

Improved Busch formula and look for a unified calculation of nuclear scattering and structure

Xilin Zhang

University of Washington

Progress in Ab Initio Techniques in Nuclear Physics, TRIUMF, Vancouver BC, Feb 2018

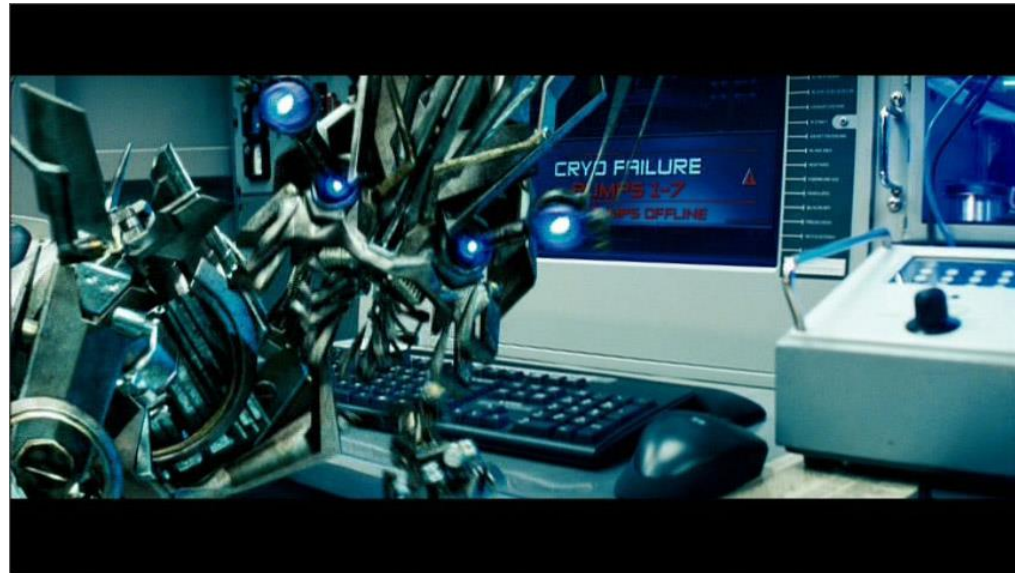
What if aliens interfere ab initio calculation?

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Outline

- Need to understand ab initio calculation on a macroscopic level: self-consistency, *global* uncertainty analysis, and **infrared extrapolation**
- Busch formula relates two-cluster spectrum in a harmonic trap to the two-cluster scattering
- Improve Busch formula: a toy model and effective field theory (EFT) generalization
- Test the formula and do a proof of principle calculation by studying He-5 system
- Summary and outlook

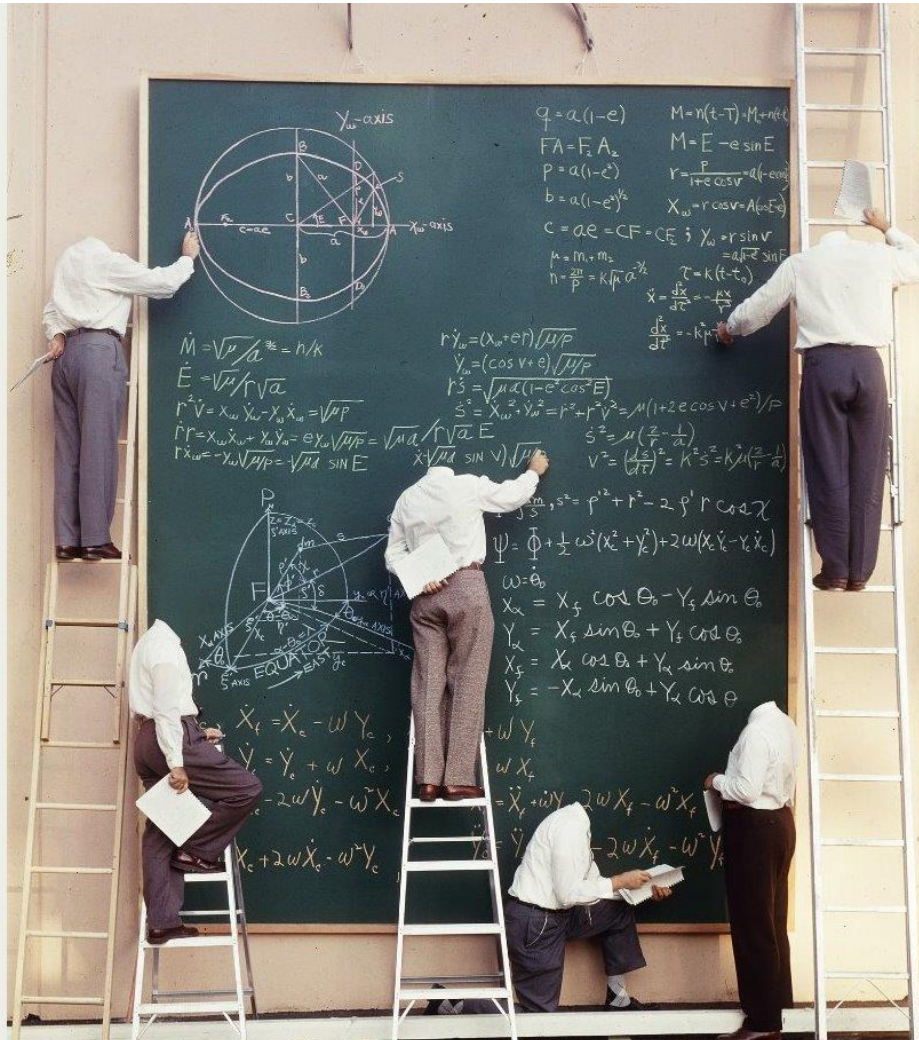
Understand complexity on a macroscopic level

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Can A.I. Be Taught to Explain Itself?

As machine learning becomes more powerful, the field's researchers increasingly find themselves unable to account for what their algorithms know — or how they know it.

