

# Update on QMC calculations with local chiral interactions



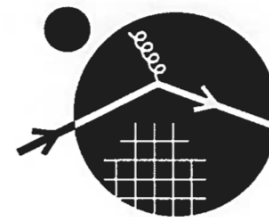
Ingo Tews

In collaboration with J. Carlson, S. Gandolfi, A. Gezerlis,  
L. Huth, D. Lonardoni, J. Lynn, A. Schwenk

Progress in Ab-Initio Techniques in Nuclear Physics,  
February 28, 2018, TRIUMF



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INSTITUTE for  
NUCLEAR THEORY

# Outline

- Quantum Monte Carlo method
  - **Very precise** for strongly interacting systems
  - Need of **local interactions** (depend only on relative distance  $r$ )
  
- Local chiral interactions
  - Can be constructed **up to  $N^2LO$**
  
- Artifacts for local regulators: Ambiguity for NN and 3N contact interactions
  
- Results for nuclei with  $A \leq 16$ 
  - Excellent description of binding energies and charge radii
  
- Summary

# Quantum Monte Carlo method

Cast many-body Schrödinger equation as diffusion equation:

$$\lim_{\tau \rightarrow \infty} e^{-H\tau} |\Psi_T\rangle \rightarrow |\Psi_0\rangle$$

$$\psi(R, \tau) = \int dR'^{3N} \langle R | e^{-(T+V)\tau} | R' \rangle \psi(R', 0)$$

Basic steps:

- Choose **trial wavefunction** which overlaps with the ground state

$$|\psi(R, 0)\rangle = |\psi_T(R, 0)\rangle = \sum_i c_i |\phi_i\rangle \rightarrow \sum_i c_i e^{-(E_i - E_0)\tau} |\phi_i\rangle$$

- **Evaluate propagator** for small timestep  $\Delta\tau$ , in practice **only for local potentials**
- Make **consecutive small time steps** using Monte Carlo techniques to project out ground state

$$|\psi_T(R, \tau)\rangle \rightarrow |\phi_0\rangle \quad \text{for } \tau \rightarrow \infty$$

More details:

Carlson, Gandolfi, Pederiva, Pieper, Schiavilla, Schmidt, Wiringa, RMP (2015)

# Quantum Monte Carlo method

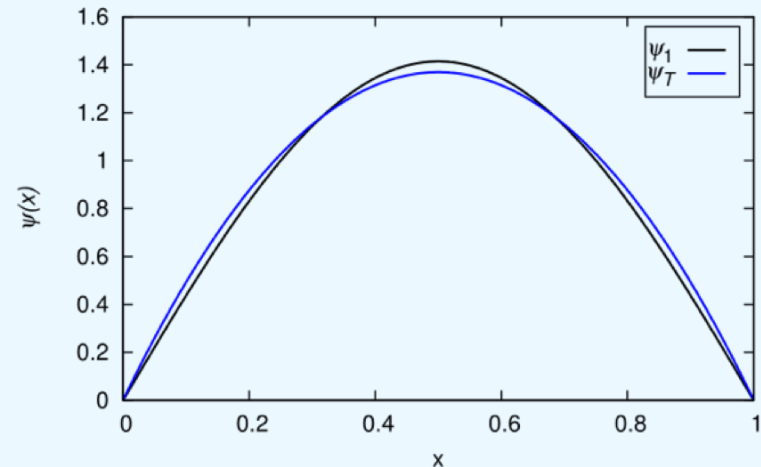
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2} \sin(n \pi x), \quad E_n = \frac{n^2 \pi^2}{2}$$

Basic steps:

- Choose parabolic **trial wavefunction** which overlaps with the ground state

Animation by Joel Lynn, TU Darmstadt



# Quantum Monte Carlo method

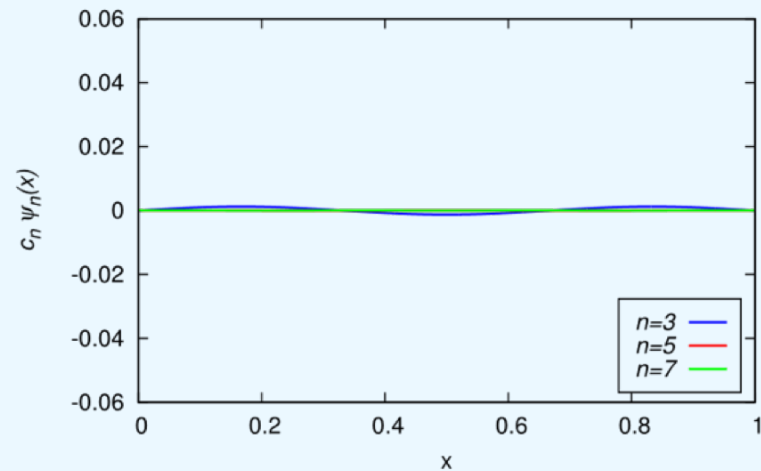
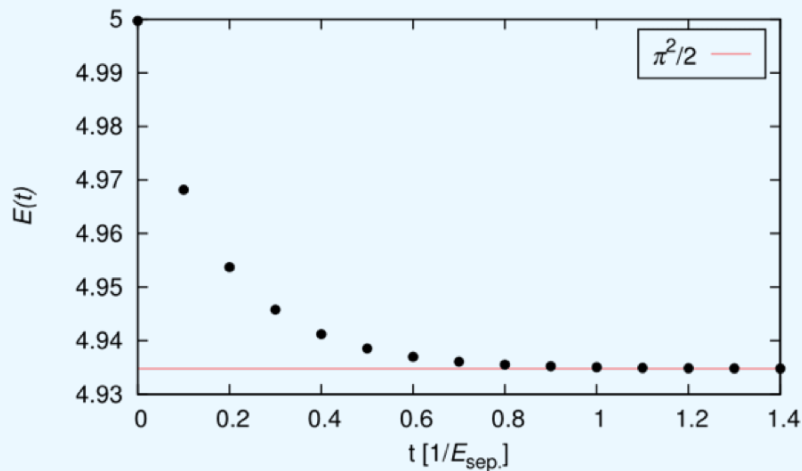
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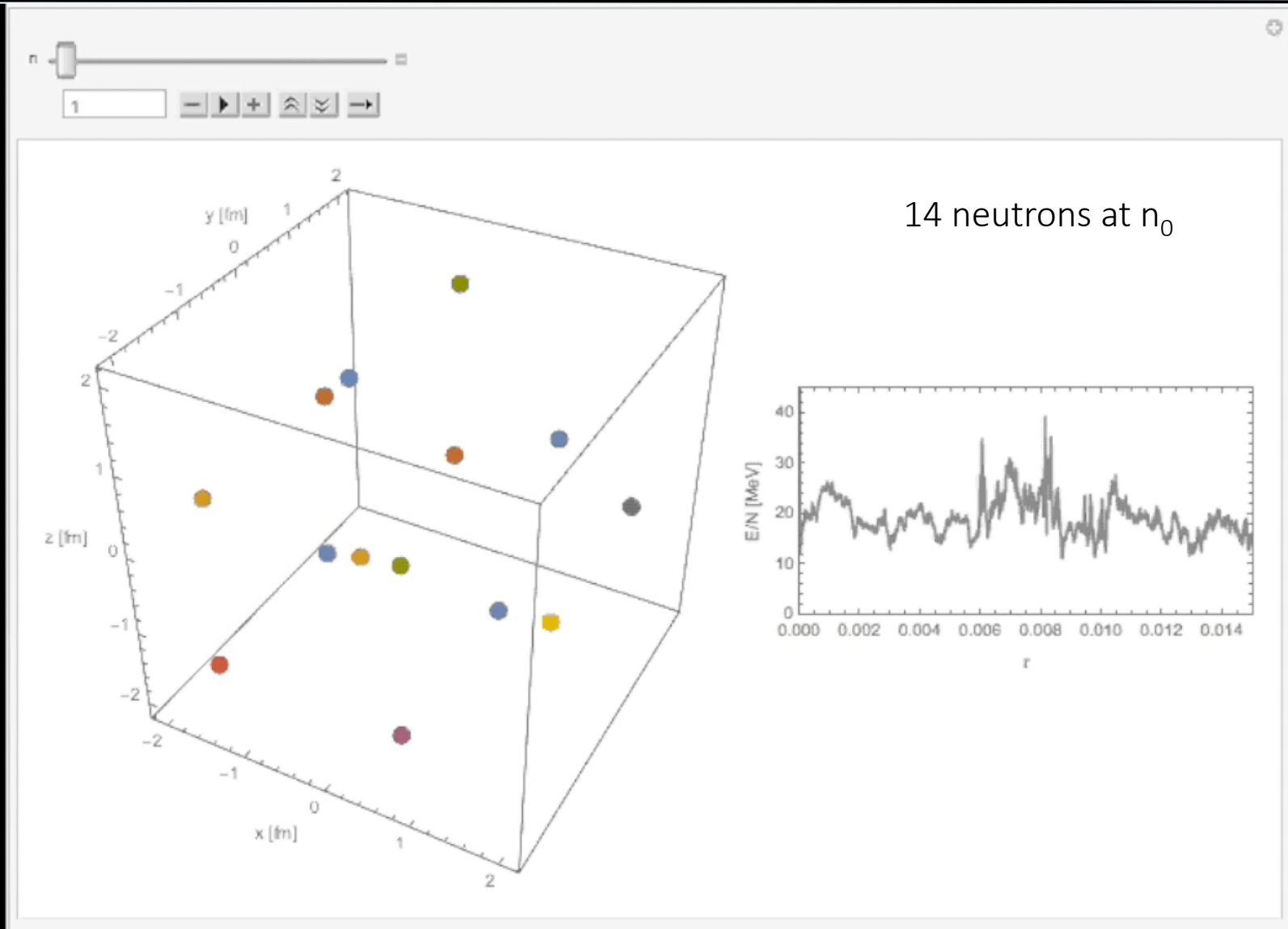
Basic steps:

