Improved Busch formula and look for a unified calculation of nuclear scattering and structure

Xilin Zhang
University of Washington

Progress in Ab Initio Techniques in Nuclear Physics, TRIUMF, Vancouver BC, Feb 2018

What if aliens interfere ab initio calculation?

Xilin Zhang
University of Washington

Progress in Ab Initio Techniques in Nuclear Physics, TRIUMF, Vancouver BC, Feb 2018

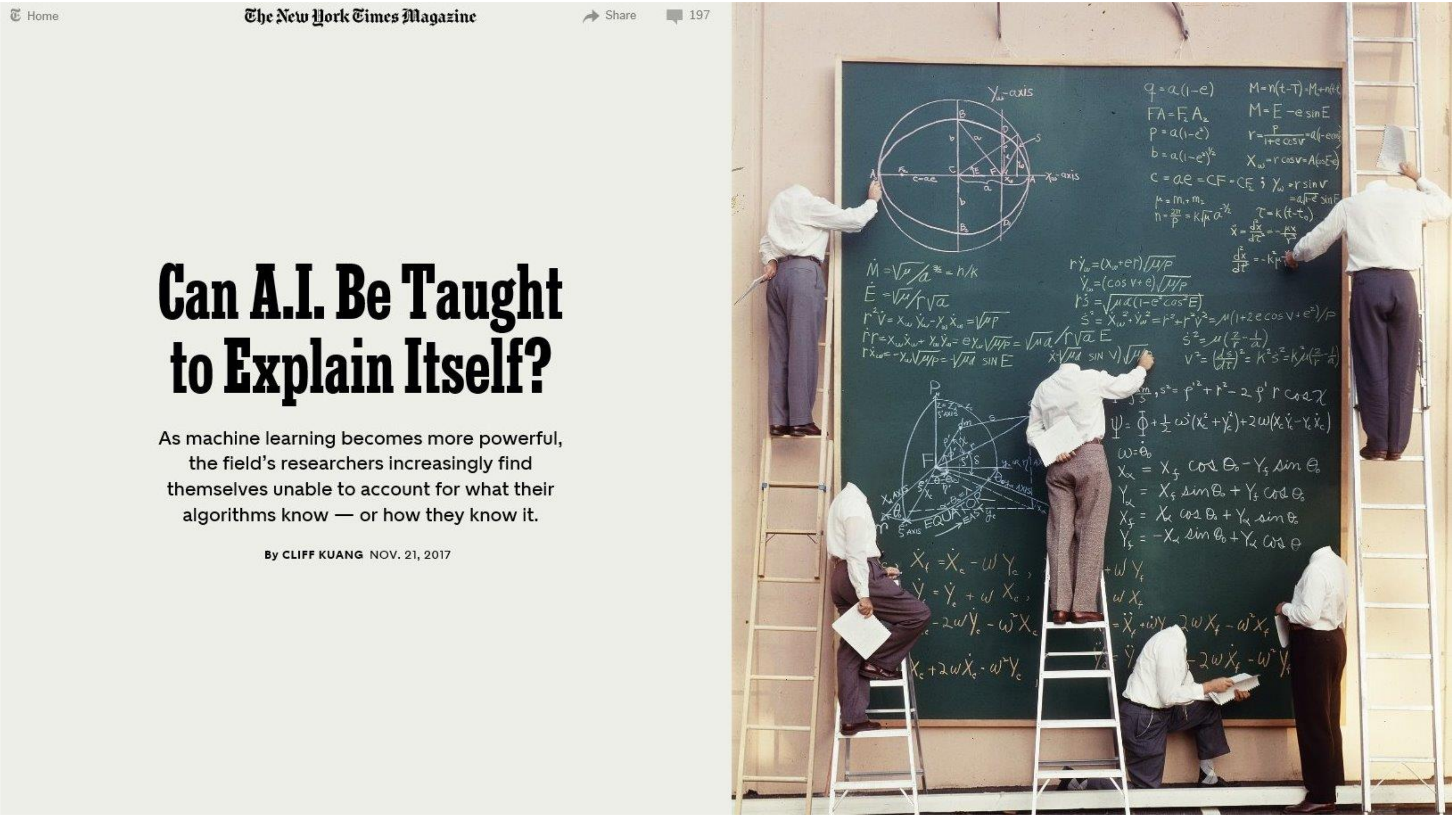
What if aliens interfere ab initio calculation?

Progress in Ab Initio Techniques in Nuclear Physics, TRIUMF, Vancouver BC, Feb 2018