By CLIFF KUANG NOV. 21, 2017



$$Q = a(1-e) \quad M = n(t-T) + t + n(t)$$
$$FA = F_z A_z \quad M = E - e \sin E$$
$$P = a(1-e^2) \quad r = \frac{P}{1+e \cos v}$$
$$b = a(1-e^2)^{1/2} \quad X_{av} = r \cos v = A(1-e \cos v)$$
$$C = ae - CF = CE \quad Y_w = r \sin v$$
$$A = m_1 + m_2 \quad \tau = k(t-t_0)$$
$$n = \frac{2\pi}{P} = k \sqrt{a^3} \quad \dot{x} = \frac{dx}{dt} = -\frac{Ax}{r^2}$$
$$\dot{y} = \frac{dy}{dt} = -\frac{Ay}{r^2}$$
$$\dot{r} = \frac{dr}{dt} = \frac{r \dot{v}}{v} \sin v$$
$$\dot{\theta} = \frac{d\theta}{dt} = \frac{r \dot{v}}{v} \cos v$$
$$M = \sqrt{\mu a} \quad \dot{M} = \frac{h}{k}$$
$$E = \sqrt{\mu a} / r \sqrt{a}$$
$$r^2 \dot{v} = X_w \dot{X}_w - X_w \dot{X}_w = \sqrt{\mu P}$$
$$r \dot{P} = X_w \dot{X}_w + X_w \dot{X}_w = \theta Y_w \sqrt{\mu P} = \sqrt{\mu a} / r \sqrt{a} E$$
$$r \dot{X}_w = -Y_w \sqrt{\mu P} = -\sqrt{\mu a} \sin v$$
$$r \dot{Y}_w = (X_w + e r) \sqrt{\mu P}$$
$$Y_w = (\cos v + e) \sqrt{\mu P}$$
$$r \dot{S} = \sqrt{\mu a (1-e \cos^2 v)}$$
$$S^2 = X_w^2 + Y_w^2 = r^2 + r^2 v^2 = \mu (1+2e \cos v + e^2) / P$$
$$S^2 = \mu \left(\frac{r}{a} - \frac{1}{a} \right)$$
$$v^2 = \left(\frac{r}{a} \right)^2 - k^2 = k^2 \left(\frac{r^2}{a^2} - \frac{1}{a^2} \right)$$
$$s^2 = p^2 + r^2 - 2 p r \cos \chi$$
$$\psi = \phi + \frac{1}{2} \omega^2 (x_c^2 + y_c^2) + 2 \omega (x_c \dot{y}_c - y_c \dot{x}_c)$$
$$\omega = \dot{\theta}_0$$
$$X_c = X_3 \cos \theta_0 - Y_3 \sin \theta_0$$
$$Y_c = X_3 \sin \theta_0 + Y_3 \cos \theta_0$$
$$X_f = X_c \cos \theta_0 + Y_c \sin \theta_0$$
$$Y_f = -X_c \sin \theta_0 + Y_c \cos \theta_0$$
$$\dot{X}_f = \dot{X}_c - \omega Y_c$$
$$\dot{Y}_f = \dot{Y}_c + \omega X_c$$
$$-2 \omega \dot{Y}_c - \omega^2 X_c$$
$$X_c + 2 \omega \dot{X}_c - \omega^2 Y_c$$
$$\dot{X}_c = \dot{X}_f + \omega Y_c$$
$$\dot{Y}_c = \dot{Y}_f - \omega X_c$$
$$-2 \omega \dot{X}_f - \omega^2 Y_f$$
$$Y_c = \dot{Y}_f + \omega X_c$$
$$-2 \omega \dot{X}_f - \omega^2 Y_f$$

