PROBABILITY FOR EFT COEFFICIENTS

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- General EFT series for observable to order k: $X = X_0 \sum_{i=0}^{k} c_i x^i$
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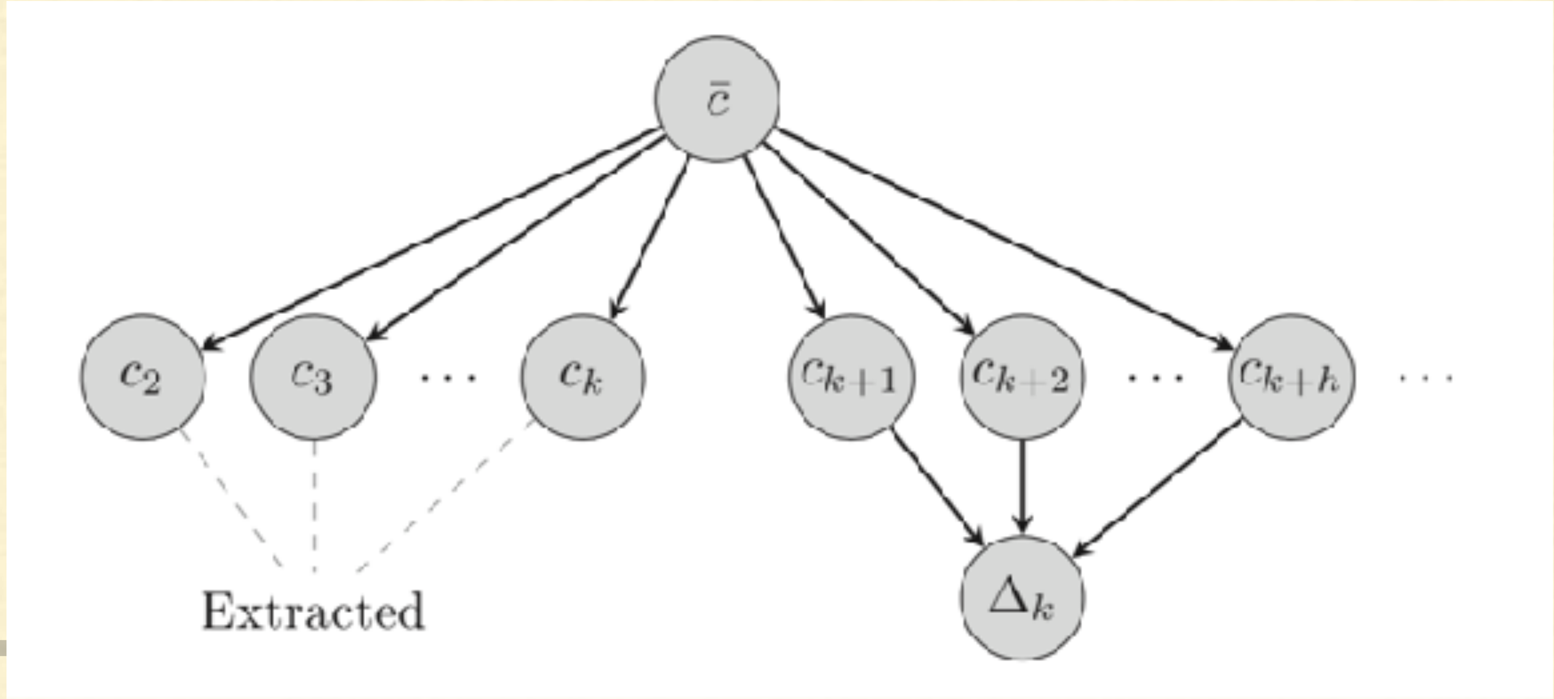
- Bayesian model:

One parameter cbar sets size of all dimensionless coefficients



WHAT DOES NATURAL MEAN TO YOU?

Choose a prior to get started:

- "Set A": coefficients are uniformly distributed up to maximum, maximum distributed uniformly in its logarithm. $\epsilon \to 0+$ at end

$$\mathrm{pr}(c_n|\bar{c}) = \frac{1}{2\bar{c}}\theta(\bar{c} - |c_n|);\ \mathrm{pr}(\bar{c}) = -\frac{1}{2\ln(\epsilon)\bar{c}}\theta\left(\frac{1}{\epsilon} - \bar{c}\right)\theta(\bar{c} - \epsilon)$$

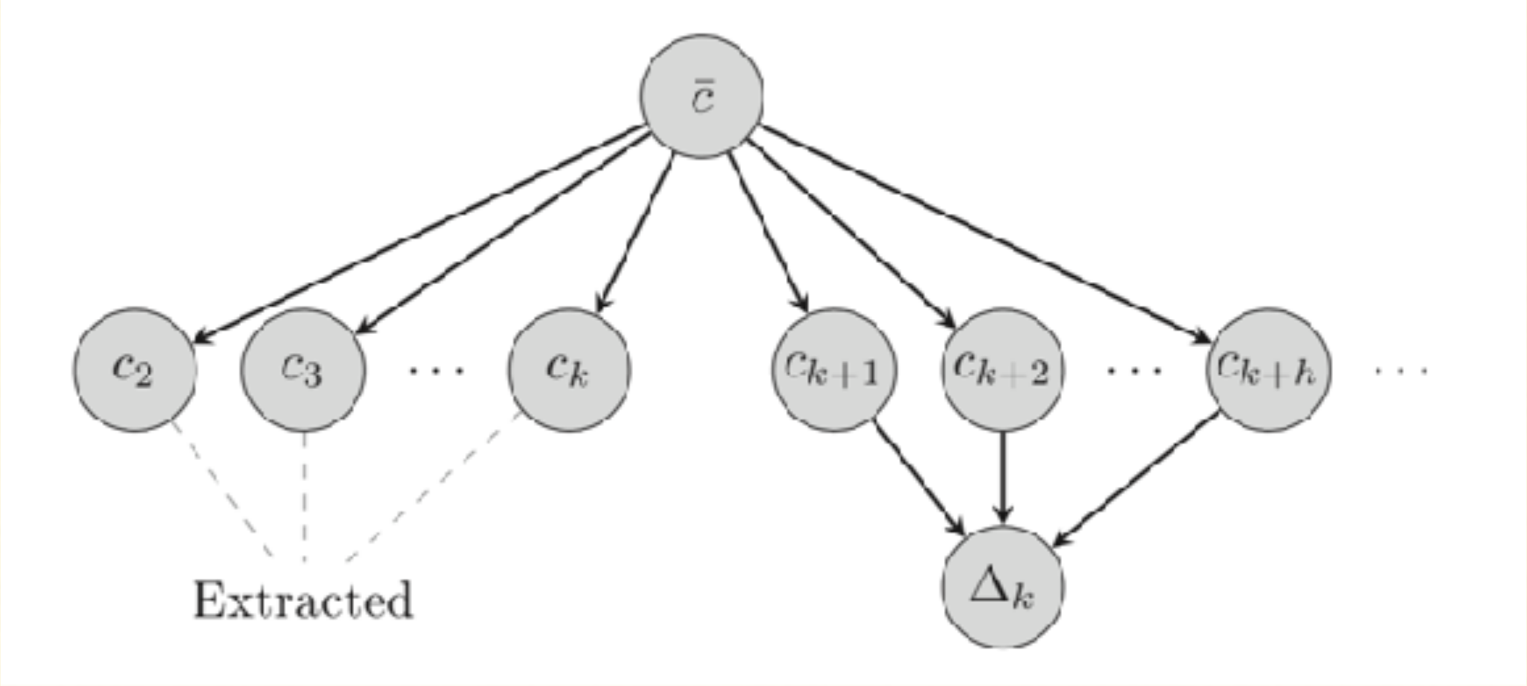
- "Set C": coefficients are normally distributed, with mean 0 and standard deviation cbar. cbar is distributed uniformly in its logarithm between some minimum and maximum values.

$$\mathrm{pr}(c_n|\bar{c}) = \frac{1}{\sqrt{2\pi}\bar{c}}e^{-c_n^2/2\bar{c}^2};\ \mathrm{pr}(\bar{c}) \propto \frac{1}{\bar{c}}\theta(\bar{c} - \bar{c}_<)\theta(\bar{c}_> - \bar{c})$$

Notes:

- Need to fix X_0 and breakdown scale to get c_n's from EFT calculation
- Just need prior to get started: will show prior dependence goes away

RESULTS FOR SET A PRIORS I

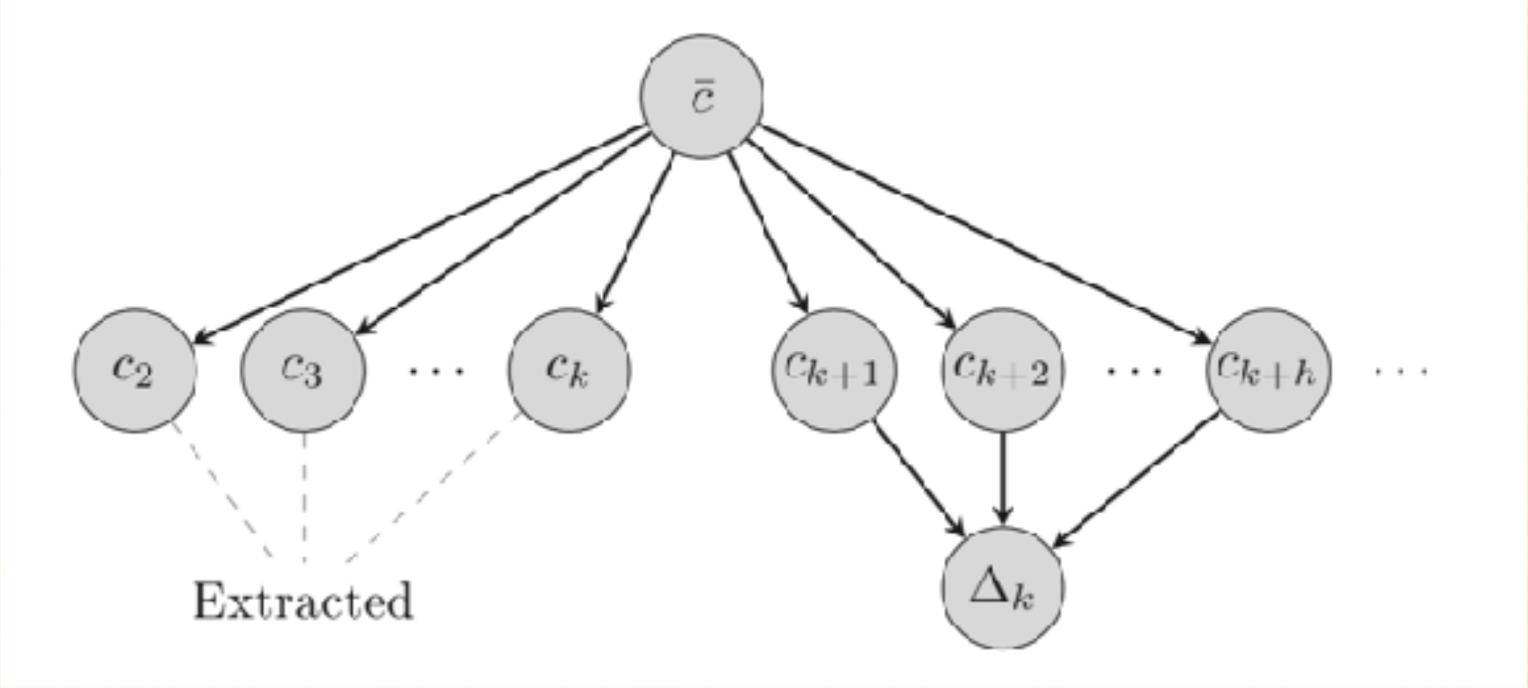


RESULTS FOR SET A PRIORS I

- Bayes theorem

$$\mathrm{pr}(\bar{c}|c_0, c_1, \ldots, c_k) = \frac{\mathrm{pr}(c_0, c_1, \ldots, c_k|\bar{c})\mathrm{pr}(\bar{c})}{\mathrm{pr}(c_0, c_1, \ldots, c_k)}$$

$$= \mathcal{N}\mathrm{pr}(\bar{c})\Pi_{n=0}^{k}\mathrm{pr}(c_n|\bar{c})$$



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$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty d\bar{c}\,\mathrm{pr}(c_{k+1}|\bar{c})\mathrm{pr}(\bar{c}|c_0, c_1, \ldots c_k)$$

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- This is generic, but the integrals are simple for "Set A" (uniform) prior

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 1 & \text{if } c_{k+1} < c_{\max} \\ \left(\frac{c_{\max}}{c_{k+1}}\right)^{k+2} & \text{if } c_{k+1} > c_{\max} \end{cases}$$

RESULTS FOR SET A PRIORS II

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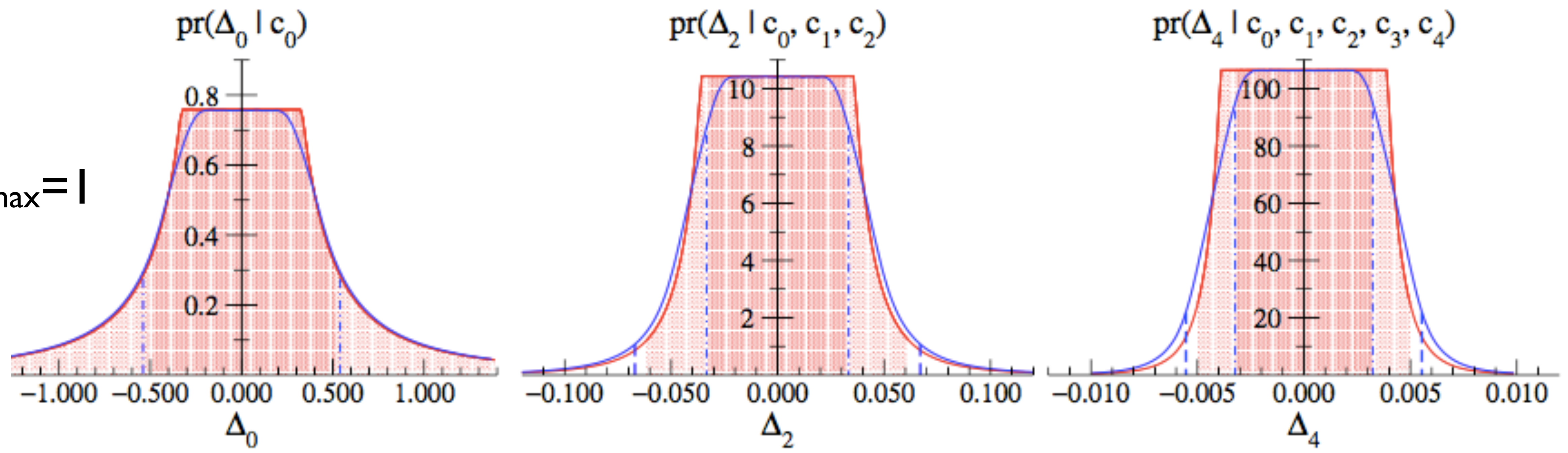
- $pr(\Delta_k) \propto X_0\, x^{k+1}\, pr(c_{k+1})$
- 68%, 95% DOB intervals from integration of probability distribution

