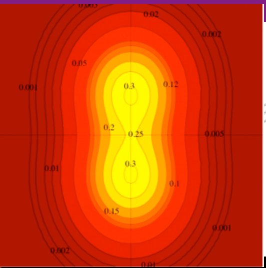
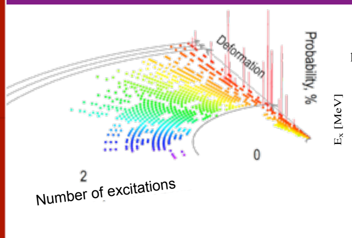
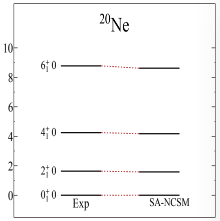
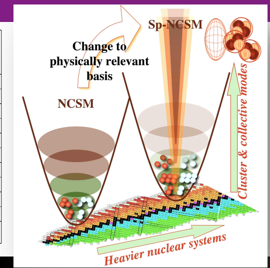
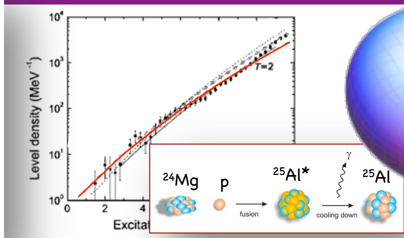
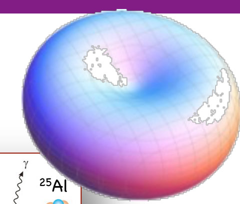


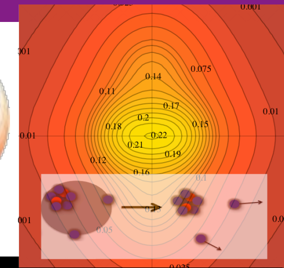












Collectivity and clustering from the ab initio symmetry-adapted no-core shell model

Kristina Launey

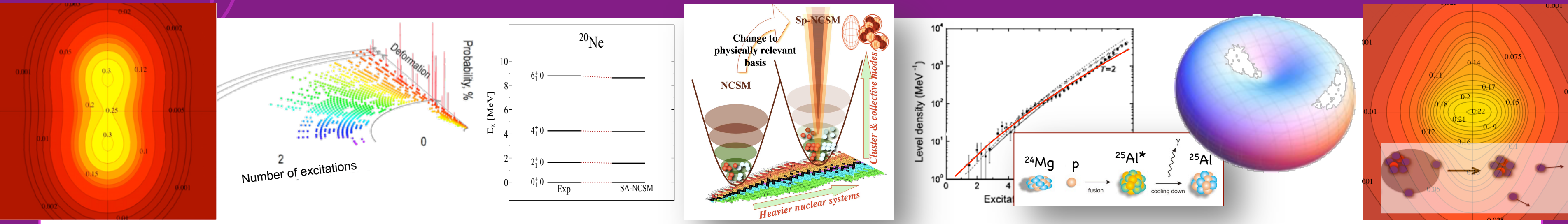
... LSU Team ... Jerry Draayer, Tomas Dytrych,
Alexis Mercenne
Robert Baker, Ali Dreyfuss,
David Kekejian, Grigor Sargsyan,
Madeleine Miora

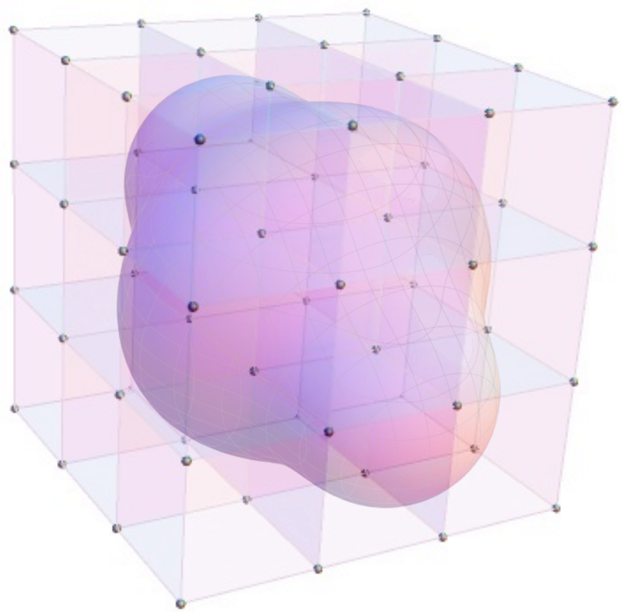
In collaboration with
Princeton U. – W. Tang & B. Wang
Czech Republic – D. Langr & T. Oberhuber

HPC Resources
NSF/U. of Illinois ...*BlueWaters*
LSU...*SuperMike-II*

Supported by NSF & DOE

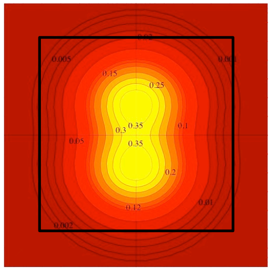
LSU





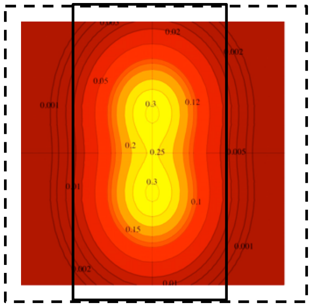
NCSM

Total HO quanta N_{max}



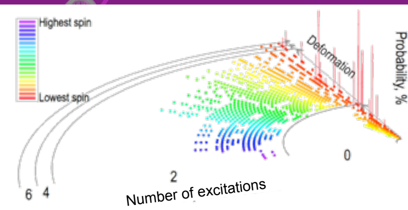
SA-NCSM

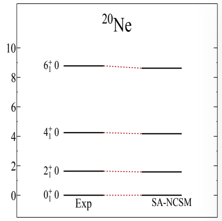
Total HO quanta N_{max} + Distribution: z, x, y

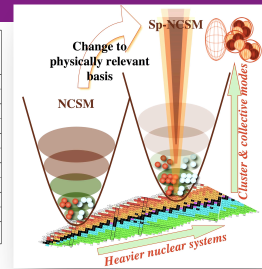


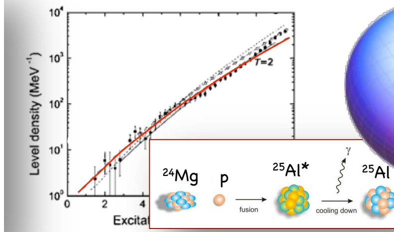
Dytrych et al., Phys. Rev. Lett. 111 (2013) 252501

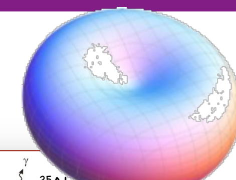
Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101

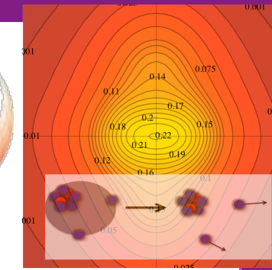


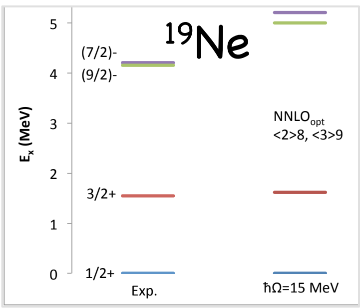


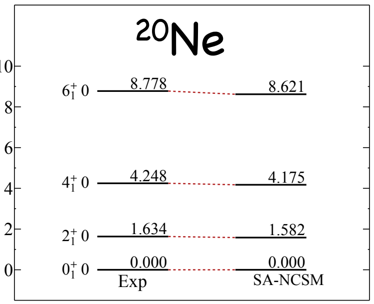








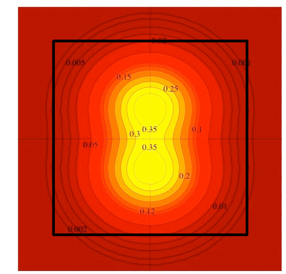


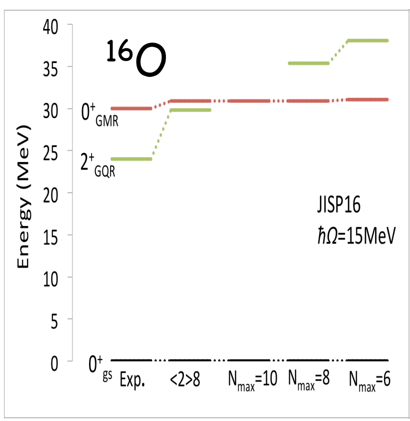


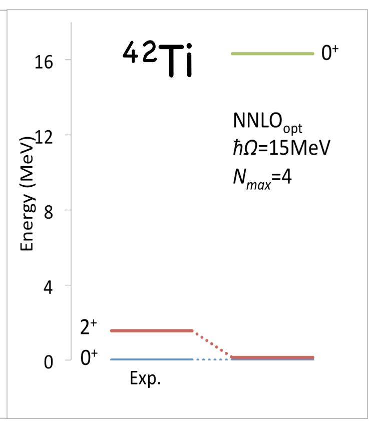
SU(3) basis

NCSM

Total HO quanta
N_{max}



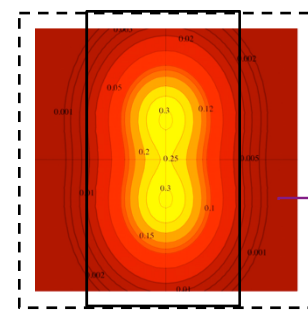




Symplectic
Sp(3,R) basis

SA-NCSM

Total HO quanta
N_{max}
+
Distribution:
z, x, y



Symmetry-adapted:
SU(3), Sp(3,R)

Deformation, rotations… …& vibrations

Guided by Symplectic symmetry

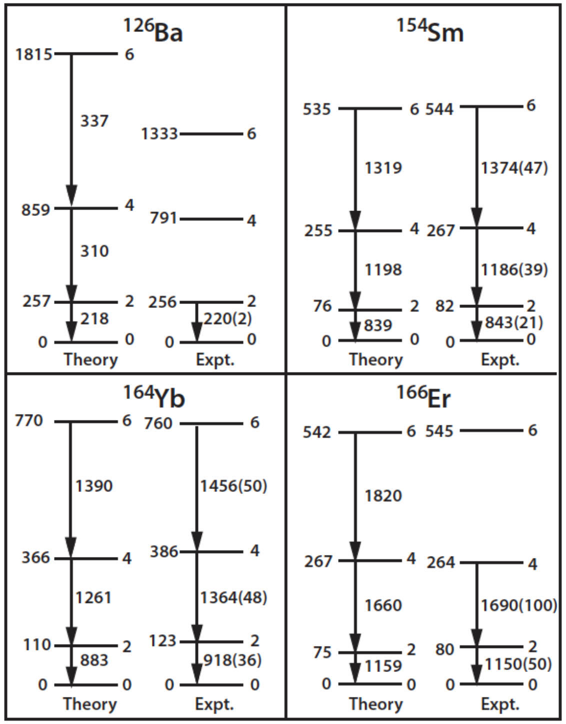
LSU code (LSU3shell): sourceforge.net/projects/lsu3shell