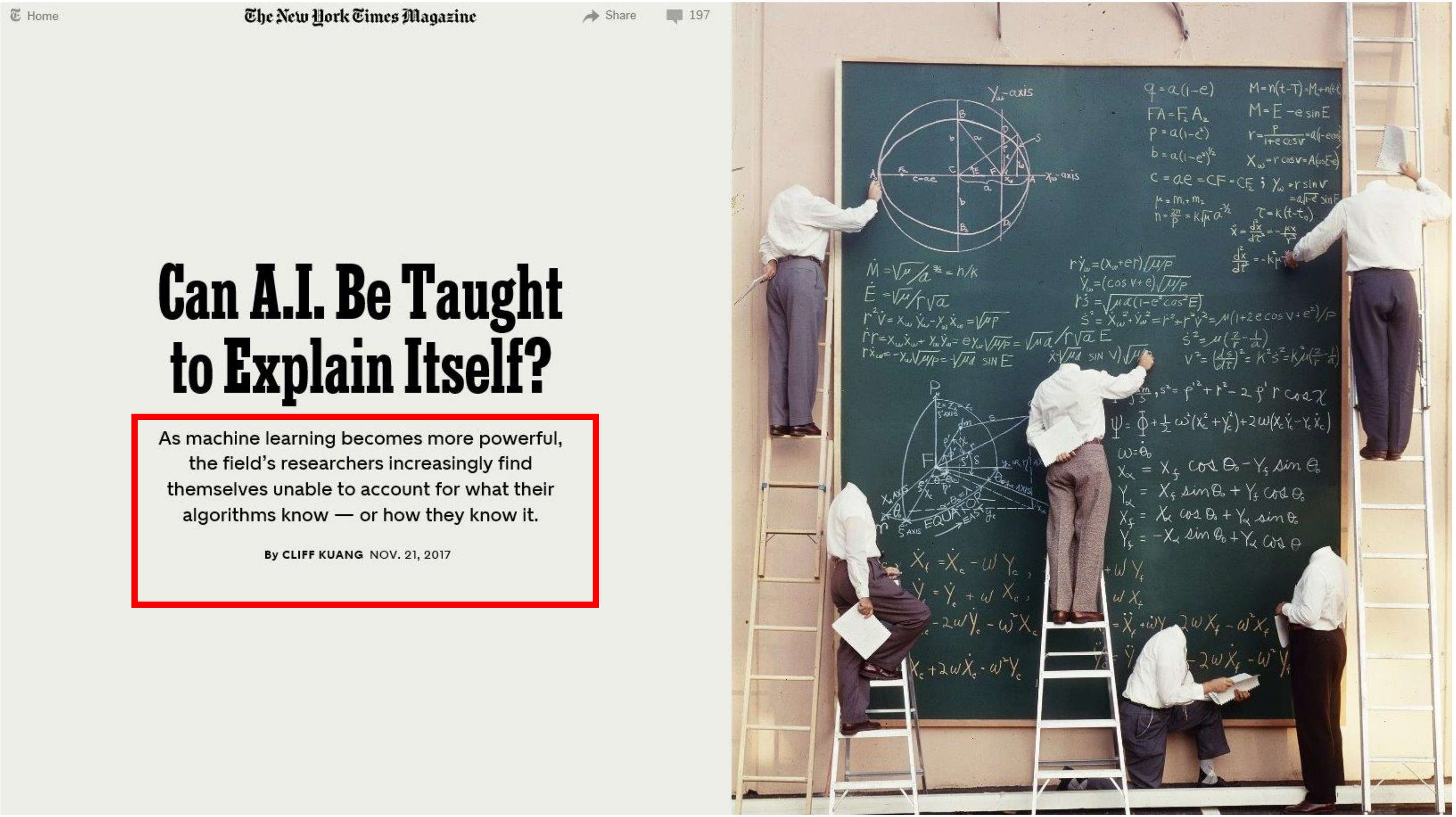
Outline

- Need to understand ab inito calculation on a macroscopic level: self-consistency, *global* uncertainty analysis, and infrared extrapolation
- Busch formula relates two-cluster spectrum in a harmonic trap to the two-cluster scattering
- Improve Busch formula: a toy model and effective field theory (EFT) generalization
- Test the formula and do a proof of principle calculation by studying He-5 system
- Summary and outlook

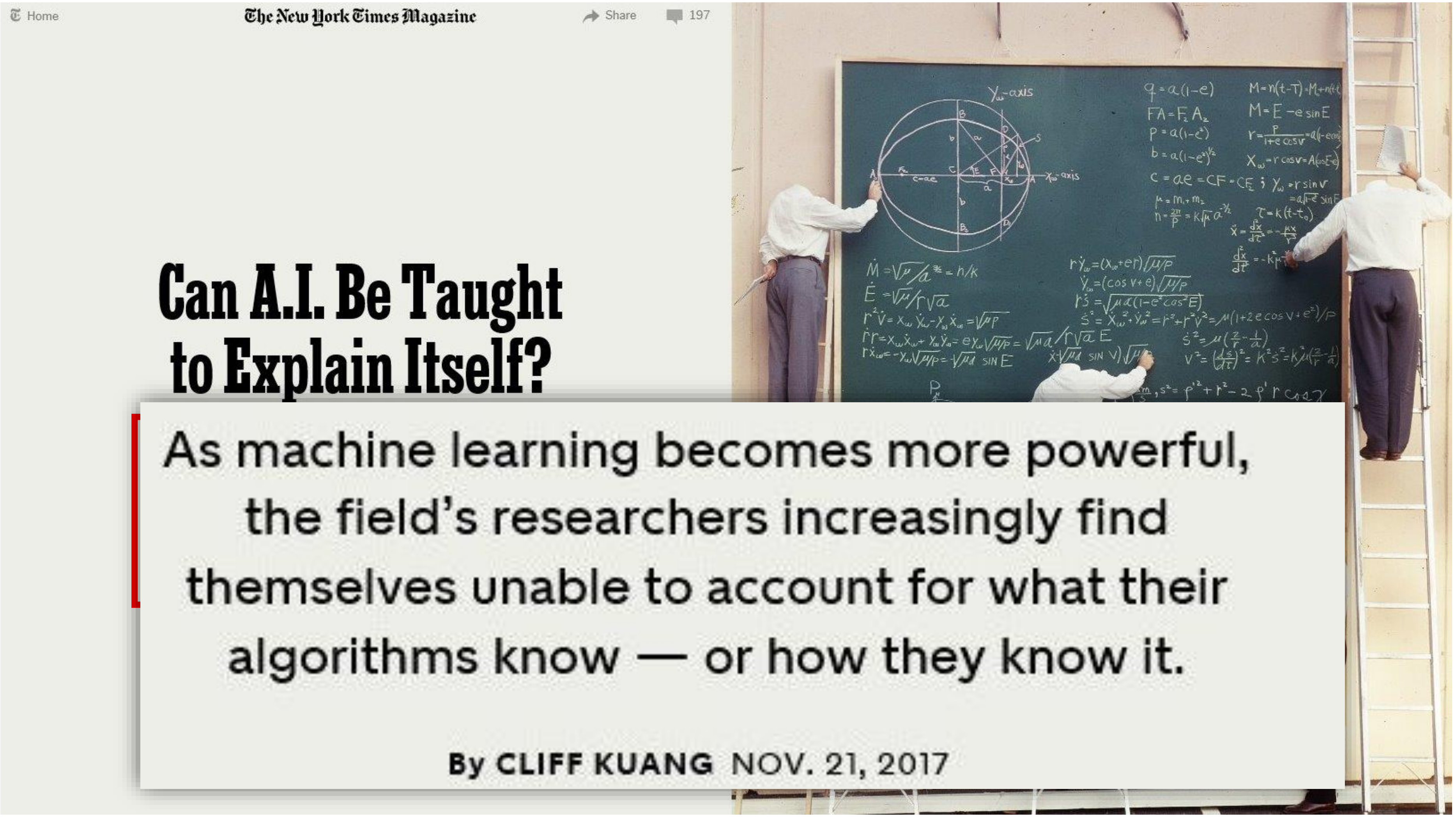
Understand complexity on a macroscopic level