$x=0.33$; $c_{max}=1$



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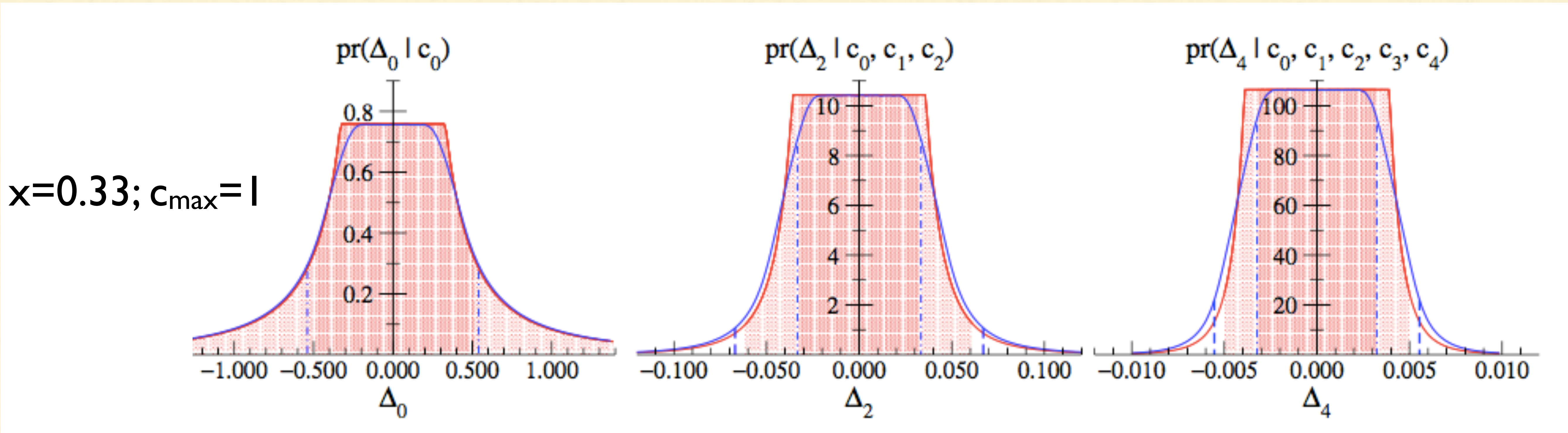

- Main feature is reduction by factor of x for each order; but tails also become steeper as more information on coefficients acquired
- Not Gaussian!
- $[-c_{max} X_0 x^{k+1}, c_{max} X_0 x^{k+1}]$ is a $\frac{k+1}{k+2} * 100\%$ DoB interval

NORMAL NATURALNESS

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty d\bar{c}\, \mathrm{pr}(c_{k+1}|\bar{c})\mathrm{pr}(\bar{c}|c_0, c_1, \ldots c_k)$$

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty \frac{d\bar{c}}{\bar{c}^{k+3}} \exp\left(-\frac{c_{k+1}}{2\bar{c}^2}\right) \exp\left(-\frac{(k+1)\langle c^2\rangle}{2\bar{c}^2}\right)$$

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \frac{\Gamma\left(\frac{k+2}{2}\right)}{\Gamma\left(\frac{k+1}{2}\right)} \left(\frac{(k+1)\langle c^2\rangle}{(k+1)\langle c^2\rangle + c_{k+1}^2}\right)^{(k+2)/2}$$

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- For "Set C" (Gaussian) priors:

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty \frac{d\bar{c}}{\bar{c}^{k+3}} \exp\left(-\frac{c_{k+1}}{2\bar{c}^2}\right) \exp\left(-\frac{(k+1)\langle c^2\rangle}{2\bar{c}^2}\right)$$

- Student's t-distribution results:

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \frac{\Gamma\left(\frac{k+2}{2}\right)}{\Gamma\left(\frac{k+1}{2}\right)} \left(\frac{(k+1)\langle c^2\rangle}{(k+1)\langle c^2\rangle + c_{k+1}^2}\right)^{(k+2)/2}$$

- DoB intervals computed using known results for this distribution. Size of error bar set by $<c^2>$, x^{k+1} (i.e. Q^{k+1}), and X_0.

NN SCATTERING WITH SEMI-LOCAL POTENTIALS

Epelbaum, Krebs, Meissner, PRC, 2015

χEFT: $\mathscr{L}$(N,π)→V^(k)→δ

$$\sigma_{np}(E_{\rm lab}) = \sigma_{\rm LO} \sum_{n=0}^{k} c_n(p_{\rm rel}) \left(\frac{p_{\rm rel}}{\Lambda_b}\right)^n$$

$$x = \frac{p_{\rm rel}}{\Lambda_b}$$

NN SCATTERING WITH SEMI-LOCAL POTENTIALS

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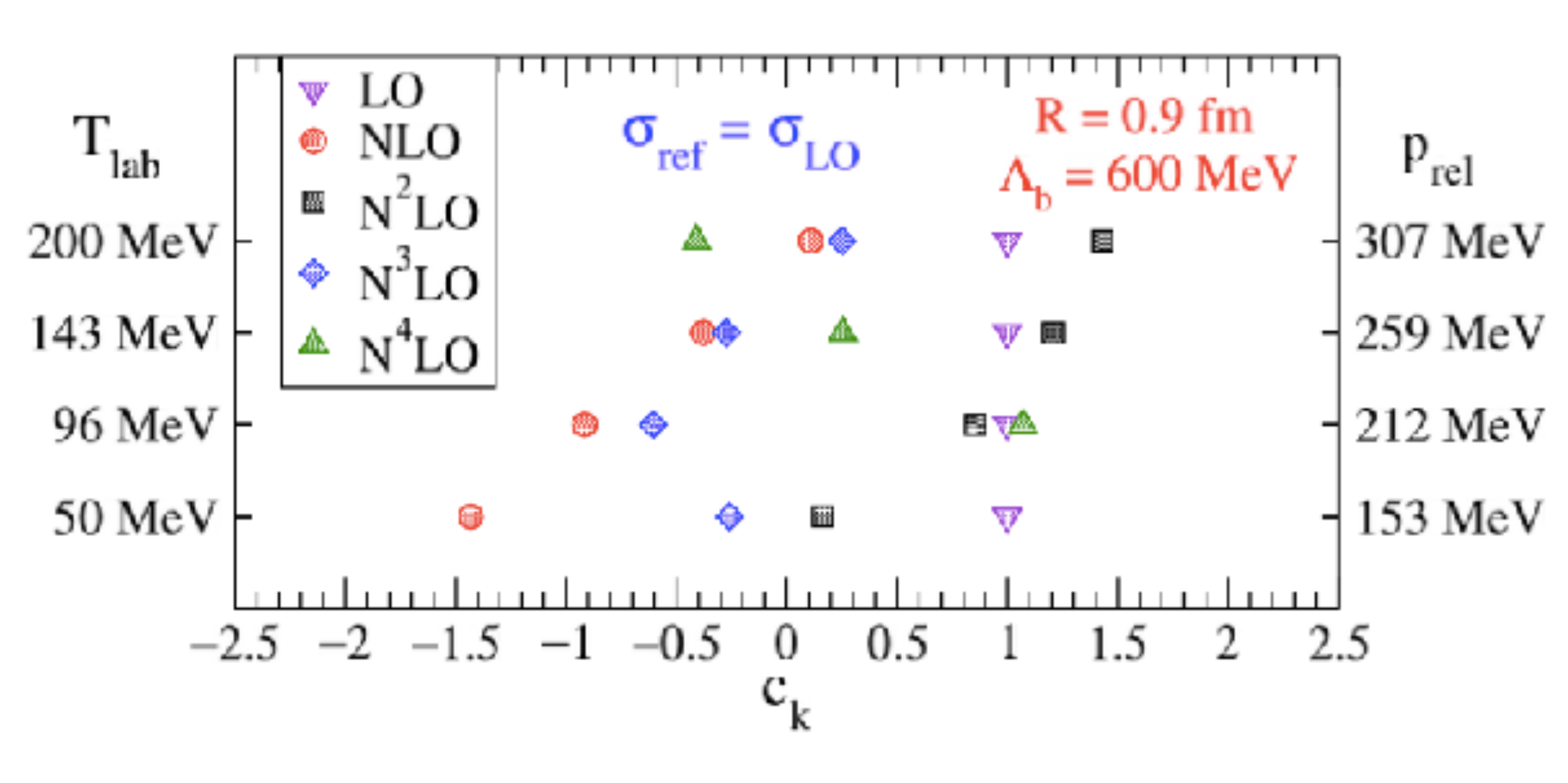
χEFT: $\mathscr{L}(N,\pi) \rightarrow V^{(k)} \rightarrow \delta$

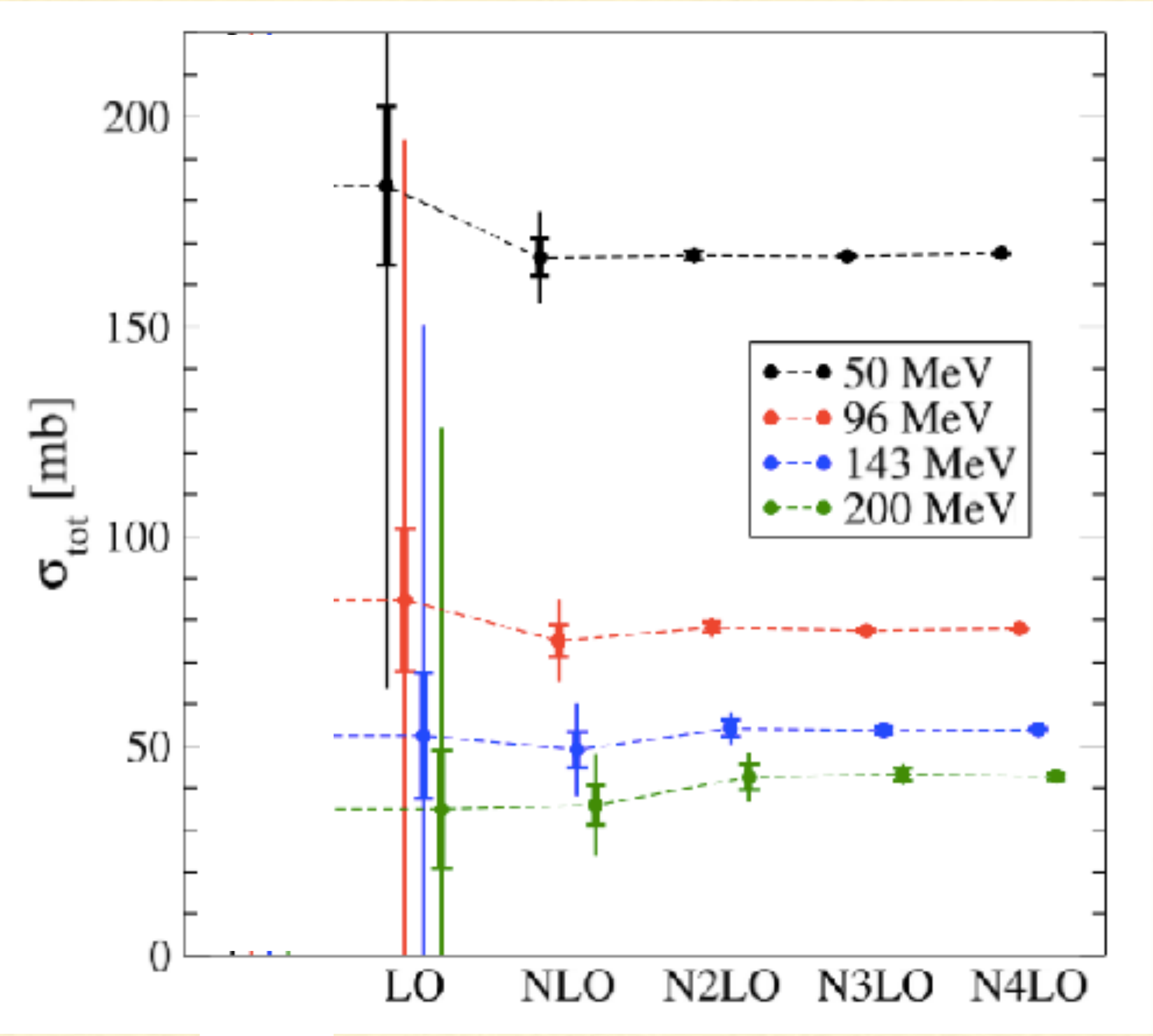
- NN cross section at T_{lab}=50, 96, 143, 200 MeV
- Potential regulated by local function, parameterized by R
- EKM identify Λ_b=600 MeV for smaller R values
- Here: R=0.9 fm data
- Results at LO, NLO, N²LO, N³LO, N⁴LO (k=0, 2, 3, 4, 5)

$$\sigma_{np}(E_{\rm lab}) = \sigma_{\rm LO} \sum_{n=0}^{k} c_n(p_{\rm rel}) \left(\frac{p_{\rm rel}}{\Lambda_b}\right)^n$$

