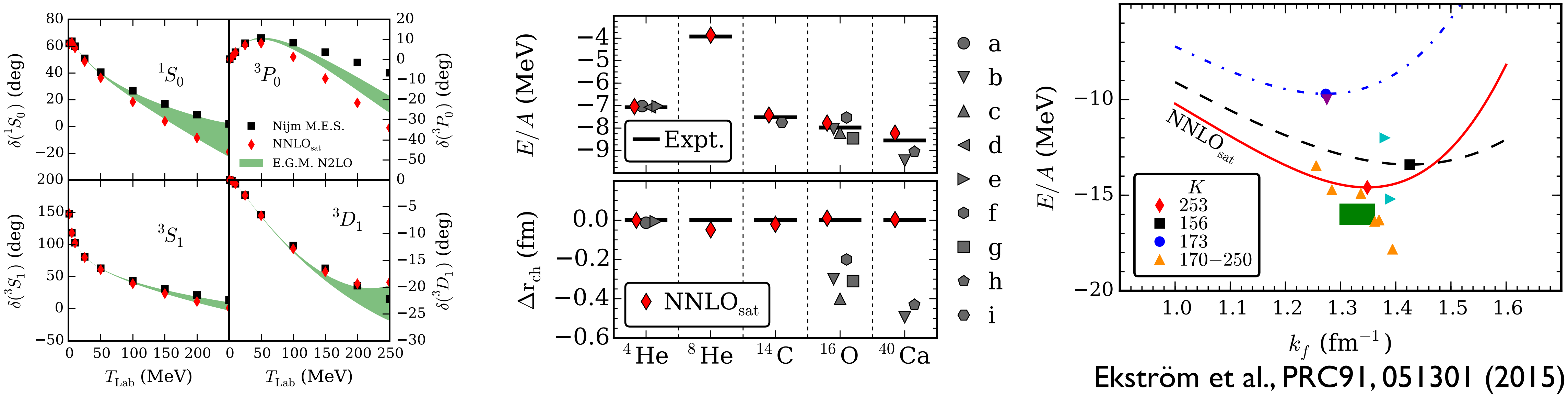


Carlsson et al.,
PRX 6, 011019 (2016)

Recent and current developments of novel nuclear interactions

3. fits of NN plus 3N forces to two-, few- and many-body observables

status: NN plus 3N up to N2LO, NN phase shifts fitted up to T_{lab}~35 MeV



Ekström et al., PRC91, 051301 (2015)

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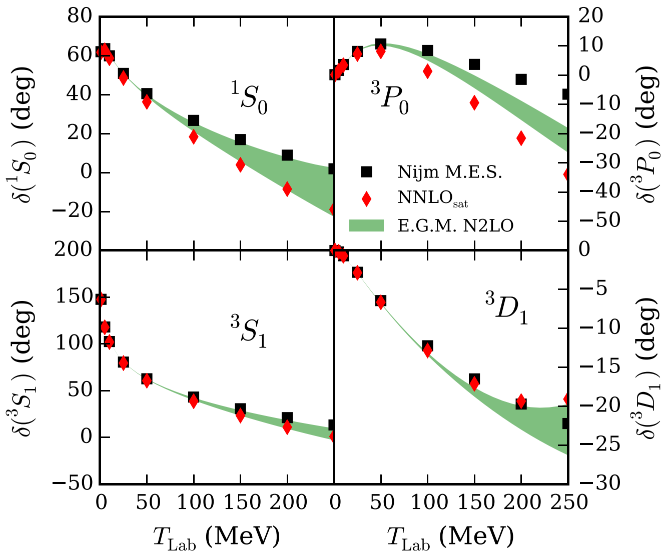
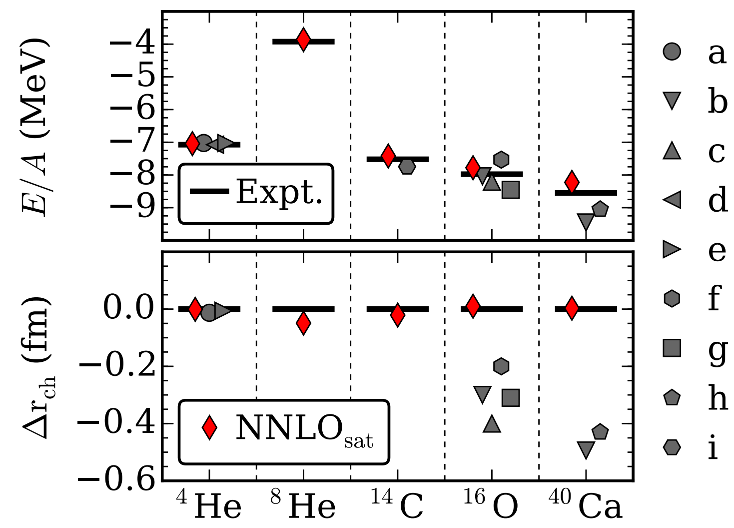
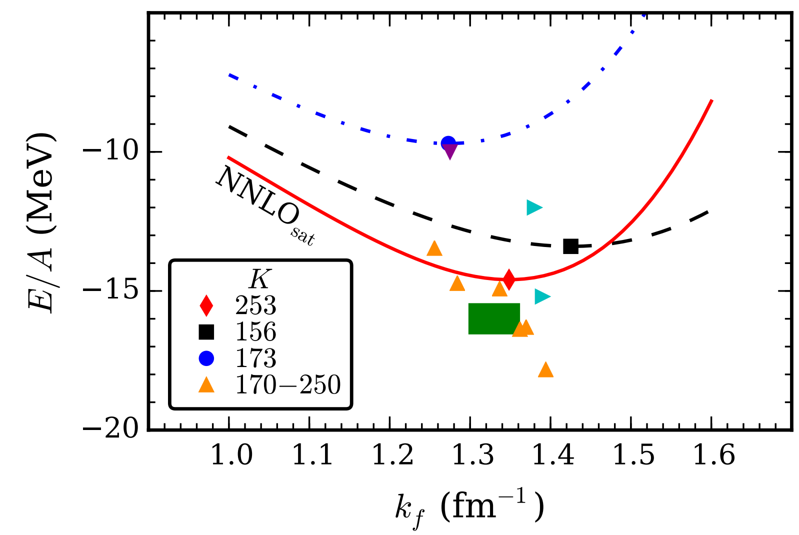


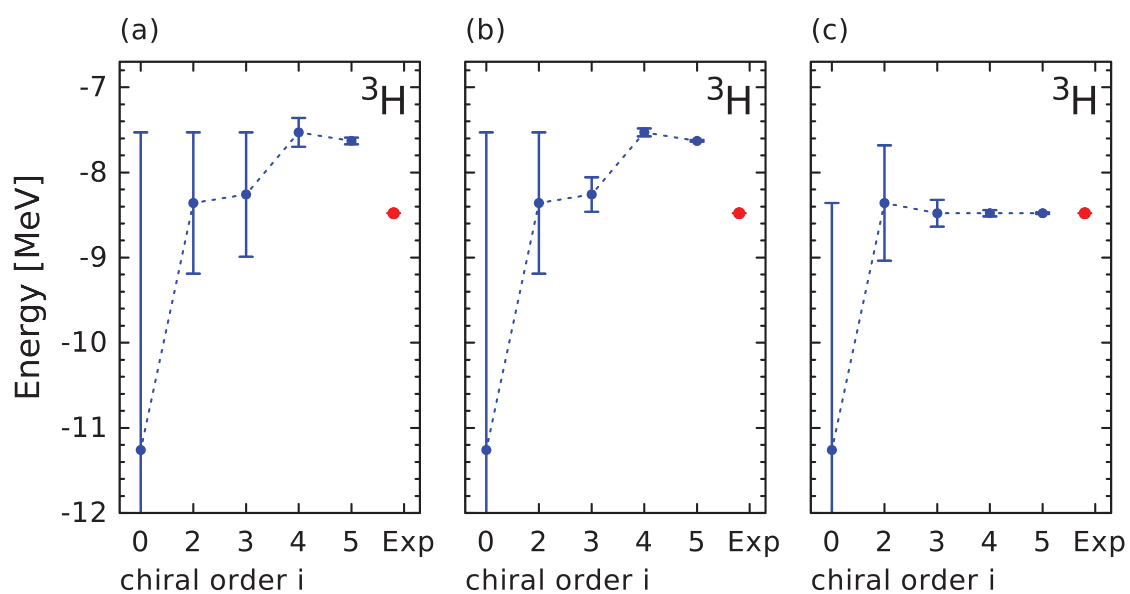


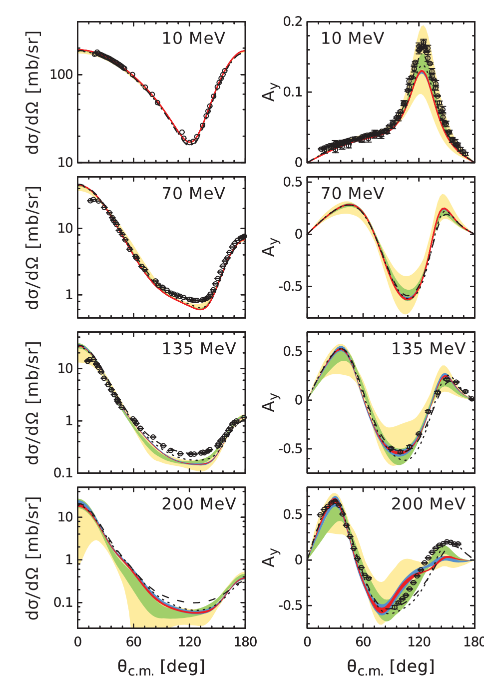


Ekström et al., PRC91, 051301 (2015)

4. semilocal NN forces, development of improved method to estimate uncertainties

status: NN up to N4LO, 3N interactions up to N3LO





LENPIC

Epelbaum, Krebs, Meißner,
PRL 115, 122301 (2015)

Binder et al.,
PRC 93, 044002 (2016)