Dytrych et al., Phys. Rev. Lett. 111 (2013) 252501

Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101

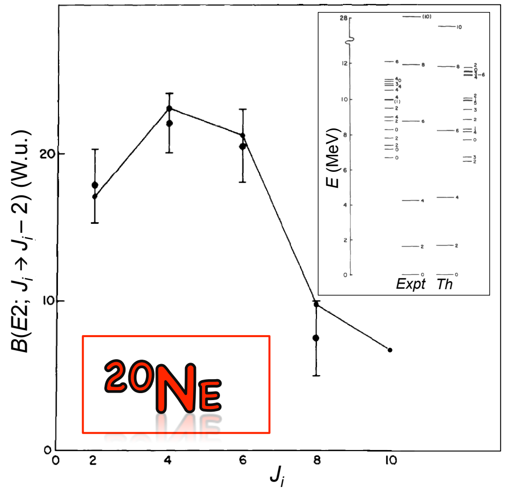
Earlier studies … algebraic models

Quite successful, but symmetries are assumed *a priori:*
Typically 1 (a few) irrep(s) + symmetry-preserving interaction



P. Park et al., Nucl. Phys. A. 414, 93 (1984)

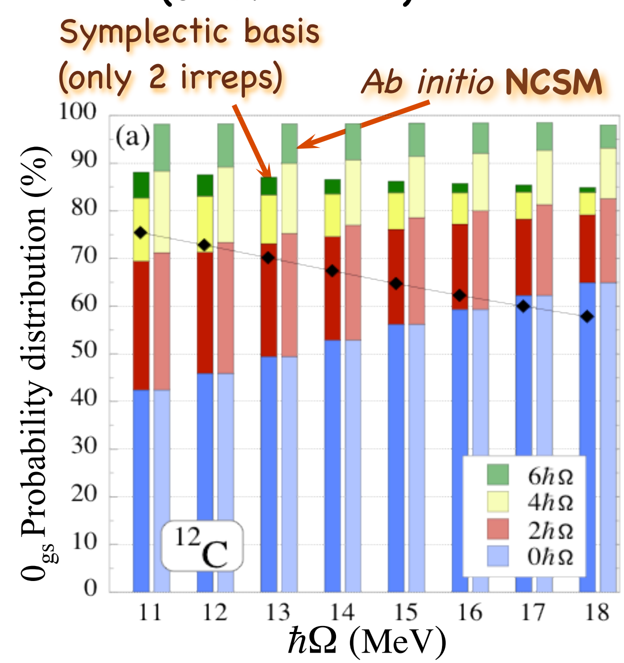
D. J. Rowe, Rep. Prog. Phys. 48, 1419 (1985)



J. Draayer, et al., Nucl. Phys. A419, 1 (1984)

No effective charges!

Ab initio results:
No *a priori* symmetry assumptions (JISP16 *NN*)



Dytrych, Launey, Bahri, Draayer, Vary, Phys. Rev. Lett. 98 (2007) 162503

SYMPLECTIC SYMMETRY, SP(3,R)

Rosensteel & Rowe, PRL 38 (1977) 10

Rowe, Rosensteel, Draayer, Hecht, Suzuki, Escher, Bahri, ….

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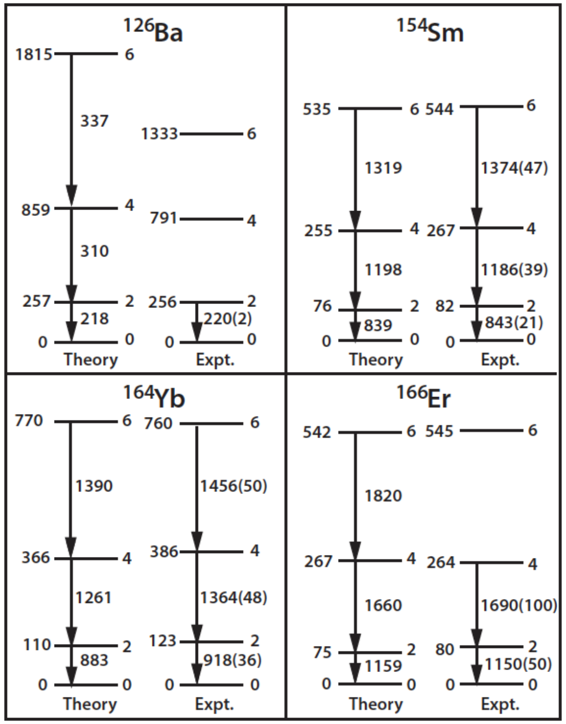
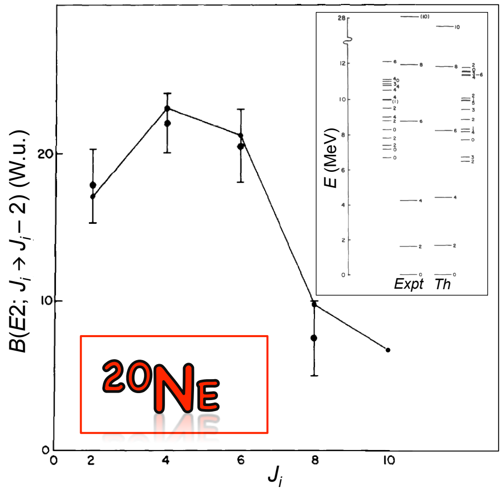


P. Park et al., Nucl. Phys. A. 414, 93 (1984)
D. J. Rowe, Rep. Prog. Phys. 48, 1419 (1985)



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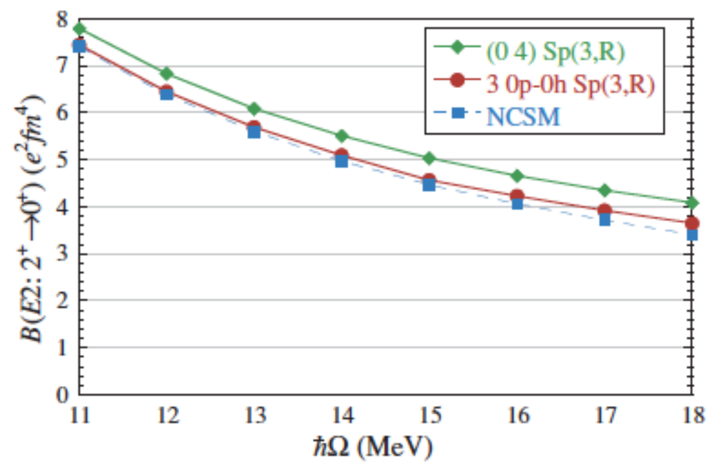


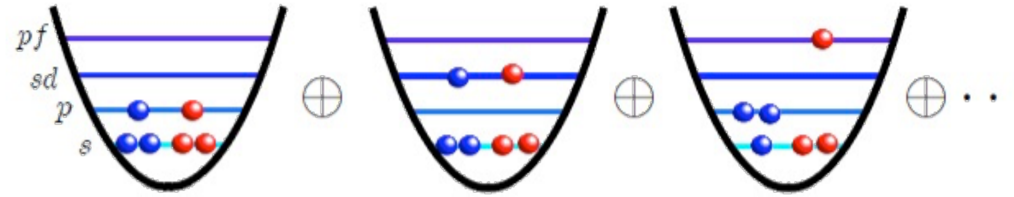
Dytrych, Launey, Bahri, Draayer, Vary, Phys. Rev. Lett. 98 (2007) 162503

SYMPLECTIC SYMMETRY, Sp(3,R)

Rosensteel & Rowe, PRL 38 (1977) 10
Rowe, Rosensteel, Draayer, Hecht, Suzuki, Escher, Bahri, ….

Ab initio symmetry-adapted theory

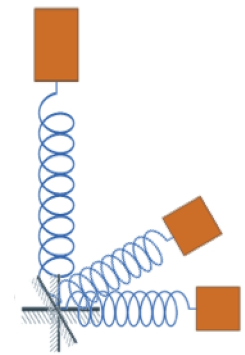
Distributions of nucleon over HO shells ($0\hbar\Omega$, $2\hbar\Omega$, …; 0p-0h, 2p-2h, …)

$N_{\max}$ = 

SU(3) basis states/Sp(3,R) basis states: reorganization of model space

(unitary transformation from *m*-scheme), e.g. for A=2:

$\lambda = n_z - n_x; \quad \mu = n_x - n_y$



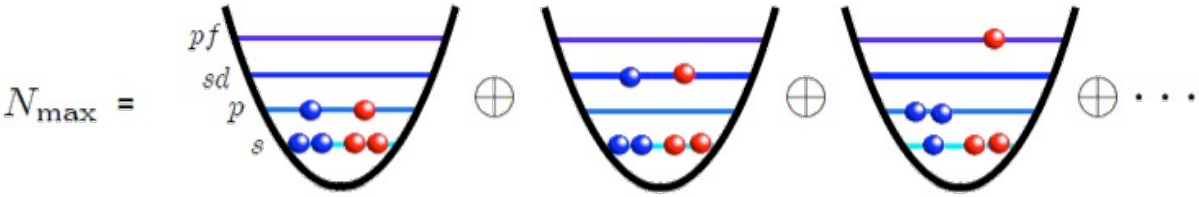
Model space scheme	Two-particle basis state
L-S scheme	$\{a^\dagger_{\eta \underline{l}\, \underline{s} t_z} \times a^\dagger_{\eta' \underline{l'}\, \underline{s'} t'_z}\}^{(LS)JM}\vert 0\rangle$
M scheme	$a^\dagger_{\eta l j m t_z} a^\dagger_{\eta' l' j' m' t'_z}\vert 0\rangle$, with $m + m' = M$
J scheme	$\{a^\dagger_{\eta l \underline{j} t_z} \times a^\dagger_{\eta' l' \underline{j'} t'_z}\}^{JM}\vert 0\rangle$
SU(3) scheme	$\{a^\dagger_{\underline{(\eta\,0)}\, \underline{s} t_z} \times a^\dagger_{\underline{(\eta' 0)}\, \underline{s'} t'_z}\}^{(\lambda\,\mu)\kappa(LS)JM}\vert 0\rangle$
Sp(3, $\mathbb{R}$) scheme*	$\{\{\hat{A}^{(2\,0)} \times \hat{A}^{(2\,0)} \dots \times \hat{A}^{(2\,0)}\}^{\underline{(\lambda_n\,\mu_n)}} \times \underbrace{\left[a^\dagger_{(\eta_1\,0)} \times a^\dagger_{(\eta_2\,0)}\right]^{\underline{(\lambda_\sigma\,\mu_\sigma)}}}_{A=2 \text{ bandhead}}\}^{\rho(\lambda_\omega\,\mu_\omega)}_{\kappa L M_L}\vert 0\rangle$