$$x = \frac{p_{\rm rel}}{\Lambda_b}$$

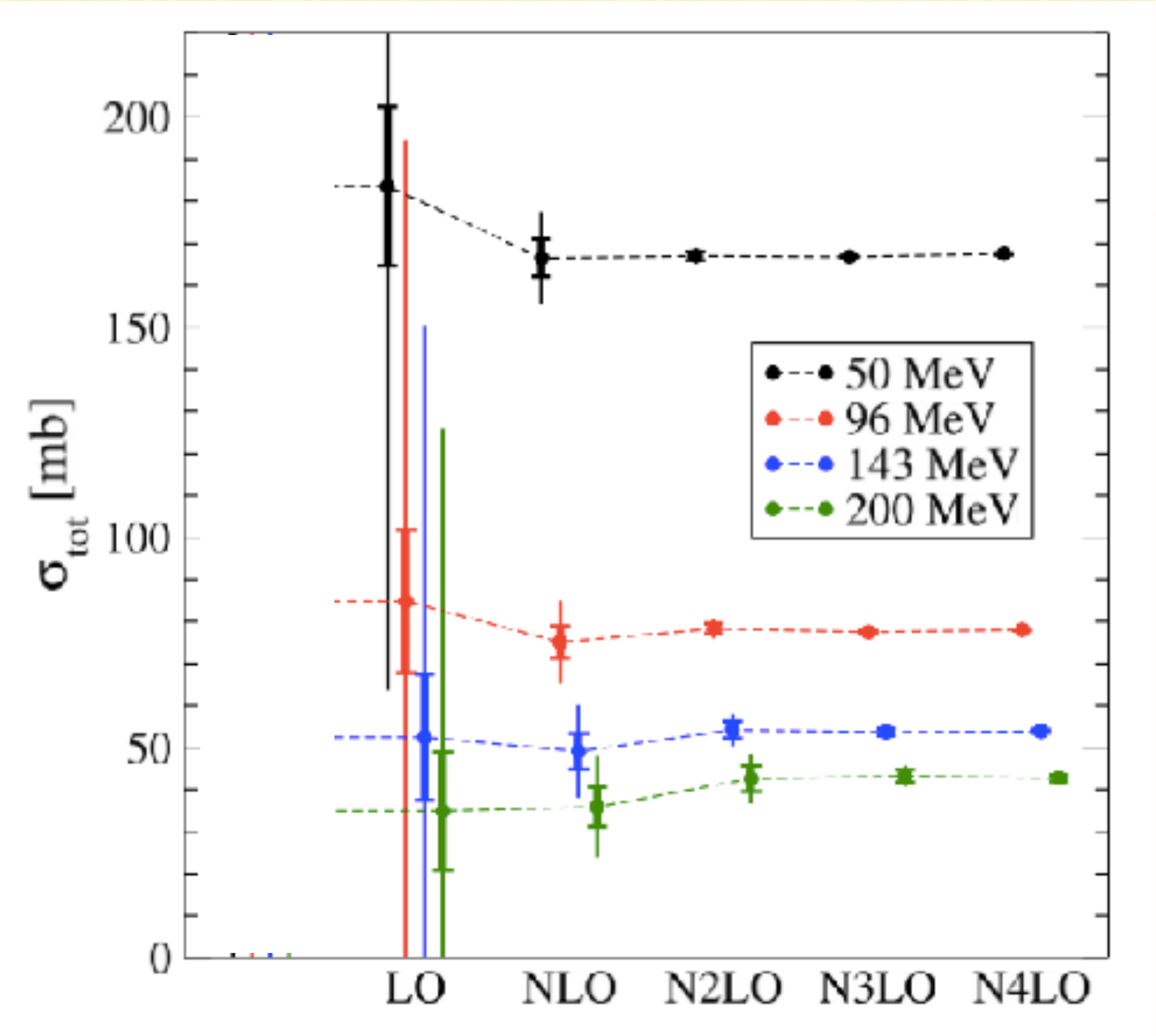


RESULTS



200
150
100
50
0
σ_{tot} [mb]
LO
NLO
N2LO
N3LO
N4LO
50 MeV
96 MeV
143 MeV
200 MeV

RESULTS



200
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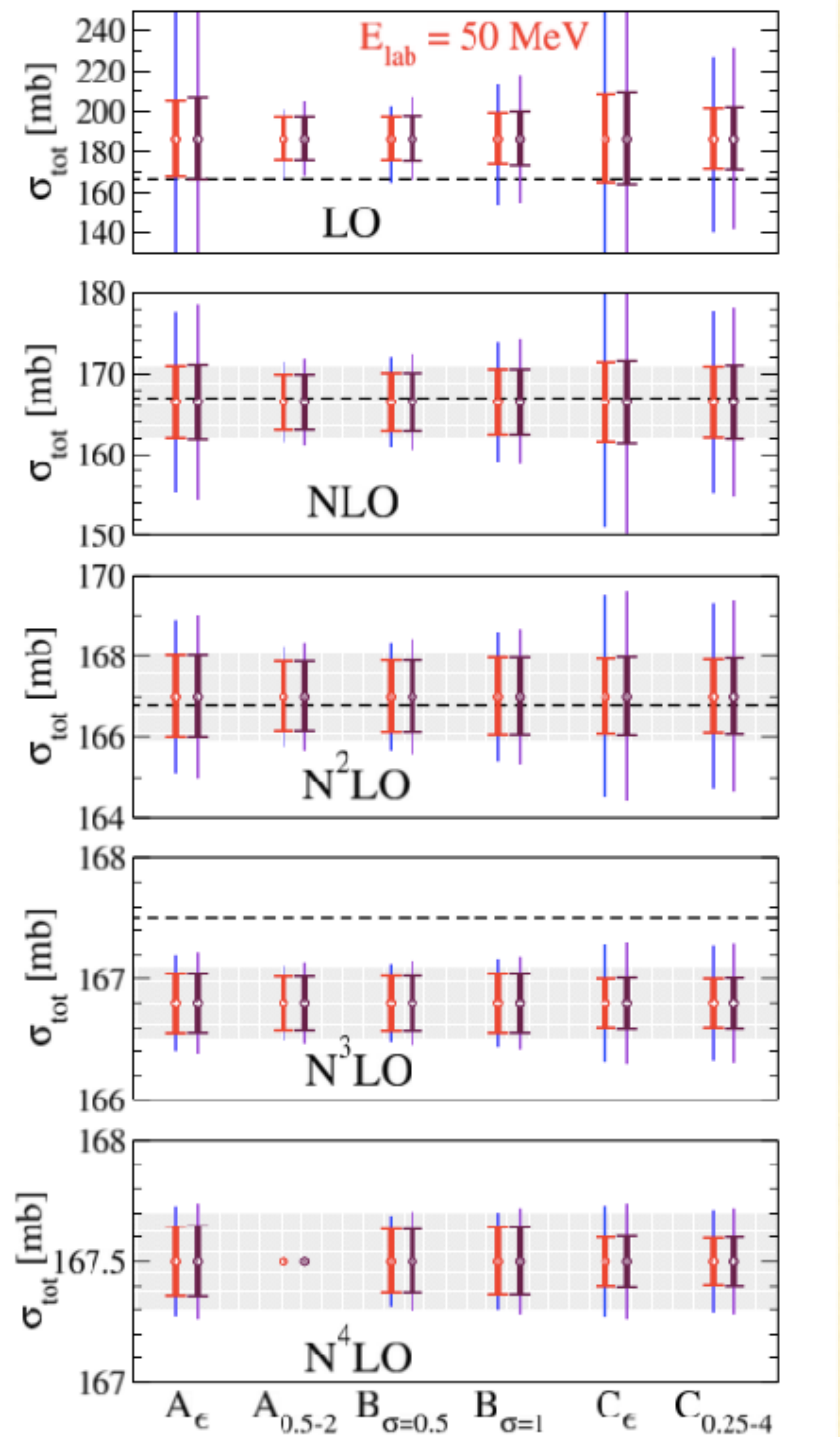
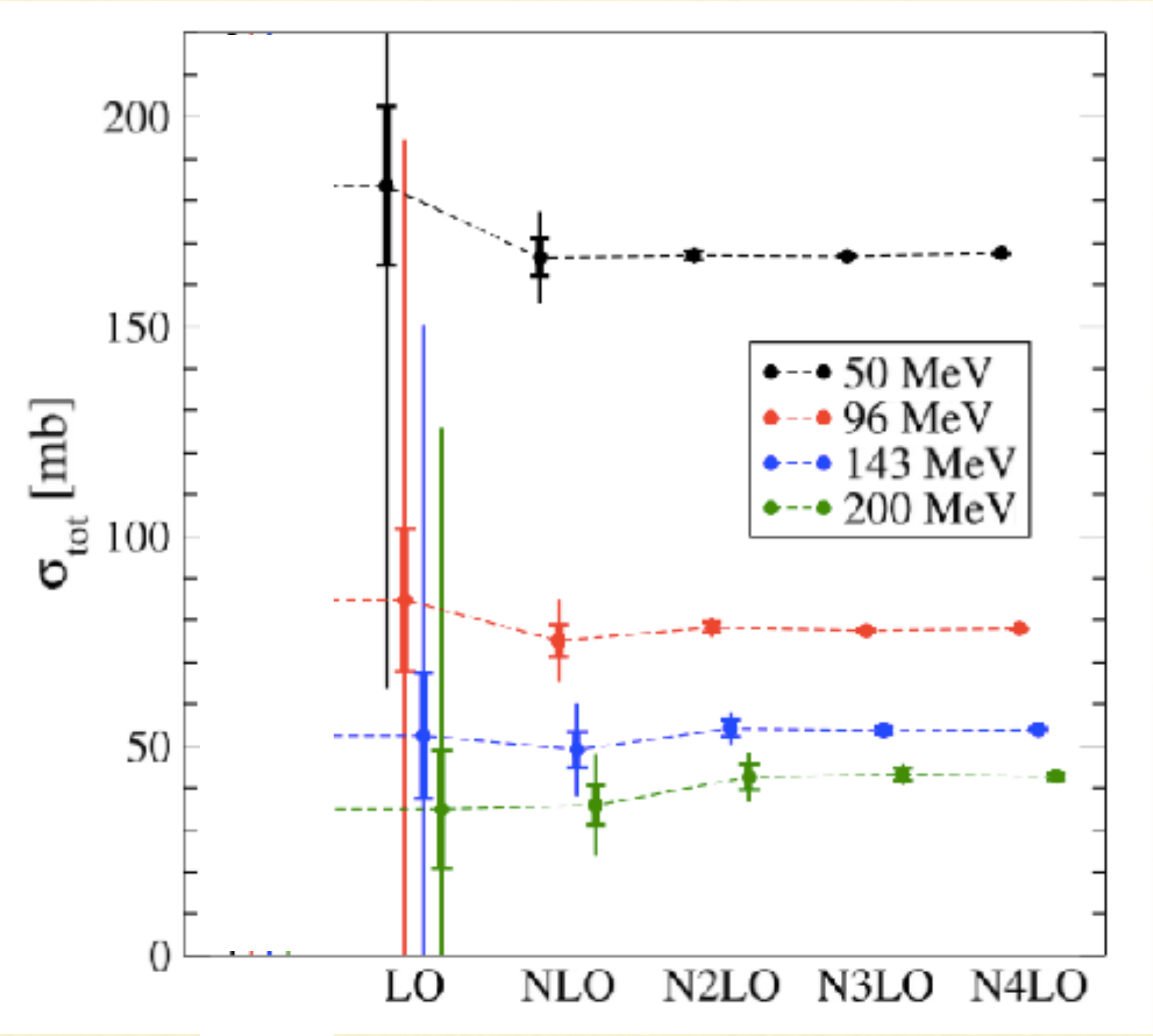


E_{lab} = 50 MeV
σ_{tot} [mb]
LO
NLO
N^2LO
N^3LO
N^4LO
A_ϵ
$A_{0.5\text{-}2}$
$B_{\sigma=0.5}$
$B_{\sigma=1}$
C_ϵ
$C_{0.25\text{-}4}$