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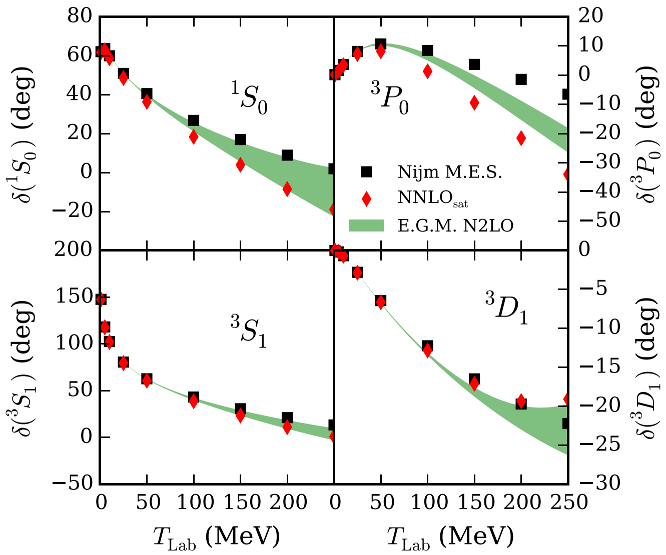

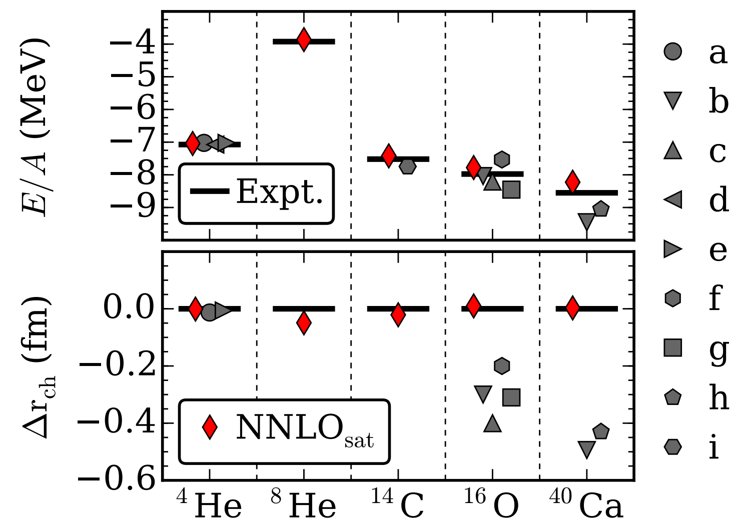

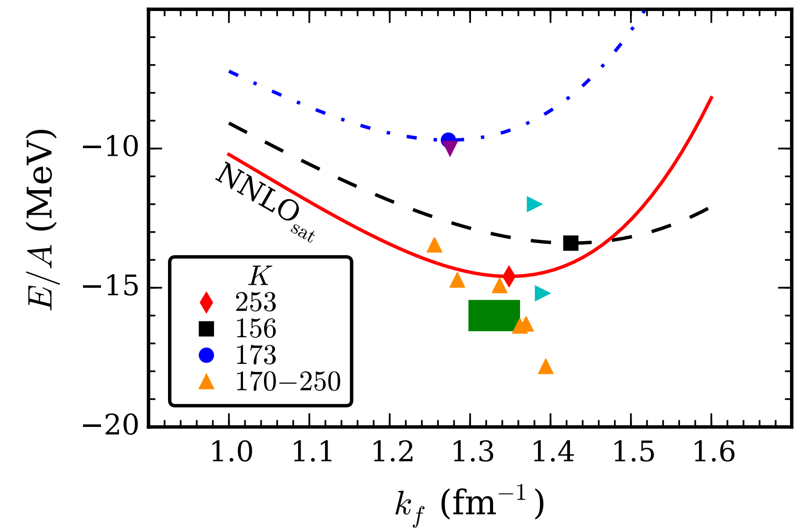

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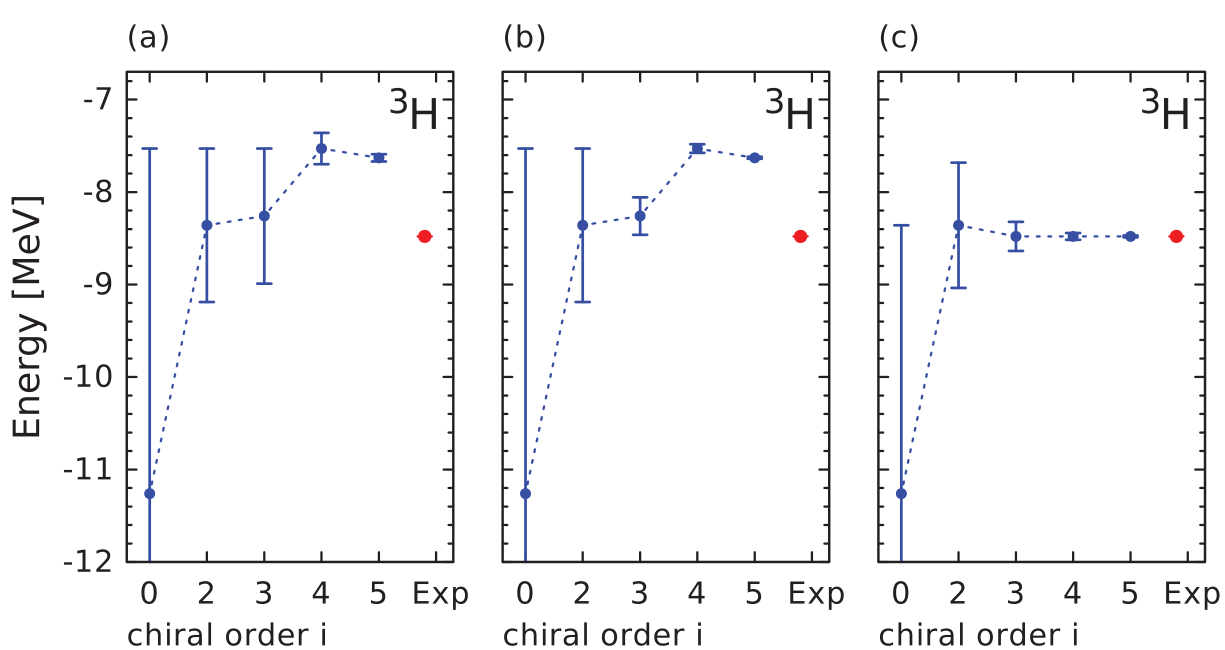
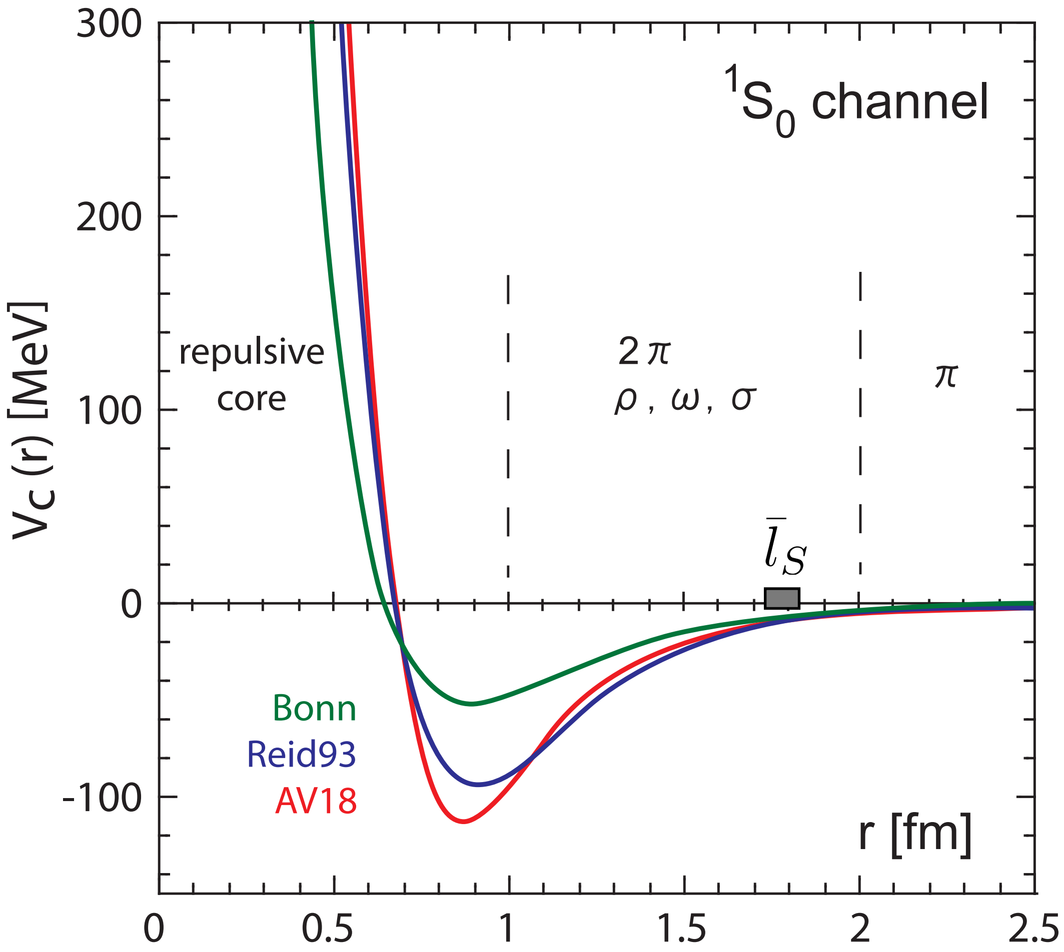