Understand complexity on a macroscopic level



Understand complexity on a macroscopic level


Home

The New York Times Magazine

Share 197

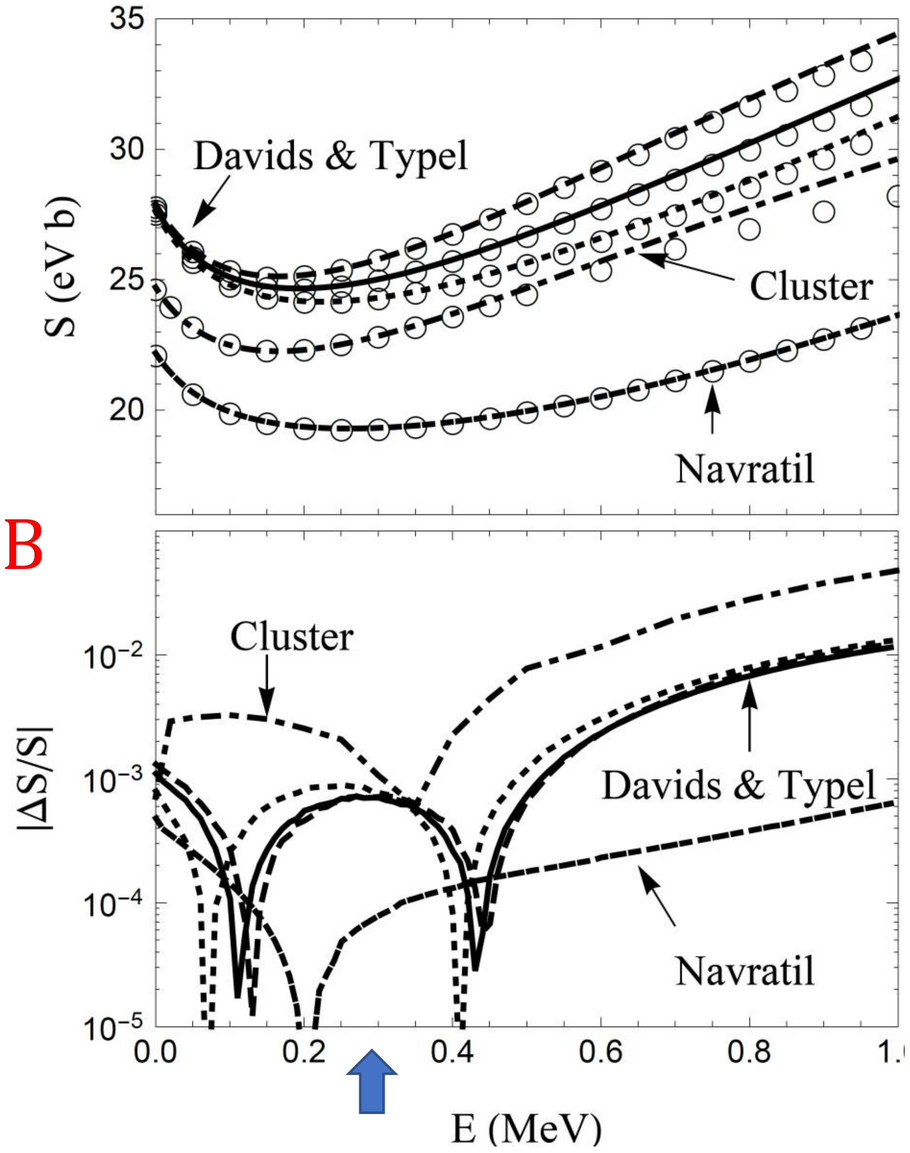
Can A.I. Be Taught to Explain Itself?

As machine learning becomes more powerful, the field's researchers increasingly find themselves unable to account for what their algorithms know — or how they know it.

By CLIFF KUANG NOV. 21, 2017

Understand ab inito calculation on a macroscopic level: consistency and global uncertainty analysis

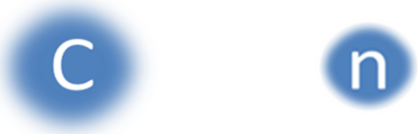
$^{7}_{4}\mathrm{Be}(\mathrm{p},\gamma)^{8}_{5}\mathrm{B}$



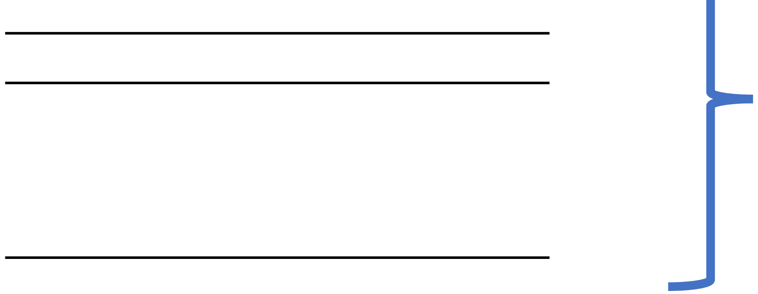
1) Halo-EFT: scattering lengths, effective ranges, ANCs,,,
2) Computational "data" on different observables→ macroscopic picture: **a systematical approach is needed**
3) Various information sources: experimental, computational, and empirical. The EFT approach can be used to combine these information and produce global uncertainty propagation