RESULTS



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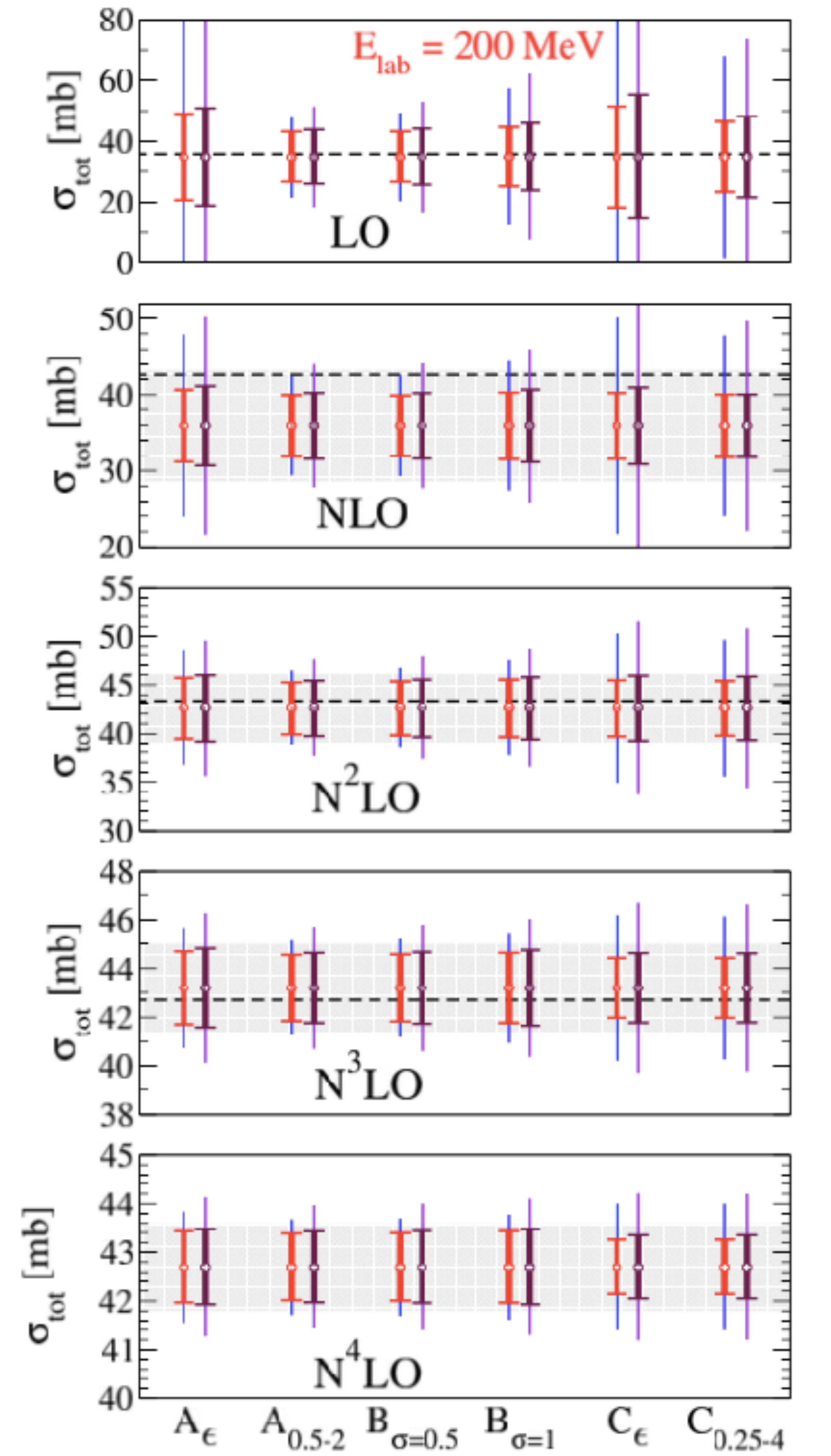

E_{lab} = 200 MeV
σ_{tot} [mb]
LO
NLO
N^2LO
N^3LO
N^4LO
A_ϵ
$A_{0.5\text{-}2}$
$B_{\sigma=0.5}$
$B_{\sigma=1}$
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$C_{0.25\text{-}4}$

CAVEATS

- Naturalness of c_i's in x-expansion for NN cross section assumed. Justified for perturbative process; not so clear why this should be so for NN
- m_π not included in x: fine at these energies
- We took EKM's LECs as given. LECs themselves have statistical errors, but we did not incorporate those in this analysis

Sarah's talk

- LECs also have truncation errors, which should be included in their quoted errors

Sarah's talk

THE WELL-CALIBRATED EFTER

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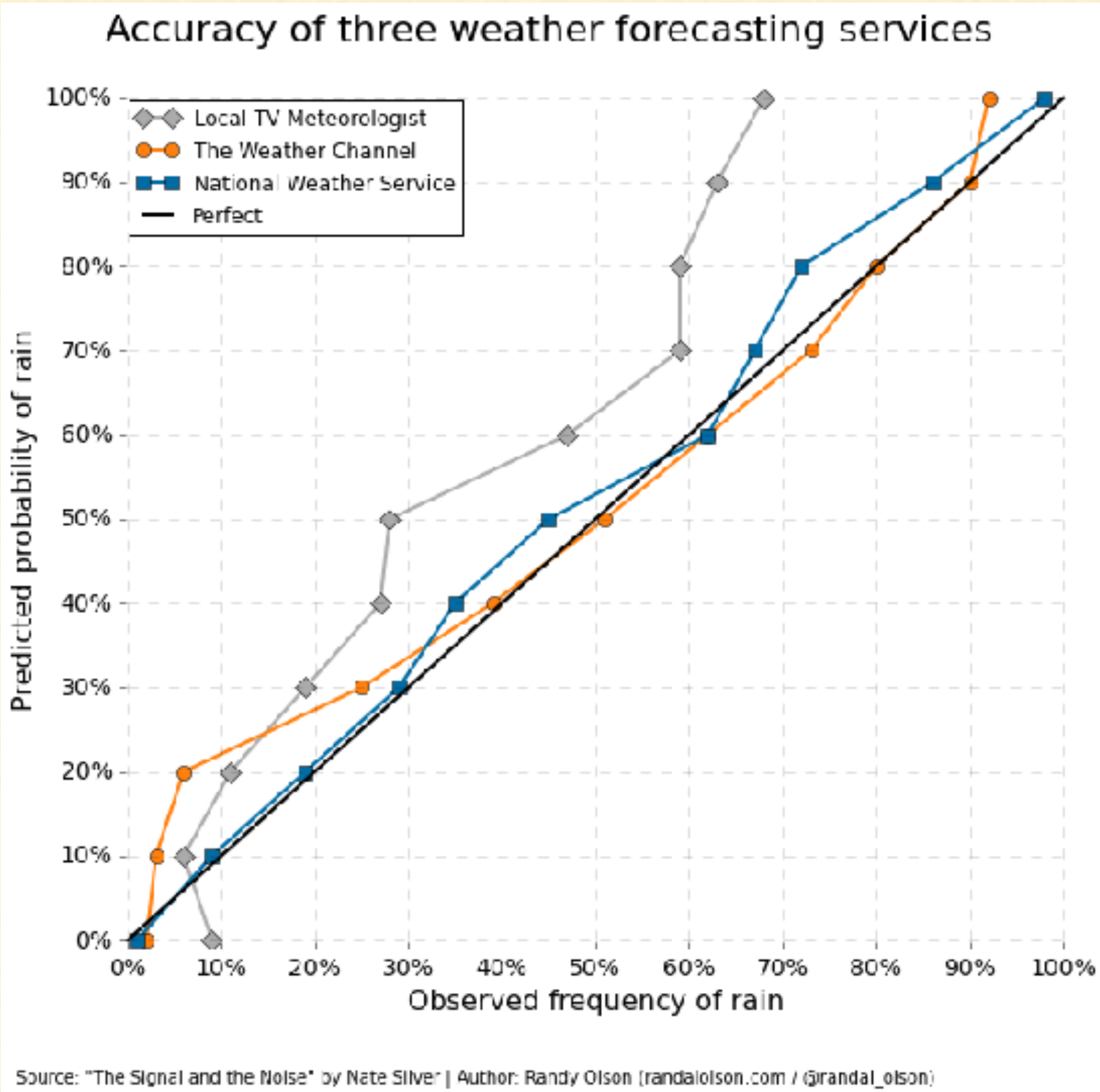

Melendez, Furnstahl, Wesolowski, PRC, 2017
after Furnstahl, Klco, DP, Wesolowski, PRC, 2015
after: Bagnaschi, Cacciari, Guffanti, Jenniches, 2015

- Consider predictions at each order, with their error bars, as data and test them to see if the procedure is consistent
- Fix a given DOB interval: compute success ratio, compare

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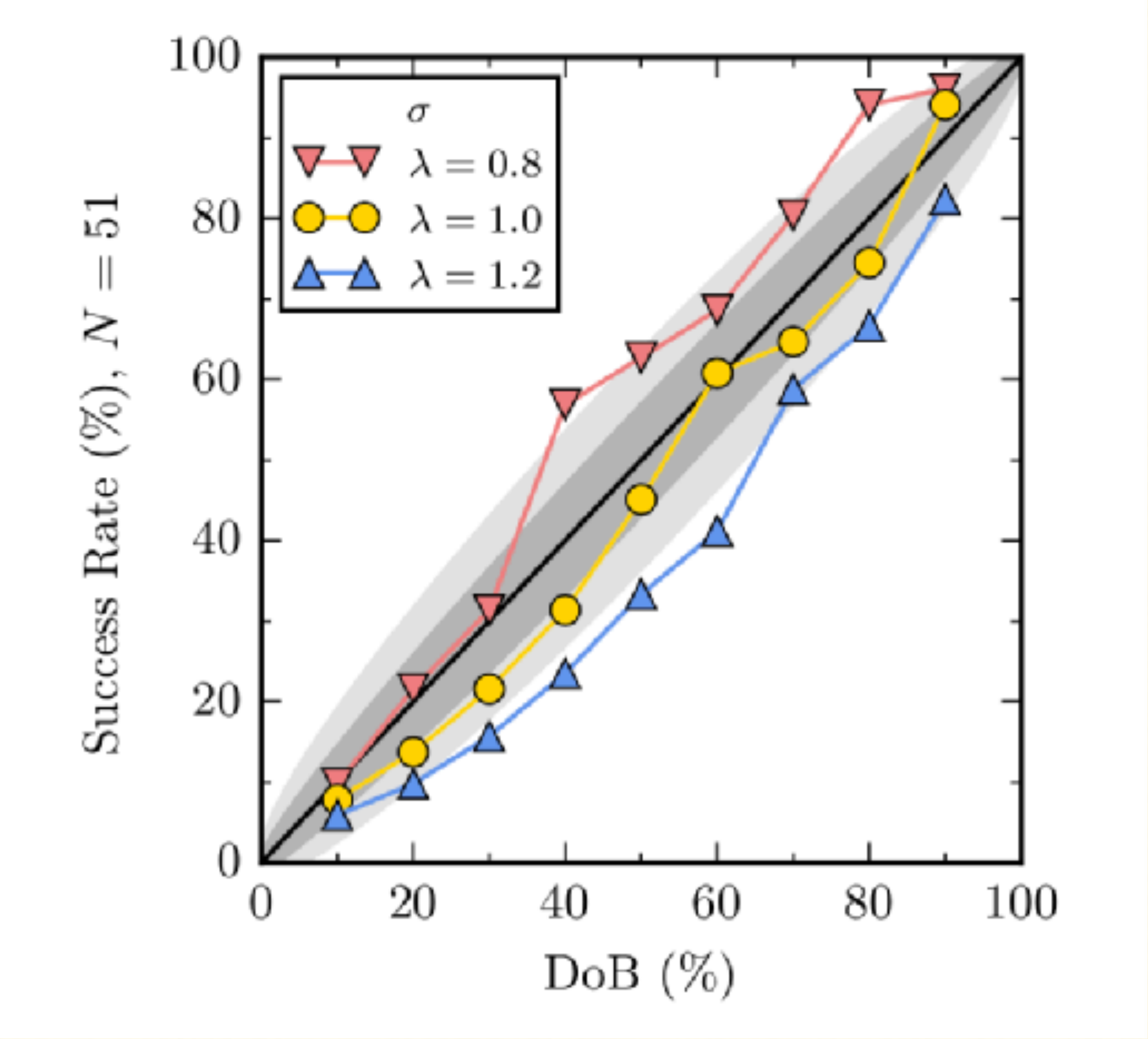

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- Interpret in terms of rescaling of Λ_b by a factor λ

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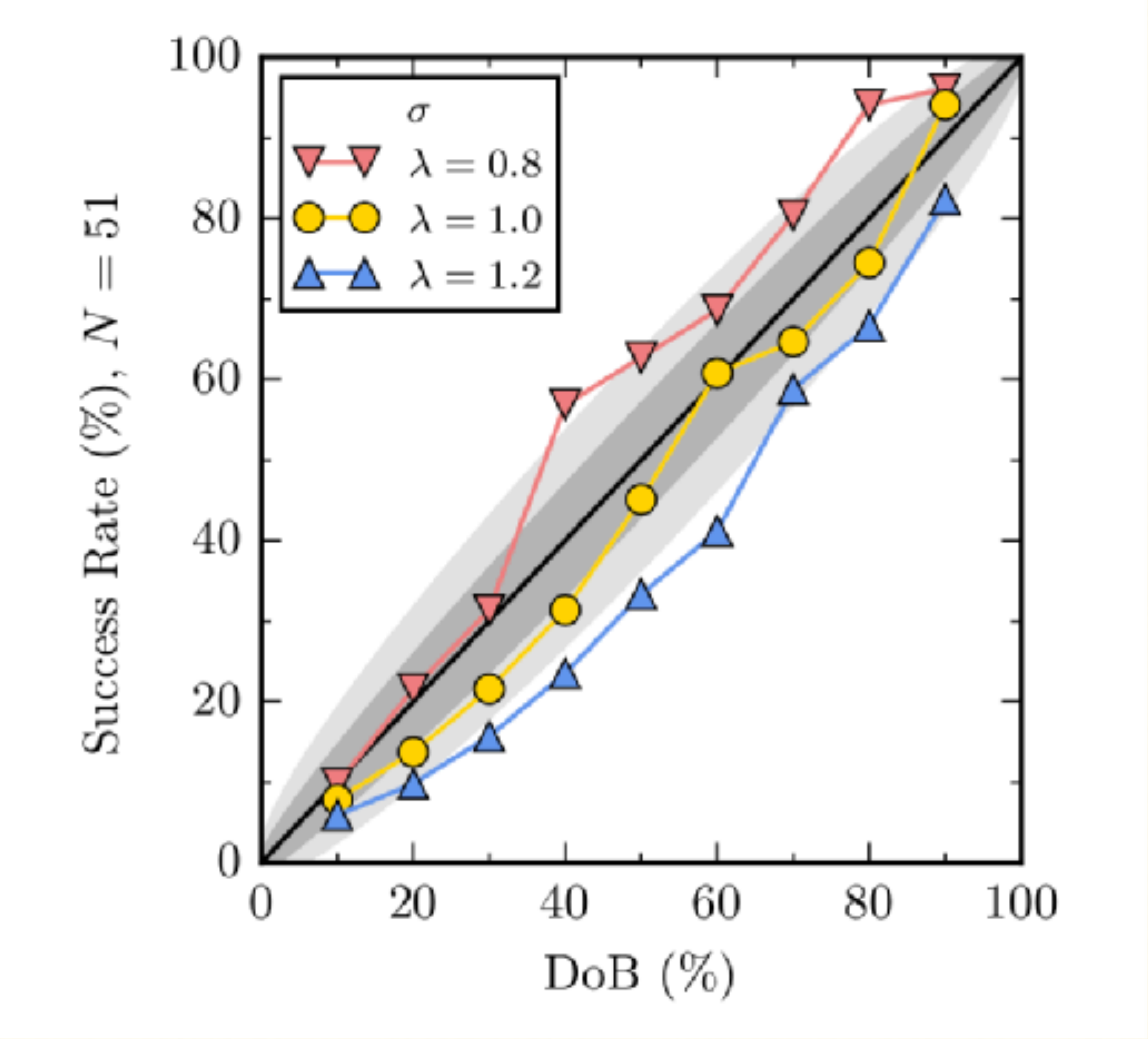