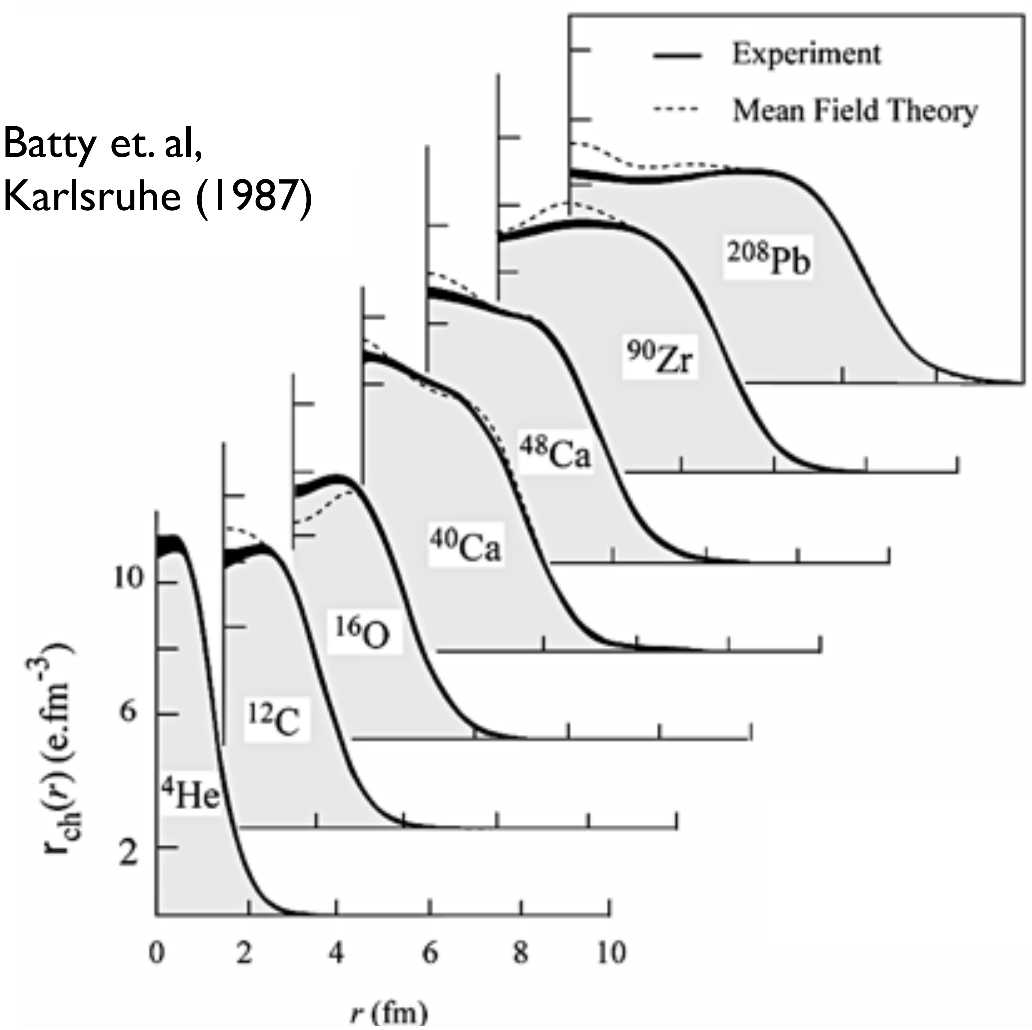
Epelbaum, Krebs, Meißner,
PRL 115, 122301 (2015)

Binder et al.,
PRC 93, 044002 (2016)

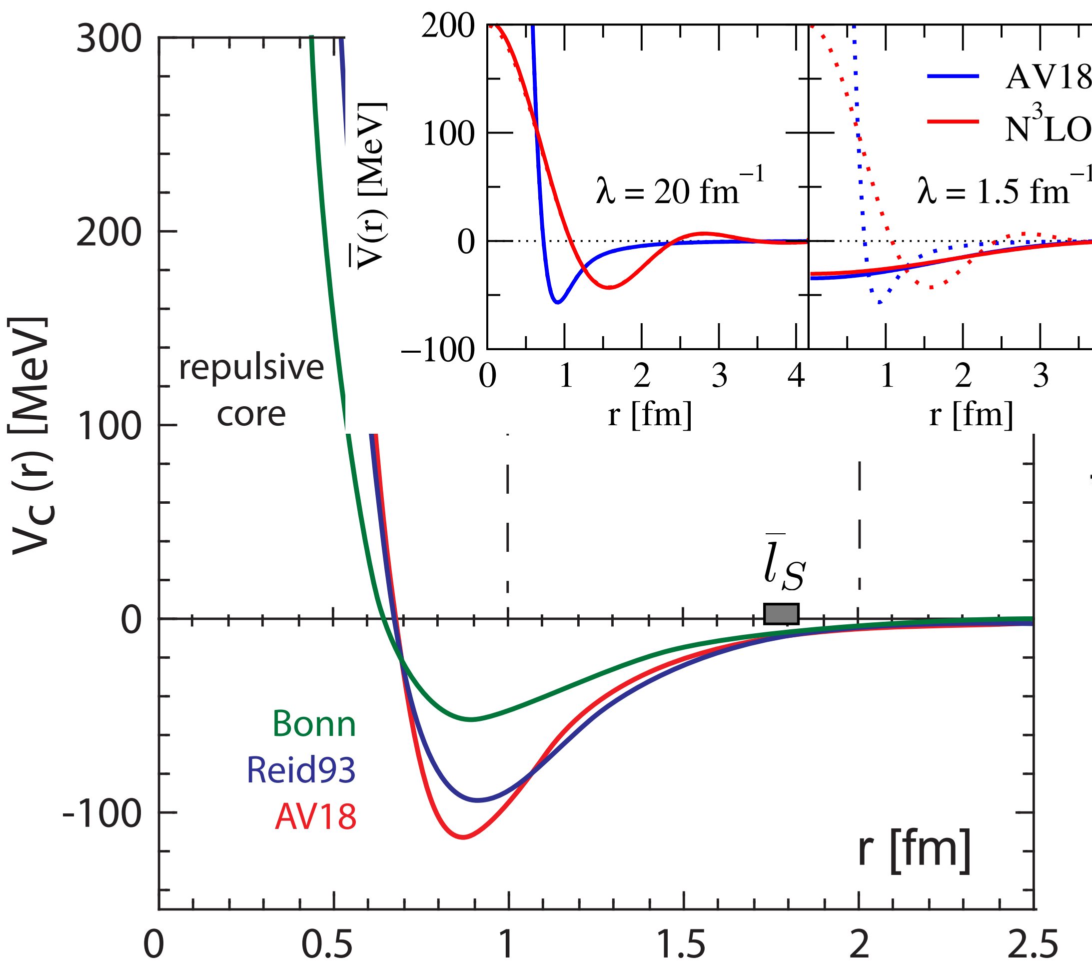
Equation of state of symmetric nuclear matter: nuclear saturation

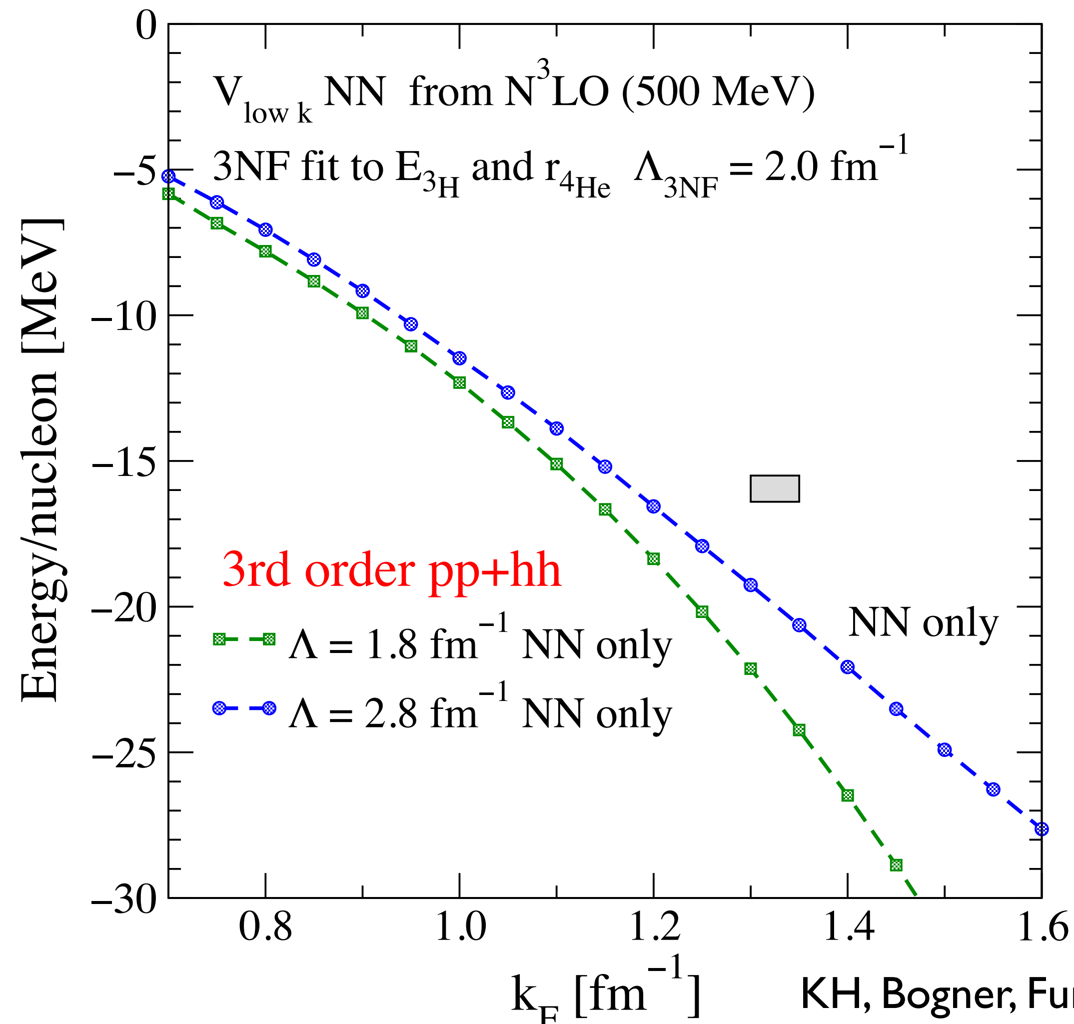


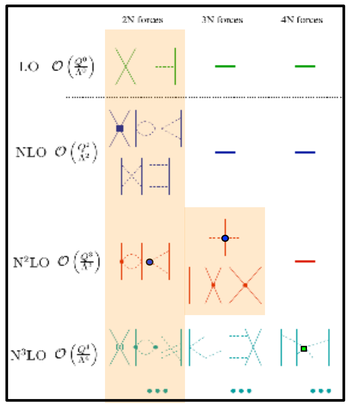
Batty et. al, Karlsruhe (1987)



Fitting the 3NF LECs at low resolution scales







KH, Bogner, Furnstahl, Nogga,
PRC(R) 83, 031301 (2011)