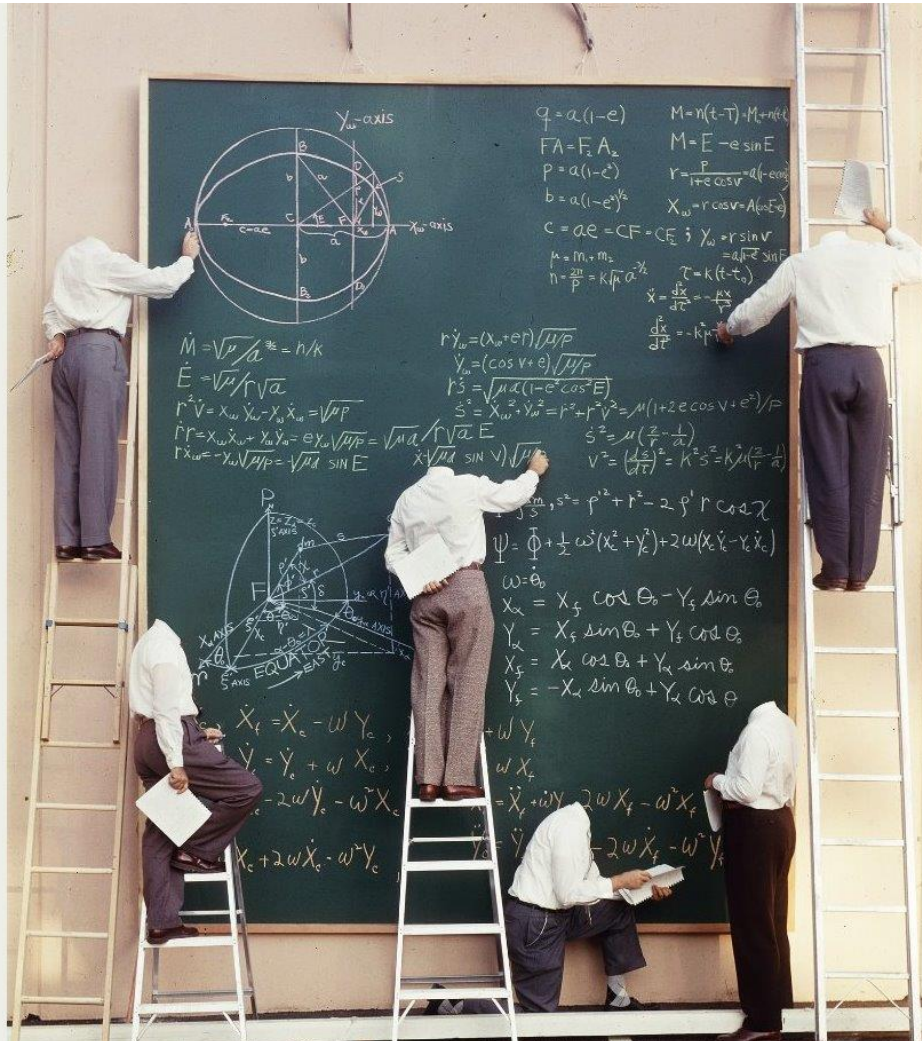
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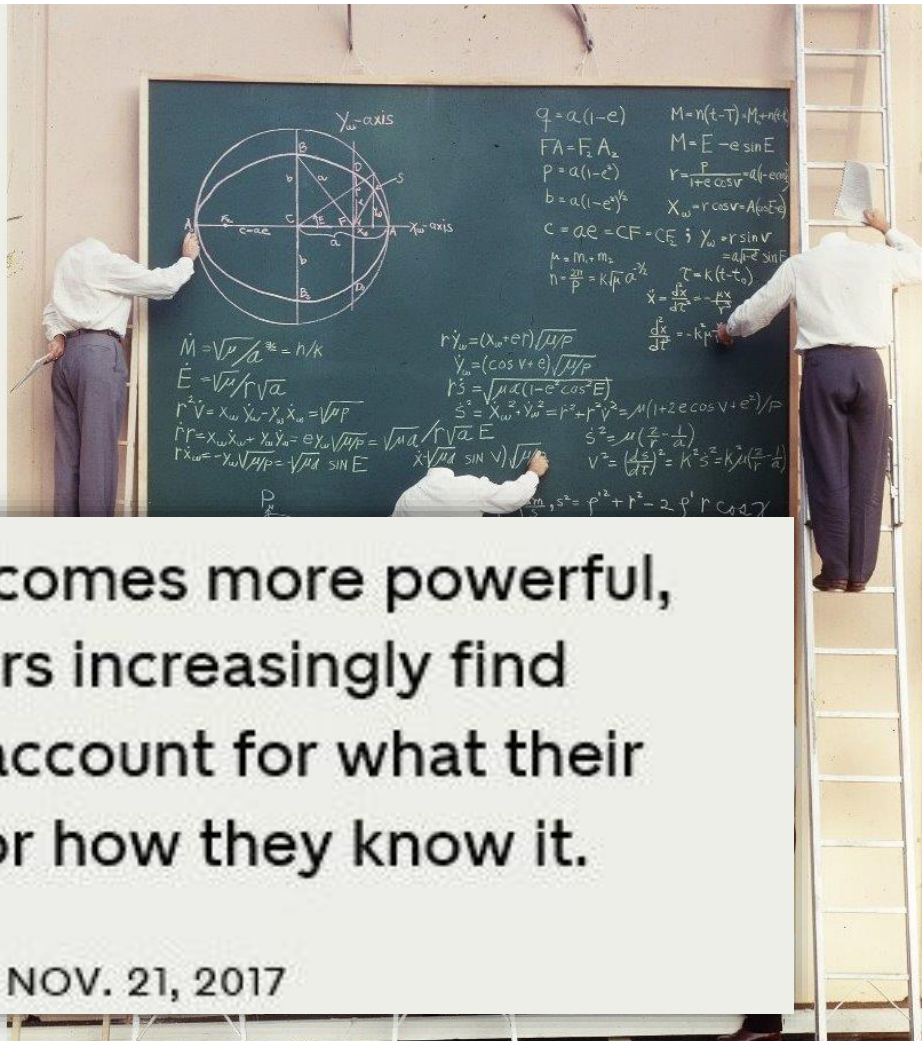
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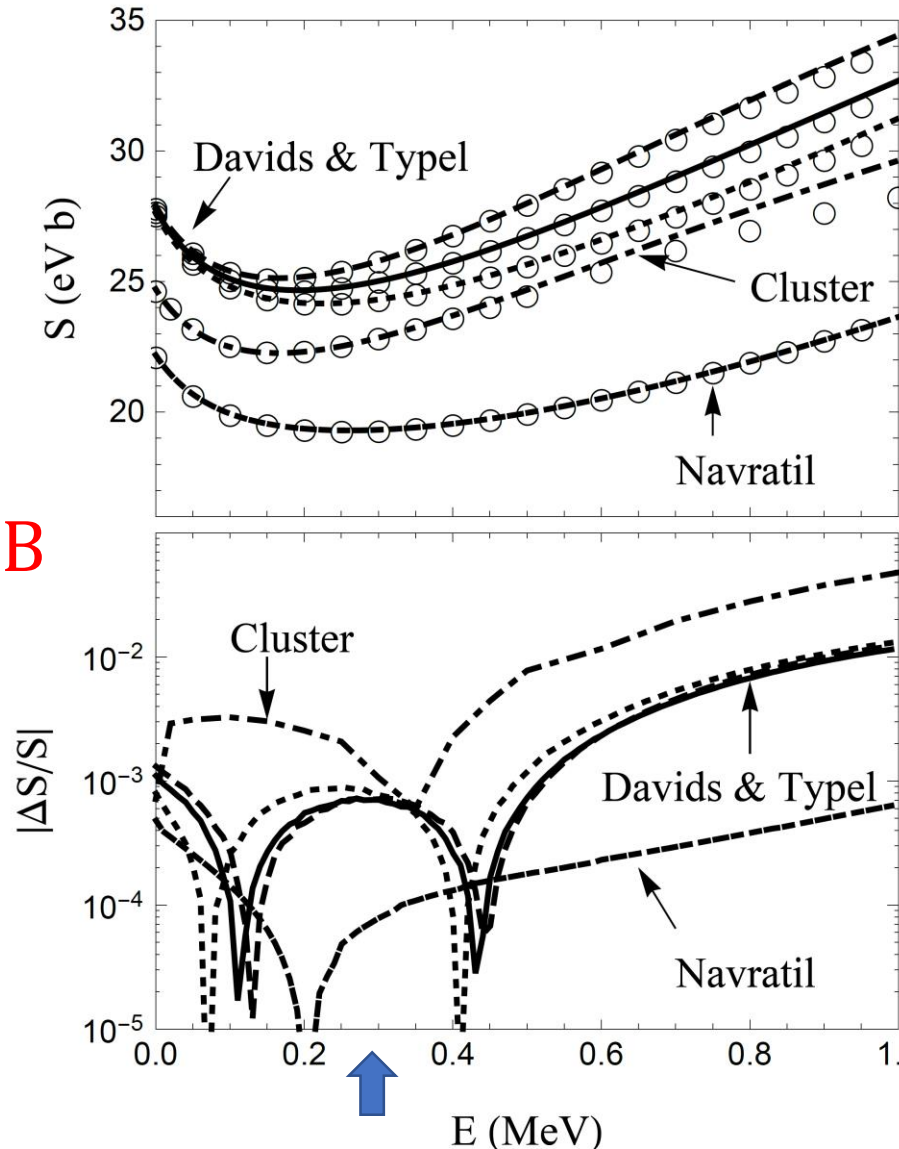
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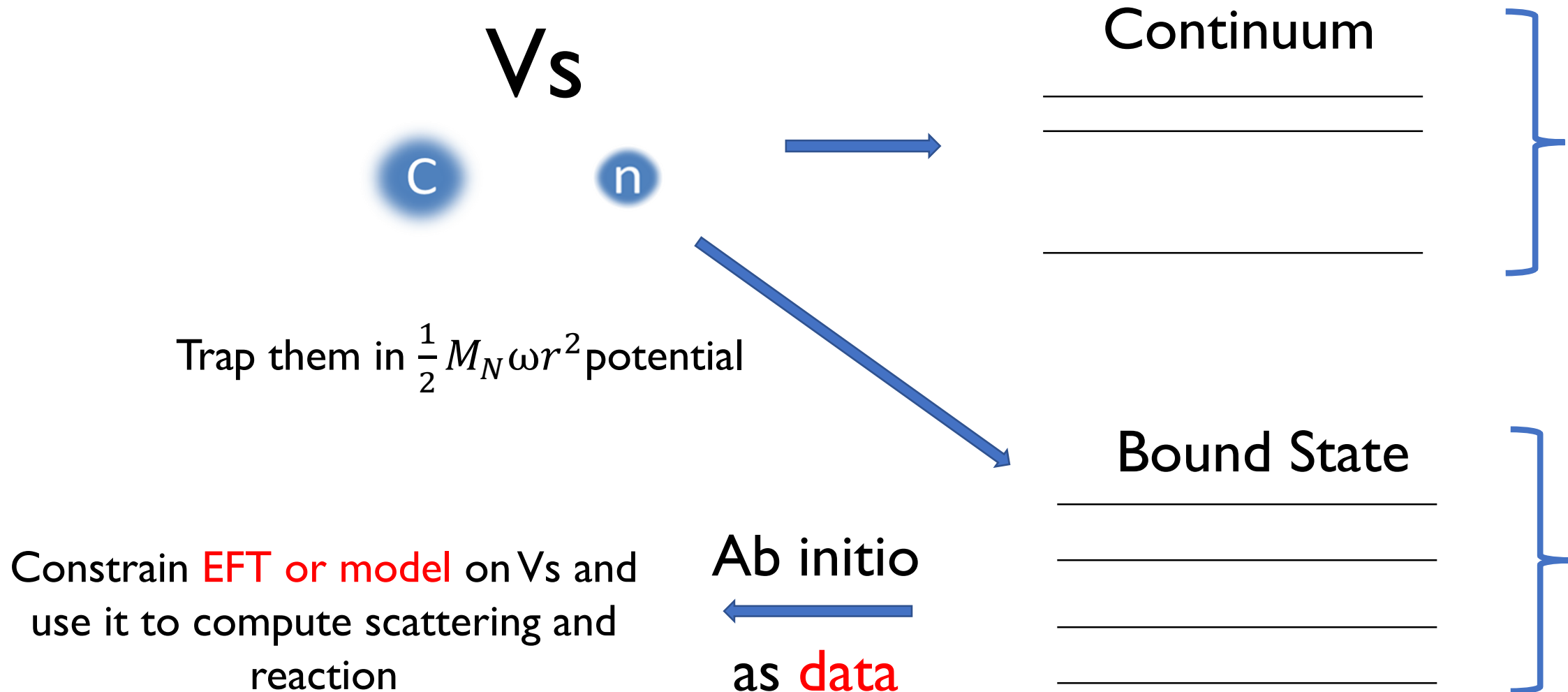
The image shows a chalkboard with a diagram of a circle with axes labeled x_0 -axis and y_0 -axis. The diagram includes points A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. The board is filled with mathematical equations, including:
 $M = \sqrt{\mu/a^*} = h/k$
 $E = \sqrt{\mu}/r\sqrt{a}$
 $r^2\dot{V} = X_w\dot{X}_w - X_w\dot{X}_w = \sqrt{\mu}P$
 $\dot{r}P = X_w\dot{X}_w + X_w\dot{Y}_w = \theta Y_w\sqrt{\mu}P = \sqrt{\mu}d/r\sqrt{a}E$
 $r\dot{X}_w = -Y_w\sqrt{\mu}P = -\sqrt{\mu}d \sin V$