- Make consecutive small timesteps,  $\tau = 1.4 \left( \frac{1}{E_{\text{sep}}} \right)$

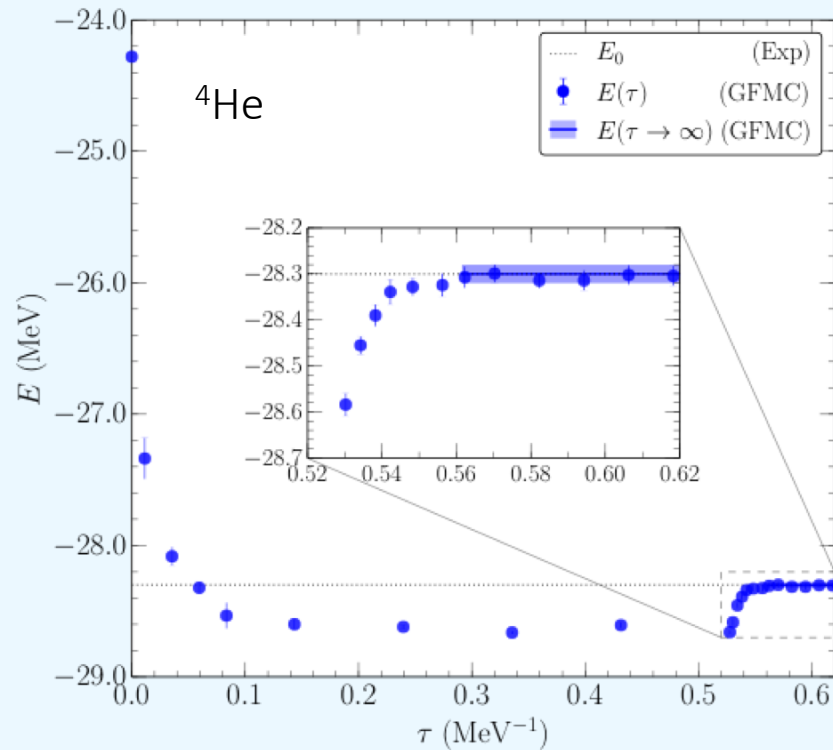
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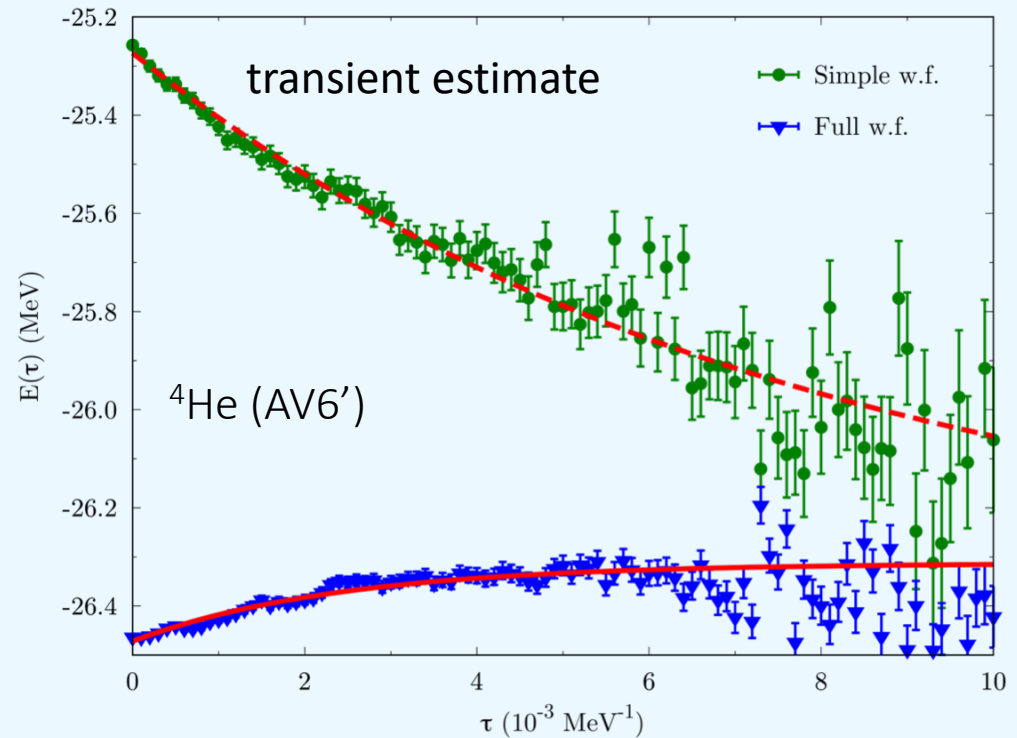
# Quantum Monte Carlo method



# Quantum Monte Carlo method






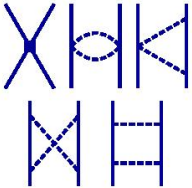



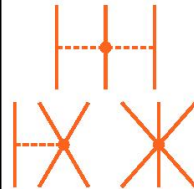


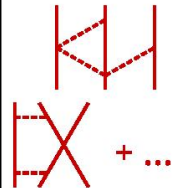

Lynn, IT, et al. PRC (2017)



Lonaroni et al., arXiv:1802.08932

- Very precise method for strongly interacting systems.
- With transient estimates, stochastically exact.
- Needs as input **local interactions** but **chiral EFT generally nonlocal!**

# Local chiral interactions

		NN	3N	4N
LO	$O\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO	$O\left(\frac{Q^2}{\Lambda^2}\right)$			
N <sup>2</sup> LO	$O\left(\frac{Q^3}{\Lambda^3}\right)$			
N <sup>3</sup> LO	$O\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	 + ...	 + ...

Weinberg, van Kolck, Kaplan, Savage, Wise,  
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

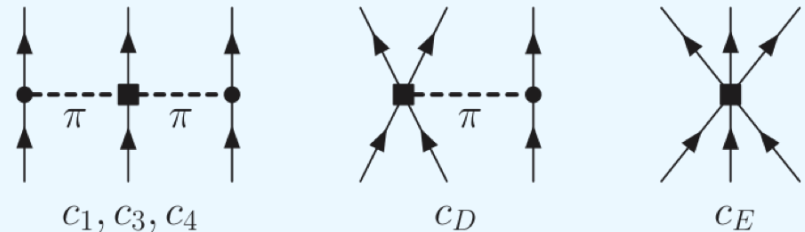
- Can be constructed up to **N<sup>2</sup>LO**

Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)

Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

- Two-body LECs fit to phase shifts

- Include **leading 3N forces**:



**3N LECs** fit to uncorrelated observables:

- Probe properties of light nuclei:  ${}^4\text{He } E_B$
- Probe spin-orbit splitting:  $n-\alpha$  scattering



# Local chiral interactions

To evaluate the propagator for small timesteps  $\Delta\tau$  we need **local potentials**:

$$\langle r' | V | r \rangle = \begin{cases} V(r)\delta(r - r') & \text{if local} \\ V(r', r) & \text{if nonlocal} \end{cases}$$

Chiral Effective Field Theory interactions generally nonlocal:

- Momentum transfer  $\mathbf{q} = \mathbf{p}' - \mathbf{p}$
- Momentum transfer in the exchange channel  $\mathbf{k} = \frac{1}{2}(\mathbf{p} + \mathbf{p}')$
- **Fourier transformation**:  $\mathbf{q} \rightarrow \mathbf{r}$ ,  $\mathbf{k} \rightarrow \nabla_{\mathbf{r}}$

Sources of nonlocalities:

- Usual **regulator** in rel. momenta

$$f(p) = e^{-(p/\Lambda)^{2n}}$$

- k-dependent **contact operators**

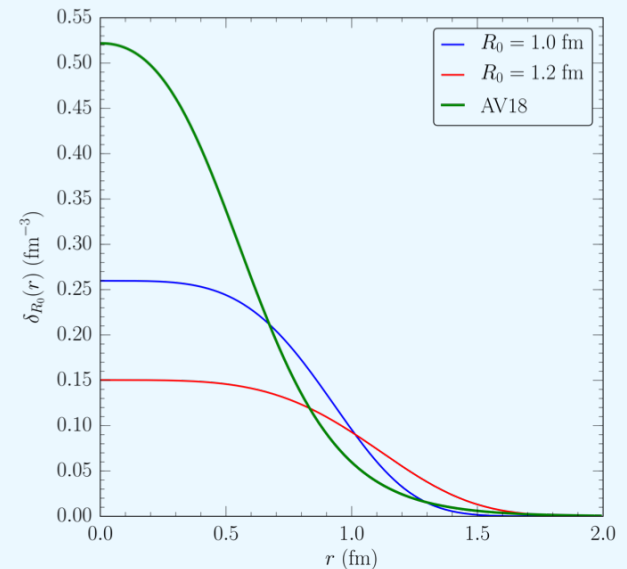
**Solutions:**

- Choose **local regulators**:

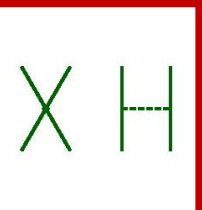
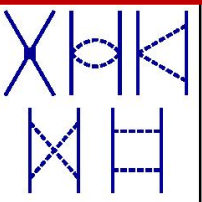
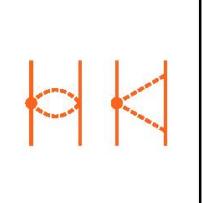
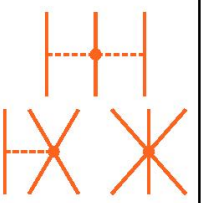
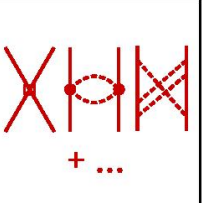
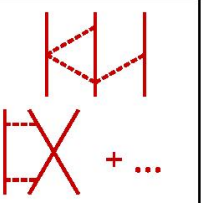
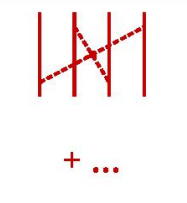
$$V_{\text{long}}(r) \rightarrow V_{\text{long}}(r) \left(1 - e^{-(r/R_0)^4}\right)$$

$$\delta(r) \rightarrow \delta_{R_0}(r) = \alpha e^{-(r/R_0)^4}$$

- Use Fierz freedom to choose **local set of contact operators**.