Busch formula (infrared extrapolation)

Vs

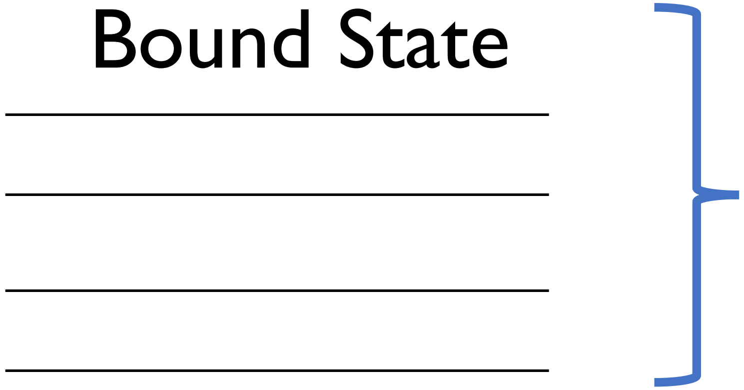


Continuum



Trap them in $\frac{1}{2} M_N \omega r^2$ potential

Bound State



Constrain EFT or model on Vs and use it to compute scattering and reaction

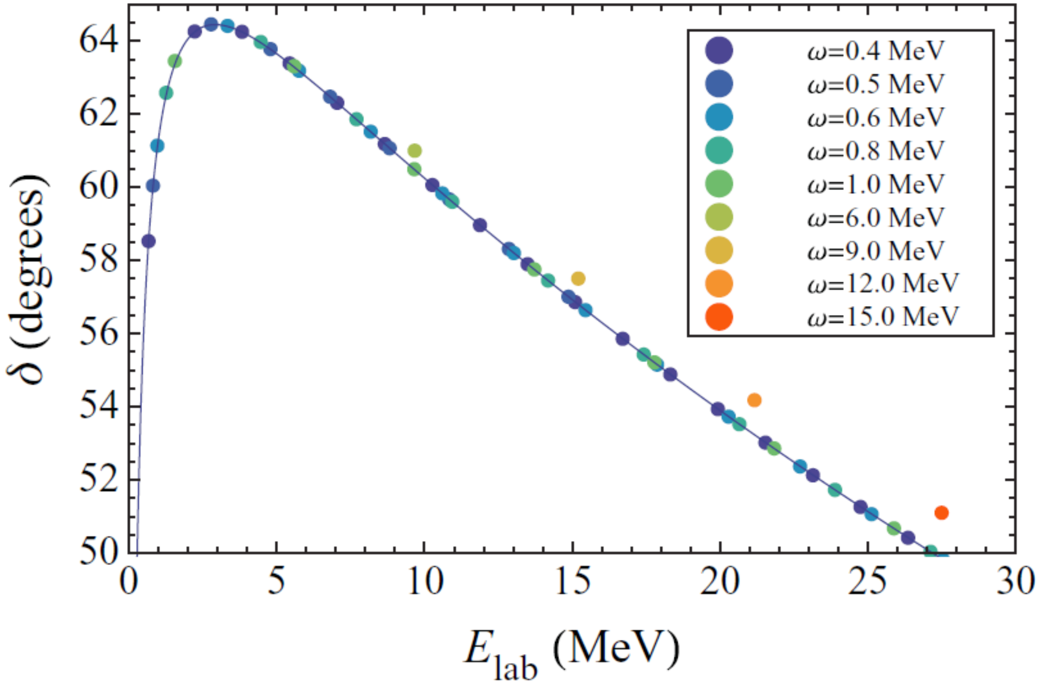
Ab initio

as data

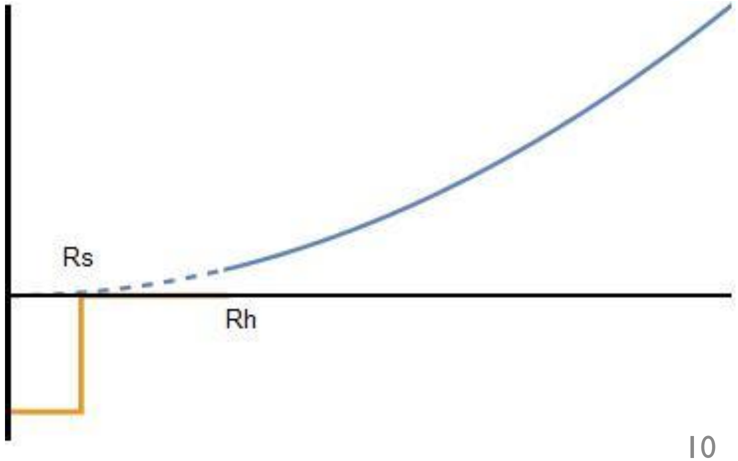
Busch formula

$$p^{2l+1}\cot\delta_l(p) = (-1)^{l+1}(4M_{\mathrm{R}}\omega)^{l+\frac{1}{2}}\frac{\Gamma\left(\frac{3}{4}+\frac{l}{2}-\frac{\epsilon}{2}\right)}{\Gamma\left(\frac{1}{4}-\frac{l}{2}-\frac{\epsilon}{2}\right)}$$

$$\text{with} \quad \epsilon \equiv \frac{E}{\omega}, \;\; E \equiv \frac{p^2}{2M_{\mathrm{R}}}$$



1) T. Luu, M. Savage, A. Schwenk, and J. Vary, PRC (2010): N-N phase shift
2) J. Rotureau, I. Stetcu, B.R. Barrett, and U. van Kolck, PRC (2012): N-D phase shift