 $r\dot{Y}_w = (X_w + eP)/\mu P$
 $Y_w = (\cos V + e)/\sqrt{\mu}P$
 $rS = \sqrt{\mu}d(1 - e^2 \cos^2 E)$
 $S^2 = X_w^2 + Y_w^2 = r^2 + \dot{r}^2 V^2 = \mu(1 + 2e \cos V + e^2)/P$
 $S^2 = \mu(\frac{r}{a} - \frac{1}{a})$
 $V^2 = (\frac{\dot{r}}{r})^2 = K^2 S^2 = K^2 \mu (\frac{r}{a} - \frac{1}{a})$
 $m, s^2 = P^2 + \dot{r}^2 - 2P\dot{r} \cos V$
Other equations include:
 $Q = a(1 - e)$
 $FA = F_z A_z$
 $P = a(1 - e^2)$
 $b = a(1 - e^2)^{1/2}$
 $C = ae - CF = CE$
 $A = m_1 + m_2$
 $n = \frac{2\pi}{P} = K\sqrt{\mu} a^{-3/2}$
 $M = n(t - T) + M_0 + n(t - T)$
 $M = E - e \sin E$
 $r = \frac{P}{1 + e \cos V} = a(1 - e \cos^2 V)$
 $X_w = r \cos V = A(1 - e \cos V)$
 $Y_w = r \sin V = a \sqrt{1 - e^2} \sin V$
 $\tau = k(t - t_0)$
 $\dot{x} = \frac{dx}{dt} = -\frac{Px}{r^2}$
 $\frac{dx}{dt} = -k\sqrt{\mu} \frac{x}{r^2}$