*"Very soft potentials must be excluded because they do not give saturation;
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Hans Bethe (1971)

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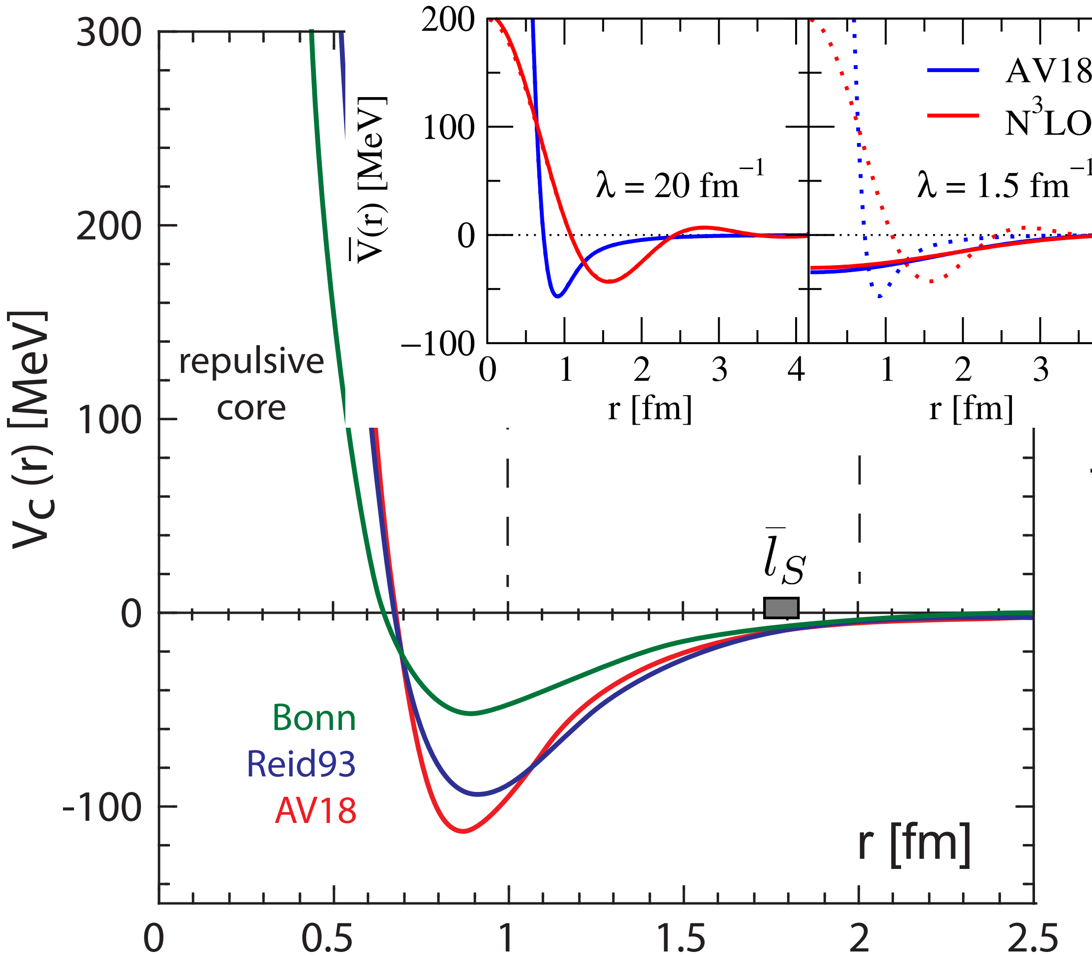
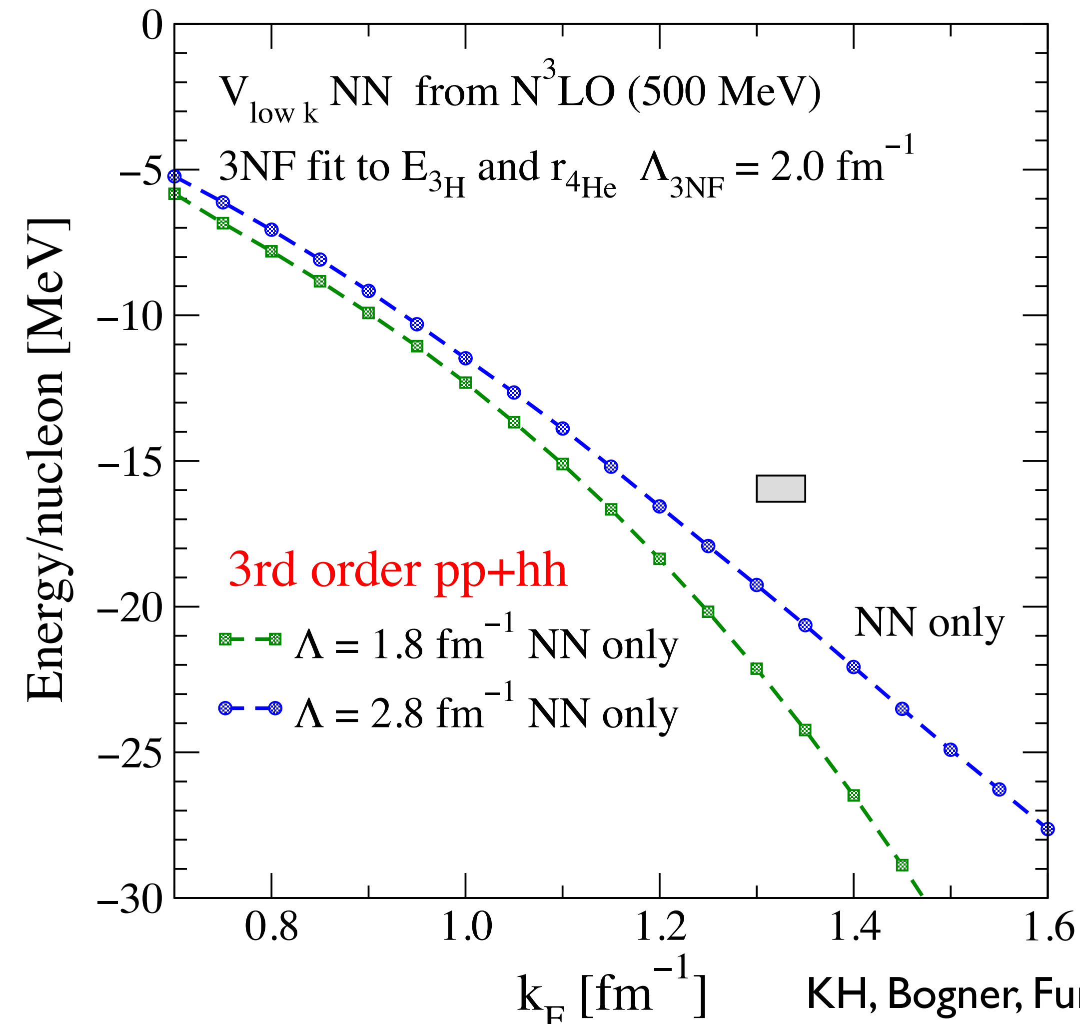
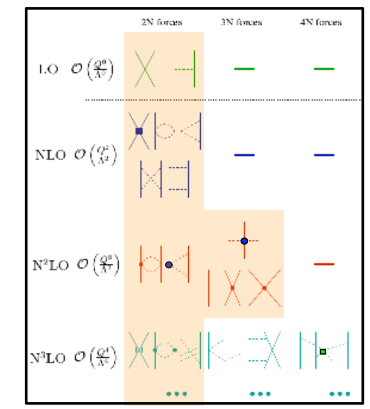
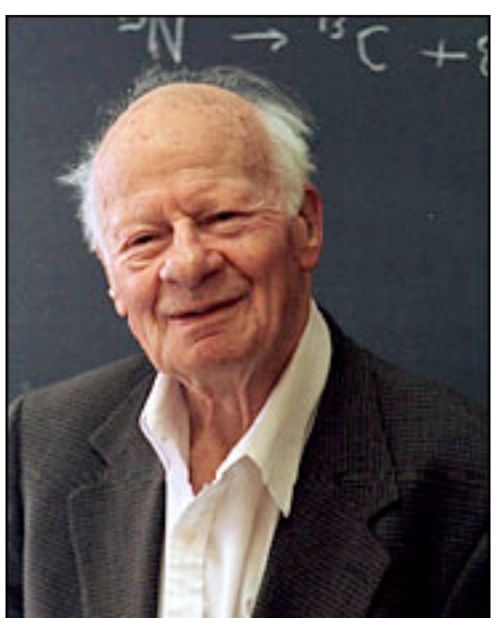






KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

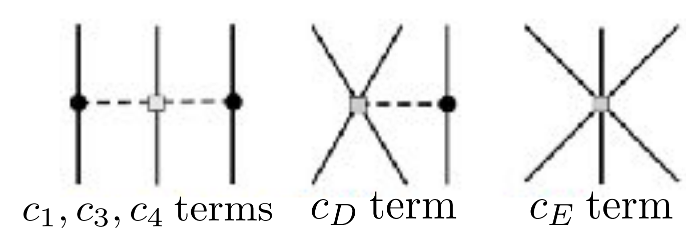


"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."