Improve Busch Formula: a model

$$V_s(r) = \begin{cases} +\infty & \text{when } r \le r_c \\ 0 & \text{when } r > r_c \ , \end{cases}$$

$$p^{2l+1}\cot\delta_l(p) - (-1)^{l+1}(4M_{\mathrm{R}}\omega)^{l+\frac{1}{2}}\frac{\Gamma\left(\frac{3}{4}+\frac{l}{2}-\frac{\epsilon}{2}\right)}{\Gamma\left(\frac{1}{4}-\frac{l}{2}-\frac{\epsilon}{2}\right)}$$

$$= -\frac{(2l+1)!!(2l-1)!!}{r_c^{2l+1}}\left[\frac{(2l+1)\left(\frac{r_c}{b}\right)^4}{2(2l-3)(2l+5)} + \frac{(2l+1)(6l+25)\frac{1}{2}(pr_c)^2\left(\frac{r_c}{b}\right)^4}{3(2l-5)(2l+3)(2l+5)(2l+7)} + O\left[\left(\frac{r_c}{b}\right)^8, (pr_c)^4\left(\frac{r_c}{b}\right)^4\right]\right]$$

$$\equiv -L_{a_l}\frac{1}{b^4 r_c^{2l-3}} - L_{r_l}\frac{p^2}{b^4 r_c^{2l-1}} + \ldots$$

Note: $b = \sqrt{\frac{1}{M_R\omega}}$

Effective range expansion (ERE):

$$p^{2l+1}\cot\delta_l(p) = -\frac{\Lambda^{2l+1}}{a_l} + \frac{1}{2}r_l\Lambda^{2l-1}p^2 + \frac{1}{4}\tilde{r}_l^{(1)}\Lambda^{2l-3}p^4$$

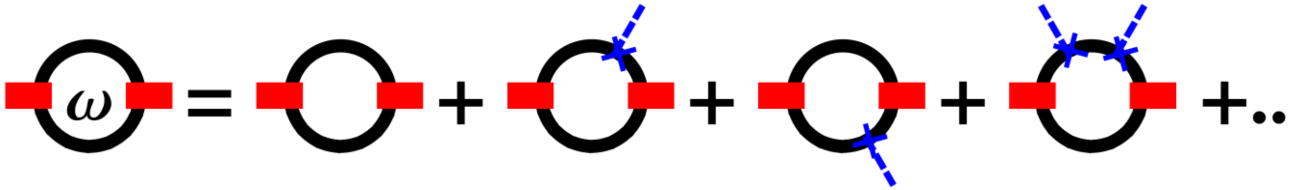
Improve Busch Formula: EFT

$$\mathcal{L}_0 = \begin{pmatrix} c^* & n^* & -\phi^* \end{pmatrix} \mathrm{diag}\left(i\partial_t - \hat{m}_c\psi + \frac{\boldsymbol{\partial}^2}{2M_c}, i\partial_t - \hat{m}_n\psi + \frac{\boldsymbol{\partial}^2}{2M_n}, i\partial_t - \hat{m}_\phi\psi + \frac{\boldsymbol{\partial}^2}{2M_{nc}} + \Delta_0\right) \begin{pmatrix} c & n & \phi \end{pmatrix}^T$$

$$\mathcal{L}_{I0} = g_0\phi^* c\, n - \phi^* \left[\sum_{j=2} d_j^{(0)} \left(i\partial_t - \hat{m}_\phi\psi + \frac{\boldsymbol{\partial}^2}{2M_{nc}}\right)^j\right] \phi + \mathrm{C.C}\,.$$

Note: $\psi = \frac{1}{2}\, m_N\, \omega^2 r^2$

Self-energy bubble:



Dimer-field propagator:

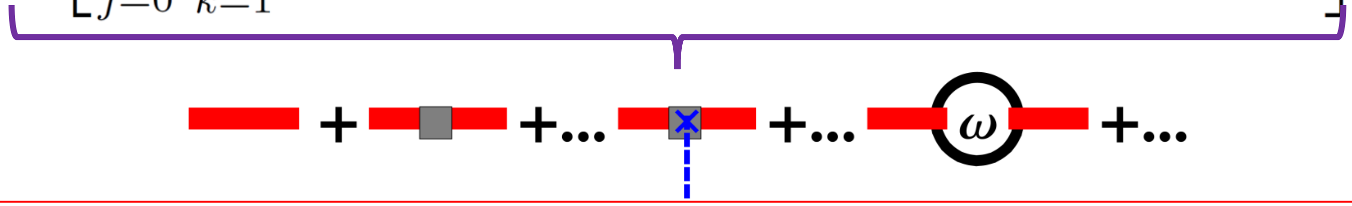
$$p_{\tilde{E}} \cot\delta_0(\tilde{E}) = -\frac{2\pi}{g_0^2 M_{\mathrm{R}}}\left(\Sigma_\omega(\tilde{E}) - \Sigma(\tilde{E})\right)$$ reproduces the Busch formula.

Then what went wrong?

Improve Busch Formula: EFT

$$\mathcal{L}_{I0} = g_0\phi^* c\, n - \phi^*\left[\sum_{j=2} d_j^{(0)}\left(i\partial_t - \hat{m}_\phi \psi + \frac{\boldsymbol{\partial}^2}{2M_{nc}}\right)^j\right]\phi + \text{C.C}\,.$$

$$-\phi^*\left[\sum_{j=0}\sum_{k=1} d_{j,k}^{(0)}\left(i\partial_t - \hat{m}_\phi\psi + \frac{\boldsymbol{\partial}^2}{2M_{nc}}\right)^j\left(\frac{M_{\mathrm{R}}^2}{3m}\boldsymbol{\partial}^2\psi\right)^k\right]\phi$$



The factorizability of CM motion severely constrains two-body current like couplings.