*spatial dof

J. P. Draayer, T. Dytrych and K. D. Launey, in "Emergent Phenomena in Atomic Nuclei …", World Scientific Co. (2017)

Ab initio symmetry-adapted theory

Distributions of nucleon over HO shells ($0\hbar\Omega$, $2\hbar\Omega$, …; 0p-0h, 2p-2h, …)



SU(3) package for SU(3) coupling/recoupling coefficients

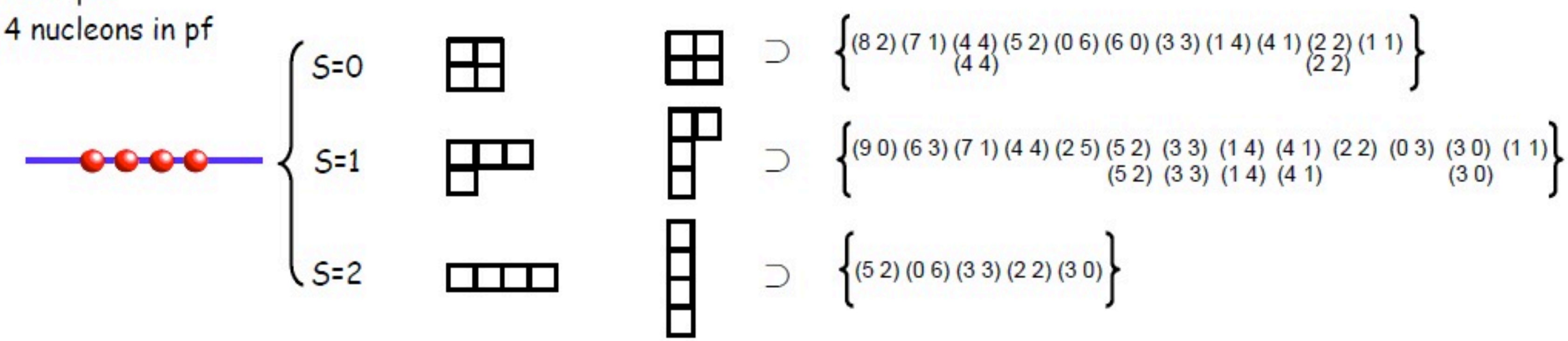
… analogous to SU(2), but outer/inner multiplicities!

Draayer & Akiyama, JMP 14 (1973) 1904
Akiyama & Draayer, Comp. Phys. Commun. 5(1973)405

SU(3) basis construction … based on Gel'fand patterns (fast and efficient!)

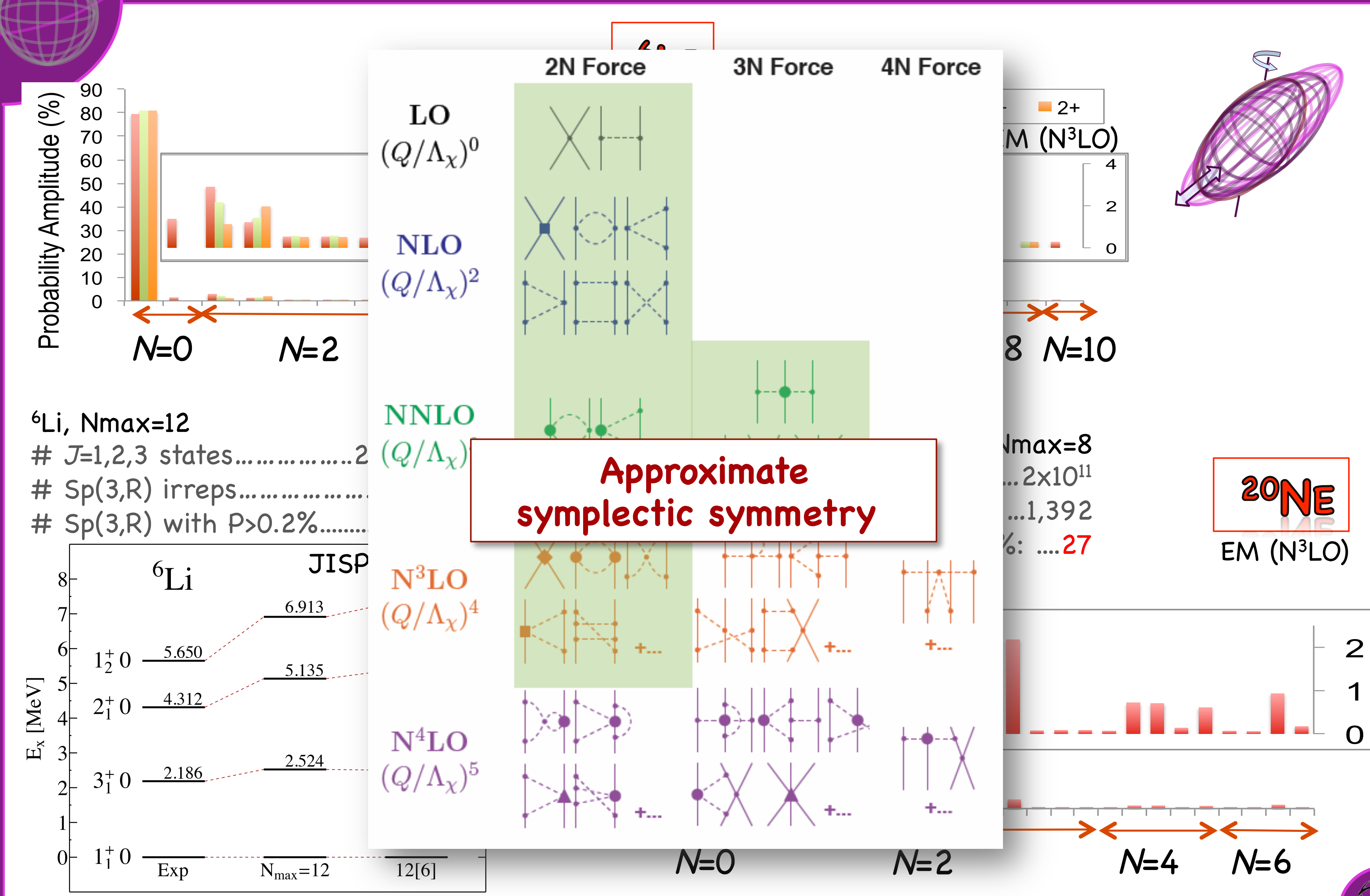
	U(2)	$\otimes$	U(10)	$\supset$	SU(3)
quantum labels:	S		[f]	α	$(\lambda\ \mu)$

- Example:
  4 nucleons in pf



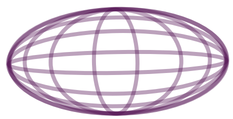
Draayer et al., "Representations of U(3) in U(N)", Comp. Phys. Commun. 56 (1989) 279

What physics can we learn?

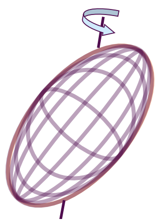


What physics can we learn?

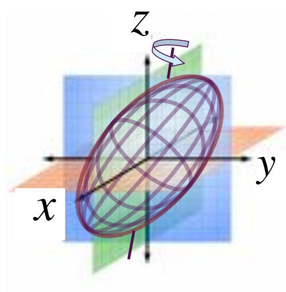
Sp(3,R) (collective) basis configuration:



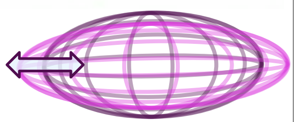
one equilibrium deformation (``shape")



rotations



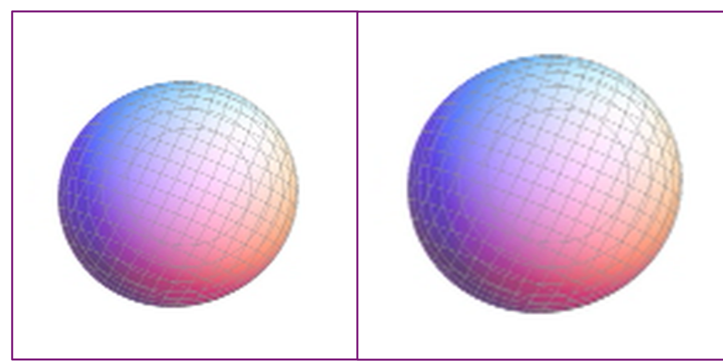
space orientation



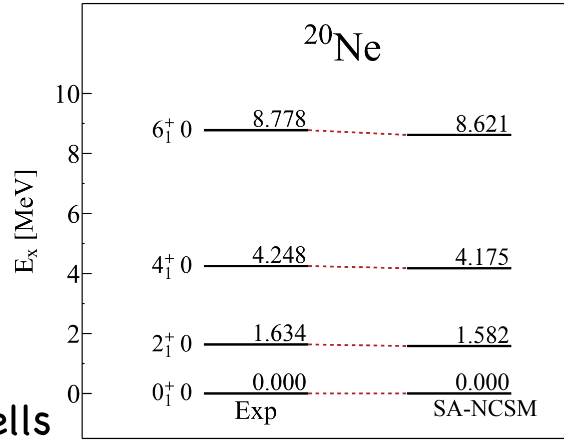
Vibrations

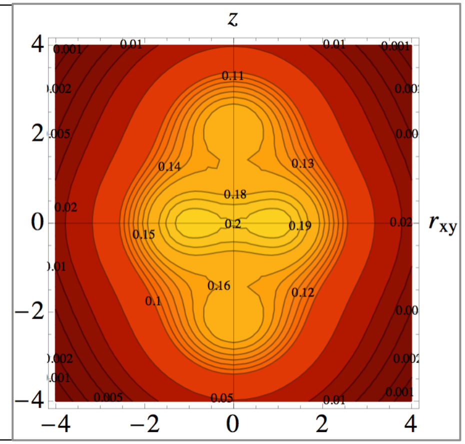
(of the giant resonance monopole (r^2)/ quadrupole (Q) type)