# Local chiral interactions

		NN	3N	4N
LO	$O\left(\frac{Q^0}{\Lambda^0}\right)$		—	—
NLO	$O\left(\frac{Q^2}{\Lambda^2}\right)$		—	—
N <sup>2</sup> LO	$O\left(\frac{Q^3}{\Lambda^3}\right)$			—
N <sup>3</sup> LO	$O\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	 + ...	 + ...

Weinberg, van Kolck, Kaplan, Savage, Wise,  
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

- Leading order  $V^{(0)} = V_{\text{cont}}^{(0)} + V_{\text{long}}^{\text{OPE}}$
- Pion exchanges local
 
$$V_{\text{long}}(r) = V_C(r) + W_C(r) \tau_1 \cdot \tau_2$$

$$+ (V_S(r) + W_S(r) \tau_1 \cdot \tau_2) \sigma_1 \cdot \sigma_2$$

$$+ (V_T(r) + W_T(r) \tau_1 \cdot \tau_2) S_{12}$$

→ local regulator

$$V_{\text{long}}(r) \rightarrow V_{\text{long}}(r) \left(1 - e^{-(r/R_0)^4}\right)$$

- Contact potential:
 
$$V_{\text{cont}}^{(0)} = \alpha_1 \mathbf{1} + \alpha_2 \sigma_1 \cdot \sigma_2 + \alpha_3 \tau_1 \cdot \tau_2$$


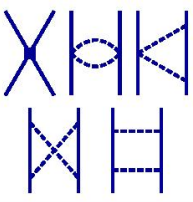
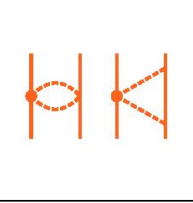
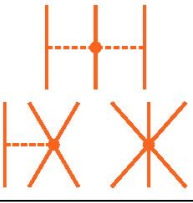

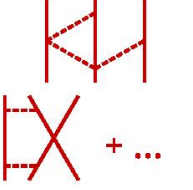
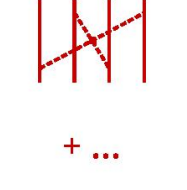
$$+ \alpha_4 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$$

→ Only two independent (Pauli principle)

$$V_{\text{cont}}^{(0)} = C_S \mathbf{1} + C_T \sigma_1 \cdot \sigma_2$$

$$\delta(\mathbf{r}) \rightarrow \delta_{R_0}(\mathbf{r}) = \alpha e^{-(r/R_0)^4}$$

# Local chiral interactions


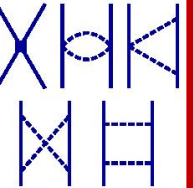

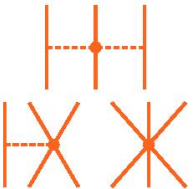

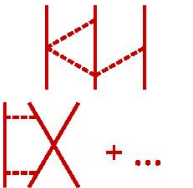

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NLO	$O\left(\frac{Q^2}{\Lambda^2}\right)$		—	—
N <sup>2</sup> LO	$O\left(\frac{Q^3}{\Lambda^3}\right)$			—
N <sup>3</sup> LO	$O\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	 + ...	 + ...

- Choose local set of short-range operators at NLO (7 out of 14)

$$\begin{aligned}
 V_{\text{cont}}^{(2)} = & \gamma_1 q^2 + \gamma_2 q^2 \sigma_1 \cdot \sigma_2 + \gamma_3 q^2 \tau_1 \cdot \tau_2 \\
 & + \gamma_4 q^2 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \\
 & + \gamma_5 k^2 + \gamma_6 k^2 \sigma_1 \cdot \sigma_2 + \gamma_7 k^2 \tau_1 \cdot \tau_2 \\
 & + \gamma_8 k^2 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \\
 & + \gamma_9 (\sigma_1 + \sigma_2)(\mathbf{q} \times \mathbf{k}) \\
 & + \gamma_{10} (\sigma_1 + \sigma_2)(\mathbf{q} \times \mathbf{k}) \tau_1 \cdot \tau_2 \\
 & + \gamma_{11} (\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q}) \\
 & + \gamma_{12} (\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q}) \tau_1 \cdot \tau_2 \\
 & + \gamma_{13} (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) \\
 & + \gamma_{14} (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) \tau_1 \cdot \tau_2
 \end{aligned}$$

Weinberg, van Kolck, Kaplan, Savage, Wise,  
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

# Local chiral interactions

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N <sup>3</sup> LO	$O\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	 + ...	 + ...

➤ Choose local set of short-range operators at NLO (7 out of 14)

➤ Pion exchanges up to N<sup>2</sup>LO are local

➤ This freedom can be used to remove all nonlocal operators up to N<sup>2</sup>LO

Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)

Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

➤ LECs fit to phase shifts

Weinberg, van Kolck, Kaplan, Savage, Wise,  
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

# Contact ambiguity: NN sector

- Contact potential at LO:

$$V_{\text{cont}}^{(0)} = C_{\mathbb{1}} \mathbb{1} + C_{\sigma} \sigma_{12} + C_{\tau} \tau_{12} + C_{\sigma\tau} \sigma_{12} \tau_{12}$$

- Construct antisymmetrized potential:

$$V_{\text{as}}(\mathbf{q}, \mathbf{k}) = \frac{1}{2} (V(\mathbf{q}, \mathbf{k}) - \mathcal{A}[V(\mathbf{q}, \mathbf{k})])$$

$$\mathcal{A}[V(\mathbf{q}, \mathbf{k})] = \frac{1}{4} (1 + \sigma_{12})(1 + \tau_{12}) V \left( \mathbf{q} \rightarrow -2\mathbf{k}, \mathbf{k} \rightarrow -\frac{1}{2}\mathbf{q} \right)$$

$$\begin{aligned} V_{\text{cont,as}}^{(0)} &= \frac{1}{2} \left( 1 - \frac{1}{4} (1 + \sigma_{12})(1 + \tau_{12}) \right) V_{\text{cont}}^{(0)} \\ &= \tilde{C}_S + \tilde{C}_T \sigma_{12} + \left( -\frac{2}{3} \tilde{C}_S - \tilde{C}_T \right) \tau_{12} + \left( -\frac{1}{3} \tilde{C}_S \right) \sigma_{12} \tau_{12} \end{aligned}$$

- Only two linearly independent contact interactions!

# Contact ambiguity: NN sector

True, only when regulator  $f$  behaves like

$$f(\mathbf{q}, \mathbf{k}) = f\left(-2\mathbf{k}, -\frac{1}{2}\mathbf{q}\right)$$

but not for local regulator  $f(\mathbf{q})$ :

$$V_{\text{cont,as}}^{(0,\text{loc})} = \tilde{C}_S + \tilde{C}_T \sigma_{12} + \left(-\frac{2}{3}\tilde{C}_S - \tilde{C}_T\right) \tau_{12} + \left(-\frac{1}{3}\tilde{C}_S\right) \sigma_{12}\tau_{12} + V_{\text{corr}}^f(\mathbf{p} \cdot \mathbf{p}')$$

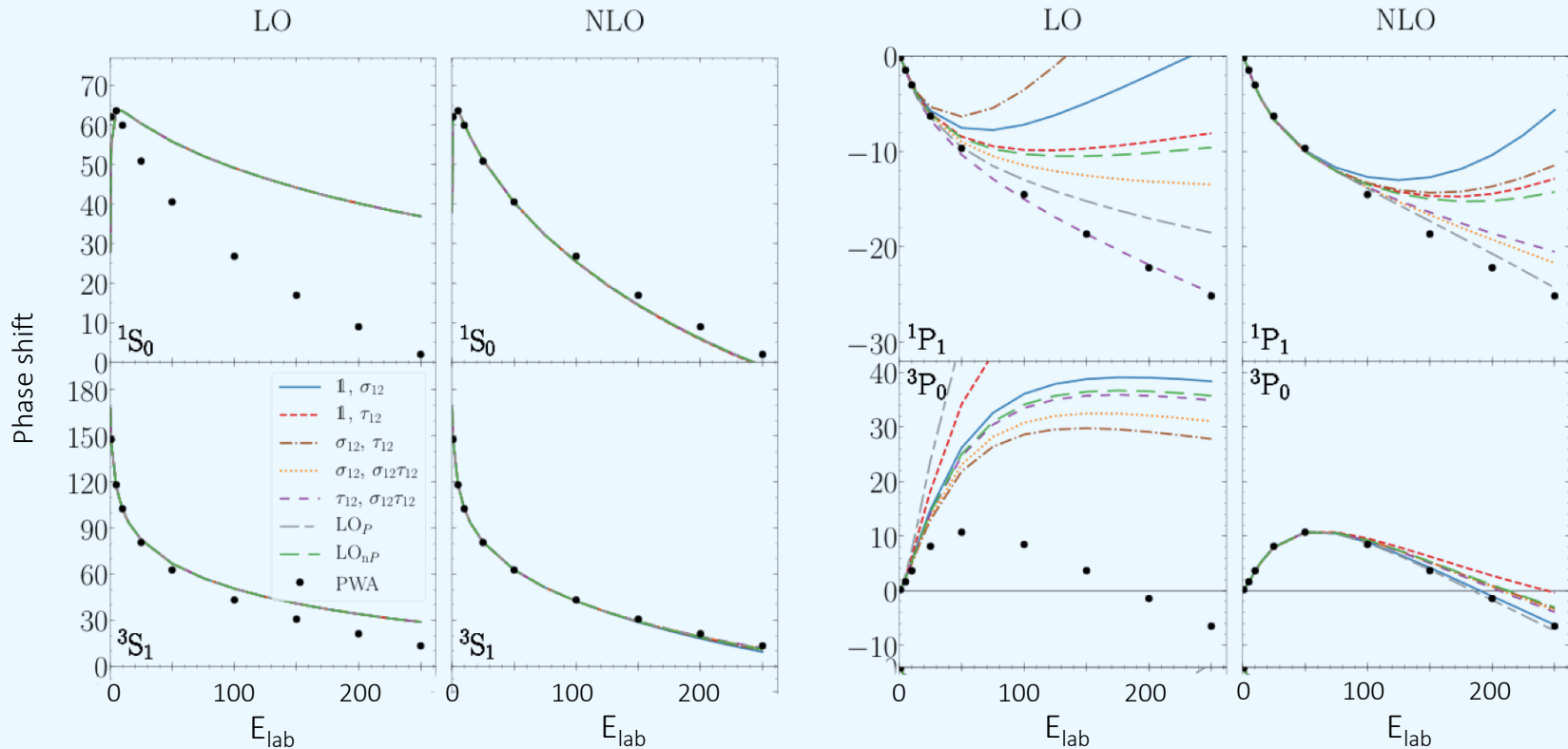
Manifestation of the fact that introducing a **regulator function affects potential terms beyond the order at which one is working**, and should be corrected at higher order.