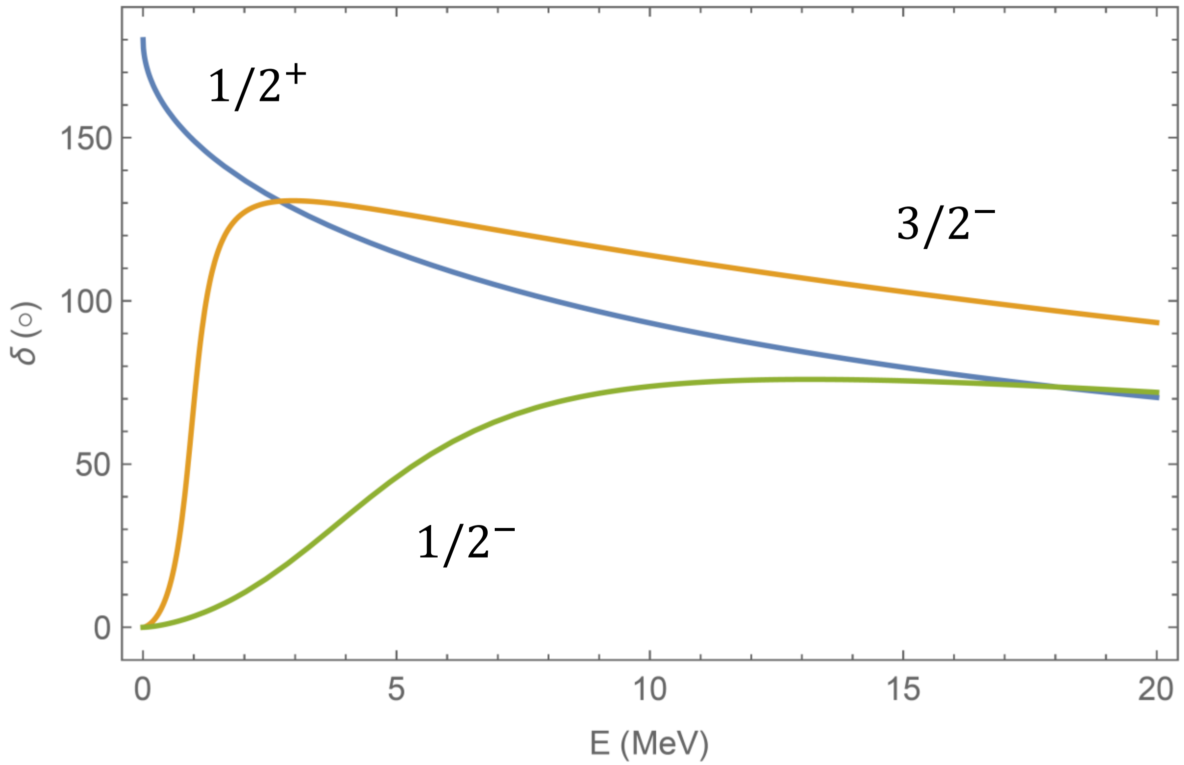
$$\Sigma_\omega \rightarrow \Sigma_\omega + \sum_{j=0}\sum_{k=1} d_{j,k}^{(0)}\tilde{E}^j b^{-4k}$$

$$p\cot\delta_0\left(p\right) \rightarrow p\cot\delta_0\left(p\right) + L_{a_0}\frac{1}{b^4\Lambda^3} + L_{r_0}\frac{p^2}{b^4\Lambda^5} + L_{\tilde{r}_0^{(1)}}\frac{p^4}{b^4\Lambda^7} + \ldots$$

$$= -\frac{\Lambda}{a_0} + \frac{1}{2}\frac{r_0}{\Lambda}p^2 + \frac{1}{4}\frac{\tilde{r}_0^{(1)}}{\Lambda^3}p^4 + L_{a_0}\frac{1}{b^4\Lambda^3} + L_{r_0}\frac{p^2}{b^4\Lambda^5} + L_{\tilde{r}_0^{(1)}}\frac{p^4}{b^4\Lambda^7}$$

Test: n − α system

$$V_s(r) = \begin{cases} V_0(1+\beta \boldsymbol{L}\cdot\boldsymbol{\sigma}) & \text{when } r < r_c \\ 0 & \text{when } r > r_c \end{cases}$$



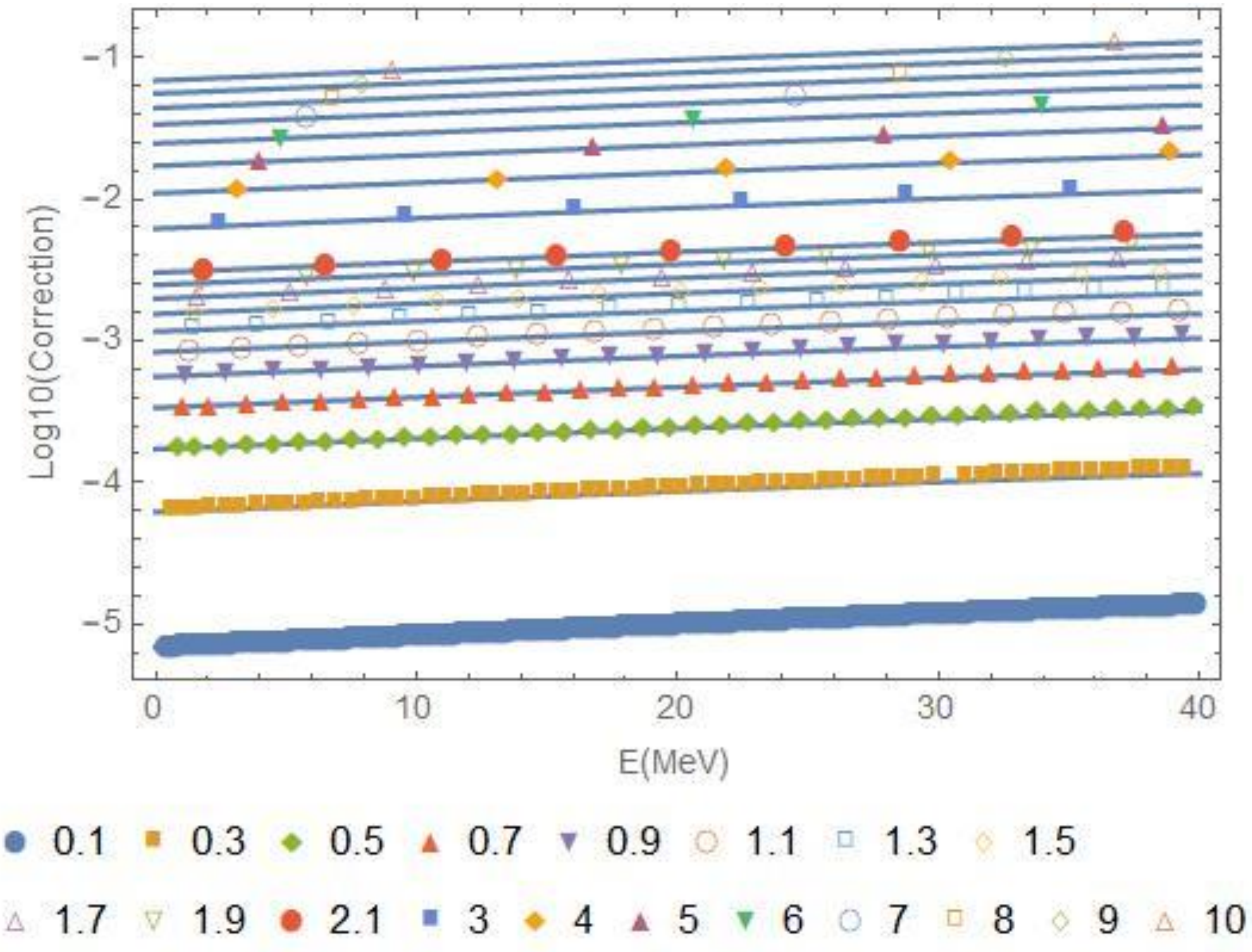
S. Ali et.al., RMP **57,** 923 (1985)