Understand ab initio calculation on a macroscopic level: consistency and global uncertainty analysis



- 1) Halo-EFT: scattering lengths, effective ranges, ANCs,,
- 2) Computational “data” on different observables \rightarrow macroscopic picture: **a systematical approach is needed**
- 3) Various information sources: experimental, computational, and empirical. The EFT approach can be used to combine these information and produce global uncertainty propagation

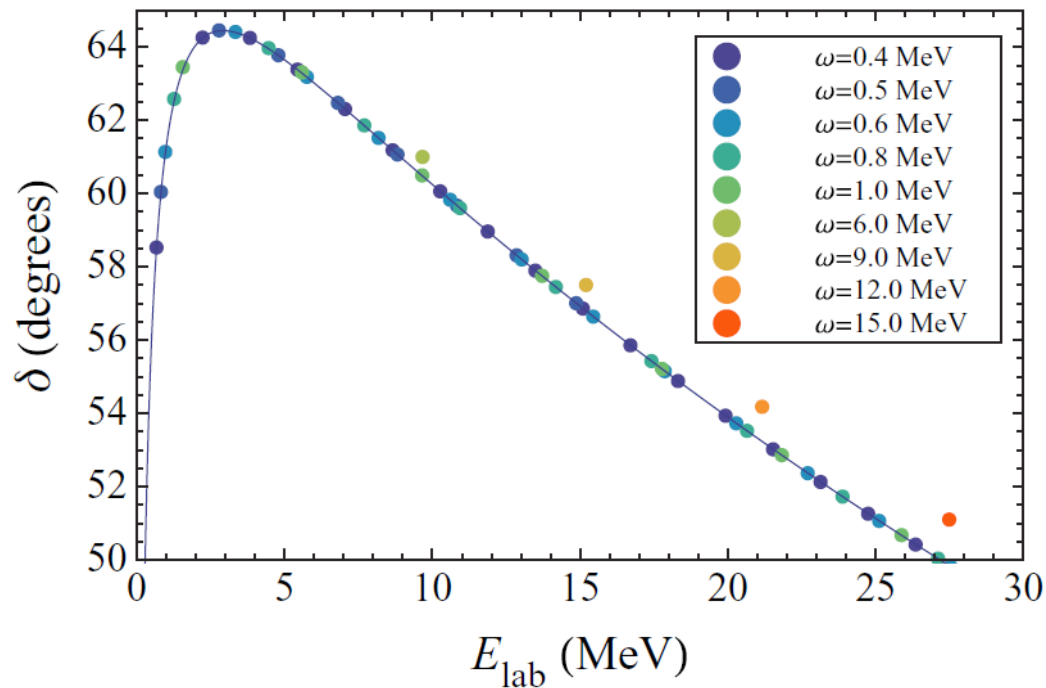
Busch formula (infrared extrapolation)



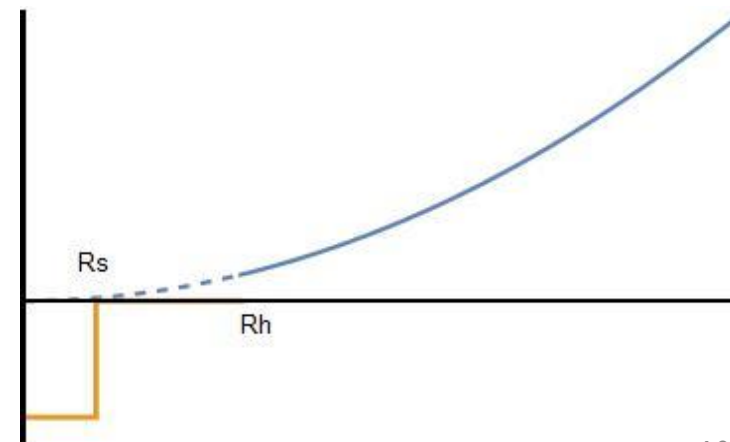
Busch formula

$$p^{2l+1} \cot \delta_l(p) = (-1)^{l+1} (4M_R \omega)^{l+\frac{1}{2}} \frac{\Gamma\left(\frac{3}{4} + \frac{l}{2} - \frac{\epsilon}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{l}{2} - \frac{\epsilon}{2}\right)}$$

with $\epsilon \equiv \frac{E}{\omega}$, $E \equiv \frac{p^2}{2M_R}$



- 1) T. Luu, M. Savage, A. Schwenk, and J. Vary, PRC (2010): N-N phase shift
- 2) J. Rotureau, I. Stetcu, B.R. Barrett, and U. van Kolck, PRC (2012): N-D phase shift



Improve Busch Formula: a model

$$V_s(r) = \begin{cases} +\infty & \text{when } r \leq r_c \\ 0 & \text{when } r > r_c, \end{cases}$$

$$p^{2l+1} \cot \delta_l(p) - (-1)^{l+1} (4M_R \omega)^{l+\frac{1}{2}} \frac{\Gamma\left(\frac{3}{4} + \frac{l}{2} - \frac{\epsilon}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{l}{2} - \frac{\epsilon}{2}\right)}$$

$$= -\frac{(2l+1)!!(2l-1)!!}{r_c^{2l+1}} \left[\frac{(2l+1)\left(\frac{r_c}{b}\right)^4}{2(2l-3)(2l+5)} + \frac{(2l+1)(6l+25)\frac{1}{2}(pr_c)^2\left(\frac{r_c}{b}\right)^4}{3(2l-5)(2l+3)(2l+5)(2l+7)} + O\left[\left(\frac{r_c}{b}\right)^8, (pr_c)^4\left(\frac{r_c}{b}\right)^4\right] \right]$$

$$\equiv -L_{a_l} \frac{1}{b^4 r_c^{2l-3}} - L_{r_l} \frac{p^2}{b^4 r_c^{2l-1}} + \dots$$

Note: $b = \sqrt{\frac{1}{M_R \omega}}$

Effective range expansion (ERE):

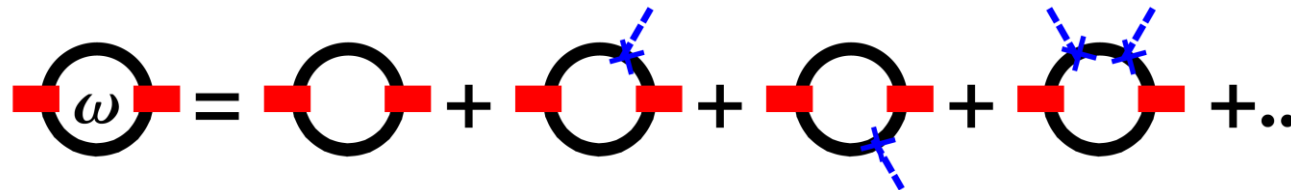
$$p^{2l+1} \cot \delta_l(p) = -\frac{\Lambda^{2l+1}}{a_l} + \frac{1}{2} r_l \Lambda^{2l-1} p^2 + \frac{1}{4} \tilde{r}_l^{(1)} \Lambda^{2l-3} p^4$$

Improve Busch Formula: EFT

$$\mathcal{L}_0 = \begin{pmatrix} c^* & n^* & -\phi^* \end{pmatrix} \text{diag} \left(i\partial_t - \hat{m}_c \psi + \frac{\partial^2}{2M_c}, i\partial_t - \hat{m}_n \psi + \frac{\partial^2}{2M_n}, i\partial_t - \hat{m}_\phi \psi + \frac{\partial^2}{2M_{nc}} + \Delta_0 \right) \begin{pmatrix} c & n & \phi \end{pmatrix}^T$$

$$\mathcal{L}_{I0} = g_0 \phi^* c n - \phi^* \left[\sum_{j=2} d_j^{(0)} \left(i\partial_t - \hat{m}_\phi \psi + \frac{\partial^2}{2M_{nc}} \right)^j \right] \phi + \text{C.C.} \quad \text{Note: } \psi = \frac{1}{2} m_N \omega^2 r^2$$

Self-energy bubble:



Dimer-field propagator:

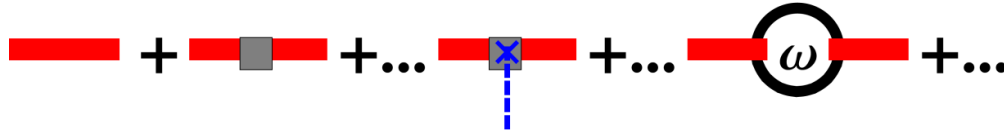


$$p_{\tilde{E}} \cot \delta_0(\tilde{E}) = -\frac{2\pi}{g_0^2 M_R} \left(\Sigma_\omega(\tilde{E}) - \Sigma(\tilde{E}) \right) \text{ reproduces the Busch formula.}$$

Then what went wrong?