But:

- Violation of Fierz ambiguity can lead to sizable contributions in 3N sector.  
Lynn, IT, et al., PRL (2016), Dyhdalo, Hebeler, Furnstahl, IT, PRC (2016)
- Leads to mixing of different partial waves.

# Contact ambiguity: NN sector

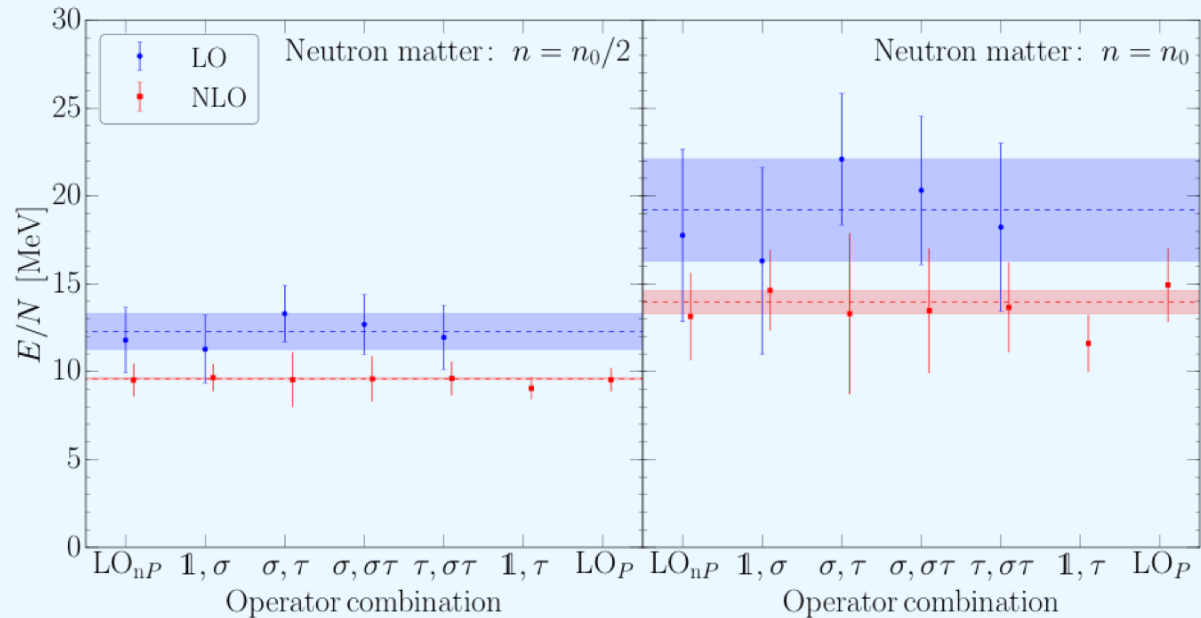
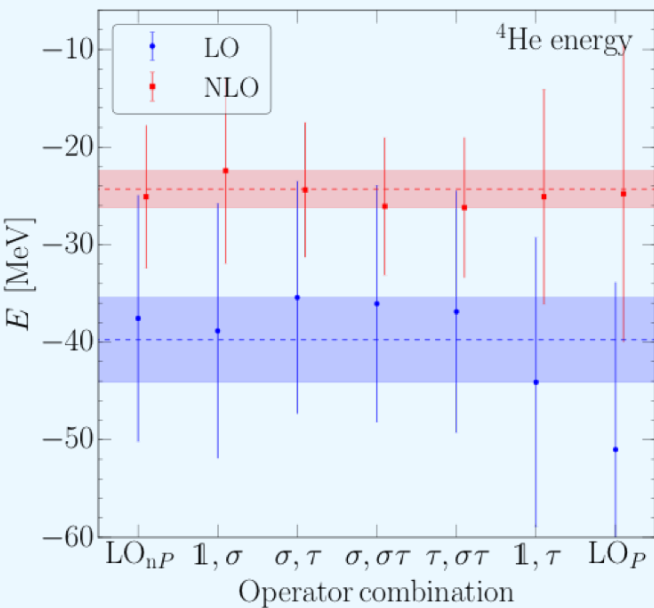
Cutoff  $R_0 = 1.0$  fm: Huth, IT, et al., PRC (2017)



- Violation of Fierz ambiguity sizable in the NN sector at LO but restored to a large extent by including subleading operators at NLO.
- In 3N sector, subleading corrections only at  $N^4LO$ .

# Contact ambiguity: NN sector

Cutoff  $R_0 = 1.0$  fm: Huth, IT, et al., PRC (2017)






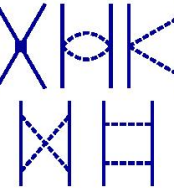



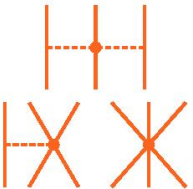


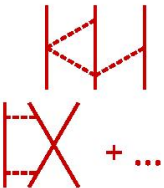

Huth, IT, et al., PRC (2017)

EKM uncertainty estimates.

- Violation of Fierz ambiguity sizable in the NN sector at LO but restored to a large extent by including subleading operators at NLO.
- In 3N sector, subleading corrections only at  $N^4\text{LO}$ .



# QMC with chiral 3N forces

		NN	3N	4N
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Weinberg, van Kolck, Kaplan, Savage, Wise,  
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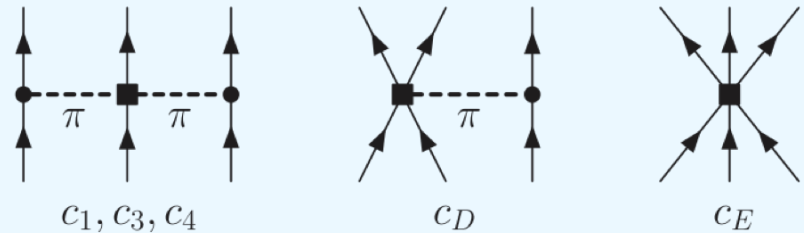
- It is possible to remove all nonlocal operators up to N<sup>2</sup>LO

Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)

Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

- Two-body LECs fit to phase shifts

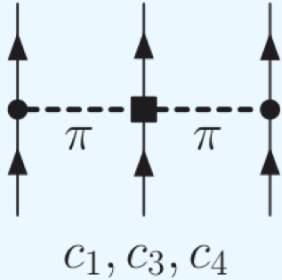
- Include **leading 3N forces**:



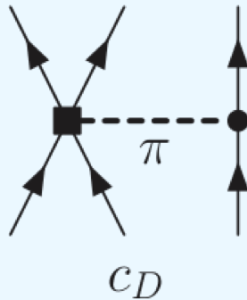
3N LECs fit to uncorrelated observables:

- Probe properties of light nuclei:  ${}^4\text{He } E_B$
- Probe spin-orbit splitting: n- $\alpha$  scattering

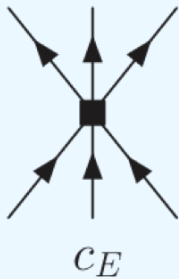
# QMC with chiral 3N forces



- Two-pion-exchange:
  - $c_1$  term: Tucson-Melbourn S-wave interaction
  - $c_{3,4}$  term: Fujita-Miyazawa interaction
- Usually only contribution to pure neutron matter.