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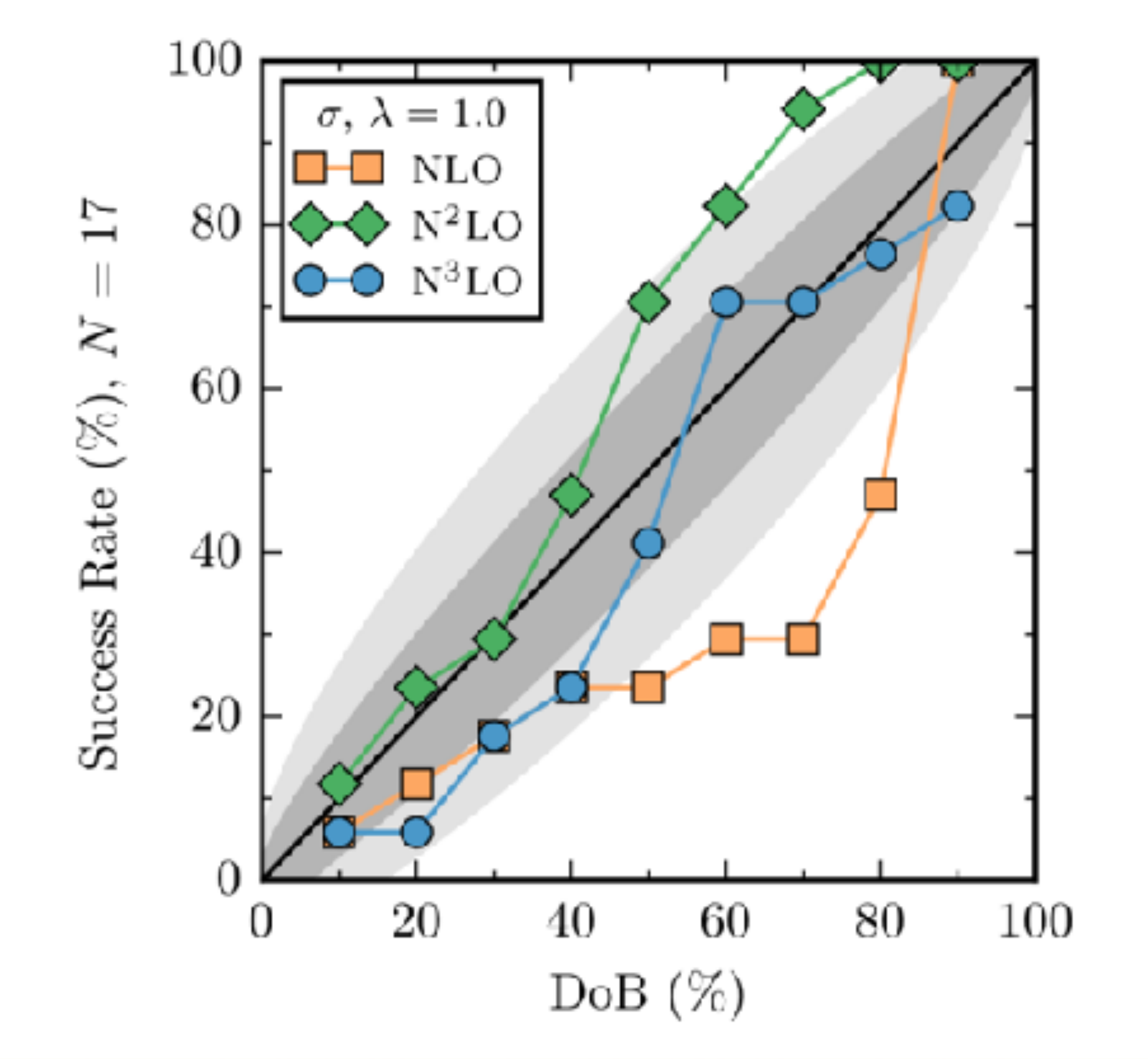
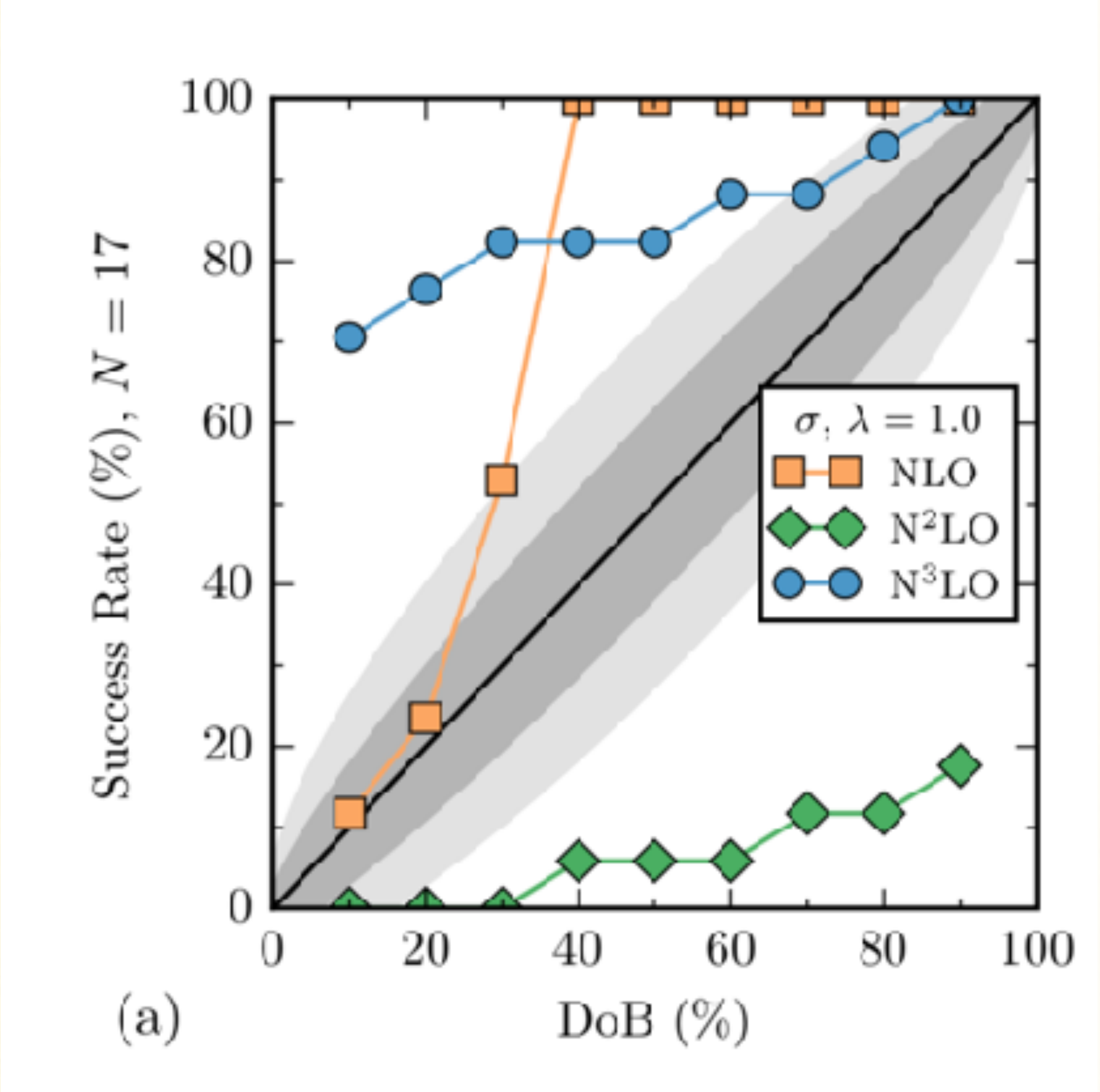
No evidence for significant rescaling of Λ_b

PHYSICS FROM CONSISTENCY PLOTS

R=0.9 fm



R=1.2 fm



- Allows assessment of order-by-order convergence
- Can look at differential cross section and spin observables too

BREAKDOWN-SCALE INFERENCE

- Λ_b determines the size of the c_n's. Choose it too big, and they'll be too big. Choose it too small, they'll be too small. And progressively so as one moves to higher and higher order.

- We have a theory for $pr(c_n|c_0, c_1, \ldots, c_k)$: now use Bayes' theorem to see how (im)probable are the c_n's that dimensionful EFT coefficients (b_n's) produce for a given Λ_b.

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At one energy:

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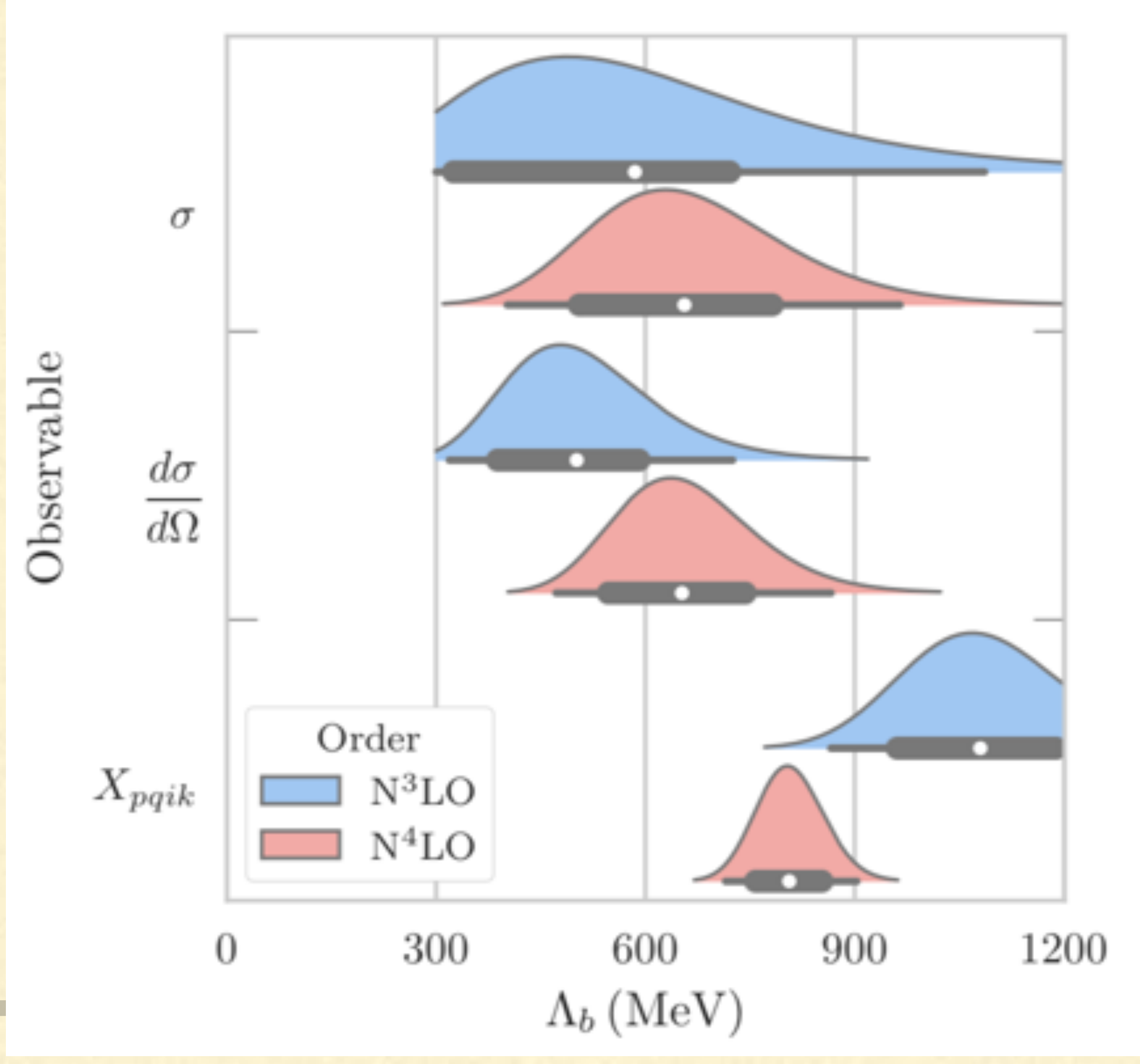
Using 5 energies (and 2 angles):