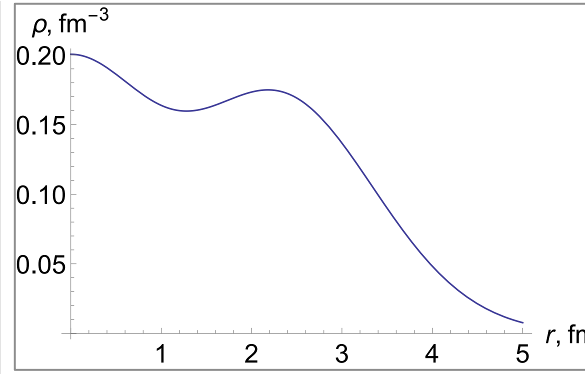
Symplectic symmetry: All states preserve the equilibrium shape



Collectivity features





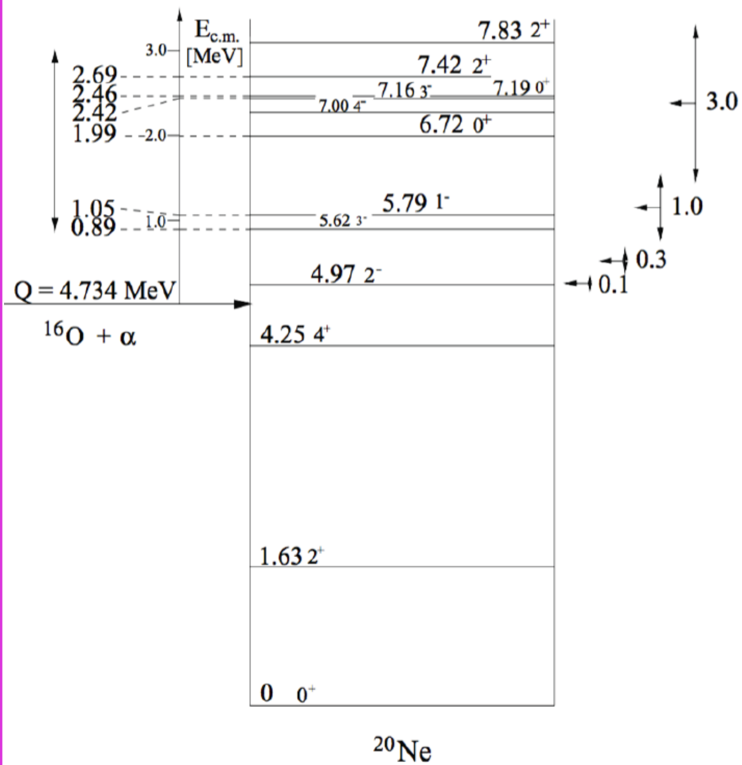


13 shells

SA-NCSM (selected model space): 50 million SU(3) states

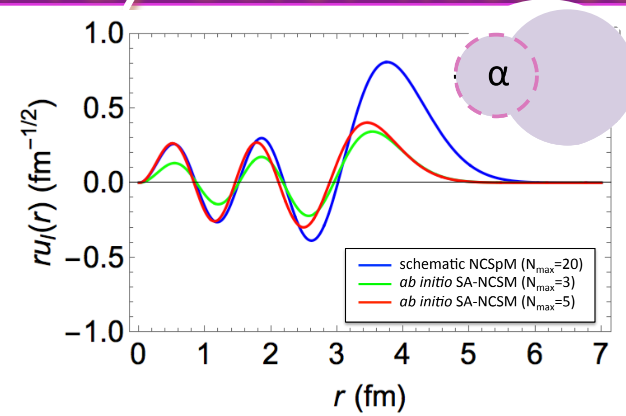
Complete model space: 1000 billion states

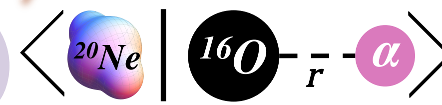
Alpha clustering and effect on X-ray burst nucleosynthesis



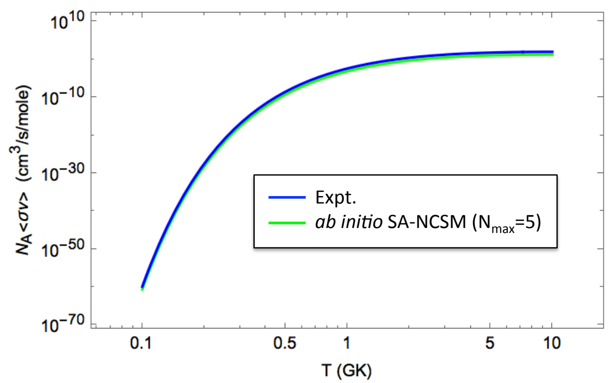
$^{16}O(\alpha,\gamma)^{20}Ne$

Ali Dreyfuss, PhD student, LSU

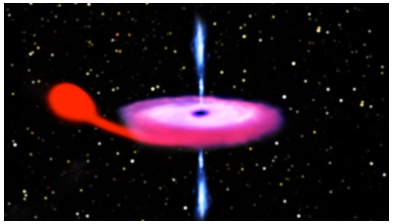




Wave functions from *ab initio* SA-NCSM



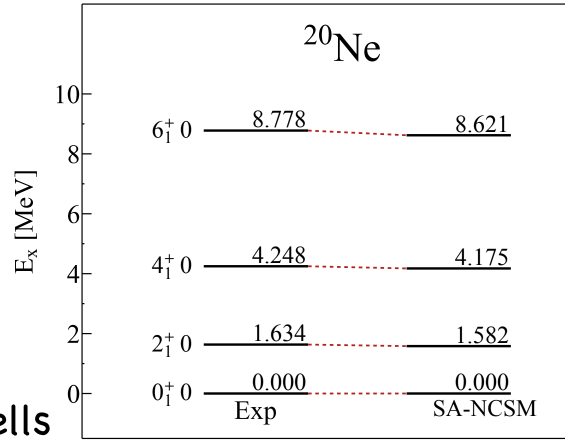
Reaction rates

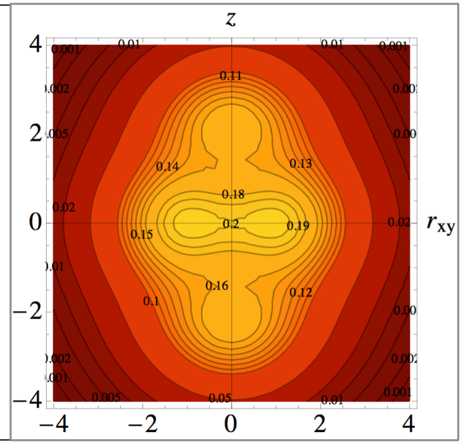


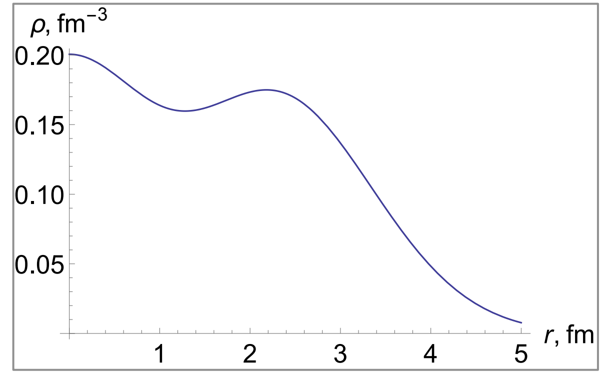
Nucleosynthesis simulations: XRB abundance pattern

Collectivity features

20NE







13 shells

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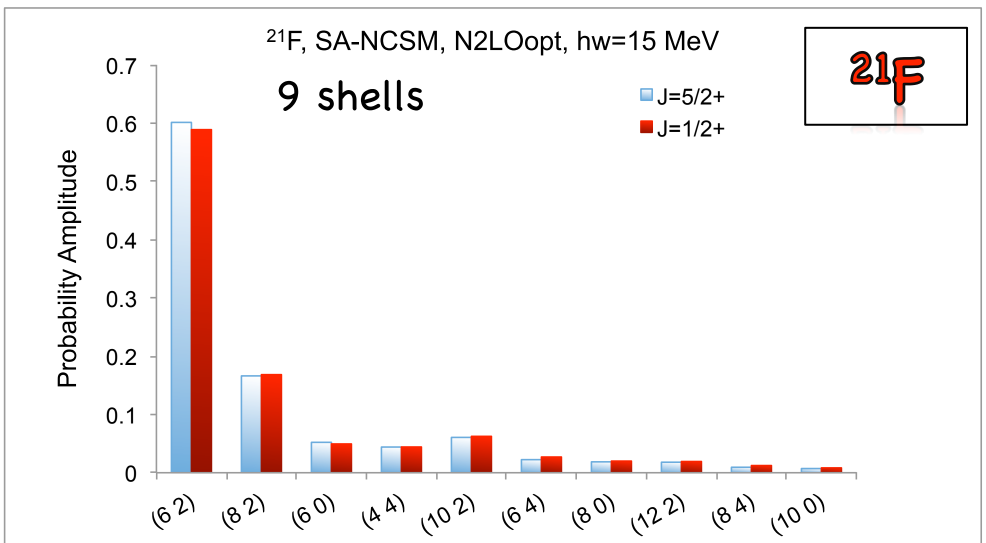
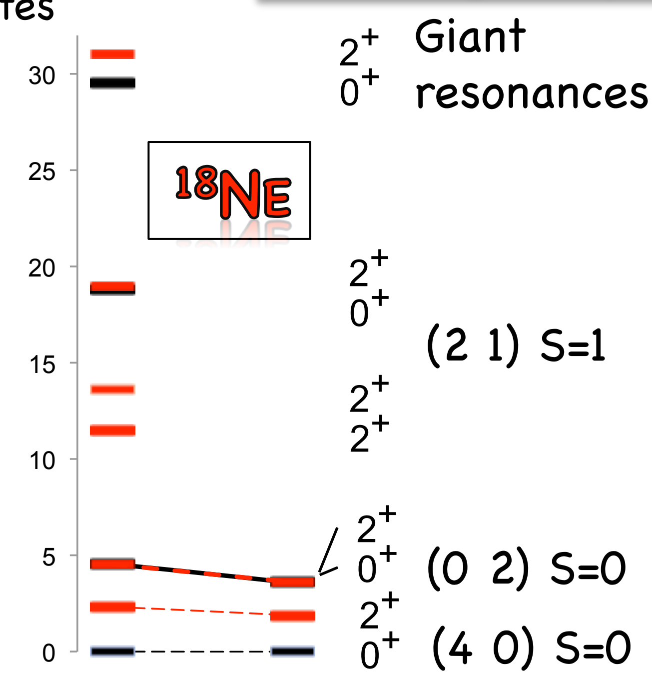
Complete model space: 1000 billion states