Improve Busch Formula: EFT

$$\mathcal{L}_{I0} = g_0 \phi^* c n - \phi^* \left[\sum_{j=2} d_j^{(0)} \left(i\partial_t - \hat{m}_\phi \psi + \frac{\partial^2}{2M_{nc}} \right)^j \right] \phi + \text{C.C.} .$$

$$- \phi^* \left[\sum_{j=0} \sum_{k=1} d_{j,k}^{(0)} \left(i\partial_t - \hat{m}_\phi \psi + \frac{\partial^2}{2M_{nc}} \right)^j \left(\frac{M_R^2}{3m} \partial^2 \psi \right)^k \right] \phi$$


The diagram below the equation shows a sequence of red horizontal bars representing terms in a sum. A blue 'x' is placed on one of the bars, with a vertical blue dashed line extending downwards from it. To the right of this sequence is a circle containing the Greek letter ω , followed by another red bar and an ellipsis.

The factorizability of CM motion severely constrains two-body current like couplings.

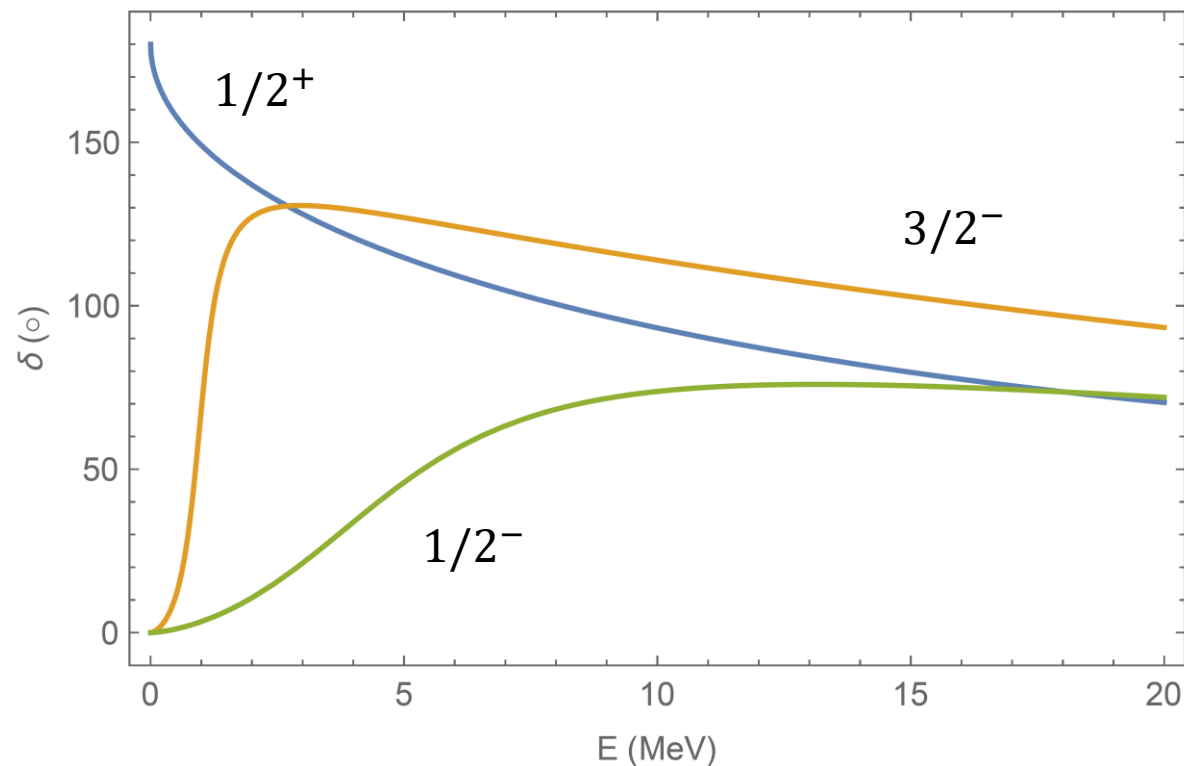
$$\Sigma_\omega \rightarrow \Sigma_\omega + \sum_{j=0} \sum_{k=1} d_{j,k}^{(0)} \tilde{E}^j b^{-4k}$$

$$p \cot \delta_0(p) \rightarrow p \cot \delta_0(p) + L_{a_0} \frac{1}{b^4 \Lambda^3} + L_{r_0} \frac{p^2}{b^4 \Lambda^5} + L_{\tilde{r}_0^{(1)}} \frac{p^4}{b^4 \Lambda^7} + \dots$$

$$= -\frac{\Lambda}{a_0} + \frac{1}{2} \frac{r_0}{\Lambda} p^2 + \frac{1}{4} \frac{\tilde{r}_0^{(1)}}{\Lambda^3} p^4 + L_{a_0} \frac{1}{b^4 \Lambda^3} + L_{r_0} \frac{p^2}{b^4 \Lambda^5} + L_{\tilde{r}_0^{(1)}} \frac{p^4}{b^4 \Lambda^7}$$

Test: n – α system

$$V_s(r) = \begin{cases} V_0(1 + \beta \mathbf{L} \cdot \boldsymbol{\sigma}) & \text{when } r < r_c \\ 0 & \text{when } r > r_c \end{cases}$$