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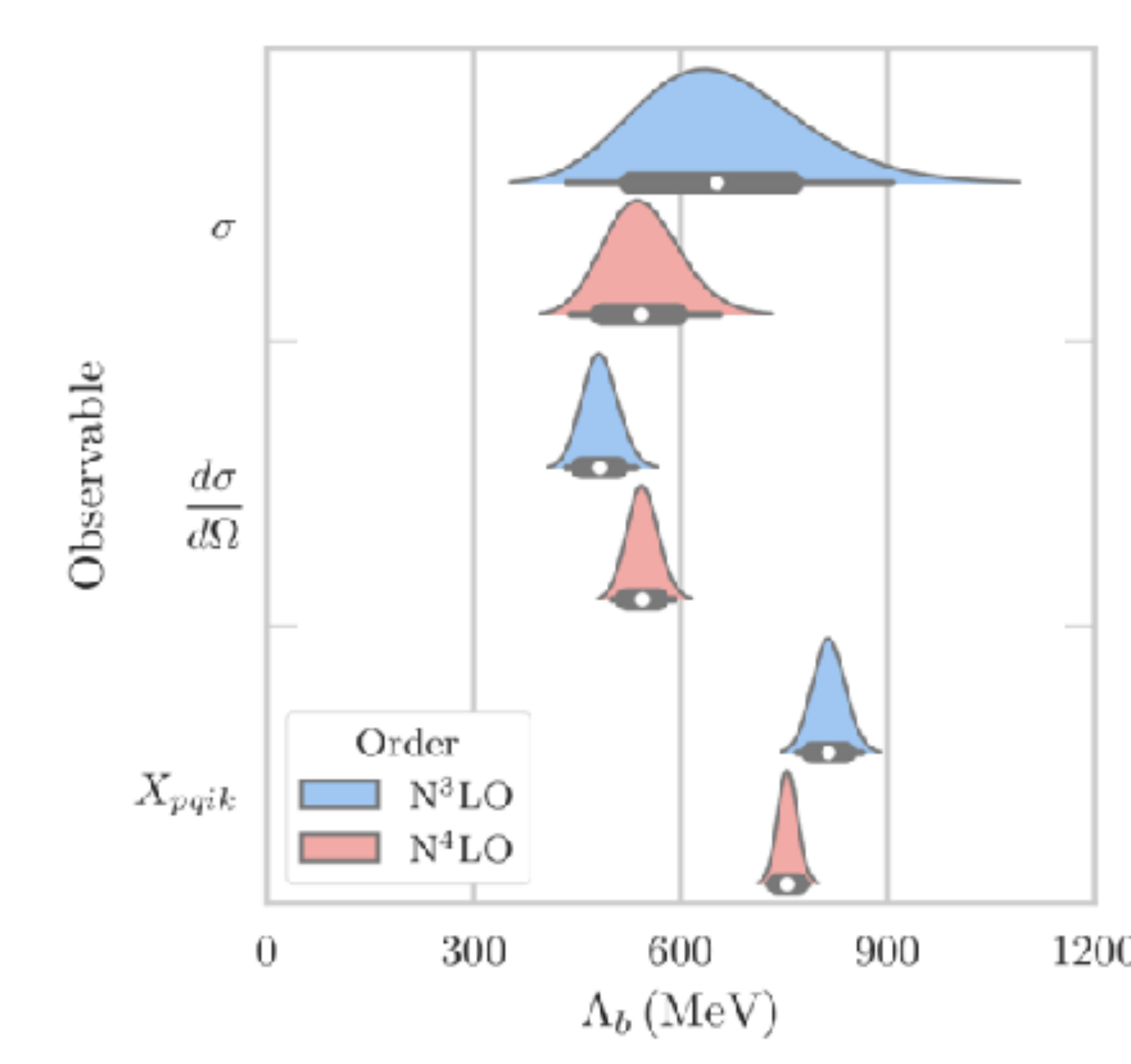

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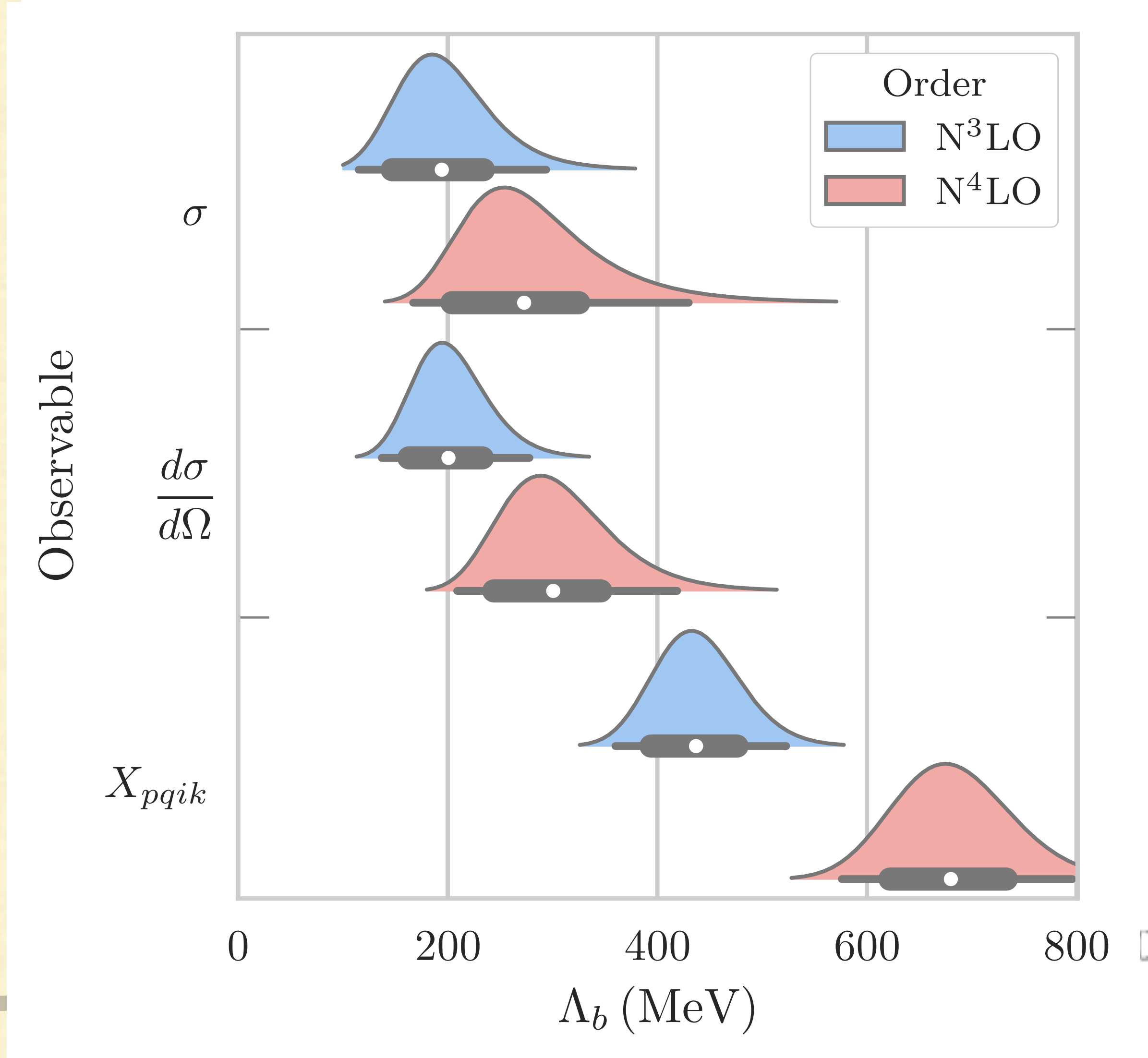
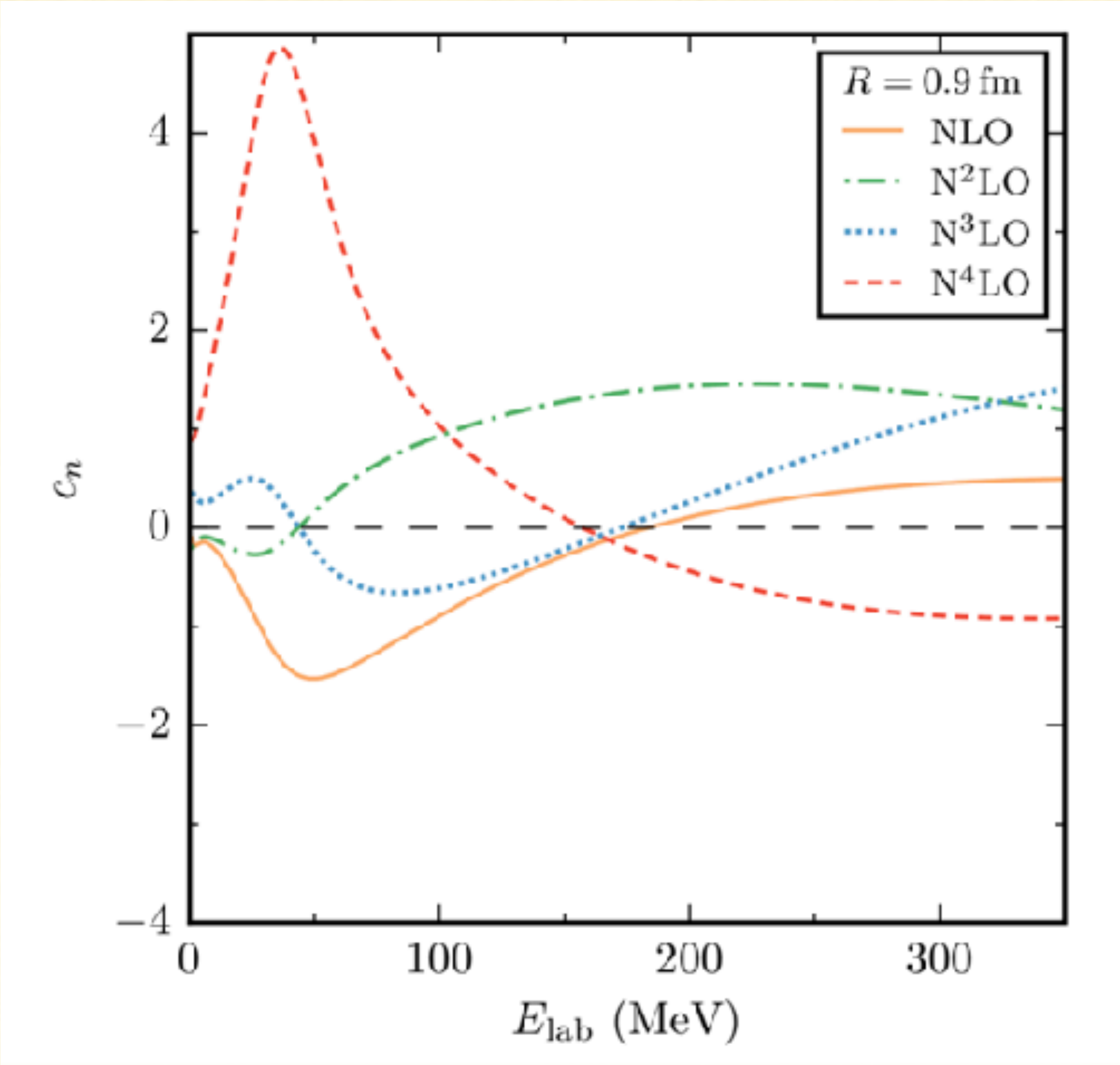


FUNCTIONAL DATA

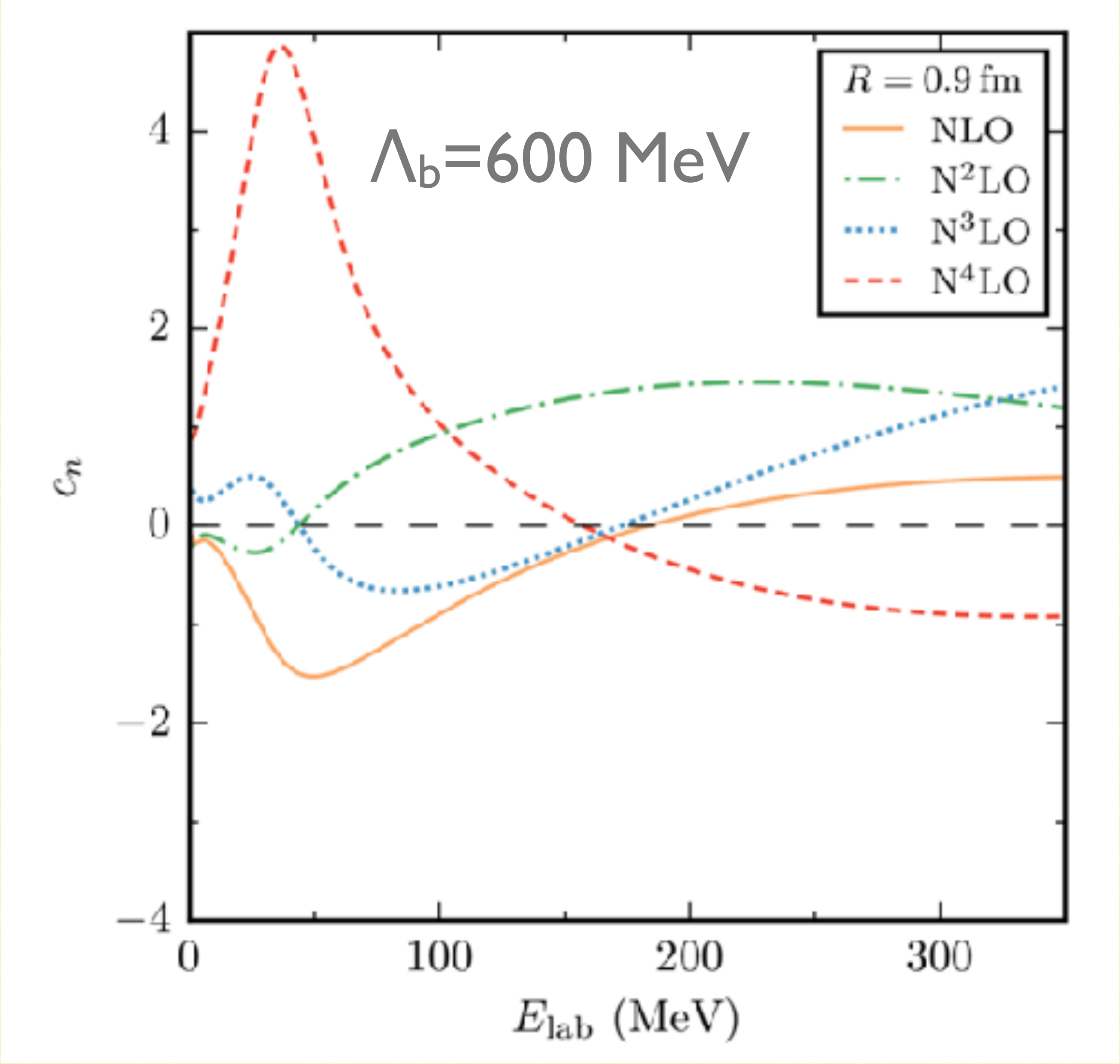
- But we don't have 119 independent data points
- We have a function for each observable at each order
- Can we understand the properties of these functions, so we can do Λ_b inference and compute success ratios rigorously?