Ne & Mg isotopes



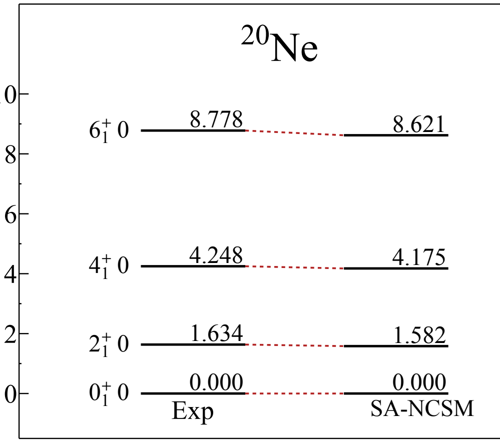
Grigor Sargsyan, PhD student, LSU

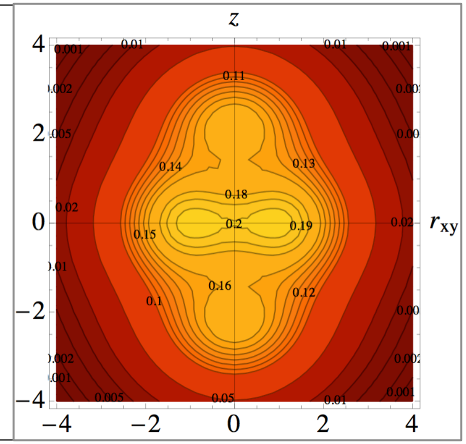


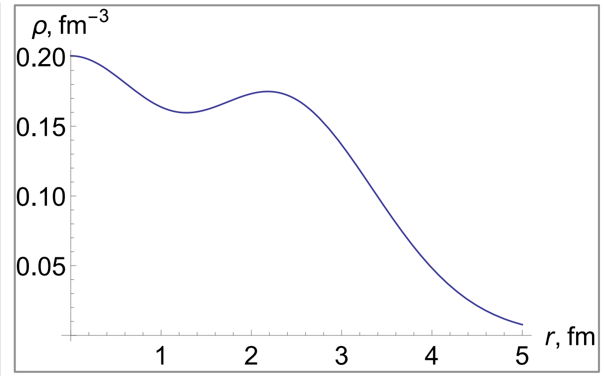
N2LOopt; 9 shells

Collectivity features

20NE







13 shells

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Complete model space: 1000 billion states

Ne & Mg isotopes

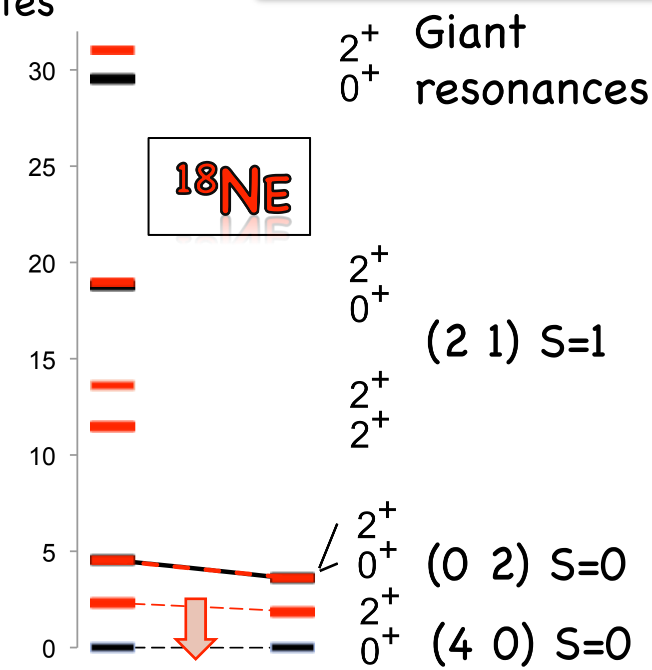
^{18}Ne, B(E2: $2^+ \to 0^+$)

Experiment……… 17.7(18) W.u.

9 shells …………… 1.13 W.u.

33 shells ………….. 13.0(7) W.u.

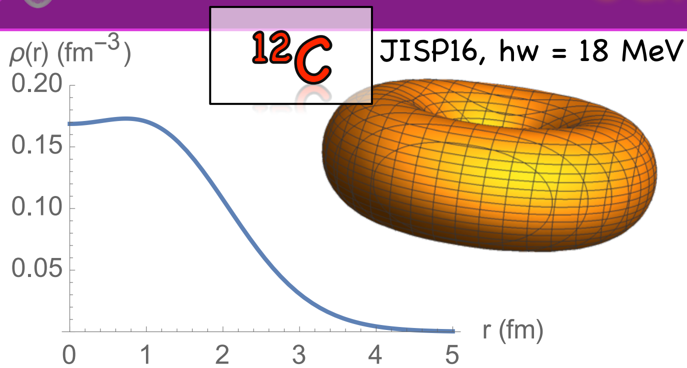
(no effective charges)