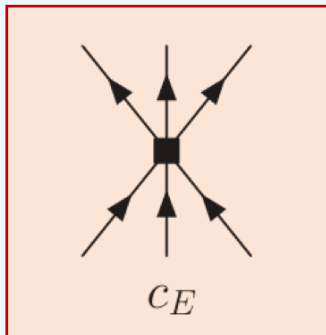
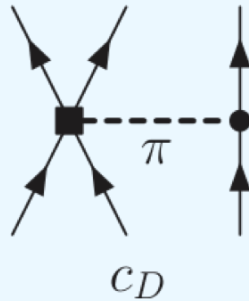
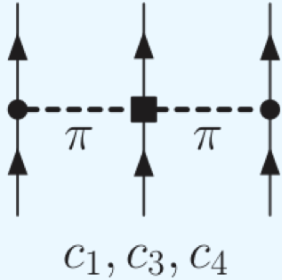


- Usually  $V_D$  and  $V_E$  vanish in  $T=3/2$  or  $S=3/2$  systems:
  - $V_D$  due to spin-isospin structure
  - $V_E$  due to Pauli principle
- see also Hebeler, Schwenk, PRC (2010)



- Only true for regulator symmetric in particle labels like commonly used nonlocal regulators, **not for local regulators!**

local 3N, see also Navratil, Few Body Syst. (2007)



- For local regulator also  $V_E$  contributes to neutron matter:

$$V_E \sim c_E \sum_{i < j < k} \sum_{\text{cyc}} \mathcal{O}_{ijk} \delta_{R_{3N}}(r_{ij}) \delta_{R_{3N}}(r_{kj})$$

- Fierz ambiguity:

$$\mathcal{O}_{ijk} = \{ \mathbb{1}, \sigma_i \cdot \sigma_j, \tau_i \cdot \tau_j, \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j, \\ \sigma_i \cdot \sigma_j \tau_i \cdot \tau_k, [(\sigma_i \times \sigma_j) \cdot \sigma_k][(\tau_i \times \tau_j) \cdot \tau_k] \}.$$

Epelbaum, Nogga, Gloeckle, Kamada, Meißner, Witala, PRC (2002)

- No Fierz rearrangement freedom for local regulators, choose different short-range structures to estimate the impact:

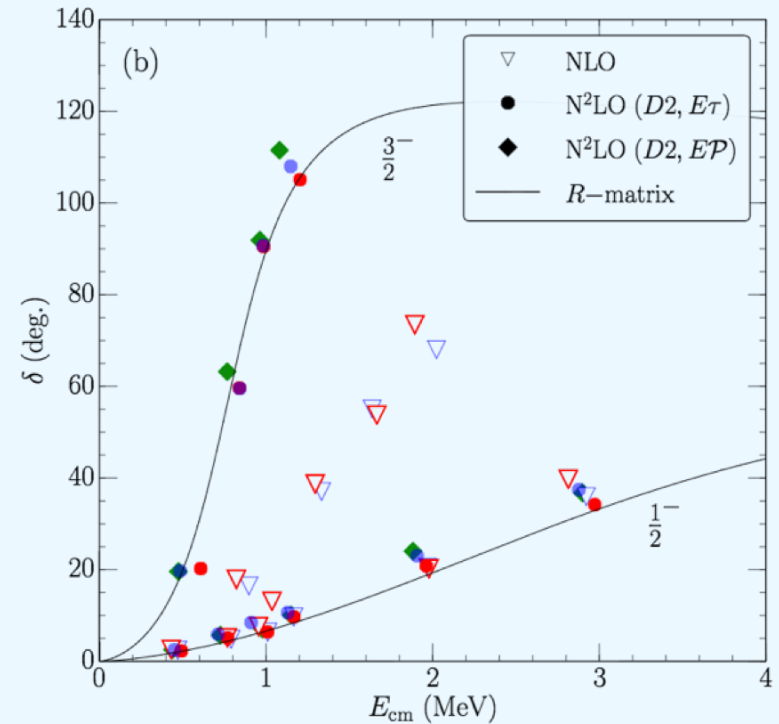
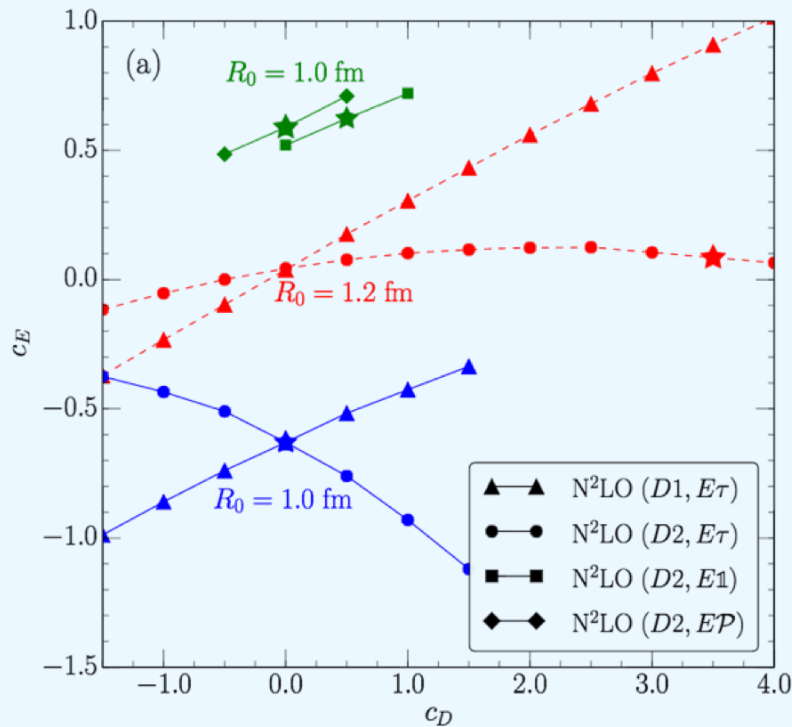
$$V_{E\tau} \sim \tau_i \cdot \tau_j$$

$$V_{E\mathbb{1}} \sim \mathbb{1}$$

$$V_{EP} \sim \mathcal{P}_{S=1/2, T=1/2}$$

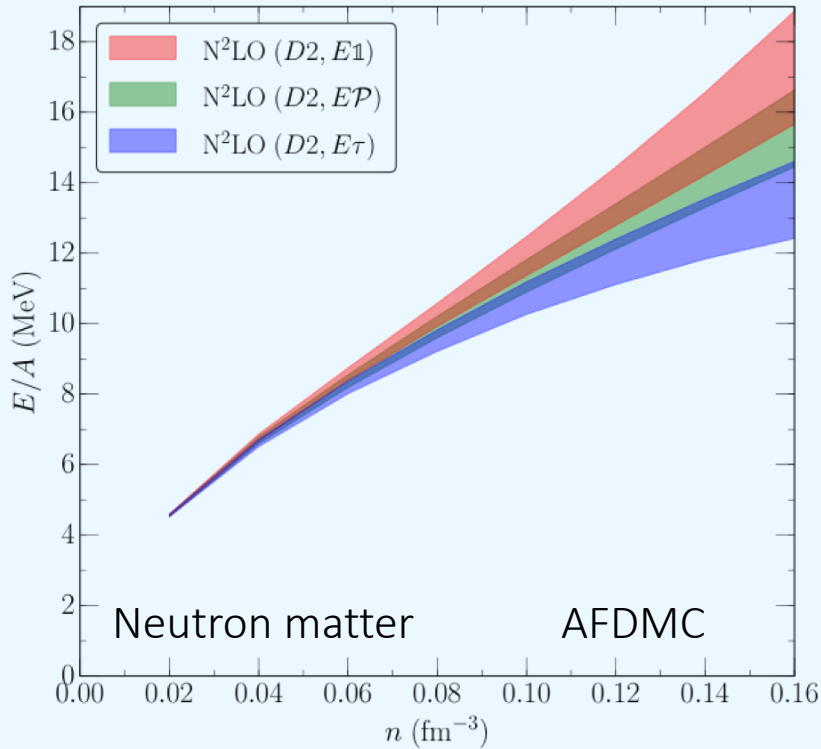
# Fits of 3N LECs

➤ Fit  $c_E$  and  $c_D$  to  ${}^4\text{He}$  binding energy and  $n\text{-}\alpha$  scattering ( $A \leq 5$ )

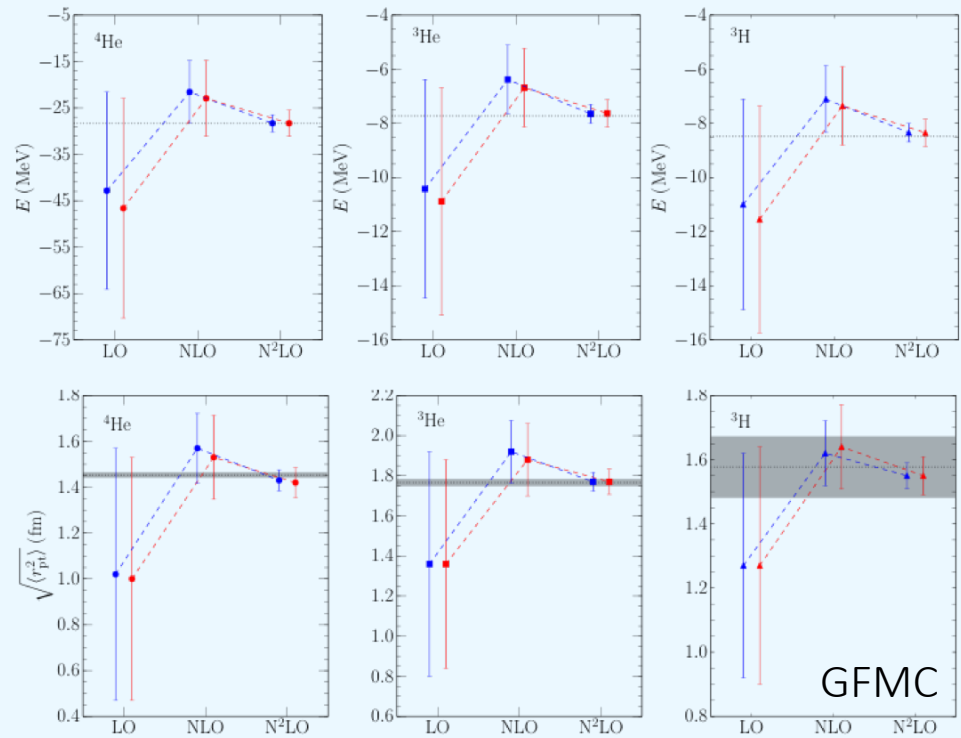


Lynn, IT, et al., PRL (2016)

# Results



Lynn, IT, et al., PRL (2016)