Test: $\mathrm{n}-\alpha$ system in p-wave

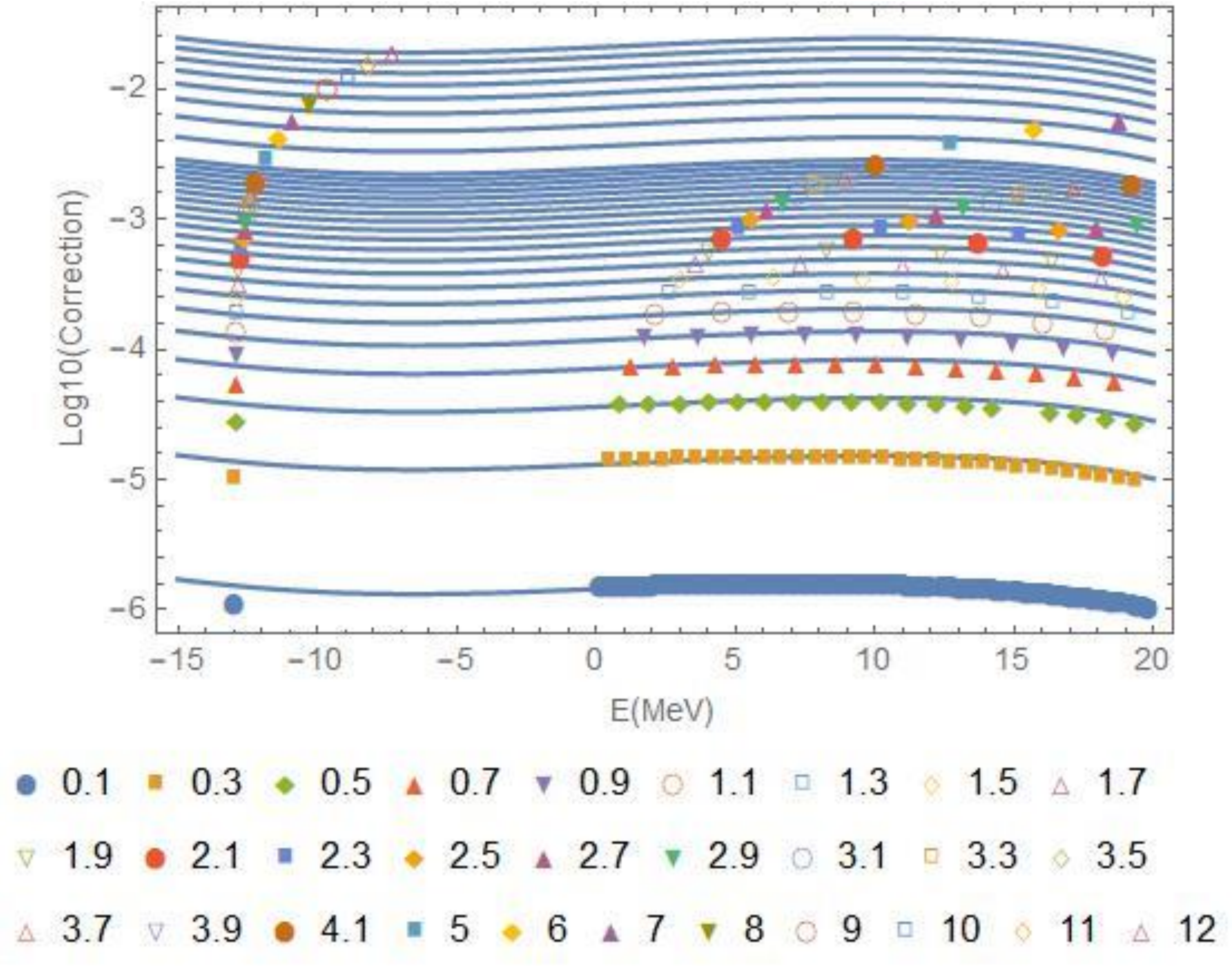
$$p^{2l+1}\cot\delta_l(p)-(-1)^{l+1}(4M_{\mathrm{R}}\omega)^{l+\frac{1}{2}}\frac{\Gamma\left(\frac{3}{4}+\frac{l}{2}-\frac{\epsilon}{2}\right)}{\Gamma\left(\frac{1}{4}-\frac{l}{2}-\frac{\epsilon}{2}\right)}$$