$$\sigma(E) = \sigma_0(E)\left[1 + c_2(E)x^2 + c_3(E)x^3 + c_4(E)x^4 + c_5(E)x^5\right]$$

OBSERVATIONS AND QUESTIONS

- c_n's do not grow or shrink with n: good Λ_b choice
- Bounded functions, mostly between -2 and 2
- Each "takes a turn" at being largest
- Not oscillating quickly in this energy range



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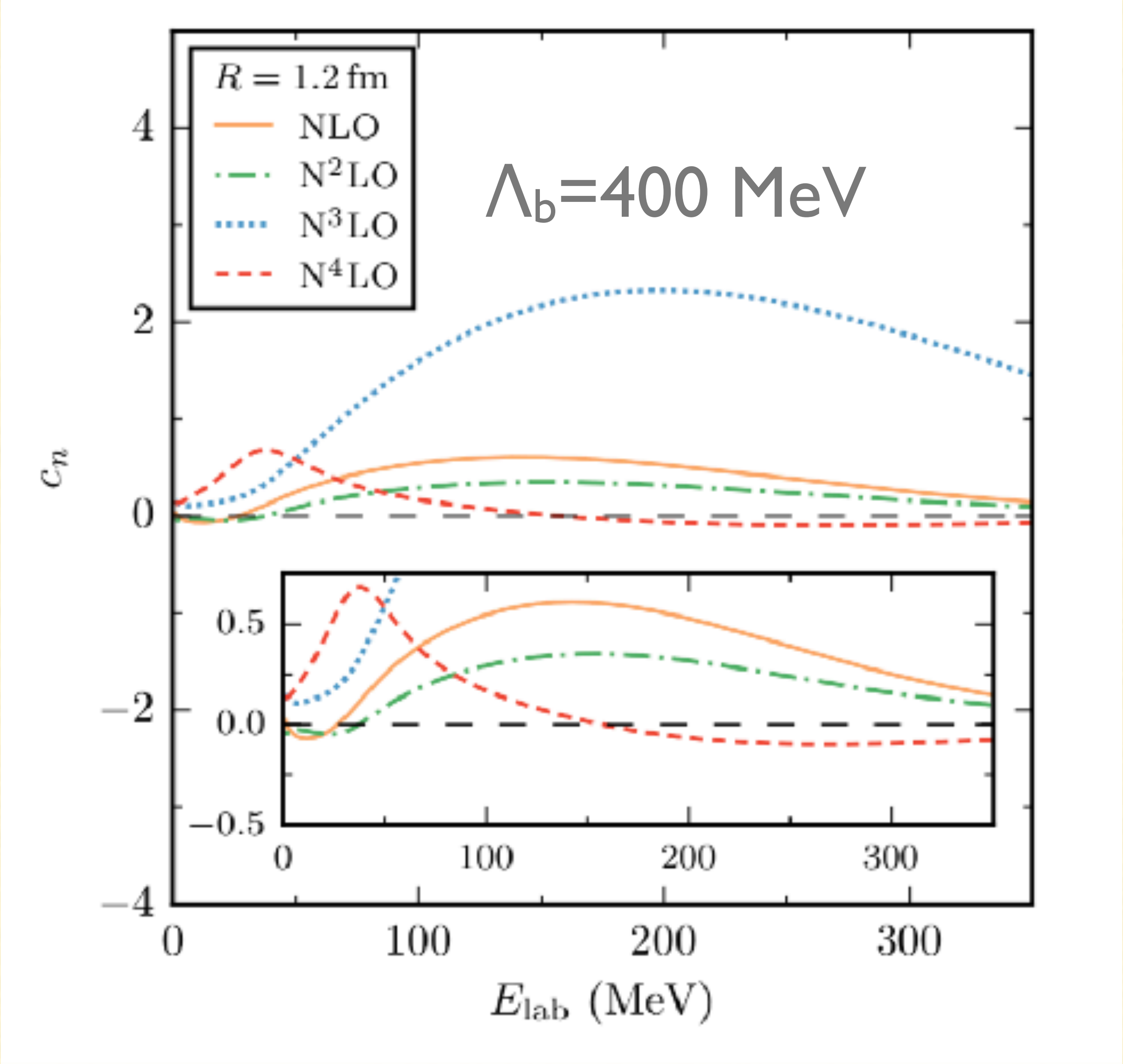

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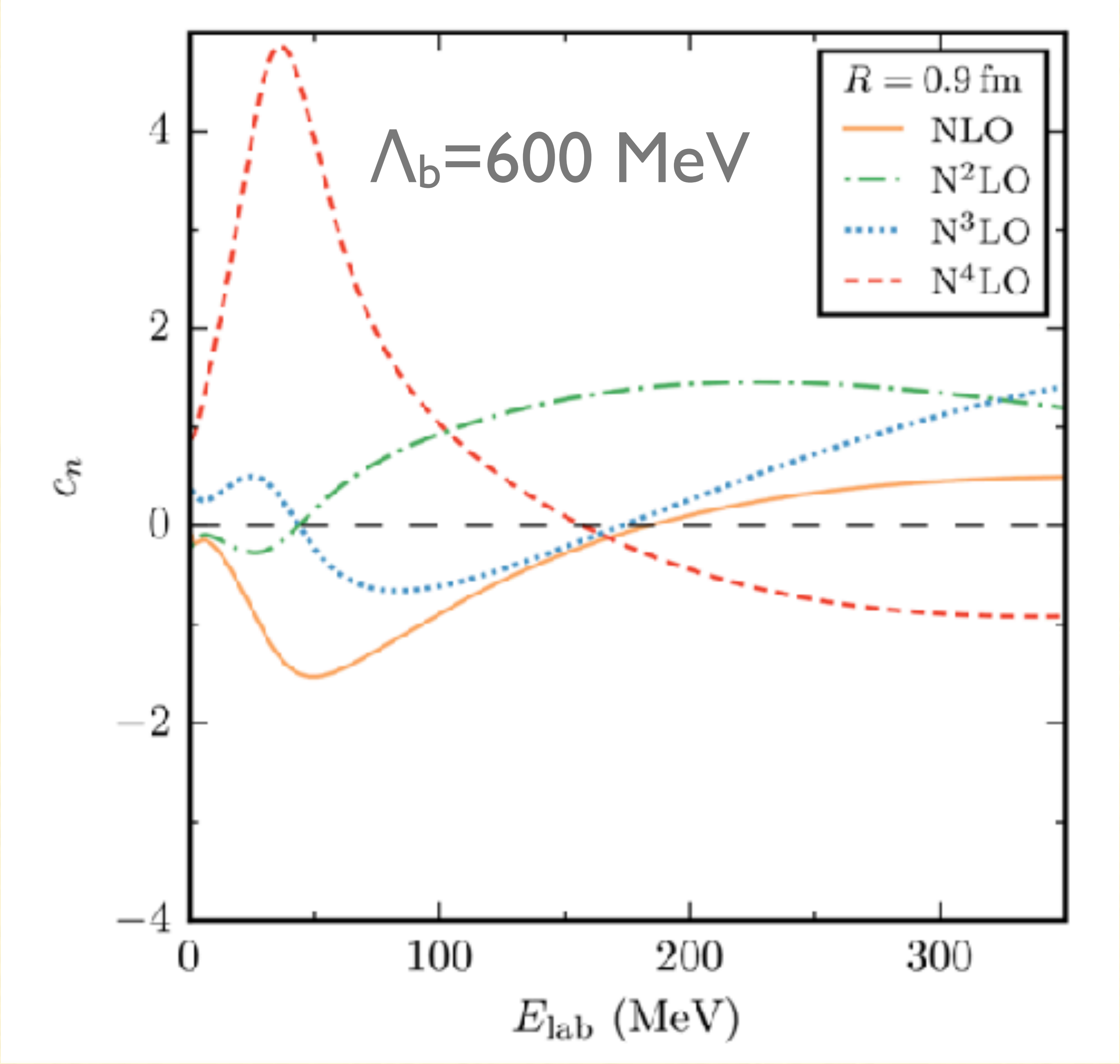

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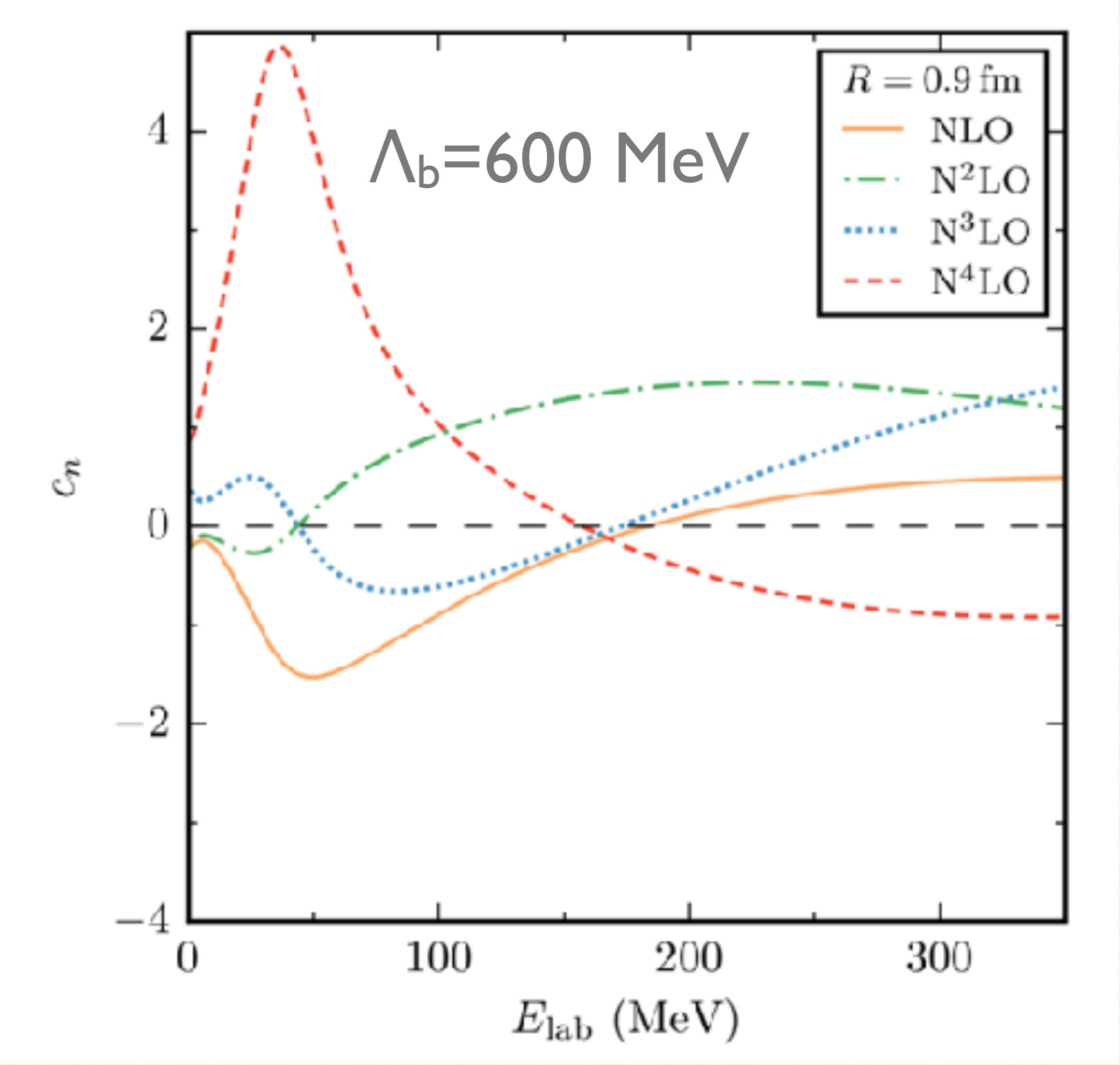


Physics questions:

- Do curves all fluctuate around zero with some common variance?
- What is the correlation length? Is it different at each order?