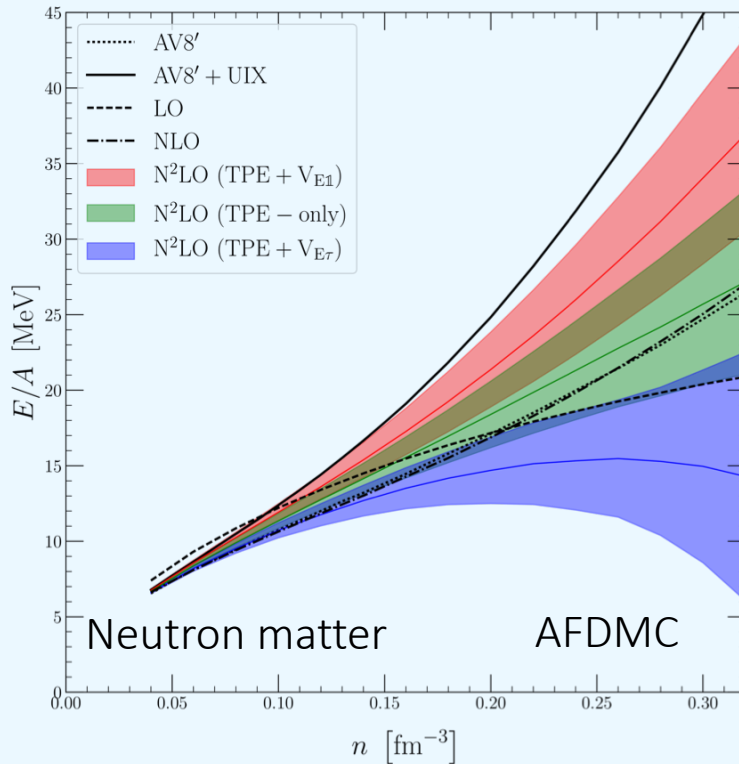


Lynn, IT, et al. PRC (2017)

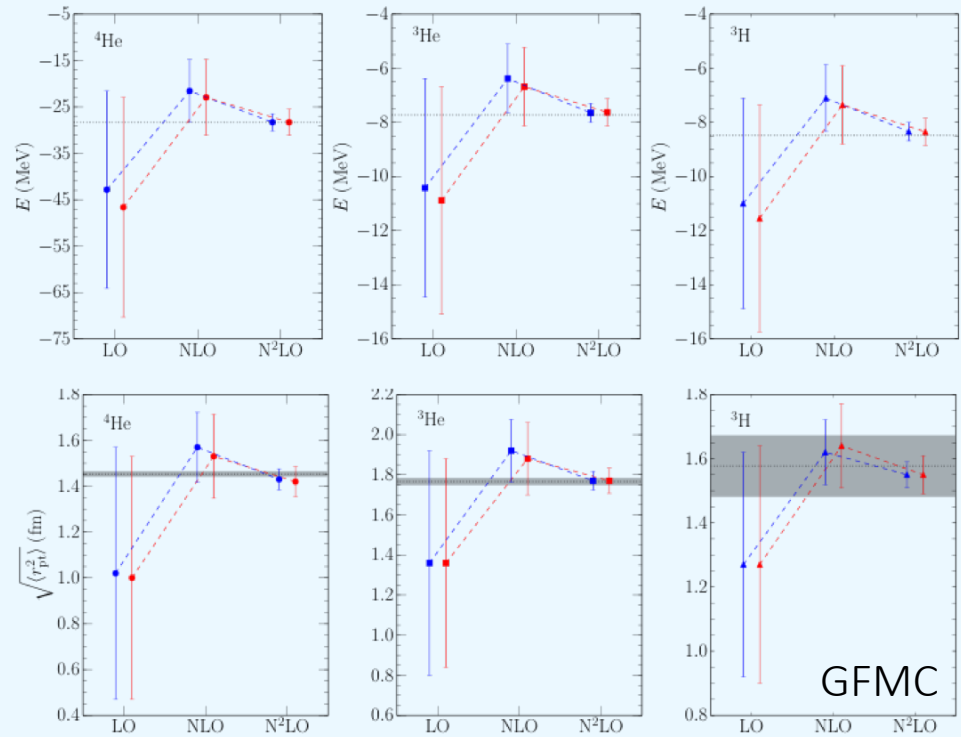
- Chiral interactions at N<sup>2</sup>LO simultaneously reproduce the properties of  $A \leq 5$  systems and of neutron matter (uncertainty estimate as in E. Epelbaum et al, EPJ (2015)).
- Commonly used phenomenological 3N interactions fail for neutron matter.

Sarsa, Fantoni, Schmidt, Pederiva, PRC (2003)

# Results



IT, Carlson, Gandolfi, Reddy, arXiv:1801.01923



Lynn, IT, et al. PRC (2017)

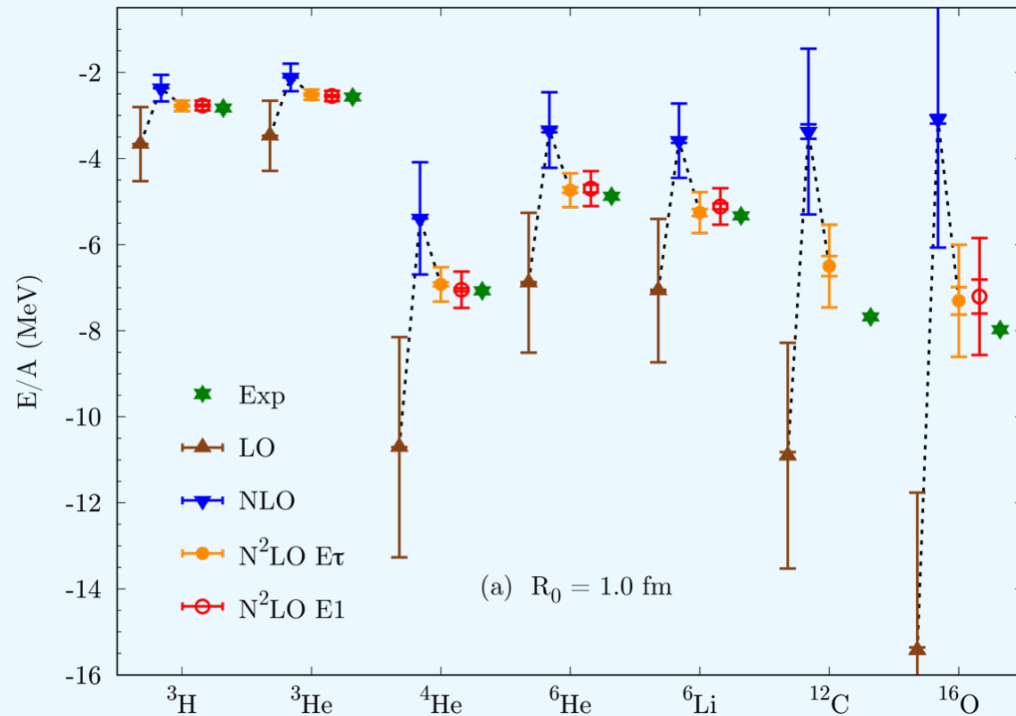
- Chiral interactions at  $\text{N}^2\text{LO}$  simultaneously reproduce the properties of  $A \leq 5$  systems and of neutron matter (uncertainty estimate as in E. Epelbaum et al, EPJ (2015)).
- Commonly used phenomenological 3N interactions fail for neutron matter.

Sarsa, Fantoni, Schmidt, Pederiva, PRC (2003)

# Results for heavier systems

Results for AFDMC calculations of heavier systems ( $R_0 = 1.0$  fm):  
(Using the same local chiral interactions)