1) Corrections $\propto\omega^2$

2) At $\omega=10$ MeV, 10% corrections



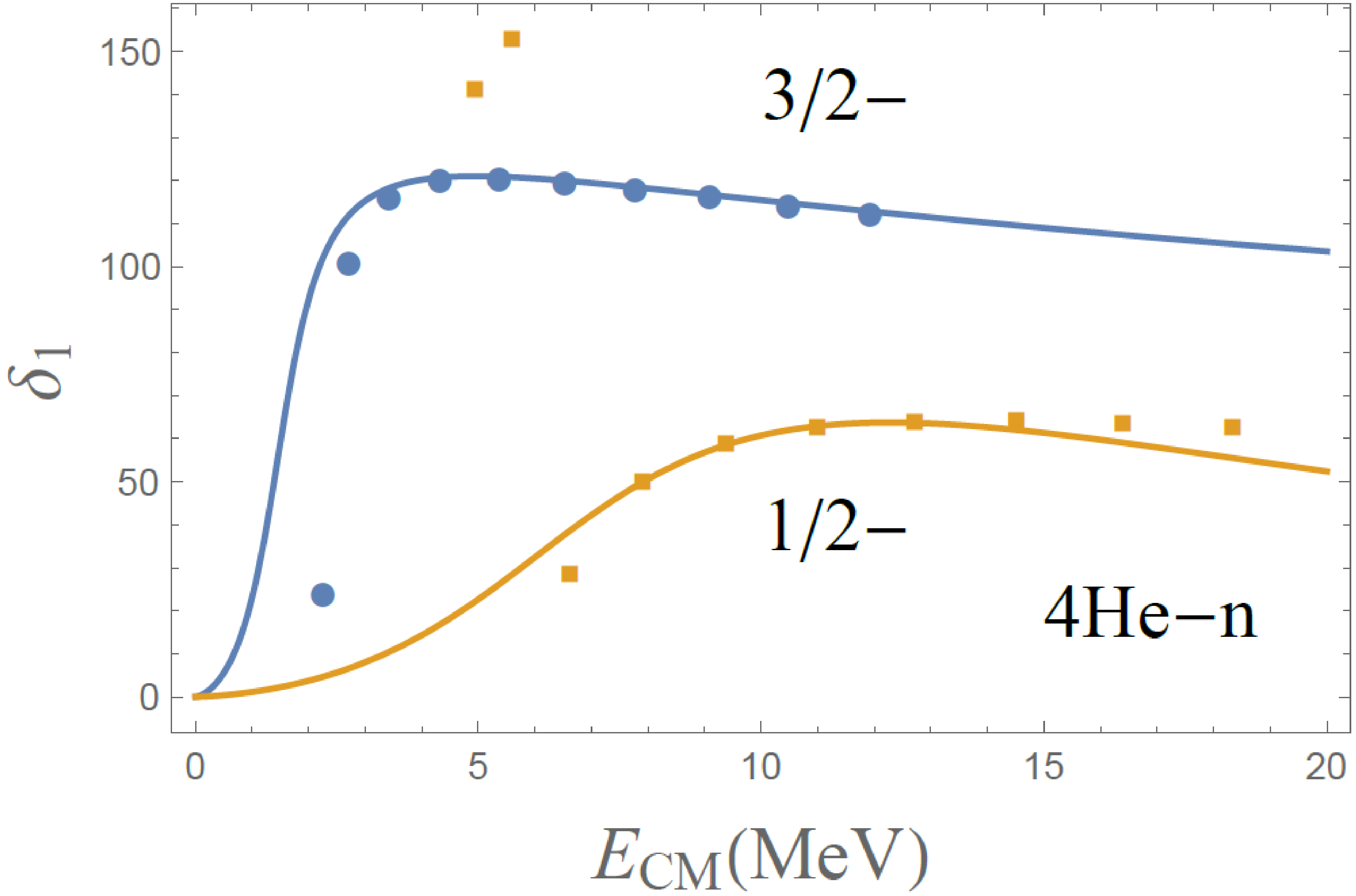
Test: $\mathrm{n} - \alpha$ system in s-wave



Test: extract $\mathrm{n} - \alpha$ ERE parameters

	a	r	$\tilde{r}^{(1)}$	$\tilde{r}^{(2)}$	L_a	L_r	$L_{\tilde{r}^{(1)}}$	$L_{\tilde{r}^{(2)}}$
P-wave fit J=3/2	-43.4101	-1.11405	2.23369	0.605012	-1.87099	-0.827113	-0.269323	0.0685459
P-wave J=3/2	-43.408	-1.11425	2.23965	0.5243				
P-wave fit J=1/2	-7.19466	-0.882978	2.44336	0.86573	-2.08059	-1.00758	-0.706915	0.257639
P-wave J=1/2	-7.19286	-0.885359	2.49157	0.668183				
S-wave fit J=1/2	2.82818	1.57966	0.700462	-0.22943	-0.405303	-0.240855	-0.172349	0.867527
S-wave fit J=1/2	2.82818	1.57967	0.700221	-0.076691				

Trial results by analyzing IM-SRG "data" from G. Chan, R. Stroberg, and J. Holt



Summary and outlook

- Need to understand ab initio calculation on a macroscopic level **in a systematical way**
- The improved Busch formula can be used to infer scattering (infrared extrapolation)
- Test on $\mathrm{n}-\alpha$ is encouraging
- Working with P. Narvatil on $\mathrm{n}-\alpha$
- Also applying it to study $\mathrm{n}-^{24}O$ with G. Chan, R. Stroberg, and J. Holt
- Consider generalizing it to study two-cluster reactions and three-cluster systems