Hans Bethe (1971)

intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \qquad r_{^4\mathrm{He}} = 1.464\ \mathrm{fm}$$



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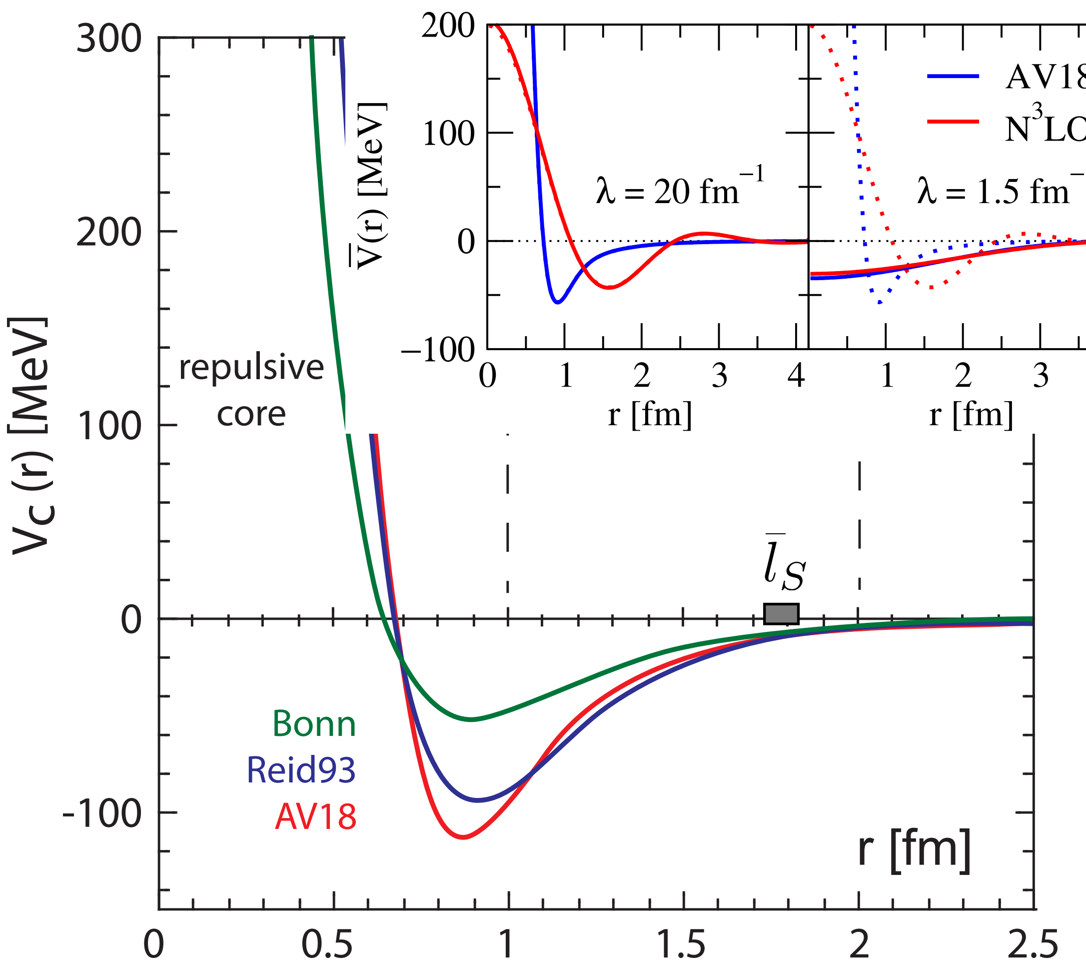
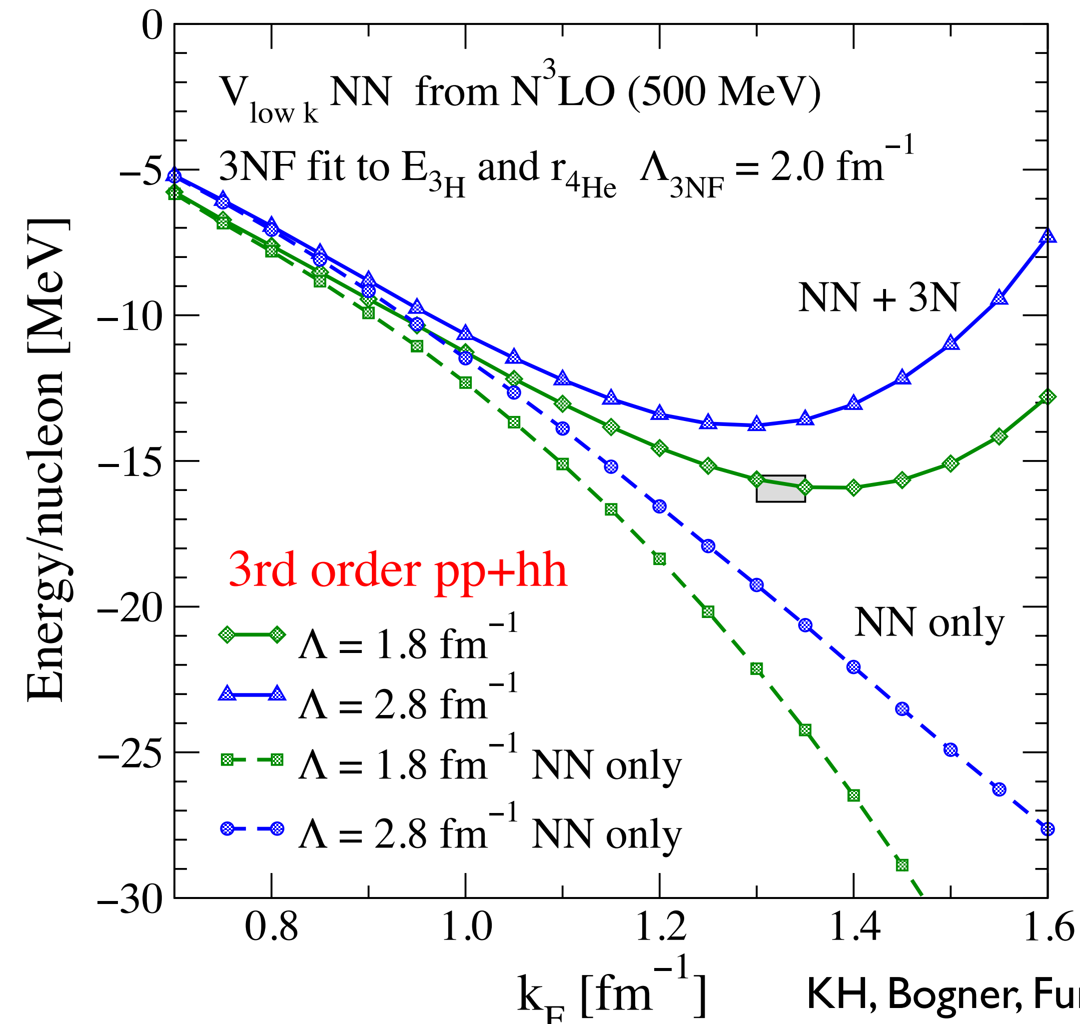
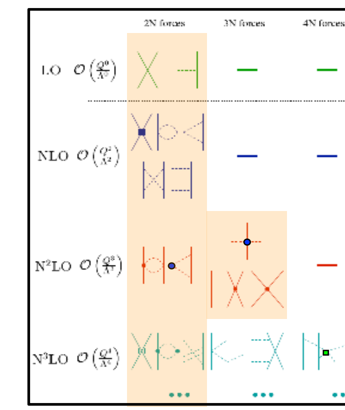
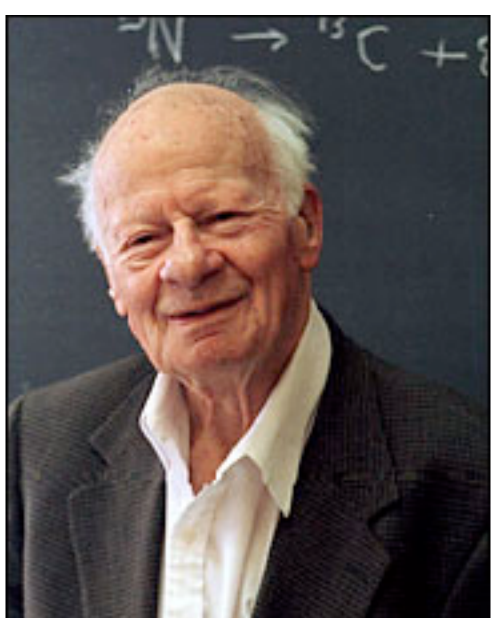






KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)