Lonardonì et al., arXiv:1709.09143 and 1802.08932

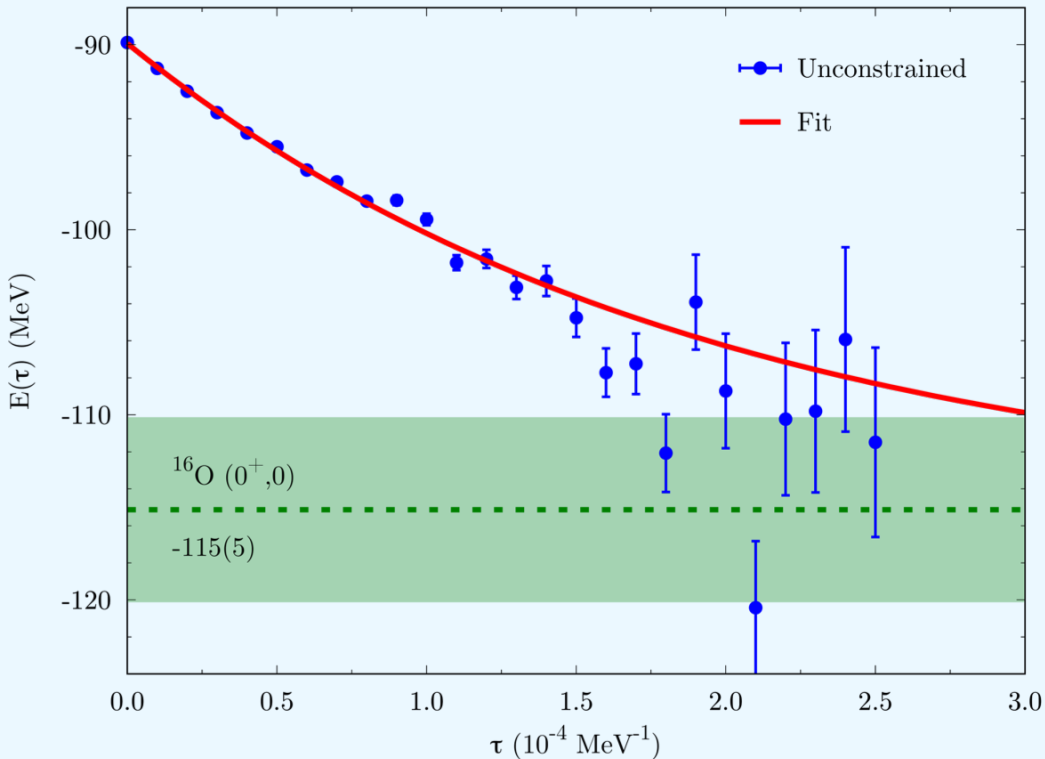


${}^AZ(J^\pi, T)$	Potential	$E_C$ (MeV)	$E$ (MeV)
${}^6\text{He}(0^+, 1)$	LO	-42.1(1)	-41.3(1)(9.6)
	NLO	-18.19(7)	-20.0(3)(5.0)
	$\text{N}^2\text{LO } NN$	-22.24(4)	-23.1(2)(1.2)
	$\text{N}^2\text{LO } 3N E\tau$	-26.58(6)	-28.4(4)(2.0)
	$\text{N}^2\text{LO } 3N E1$	-26.33(8)	-28.2(5)(1.9)
	exp		-29.3
${}^6\text{Li}(1^+, 0)$	LO	-42.8(1)	-42.4(1)(9.9)
	NLO	-19.2(2)	-21.5(3)(4.9)
	$\text{N}^2\text{LO } NN$	-24.3(1)	-25.5(4)(1.1)
	$\text{N}^2\text{LO } 3N E\tau$	-28.9(1)	-31.5(5)(2.3)
	$\text{N}^2\text{LO } 3N E1$	-28.9(1)	-30.7(4)(2.1)
	exp		-32.0
${}^{12}\text{C}(0^+, 0)$	LO	-131.5(2)	-131(1)(31)
	NLO	-31.1(2)	-41(2)(21)
	$\text{N}^2\text{LO } NN$	-63.5(2.4)	-66(3)(6)
	$\text{N}^2\text{LO } 3N E\tau$	-70.2(5)	-78(3)(9)
	$\text{N}^2\text{LO } 3N E1$	-	-
	exp		-92.2
${}^{16}\text{O}(0^+, 0)$	LO	-251.7(2)	-247(1)(58)
	NLO	-37.3(2)	-49(2)(46)
	$\text{N}^2\text{LO } NN$	-72.8(2)	-87(3)(11)
	$\text{N}^2\text{LO } 3N E\tau$	-91.8(6)	-117(5)(16)
	$\text{N}^2\text{LO } 3N E1$	-85.8(5)	-115(6)(15)
	exp		-127.6

# Results for heavier systems

Results for AFDMC calculations of heavier systems ( $R_0 = 1.0$  fm):  
(Using the same local chiral interactions)

Lonardoni et al., arXiv:1709.09143 and 1802.08932



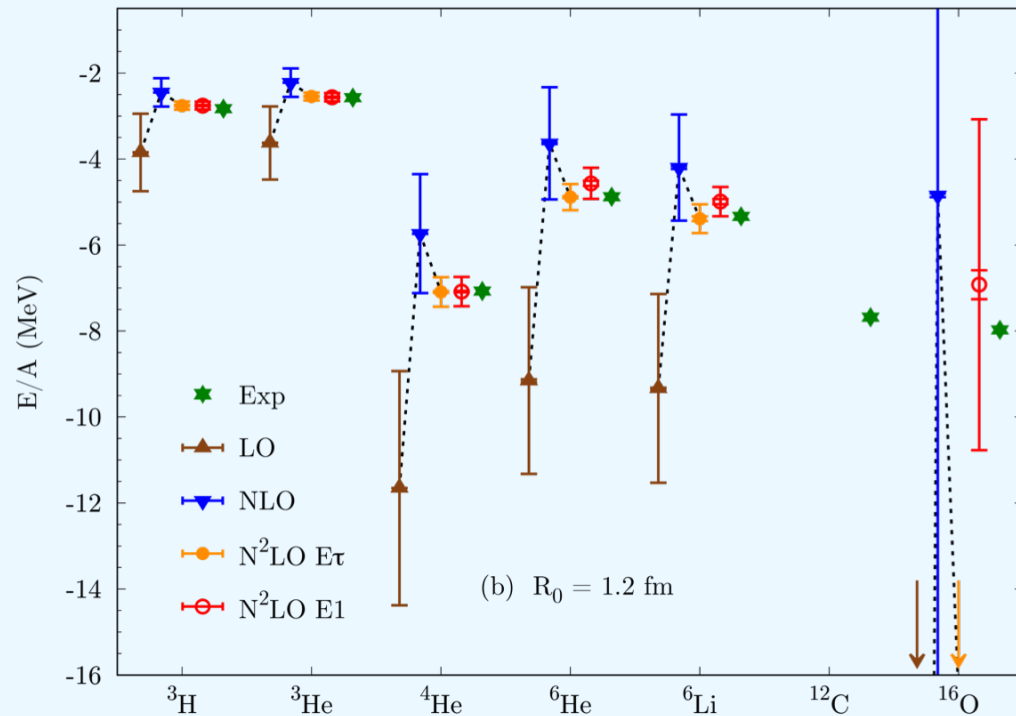
$^AZ (J^\pi, T)$	Potential	$E_C$ (MeV)	$E$ (MeV)
$^6\text{He} (0^+, 1)$	LO	-42.1(1)	-41.3(1)(9.6)
	NLO	-18.19(7)	-20.0(3)(5.0)
	$\text{N}^2\text{LO } NN$	-22.24(4)	-23.1(2)(1.2)
	$\text{N}^2\text{LO } 3N E\tau$	-26.58(6)	-28.4(4)(2.0)
	$\text{N}^2\text{LO } 3N E\perp$	-26.33(8)	-28.2(5)(1.9)
	exp		-29.3
$^6\text{Li} (1^+, 0)$	LO	-42.8(1)	-42.4(1)(9.9)
	NLO	-19.2(2)	-21.5(3)(4.9)
	$\text{N}^2\text{LO } NN$	-24.3(1)	-25.5(4)(1.1)
	$\text{N}^2\text{LO } 3N E\tau$	-28.9(1)	-31.5(5)(2.3)
	$\text{N}^2\text{LO } 3N E\perp$	-28.9(1)	-30.7(4)(2.1)
	exp		-32.0
$^{12}\text{C} (0^+, 0)$	LO	-131.5(2)	-131(1)(31)
	NLO	-31.1(2)	-41(2)(21)
	$\text{N}^2\text{LO } NN$	-63.5(2.4)	-66(3)(6)
	$\text{N}^2\text{LO } 3N E\tau$	-70.2(5)	-78(3)(9)
	$\text{N}^2\text{LO } 3N E\perp$	-	-
	exp		-92.2
$^{16}\text{O} (0^+, 0)$	LO	-251.7(2)	-247(1)(58)
	NLO	-37.3(2)	-49(2)(46)
	$\text{N}^2\text{LO } NN$	-72.8(2)	-87(3)(11)
	$\text{N}^2\text{LO } 3N E\tau$	-91.8(6)	-117(5)(16)
	$\text{N}^2\text{LO } 3N E\perp$	-85.8(5)	-115(6)(15)
	exp		-127.6



# Results for heavier systems

Results for AFDMC calculations of heavier systems ( $R_0 = 1.2$  fm):  
(Using the same local chiral interactions)

Lonardonì et al., arXiv:1709.09143 and 1802.08932

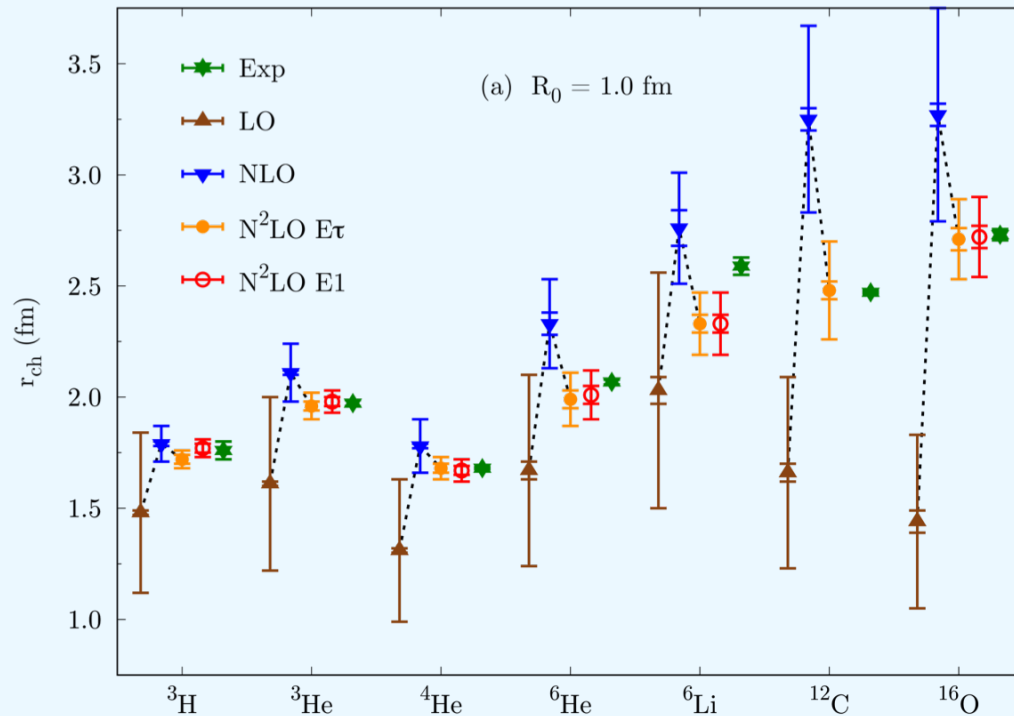


${}^AZ (J^\pi, T)$	Potential	$E_C$ (MeV)	$E$ (MeV)
${}^6\text{He} (0^+, 1)$	LO	-55.65(6)	-54.9(2)(12.8)
	NLO	-21.41(6)	-21.8(1)(7.7)
	$\text{N}^2\text{LO } NN$	-24.25(5)	-24.3(1)(1.8)
	$\text{N}^2\text{LO } E\tau$	-28.37(5)	-29.3(1)(1.8)
	$\text{N}^2\text{LO } E1$	-26.98(8)	-27.4(4)(1.8)
	exp		-29.3
${}^6\text{Li} (1^+, 0)$	LO	-56.84(3)	-56.0(1)(13.1)
	NLO	-23.64(8)	-25.2(2)(7.2)
	$\text{N}^2\text{LO } NN$	-26.76(3)	-27.0(2)(1.7)
	$\text{N}^2\text{LO } E\tau$	-30.8(1)	-32.3(3)(1.7)
	$\text{N}^2\text{LO } E1$	-29.2(1)	-29.9(4)(1.7)
	exp		-32.0
${}^{16}\text{O} (0^+, 0)$	LO	-1158.8(5)	-1110(31)(259)
	NLO	-72.3(1)	-77.5(7)(240.8)
	$\text{N}^2\text{LO } NN$	-98.6(1)	-106(4)(56)
	$\text{N}^2\text{LO } E\tau$	-169(2)	-263(26)(56)
	$\text{N}^2\text{LO } E1$	-99.5(4)	-111(5)(56)
	exp		-127.6