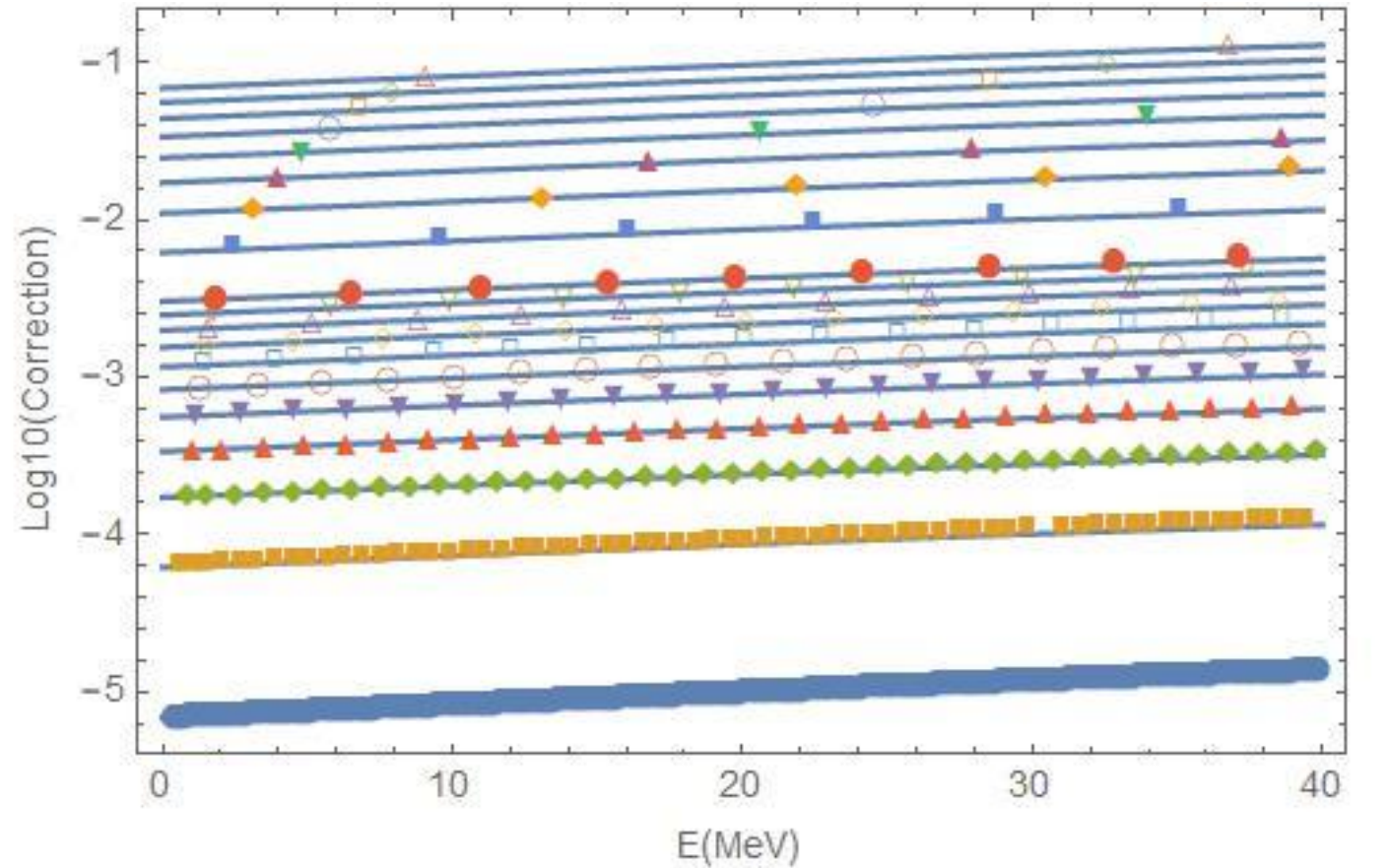


S.Ali et.al., RMP **57**, 923 (1985)