"Very soft potentials must be excluded because they do not give saturation;
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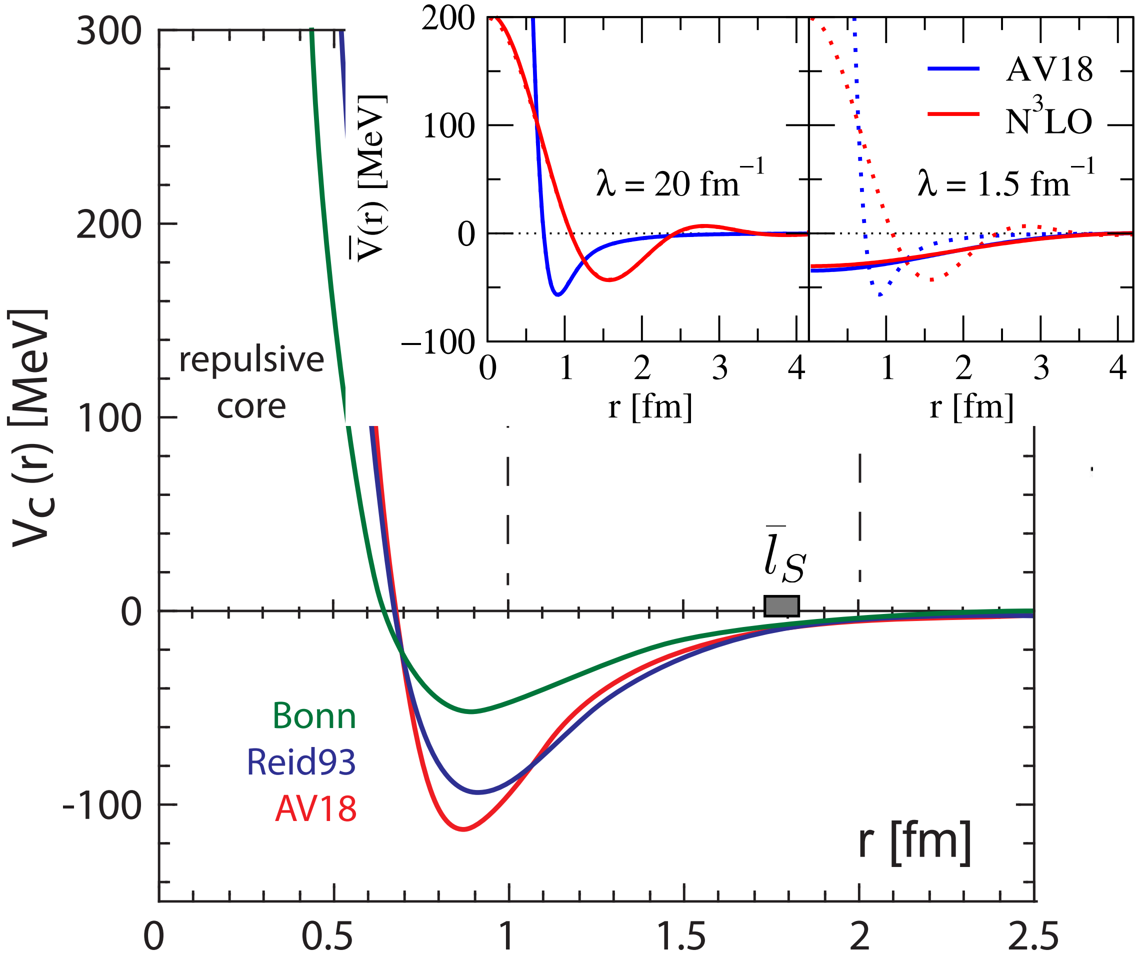
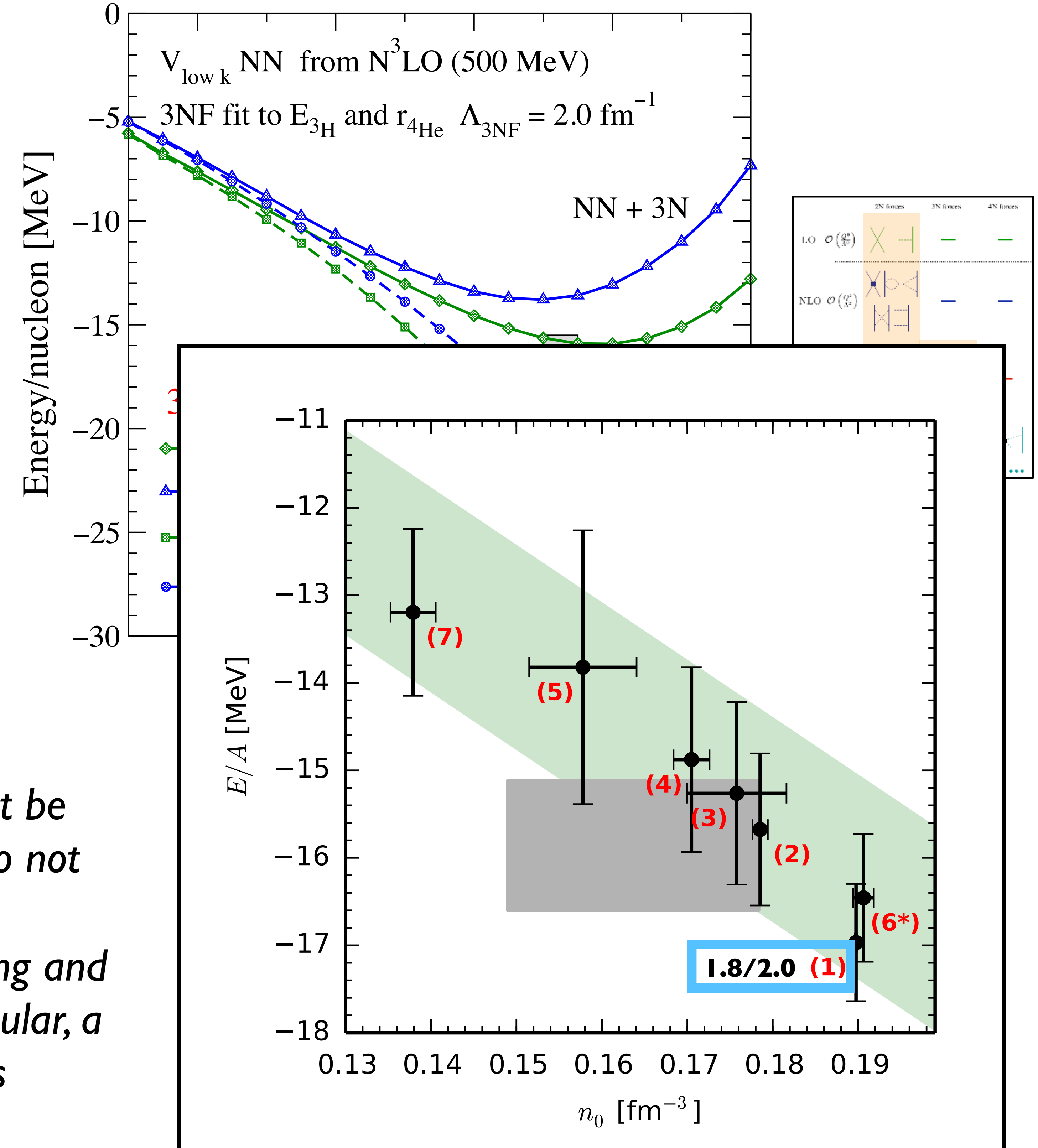
Hans Bethe (1971)

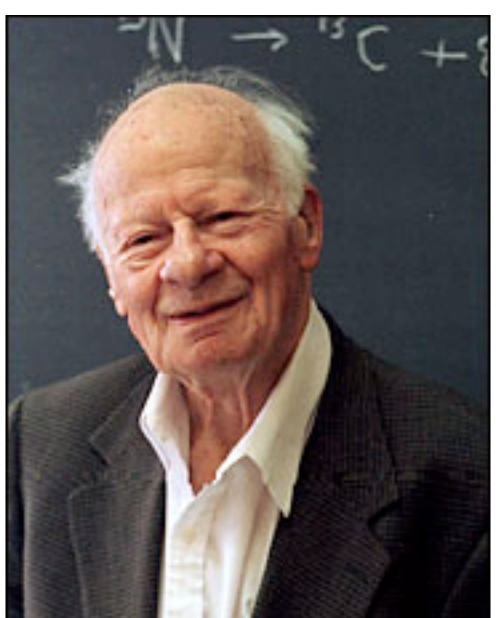
Reproduction of saturation point
without readjusting parameters!

Fitting the 3NF LECs at low resolution scales





Drischler, KH, Schwenk, PRC93, 054314 (2016)



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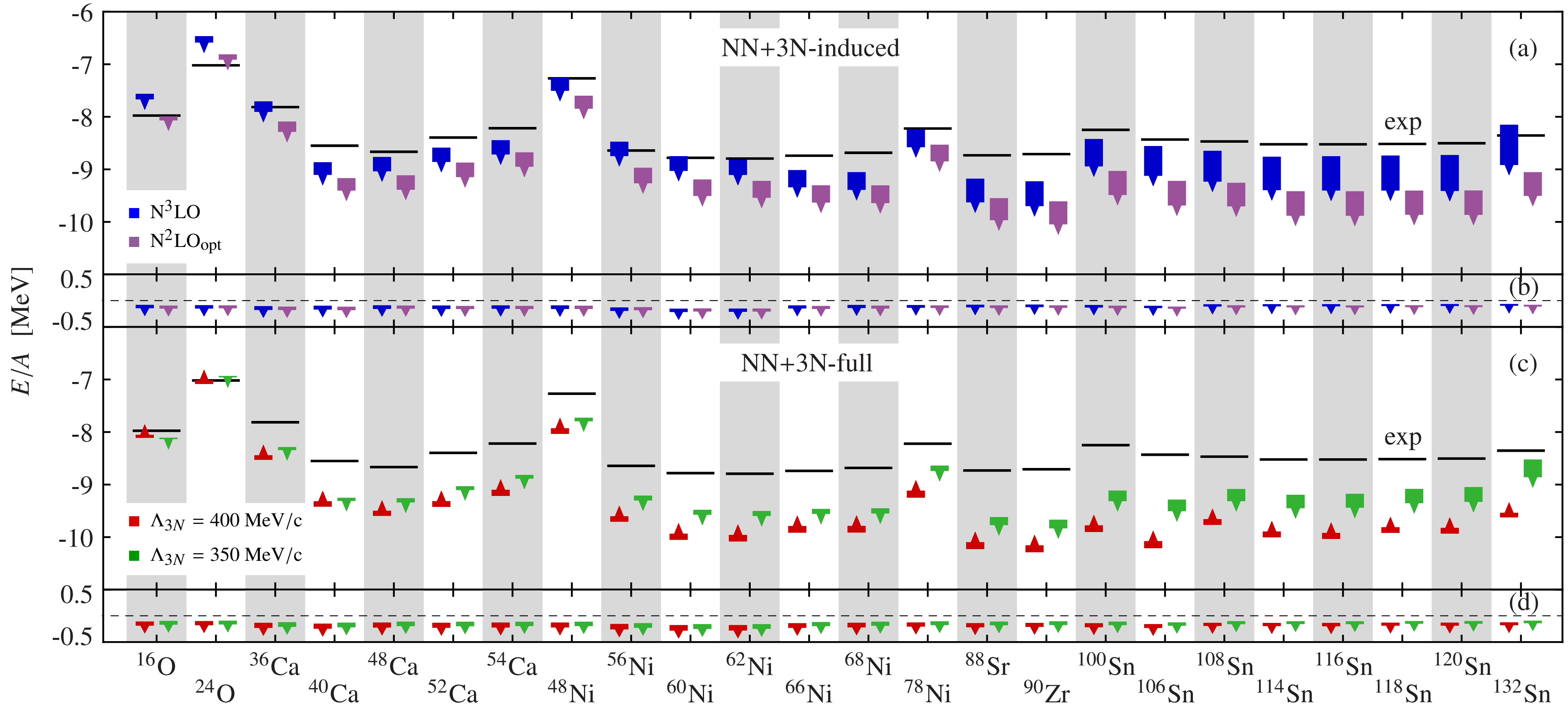
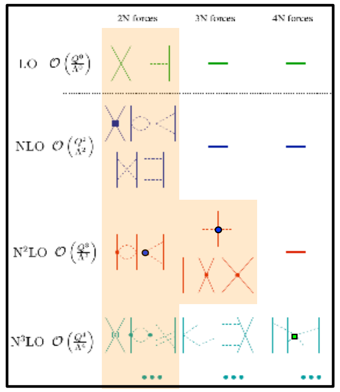
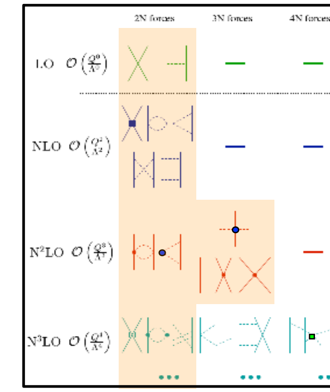
Ab initio calculations of heavier nuclei

coupled cluster (CC) framework



Binder et al., Phys. Lett B 736, 119 (2014)