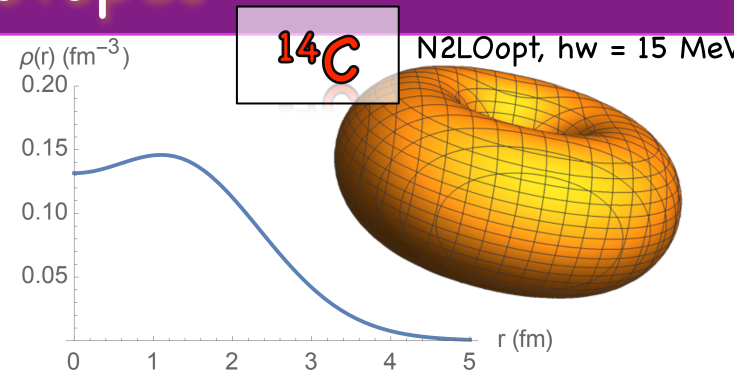


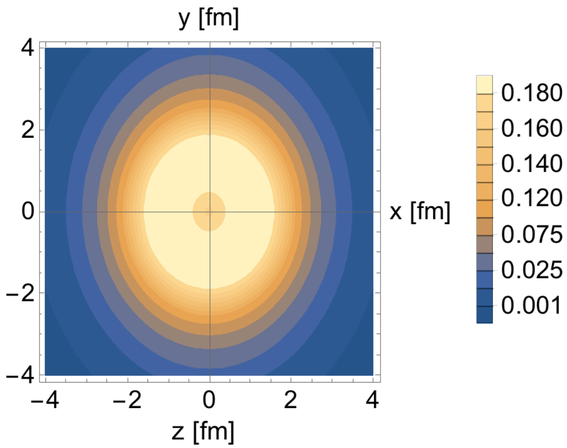
N2LOopt; 9 shells

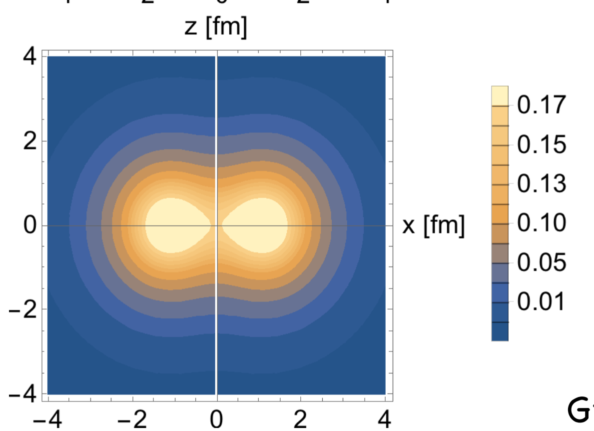
Grigor Sargsyan, PhD student, LSU

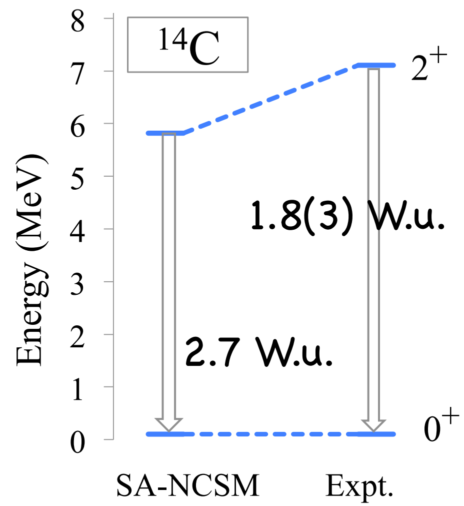
Carbon isotopes

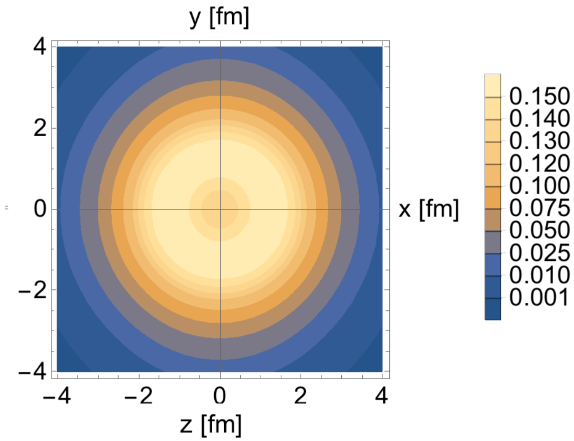


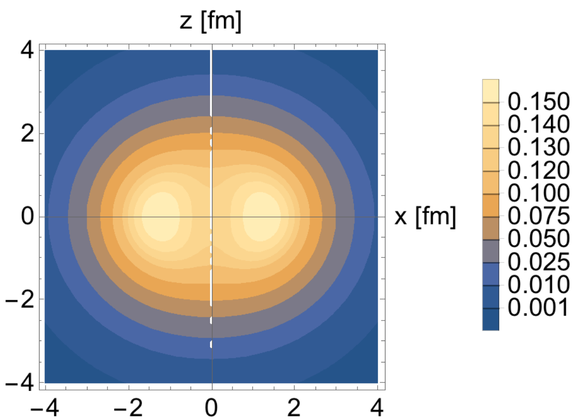










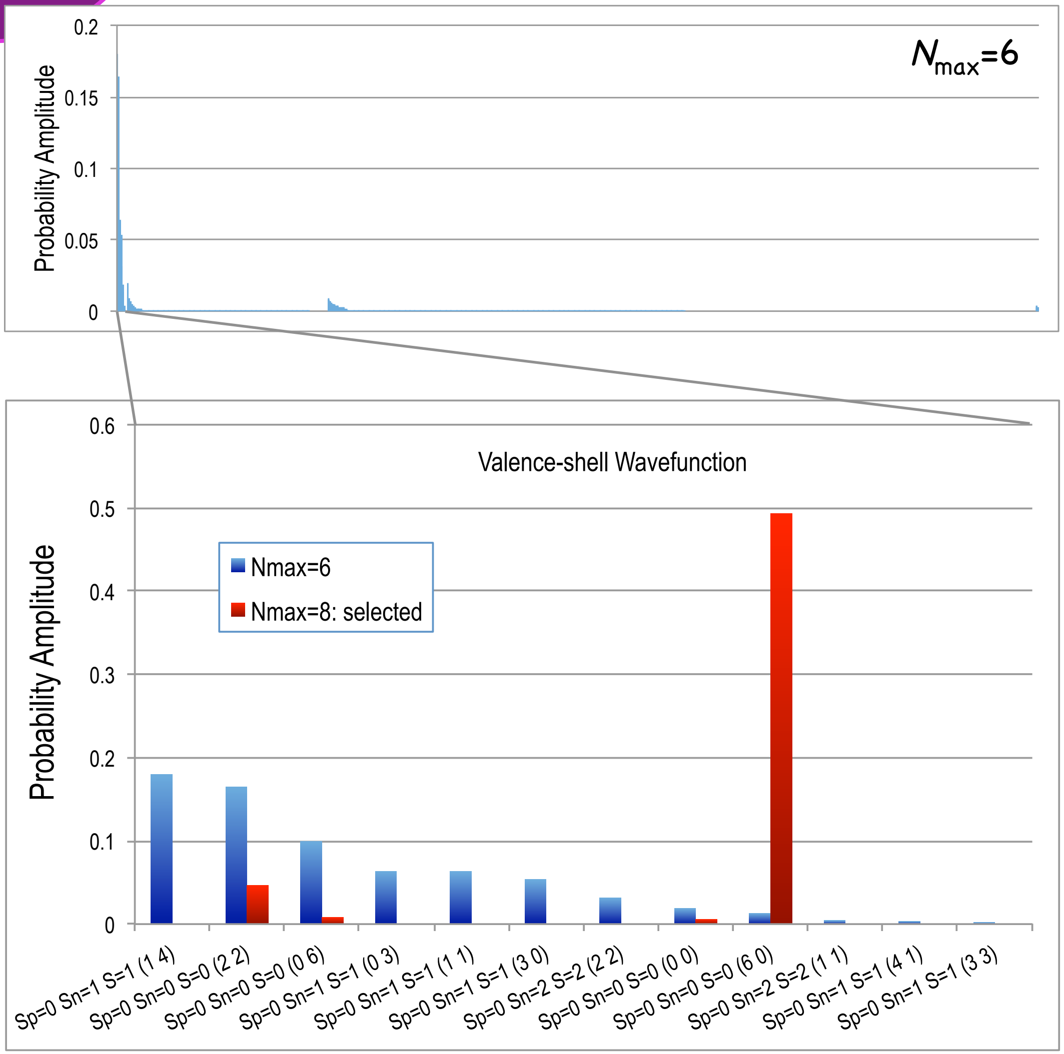


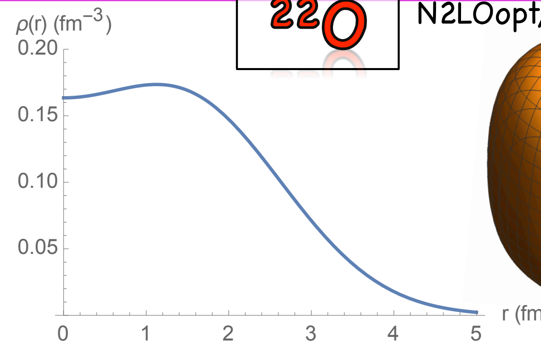
Grigor Sargsyan, PhD student, LSU

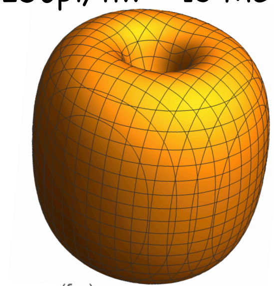
Oxygen isotopes

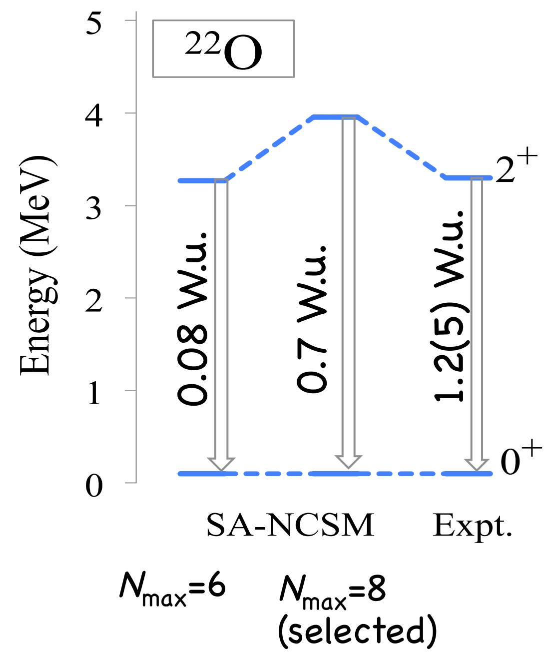
^{22}O

N2LOopt, hw = 15 MeV





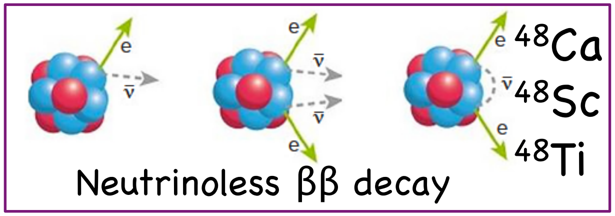




Grigor Sargsyan, PhD student, LSU

Structure of Ca-48 and Ti-48

^{48}CA



^{48}TI

^{48}Ca:

8 shells, N2LOopt

0^+

SA-NCSM (selected): ……………966,152

Complete model space: ……3,162,511,819

2^+

SA-NCSM (selected): ……………3,055,554

Complete model space: …14,522,234,982

^{48}Ti:

8 shells, N2LOopt

0^+

SA-NCSM (selected): ………………602,493

Complete model space: …..24,694,678,414

2^+

SA-NCSM (selected): ……………1,178,834

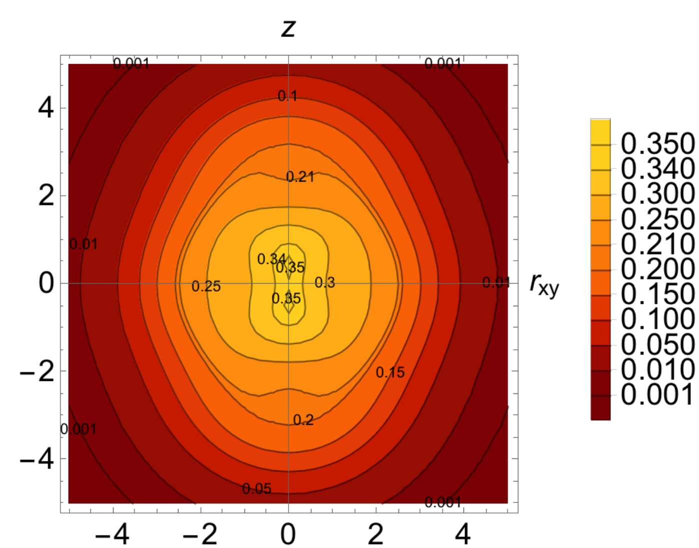
Complete model space: …113,920,316,658

^{48}Ti, $Q(2^+)$ [e fm^2]

Experiment……… -17.7

8 shells …………… -19.3

(no effective charges)



Grigor Sargsyan, PhD student, LSU

Spectral Distribution Theory: Particle-rank tensors

French (‘66), Draayer, Hecht, Kota, ...

3-body interaction (scalar partitioning)

$$H(3) = \binom{A}{3}\mathcal{H}^{(3)}(0) + \binom{A-1}{2}\mathcal{H}^{(3)}(1) + (A-2)\mathcal{H}^{(3)}(2) + \mathcal{H}^{(3)}(3)$$

Depend on *A*,
model-space dimension

Centroids

Particle-rank tensors
(effective
0-,1-,2-,3-body)

$$\mathcal{H}^{(3)}(0) = \frac{1}{3!}\frac{1}{\binom{\mathcal{N}}{3}}D^3H(3) \equiv W_c^{(3)}, \quad \mathcal{H}^{(3)}(1) = \frac{1}{2!}\frac{1}{\binom{\mathcal{N}-2}{2}}\left(D^2H(3) - \frac{A}{\mathcal{N}}D^3H(3)\right),$$

$$\mathcal{H}^{(3)}(2) = \frac{1}{\mathcal{N}-4}\left(DH(3) - \frac{A-1}{\mathcal{N}-2}D^2H(3) + \frac{1}{2}\frac{\binom{A}{2}}{\binom{\mathcal{N}-1}{2}}D^3H(3)\right),$$

$$\mathcal{H}^{(3)}(3) = H(3) - \frac{A-2}{\mathcal{N}-4}DH(3) + \frac{1}{2}\frac{\binom{A-1}{2}}{\binom{\mathcal{N}-3}{2}}D^2H(3) - \frac{1}{3!}\frac{\binom{A}{3}}{\binom{\mathcal{N}-2}{3}}D^3H(3).$$

Depends on *interaction:*

$$D^3H(3) = \left(\sum_{ijq} W_{ijqijq}\right), \quad D^2H(3) = \sum_{ir}\left(\sum_{jq} W_{ijqrjq}\right)a_i^\dagger a_r, \quad DH(3) = \frac{1}{4}\sum_{ijrs}\left(\sum_{q} W_{ijqrsq}\right)a_i^\dagger a_j^\dagger a_s a_r.$$

For *NNN*: Launey et al., Phys. Rev. C 85 (2012) 044003

For *NN*: Launey et al., Comput. Phys. Commun. 185 (2014) 254

Spectral Distribution Theory: Particle-rank tensors

3-body interaction (scalar partitioning)

$$H(3) = \binom{A}{3}\mathcal{H}^{(3)}(0) + \binom{A-1}{2}\mathcal{H}^{(3)}(1) + (A-2)\mathcal{H}^{(3)}(2) + \mathcal{H}^{(3)}(3)$$