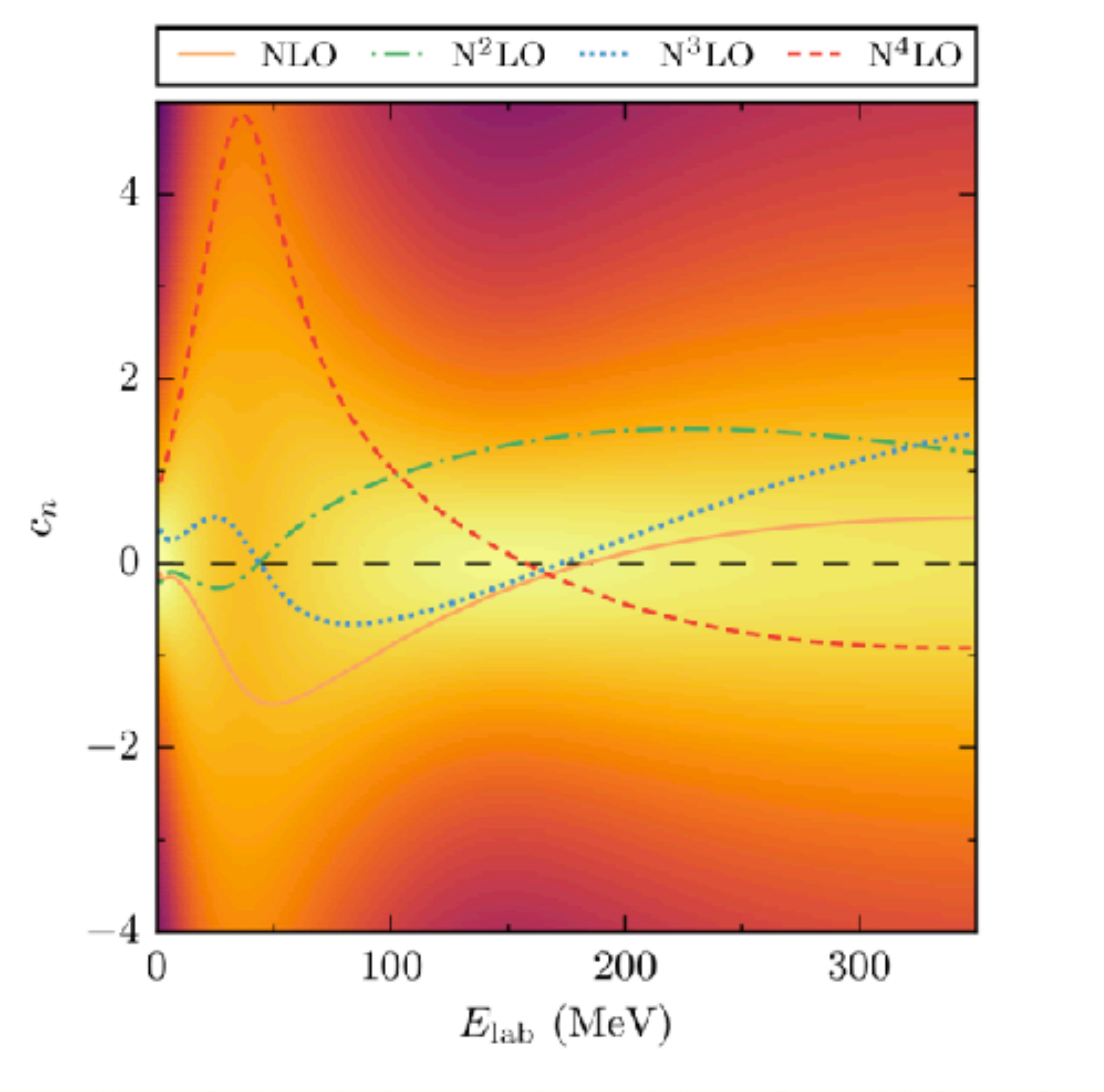
GAUSSIAN PROCESSES

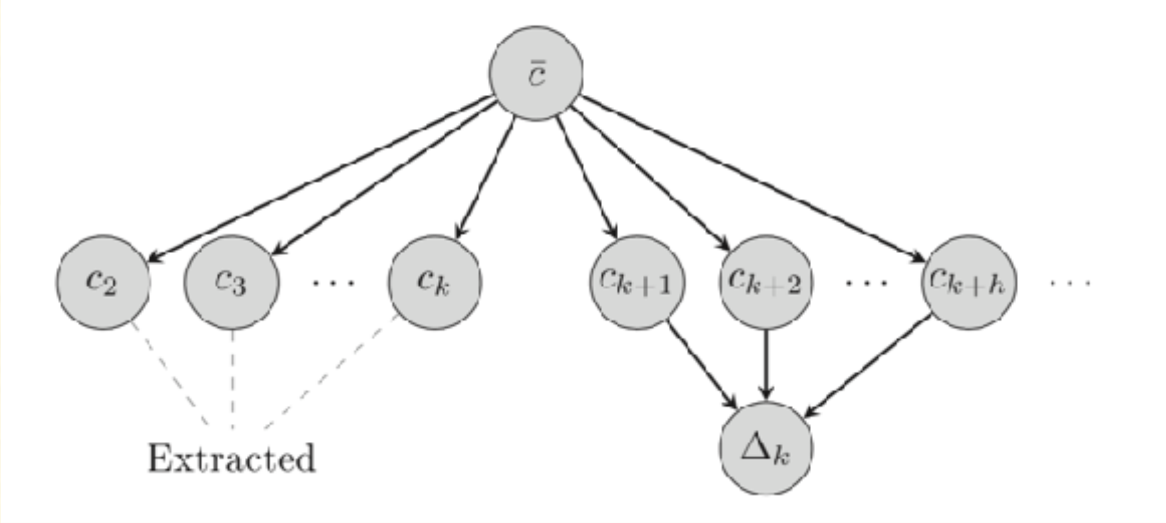
- Non-parametric, probabilistic model for a function
- Suppose we already know f at $x_1, x_2, x_3, \ldots, x_n$.
- Specify how f(y) is correlated with $f(x_1)$, $f(x_2)$, …..; don't specify underlying functional form.
- But value of f(y) is not deterministic: it's given by a probability distribution.
- Correlation decreases as points get further away from each other
- Specify correlation matrix of f at x and x', e.g.:

$$k(x, x') = \bar{c}^2 \exp\left(-\frac{(x - x')^2}{2l^2}\right)$$

- k(x,x') determines the probability of getting a particular value of f(x), if the value of f(x') is known

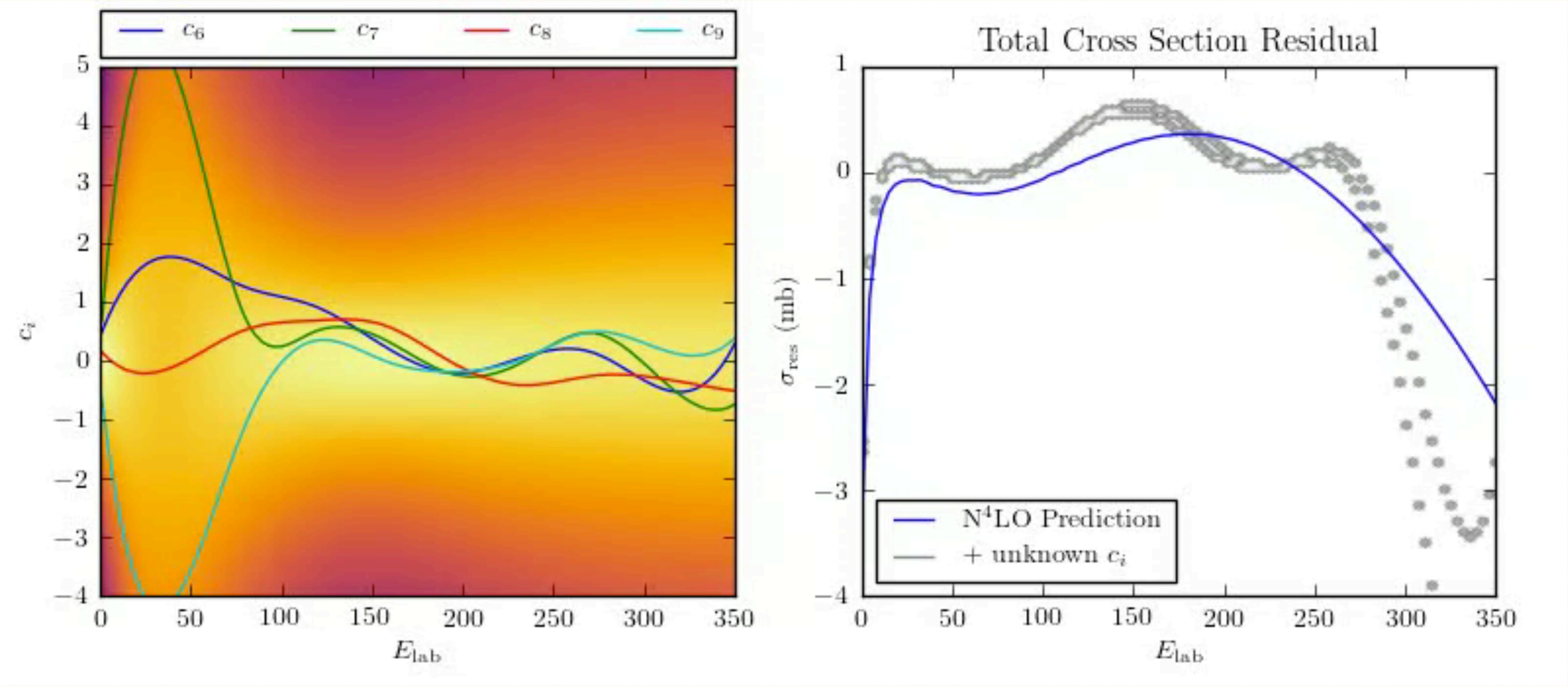
INFERRING THE NEXT COEFFICIENT





$$\Delta\sigma(E) = \sigma_0(E)\left[c_6(E)x^6 + c_7(E)x^7 + c_8(E)x^8 + c_9(E)x^9\right]$$

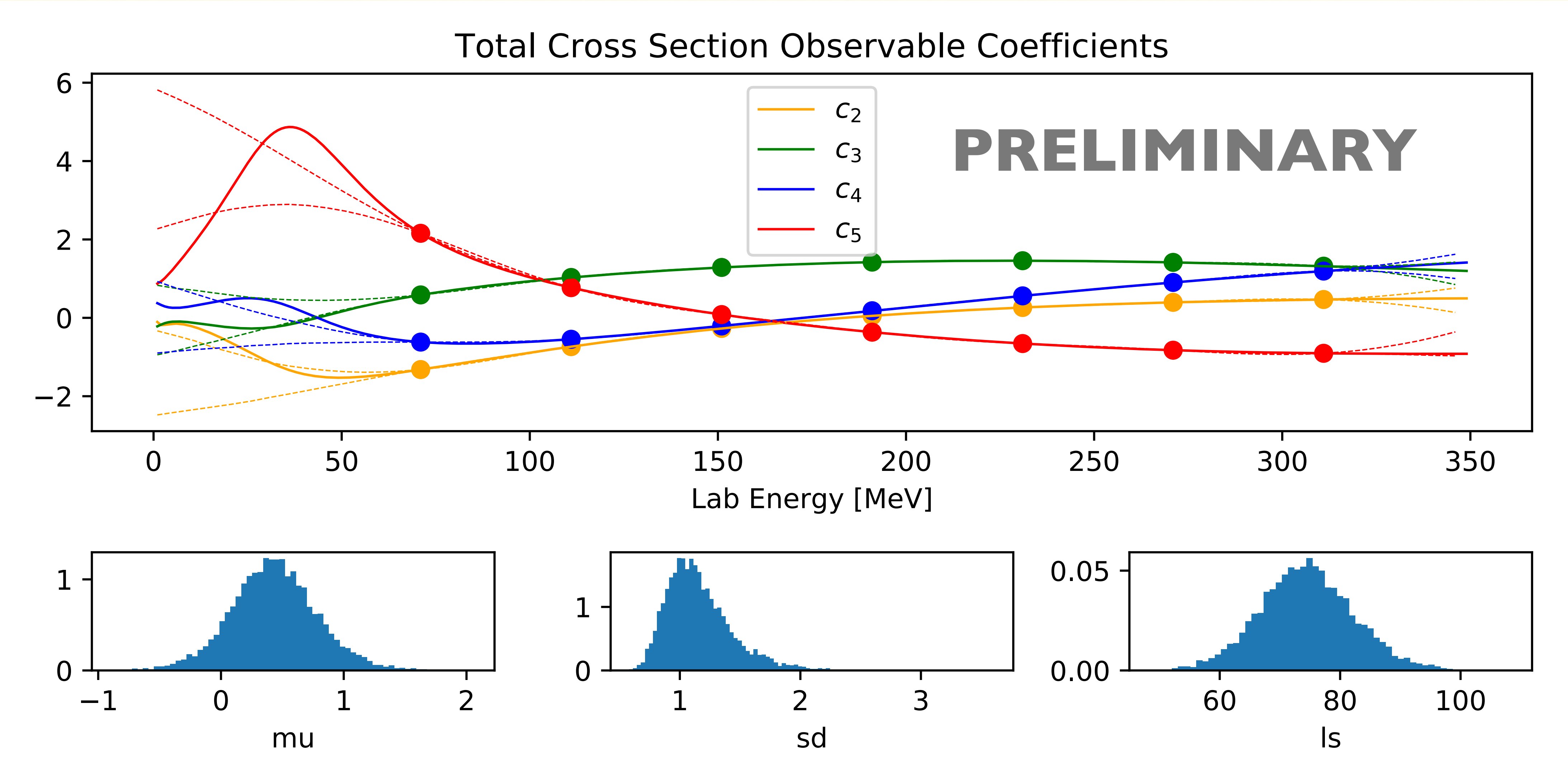
INFERRING THE NEXT COEFFICIENT



Gaussian process "model" for χEFT coefficients, trained on c_2 -c_5, can be used to predict distribution of N⁶LO corrections

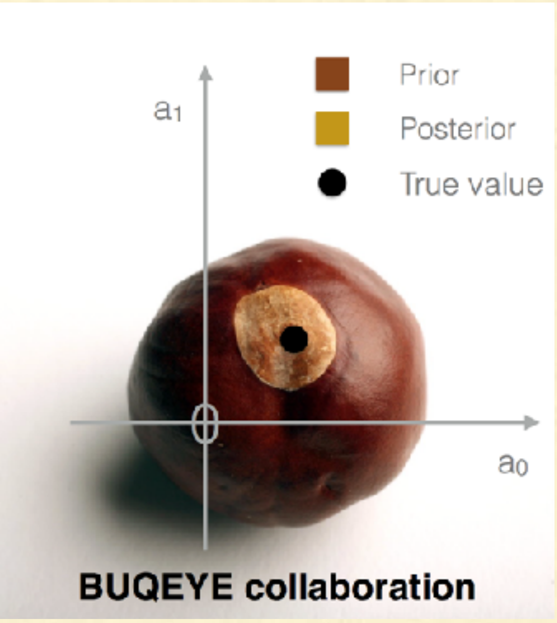
$$\Delta\sigma(E) = \sigma_0(E)\left[c_6(E)x^6 + c_7(E)x^7 + c_8(E)x^8 + c_9(E)x^9\right]$$

PARAMETERS AND PHYSICS



- For E>70 MeV, so "transition" in Q does not affect length scale
- Length scale peak around 70 MeV
- (The common) cbar peaks just above 1, average peaks slightly above 0

SUMMARY



Why I think truncation errors are interesting:

- Bayesian analysis of truncation error makes explicit what the assumptions about the EFT convergence pattern are
- The pdfs obtained thereby are easy to write down and use
- Truncation errors are stable under choices of "naturalness priors"
- Physics can be extracted: success ratios and breakdown-scale inference. Can combine pdfs with those from parameter estimation

Sarah's talk

- But need to understand which "data" from EFT calculation are and are not correlated: Gaussian process models of EFT-truncation errors

BUQEYE Github (under development): http://buqeye.github.io

BONUS MATERIAL: RKE POTENTIALS

Λ=450 MeV