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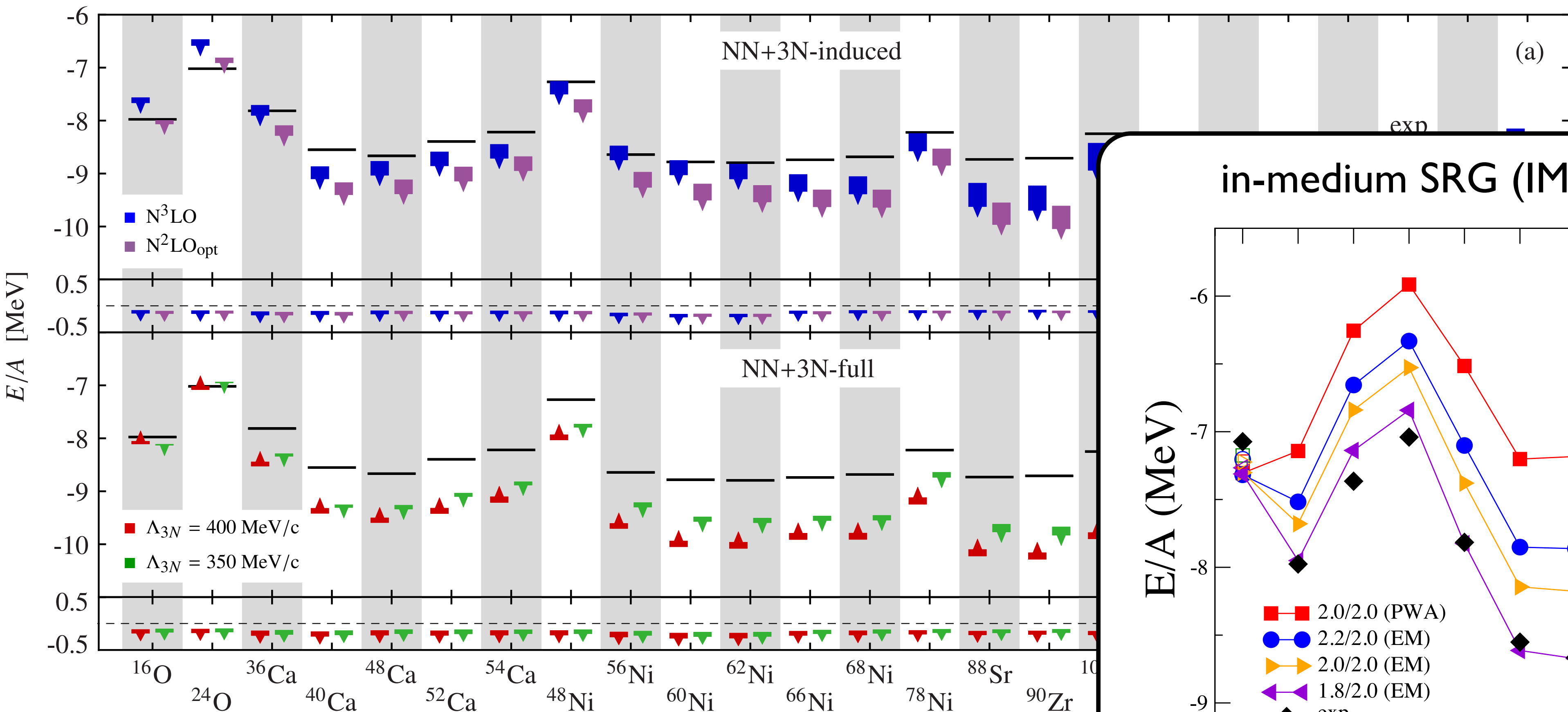
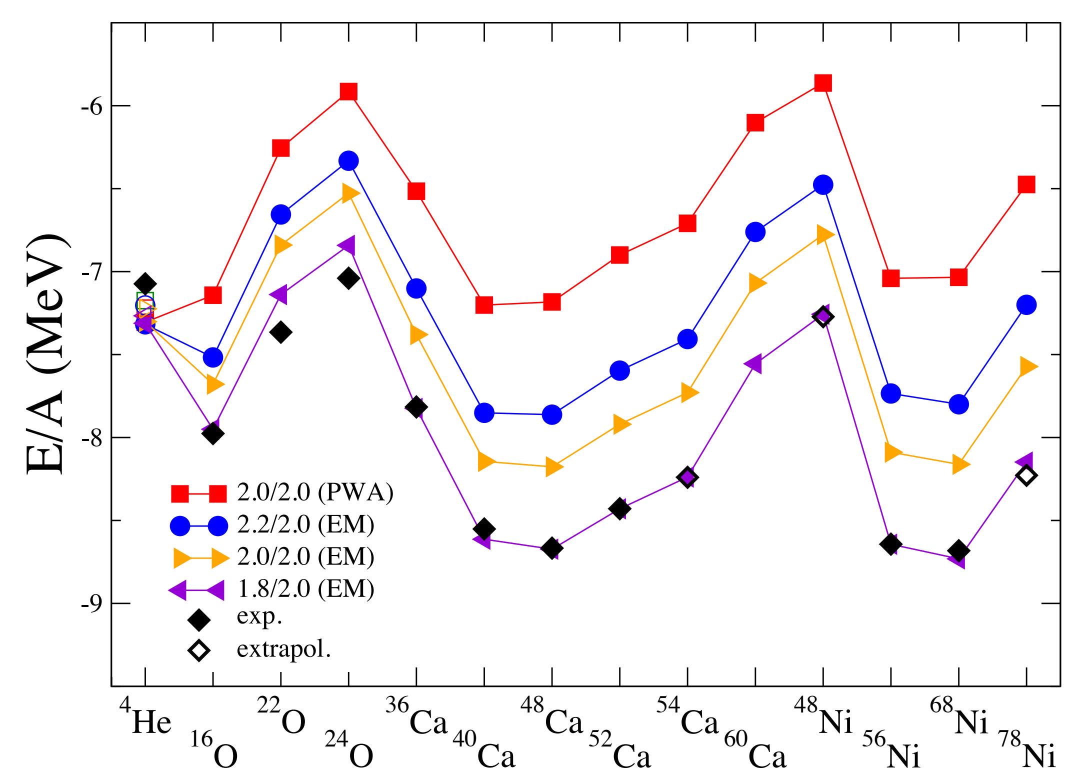


Binder et al., Phys. Lett B 736, 119 (2014)

in-medium SRG (IMSRG) framework



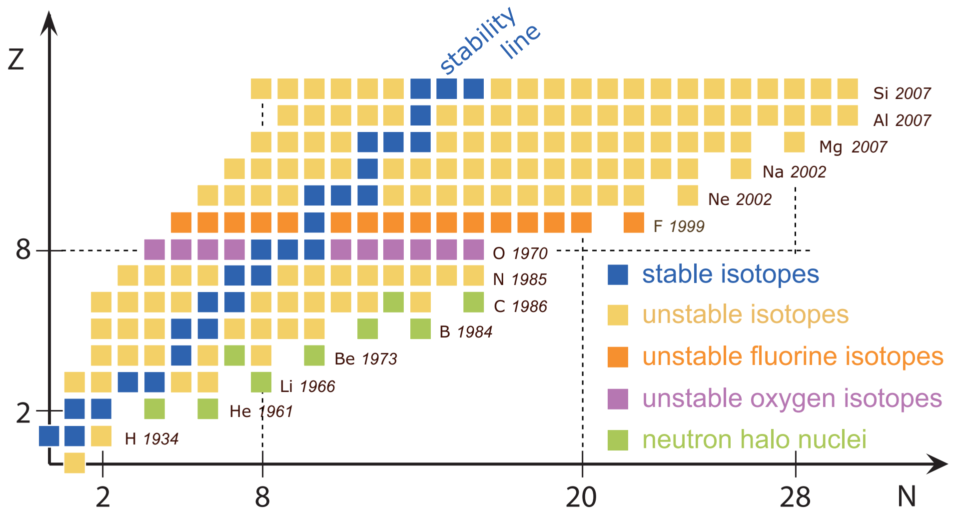
Simonis, Stroberg et al., PRC 96, 014303 (2017)

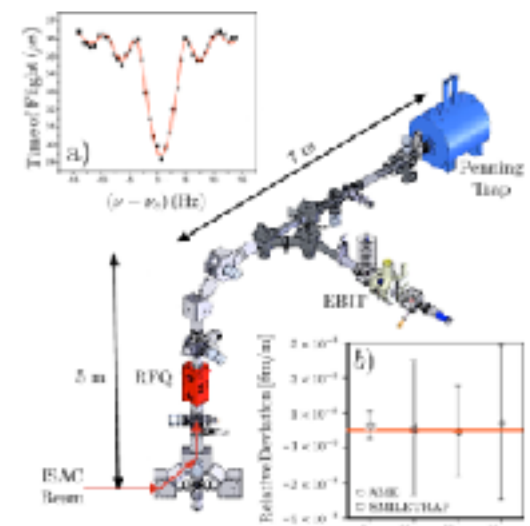
- spectacular increase in range of applicability of ab initio many body frameworks
- significant discrepancies to experimental data for heavy nuclei for