For given *T*
(isoscalar partitioning)

$$H_{mon} \equiv \frac{W_{2,0} + 3W_{2,1}}{4}\binom{\hat{n}}{2} + \frac{W_{2,1} - W_{2,0}}{2}(\mathbf{T}^2 - \frac{3}{4}\hat{n}) \Leftarrow NN$$

$$+ \frac{W_{3,\frac{1}{2}} + W_{3,\frac{3}{2}}}{2}\binom{\hat{n}}{3} + \frac{W_{3,\frac{3}{2}} - W_{3,\frac{1}{2}}}{3}(\hat{n} - 2)(\mathbf{T}^2 - \frac{3}{4}\hat{n}). \Leftarrow NNN$$

For *NNN*: Launey et al., Phys. Rev. C 85 (2012) 044003

For *NN*: Launey et al., Comput. Phys. Commun. 185 (2014) 254

Spectral Distribution Theory: Particle-rank tensors

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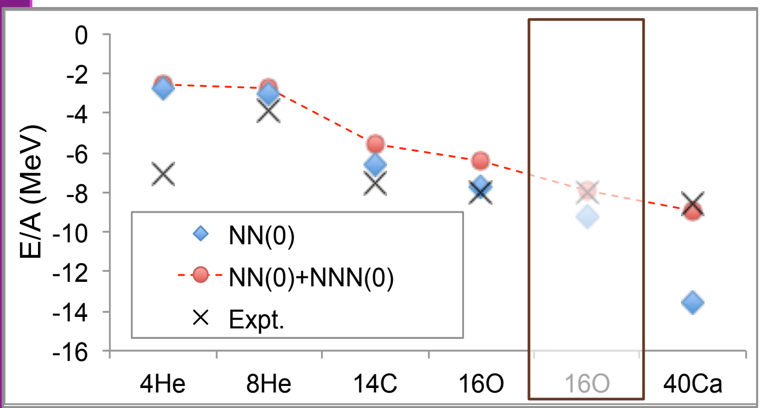
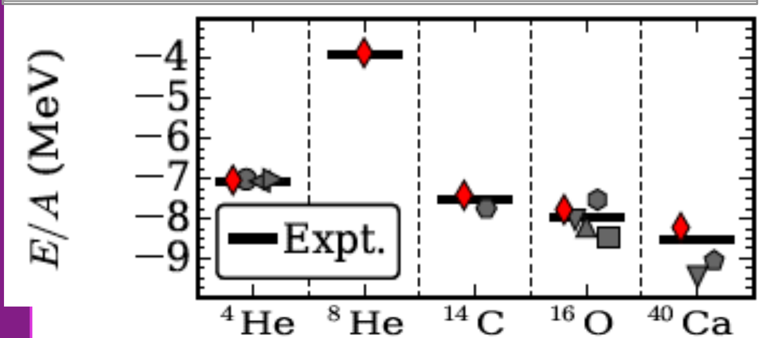
$$+ \frac{W_{3,\frac{1}{2}}+W_{3,\frac{3}{2}}}{2}\binom{\hat{n}}{3} + \frac{W_{3,\frac{3}{2}}-W_{3,\frac{1}{2}}}{3}(\hat{n}-2)\left(\mathbf{T}^2 - \frac{3}{4}\hat{n}\right). \quad NNN$$

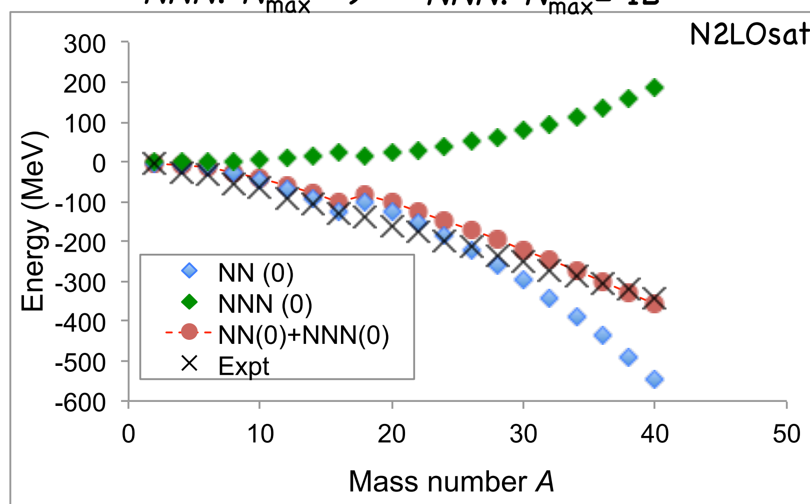




Ekström et al., Phys. Rev. C 91, 051301(R) (2015)

p shell
NN: N_{max}= 6
NNN: N_{max}= 9

sd shell
NN: N_{max}= 8
NNN: N_{max}= 12



Spectral Distribution Theory: Particle-rank tensors

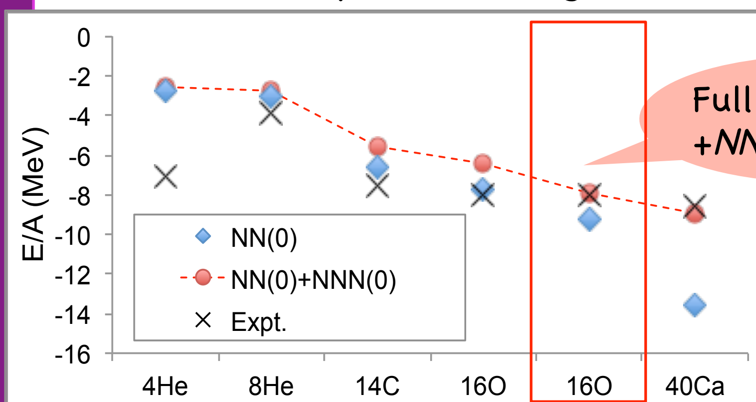
3-body interaction (scalar partitioning)

$$H(3) = \binom{A}{3}\mathcal{H}^{(3)}(0) + \binom{A-1}{2}\mathcal{H}^{(3)}(1) + (A-2)\mathcal{H}^{(3)}(2) + \mathcal{H}^{(3)}(3)$$

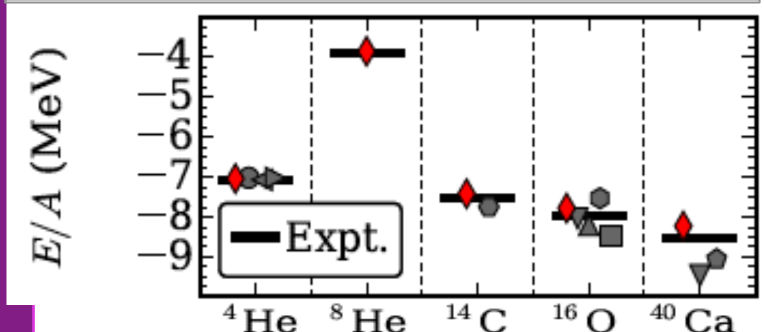
For given *T*
(isoscalar partitioning)

$$H_{mon} \equiv \frac{W_{2,0}+3W_{2,1}}{4}\binom{\hat{n}}{2} + \frac{W_{2,1}-W_{2,0}}{2}(\mathbf{T}^2 - \frac{3}{4}\hat{n}) \quad NN$$

$$+\frac{W_{3,\frac{1}{2}}+W_{3,\frac{3}{2}}}{2}\binom{\hat{n}}{3} + \frac{W_{3,\frac{3}{2}}-W_{3,\frac{1}{2}}}{3}(\hat{n}-2)(\mathbf{T}^2 - \frac{3}{4}\hat{n}). \quad NNN$$



Full *NN*, N_{max}=8 +*NNN*(0)



Ekström et al., Phys. Rev. C 91, 051301(R) (2015)

p shell
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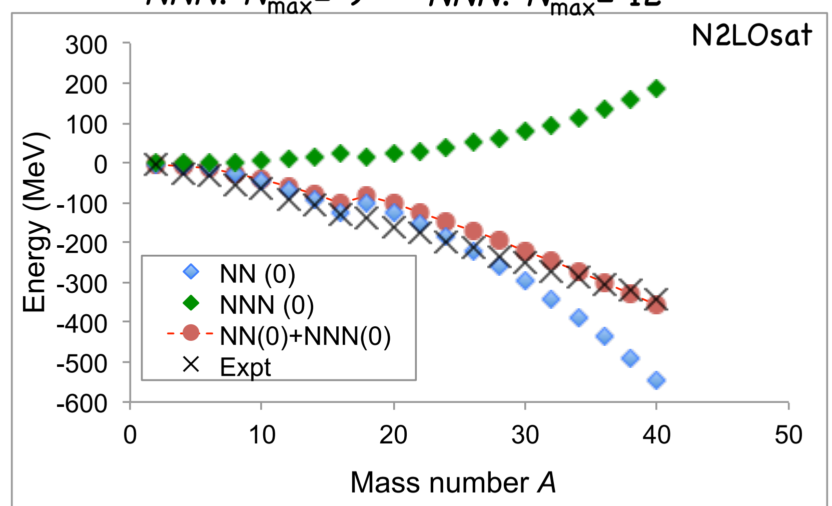
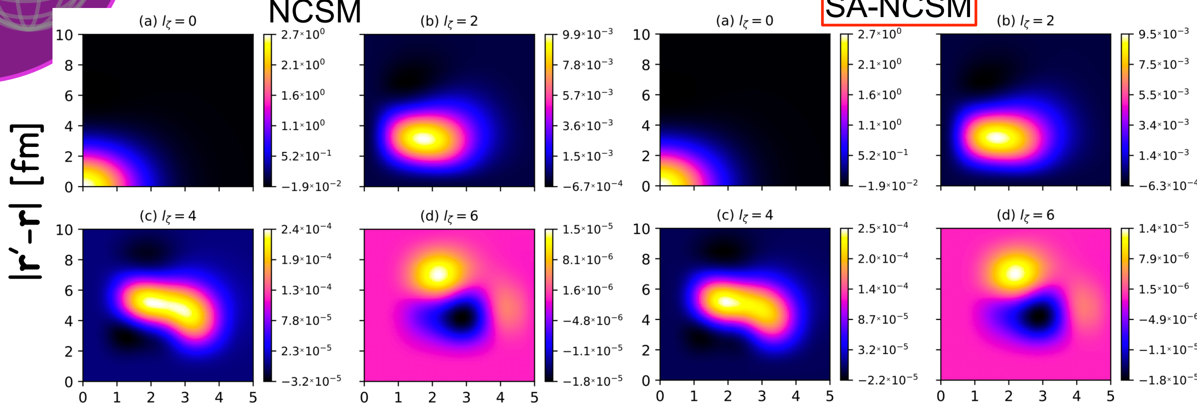