# Results for heavier systems

Results for AFDMC calculations of heavier systems ( $R_0 = 1.0$  fm):  
(Using the same local chiral interactions)

Lonardonì et al., arXiv:1709.09143 and 1802.08932



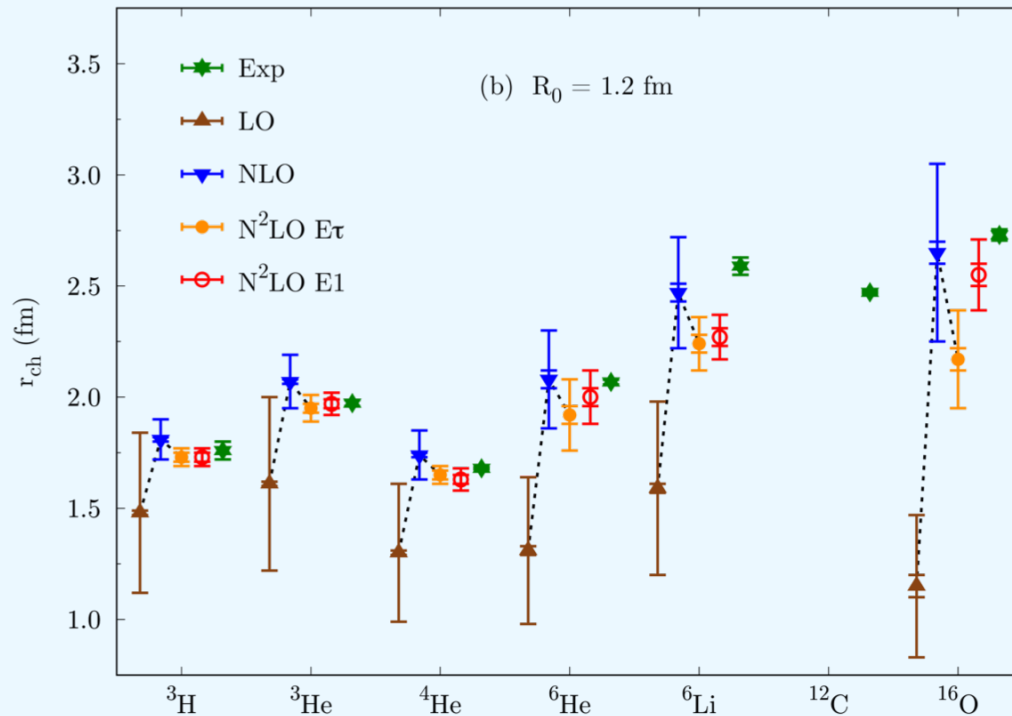
Excellent description of binding energies and  
charge radii for  $A \leq 16$ !

${}^A Z (J^\pi, T)$	Potential	$r_{\text{ch}}$ (fm)
${}^6\text{He} (0^+, 1)$	LO	1.67(4)(39)
	NLO	2.33(5)(15)
	$\text{N}^2\text{LO } NN$	2.11(4)(5)
	$\text{N}^2\text{LO } 3N E\tau$	1.99(4)(8)
	$\text{N}^2\text{LO } 3N E1$	2.01(4)(7)
	exp	2.068(11) [52]
${}^6\text{Li} (1^+, 0)$	LO	2.03(6)(47)
	NLO	2.76(8)(17)
	$\text{N}^2\text{LO } NN$	2.46(4)(7)
	$\text{N}^2\text{LO } 3N E\tau$	2.33(4)(10)
	$\text{N}^2\text{LO } 3N E1$	2.33(4)(10)
	exp	2.589(39) [53]
${}^{12}\text{C} (0^+, 0)$	LO	1.66(4)(39)
	NLO	3.25(5)(37)
	$\text{N}^2\text{LO } NN$	2.66(4)(14)
	$\text{N}^2\text{LO } 3N E\tau$	2.48(4)(18)
	$\text{N}^2\text{LO } 3N E1$	—
	exp	2.471(6) [54]
${}^{16}\text{O} (0^+, 0)$	LO	1.44(3)(34)
	NLO	3.27(5)(43)
	$\text{N}^2\text{LO } NN$	2.76(5)(12)
	$\text{N}^2\text{LO } 3N E\tau$	2.71(5)(13)
	$\text{N}^2\text{LO } 3N E1$	2.72(5)(11)
	exp	2.730(25) [55]

# Results for heavier systems

Results for AFDMC calculations of heavier systems ( $R_0 = 1.2$  fm):  
(Using the same local chiral interactions)

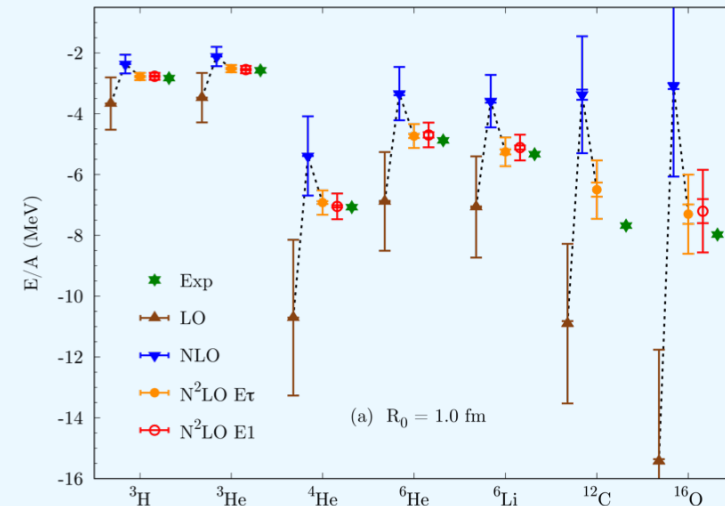
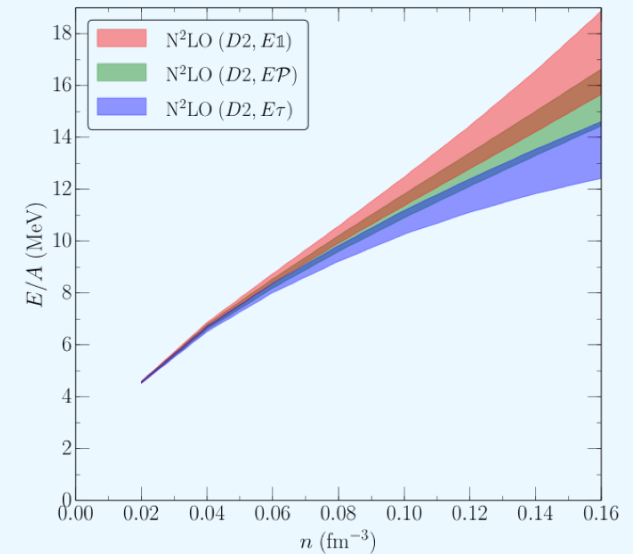
Lonardonì et al., arXiv:1709.09143 and 1802.08932



${}^AZ (J^\pi, T)$	Potential	$r_{\text{ch}}$ (fm)
${}^6\text{He} (0^+, 1)$	LO	1.31(2)(31)
	NLO	2.08(4)(18)
	N <sup>2</sup> LO $NN$	2.02(4)(4)
	N <sup>2</sup> LO $E\tau$	1.92(4)(4)
	N <sup>2</sup> LO $E1$	2.00(4)(4)
	exp	2.068(11) [52]
${}^6\text{Li} (1^+, 0)$	LO	1.59(2)(37)
	NLO	2.47(4)(21)
	N <sup>2</sup> LO $NN$	2.41(4)(5)
	N <sup>2</sup> LO $E\tau$	2.24(4)(6)
	N <sup>2</sup> LO $E1$	2.29(4)(5)
	exp	2.589(39) [53]
${}^{16}\text{O} (0^+, 0)$	LO	1.15(5)(27)
	NLO	2.65(5)(35)
	N <sup>2</sup> LO $NN$	2.47(5)(8)
	N <sup>2</sup> LO $E\tau$	2.17(5)(11)
	N <sup>2</sup> LO $E1$	2.55(5)(8)
	exp	2.730(25) [55]

# Summary

- QMC calculations of neutron matter, light nuclei, and n-alpha scattering with local chiral potentials up to  $N^2LO$  including NN and 3N forces.
- QMC methods offer access to harder chiral interactions.
- Chiral interactions at  $N^2LO$  simultaneously reproduce the properties of  $A \leq 16$  systems and of neutron matter, commonly used phenomenological 3N interactions fail. Extension to heavier, neutron-rich systems possible.
- Important regulator effects for local interactions.
- Further improvements necessary to reduce uncertainties.



# Thanks

- INT Seattle:  
S. Reddy
- Technische Universität Darmstadt:  
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- Universität Bochum:  
E. Epelbaum
- Los Alamos National Laboratory:  
J. Carlson, S. Gandolfi, D. Lonardoni
- University of Guelph:  
A. Gezerlis
- Forschungszentrum Jülich:  
A. Nogga



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Thank you for your attention.