(most of) presently used nuclear interactions

- need to quantify theoretical uncertainties

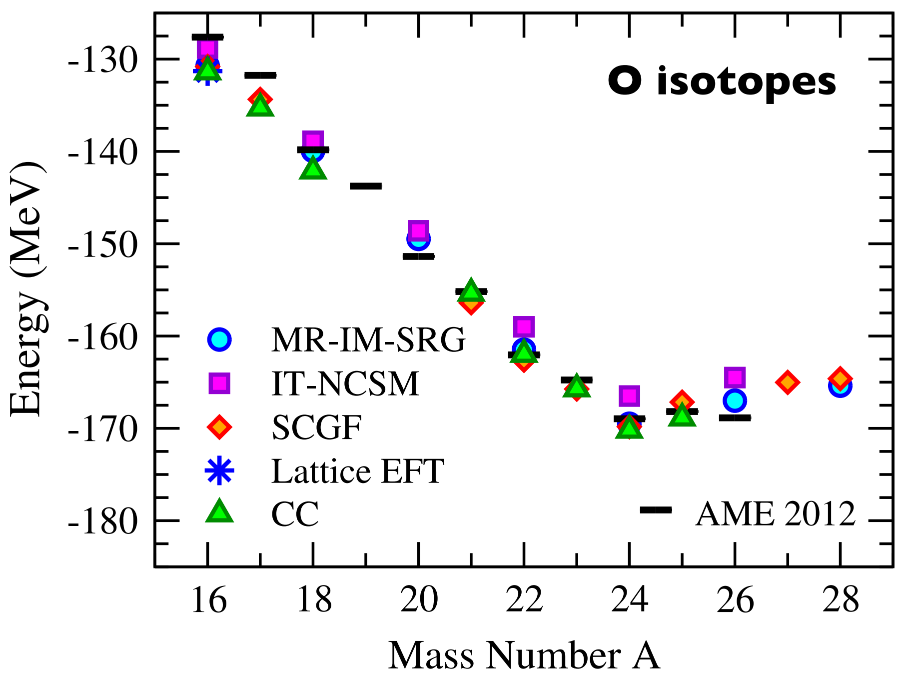
Studies of neutron-rich nuclei

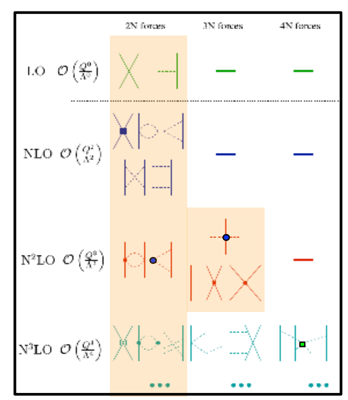




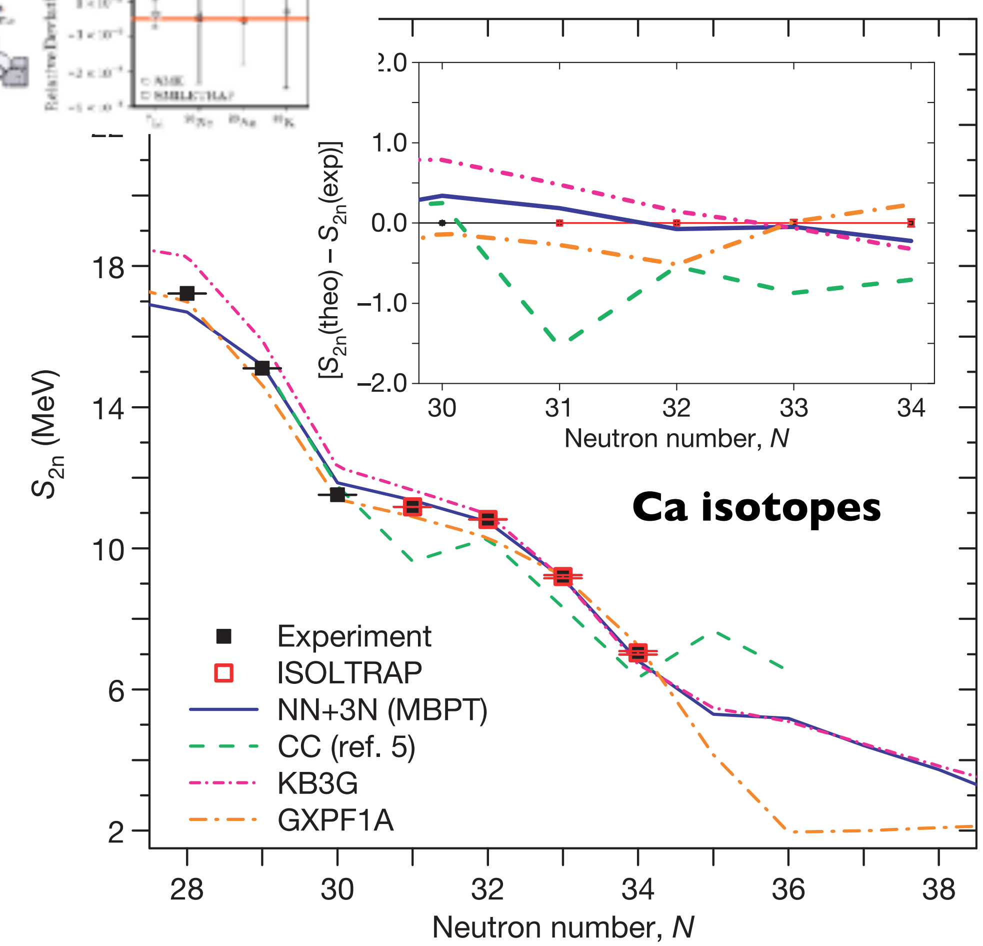
Gallant et al.
PRL 109, 032506 (2012)

Wienholtz et al.
Nature 498, 346 (2013)



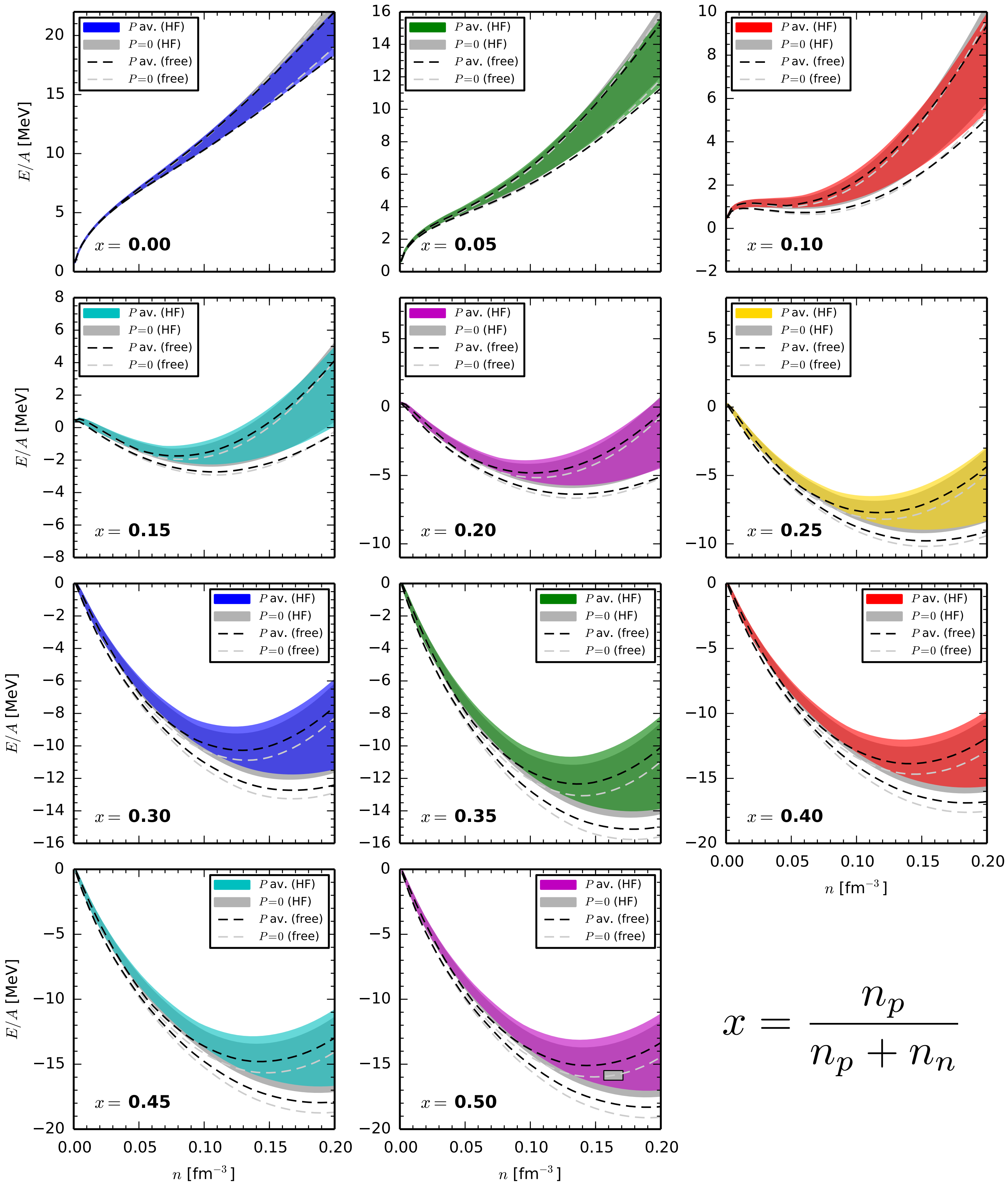


KH et al., Ann. Rev. Nucl. Part. Sci. 165, 457 (2015)



- remarkable agreement between different many-body frameworks
- excellent agreement between theory and experiment for masses of oxygen and calcium isotopes based on specific chiral interactions
- need to quantify theoretical uncertainties

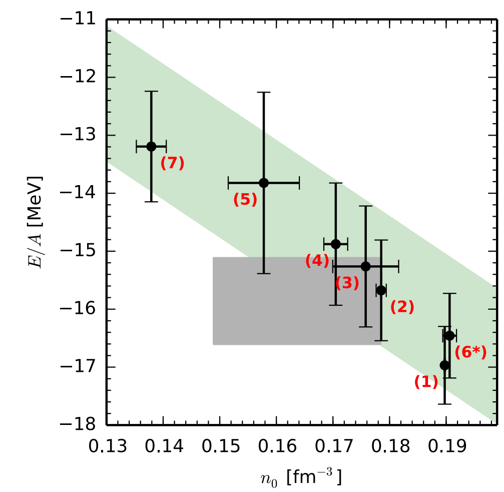
Microscopic calculations of the equation of state



$$x = \frac{n_p}{n_p + n_n}$$

- microscopic framework to calculate equation of state for general proton fractions

- uncertainty bands determined by set of 7 Hamiltonians



- many-body framework allows treatment of general 3N interaction

Drischler, KH, Schwenk,
PRC 054314 (2016)