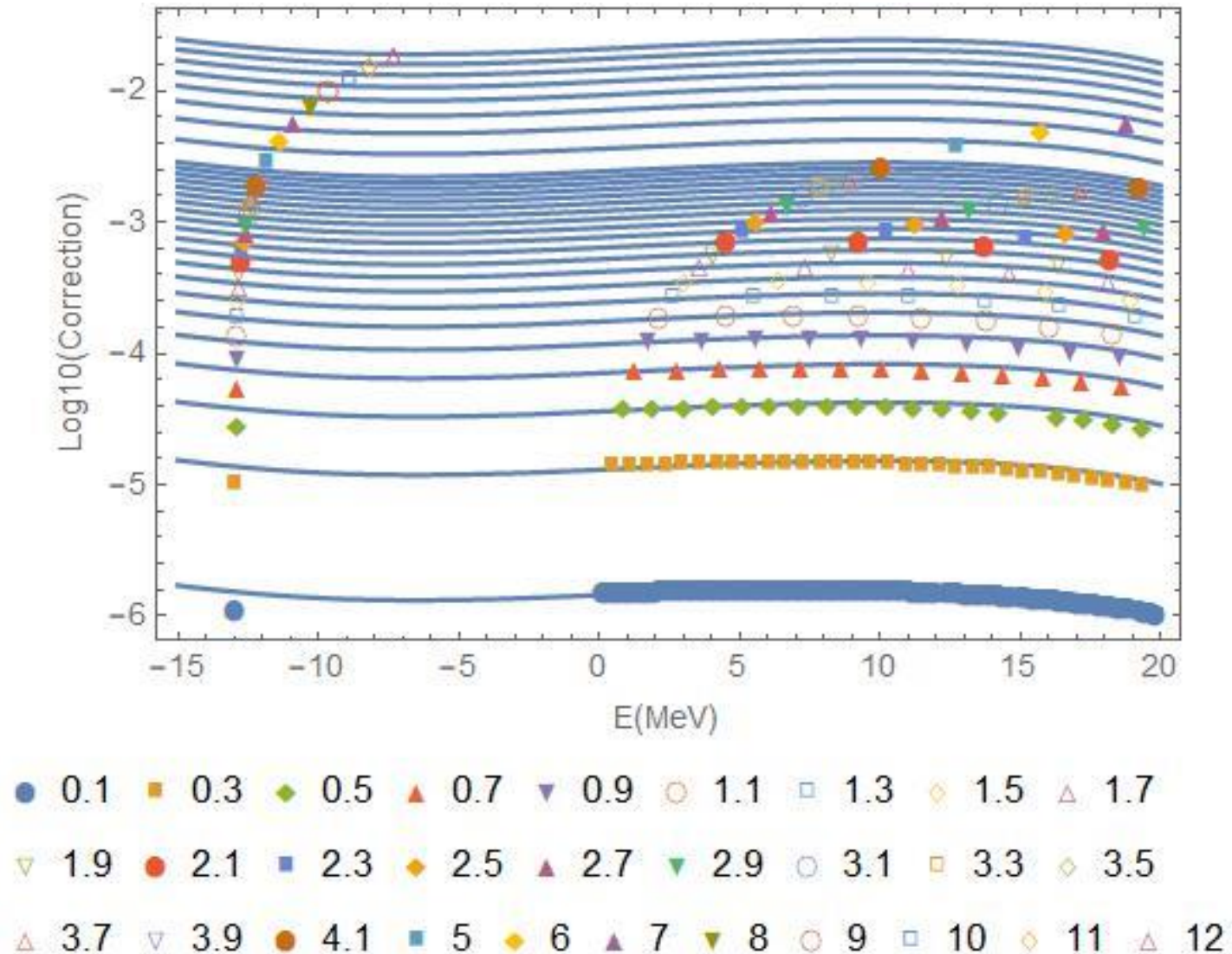
Test: n – α system in p-wave

$$p^{2l+1} \cot \delta_l(p) - (-1)^{l+1} (4M_R \omega)^{l+\frac{1}{2}} \frac{\Gamma\left(\frac{3}{4} + \frac{l}{2} - \frac{\epsilon}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{l}{2} - \frac{\epsilon}{2}\right)}$$

- 1) Corrections $\propto \omega^2$
- 2) At $\omega = 10$ MeV,
10% corrections



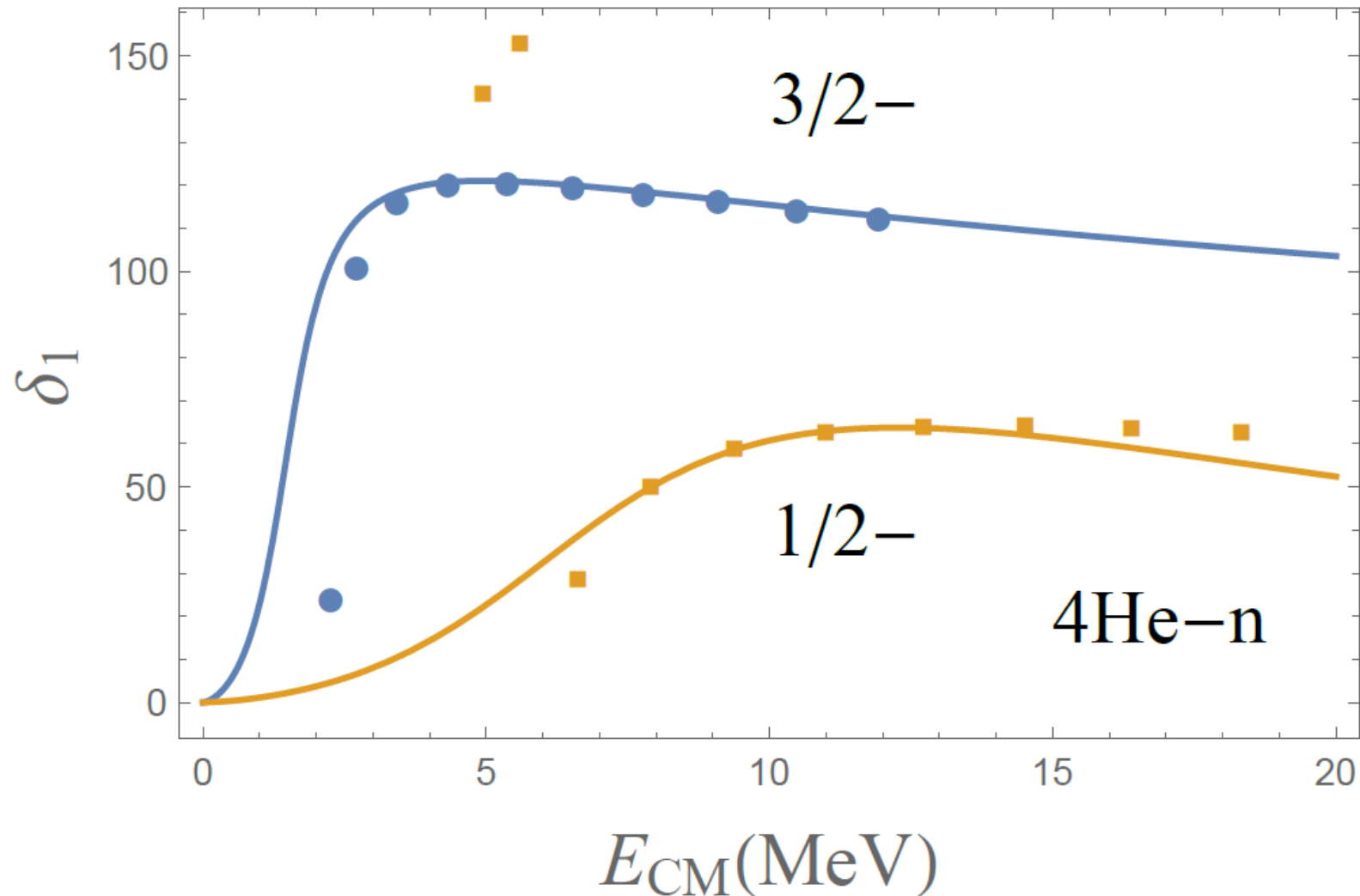
Test: $n - \alpha$ system in s-wave



Test: extract $n - \alpha$ ERE parameters

	a	r	$\tilde{r}^{(1)}$	$\tilde{r}^{(2)}$	L_a	L_r	$L_{\tilde{r}^{(1)}}$	$L_{\tilde{r}^{(2)}}$
P-wave fit J=3/2	-43.4101	-1.11405	2.23369	0.605012	-1.87099	-0.827113	-0.269323	0.0685459
P-wave J=3/2	-43.408	-1.11425	2.23965	0.5243				
P-wave fit J=1/2	-7.19466	-0.882978	2.44336	0.86573	-2.08059	-1.00758	-0.706915	0.257639
P-wave J=1/2	-7.19286	-0.885359	2.49157	0.668183				
S-wave fit J=1/2	2.82818	1.57966	0.700462	-0.22943	-0.405303	-0.240855	-0.172349	0.867527
S-wave fit J=1/2	2.82818	1.57967	0.700221	-0.076691				

Trial results by analyzing IM-SRG “data” from G. Chan, R. Stroberg, and J. Holt



Summary and outlook

- Need to understand ab initio calculation on a macroscopic level **in a systematical way**
- The improved Busch formula can be used to infer scattering (infrared extrapolation)
- Test on $n - \alpha$ is encouraging
- Working with P. Narvati on $n - \alpha$
- Also applying it to study $n -^{24} O$ with G. Chan, R. Stroberg, and J. Holt
- Consider generalizing it to study two-cluster reactions and three-cluster systems