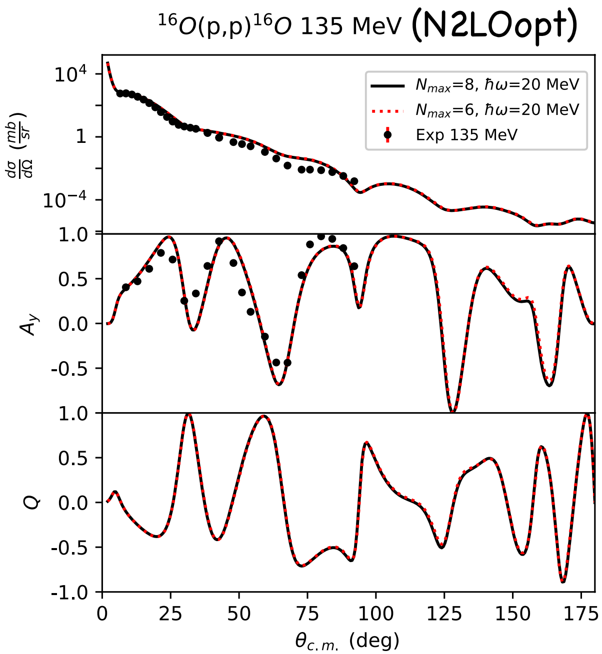
Features of *NN* interactions

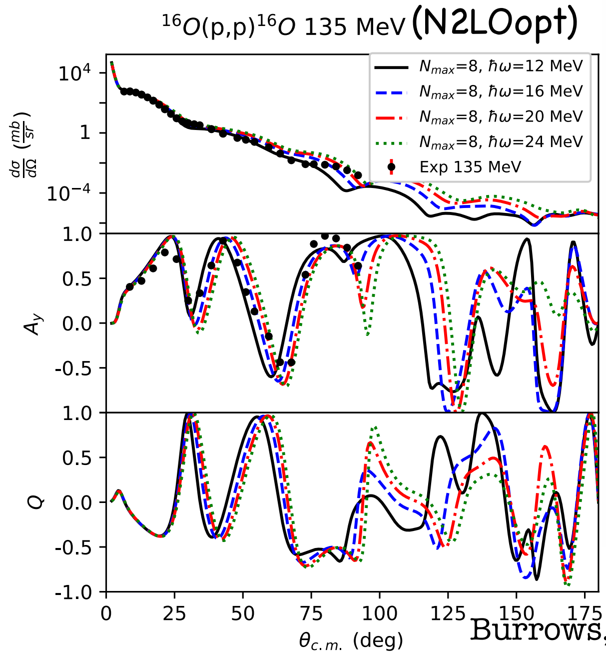
^{6}Li, Non-local densities

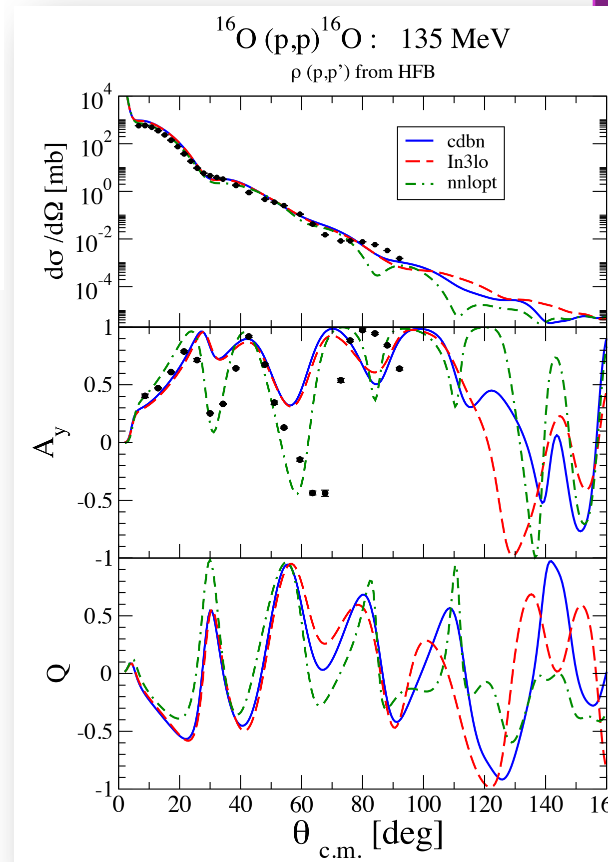


|ravg| [fm]

Proton scattering off ^{16}O

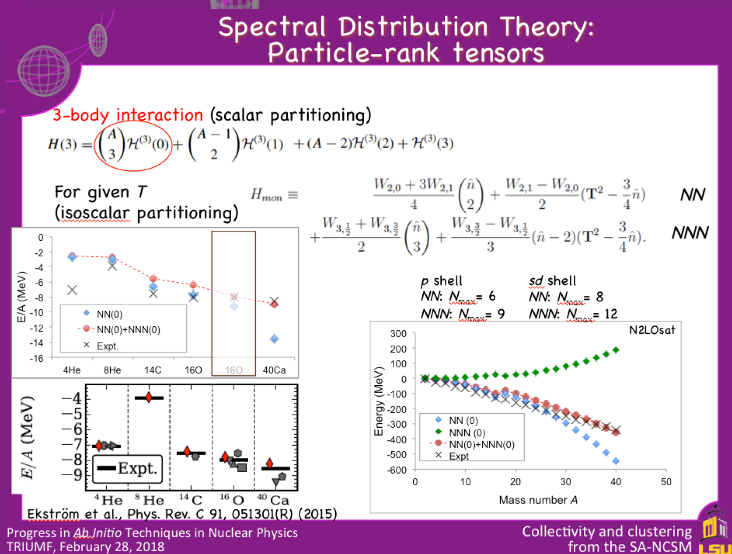




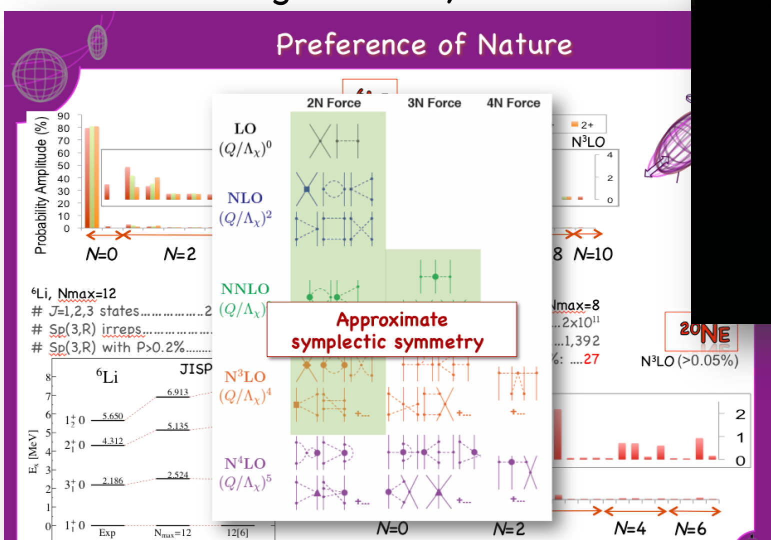


Burrows, Elster, Popa, Launey, Nogga, Maris,
Phys. Rev. C 97 (2018) 024325

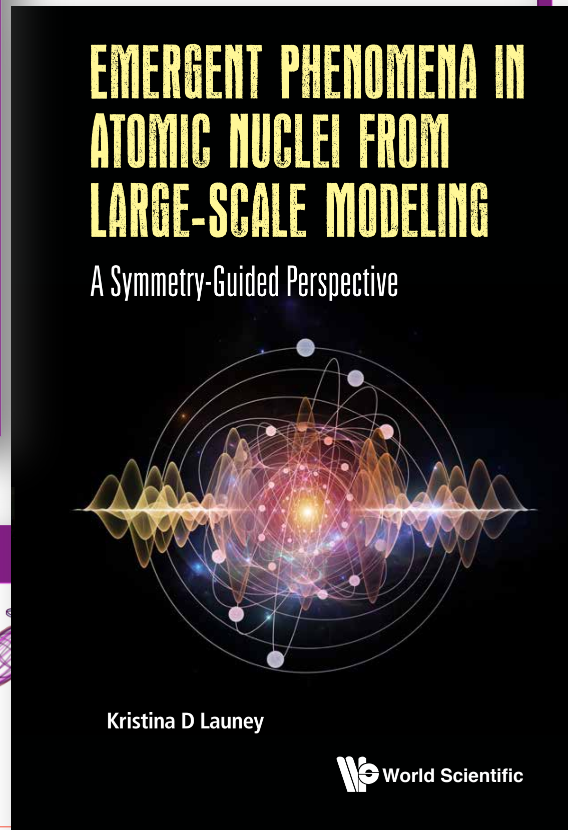
Conclusions

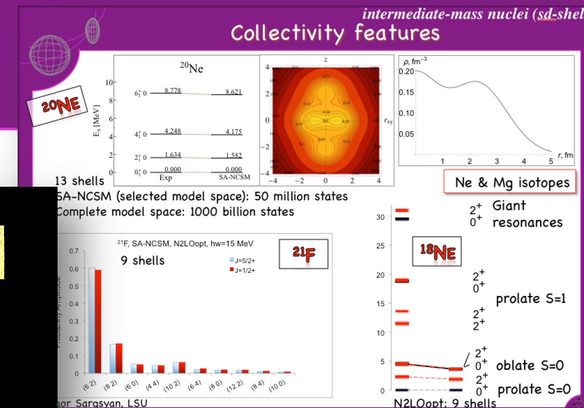


Binding energies with isospin-averaged 3-body interactions

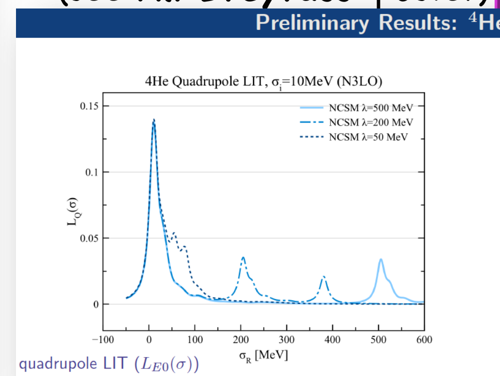


Simple physics: “shape” + vibrations + rotations





Collective and alpha clustering features in nuclei (see **Ali Dreyfuss**’ poster)



Sum rules : LSR+SA-NCSM (**Nir Nevo Dinur** ‘s talk)

J. L. Wood
C. W. Johnson
D. J. Rowe
J. E. Escher
D. Lee
Y. Alhassid

T. Luu & A. Shindler
Y. Suzuki & W. Horiuchi
J. P. Draayer & T. Dytrych
G. Rosensteel
F. Pan & X. Guan