Canada's national laboratory for particle and nuclear physics and accelerator-based science

Calculations of $0\nu\beta\beta$ with consistently evolved operators

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Goal: solve the nuclear eigenvalue problem

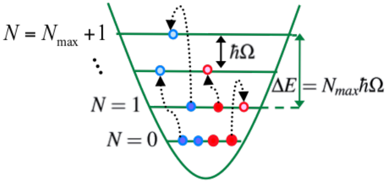
$$H\left|\Psi_k\right\rangle = E_k \left|\Psi_k\right\rangle, \text{ where } H = \sum_i^A T_i + \sum_{i<j} V_{ij} + \sum_{i<j<f} V_{ijf} + \cdots$$

with nucleons as the degrees of freedom

The No-core Shell Model

Expand in anti-symmetrized products of harmonic oscillator single-particle states

$$\left|\Psi_k\right\rangle = \sum_{N=0}^{N_{max}} \sum_j c_{Nj}^k \left|\Phi_{Nj}\right\rangle$$



Calculations should converge to the exact value as $N_{max} \to \infty$

Unitary transformation that decouples high and low momentum physics

$$H_\alpha = U_\alpha H U_\alpha^\dagger \text{ where } U_\alpha U_\alpha^\dagger = 1$$

$$\frac{\mathrm{d}H_\alpha}{\mathrm{d}\alpha} = [\eta_\alpha, H_\alpha]$$

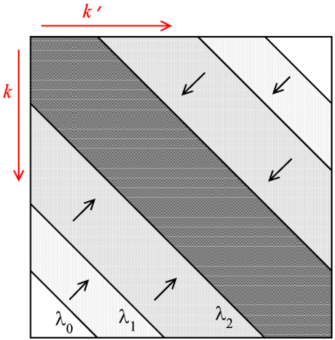
$$\eta_\alpha = \frac{\mathrm{d}U_\alpha}{\mathrm{d}\alpha} U_\alpha^\dagger = -\eta_\alpha^\dagger$$

Choose a generator, e.g. $\eta_\alpha = [T, H_\alpha]$

$$\lambda = \alpha^{-1/4}$$

$$H_{\lambda=\infty} = H, \ U_{\lambda=\infty} = 1$$

$$H_{\alpha=0} = H, \ U_{\alpha=0} = 1$$



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General Operators

$$H\left|\Psi_k\right\rangle = E_k\left|\Psi_k\right\rangle \;\rightarrow\; H_\alpha\left|\Psi_{k,\alpha}\right\rangle = E_k\left|\Psi_{k,\alpha}\right\rangle$$

General operators must also be transformed:

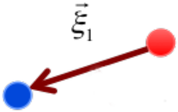
$$\left\langle\Psi_f\right|\hat{O}\left|\Psi_i\right\rangle = \left\langle\Psi_{f,\alpha}\right|\hat{O}_\alpha\left|\Psi_{i,\alpha}\right\rangle \text{ where } \hat{O}_\alpha = U_\alpha\hat{O}U_\alpha^\dagger$$

$$U_\alpha = \sum_k\left|\Psi_{k,\alpha}\right\rangle\left\langle\Psi_k\right|$$

Two-body Implementation

For $|\psi_k\rangle = |kj^\pi tt_z\rangle = \sum c^k_{n\ell s} |n\ell sj^\pi tt_z\rangle$, U_α is constructed in blocks:

$$U_\alpha^{j^\pi tt_z} = \sum_k |kj^\pi t, \alpha\rangle \langle kj^\pi t|$$



Non-scalar operators may connect states with $j^\pi tt_z$, e.g.

$$\langle f, j_f| O_\alpha |i, j_i\rangle = \langle f, j_f| U_\alpha^{j_f} O U_\alpha^{j_i \dagger} |i, j_i\rangle$$

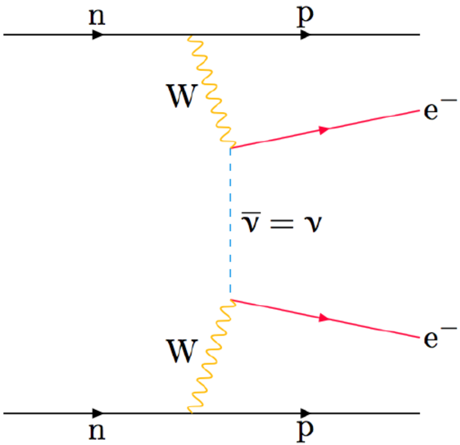
Converting to single-particle basis:

$$\langle a'b'J_{a'b'}| O_\alpha |abJ_{ab}\rangle \qquad a = n_a, \ell_a, j_a$$

$$= \sum_{if} c^{if}_{a'b'ab} \langle f, j_f| O_\alpha |i, j_i\rangle$$

Neutrino-less Double-Beta Decay

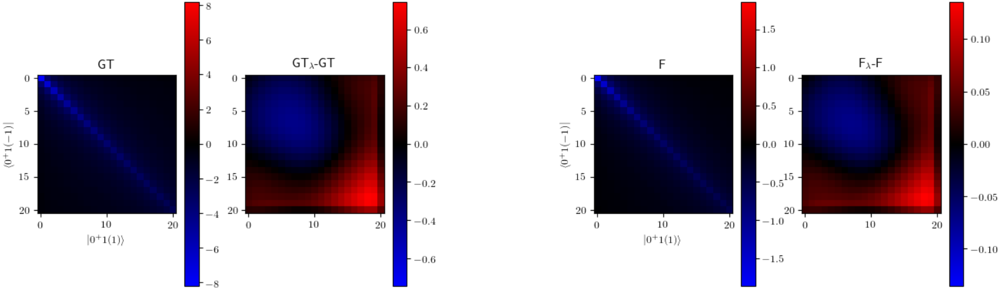
- discovery would confirm violation of Lepton number
- determine Majorana character of neutrinos
- determine neutrino masses
- candidates include ^{48}Ca, ^{76}Ge and ^{136}Xe



(C. Payne, MSc Thesis)

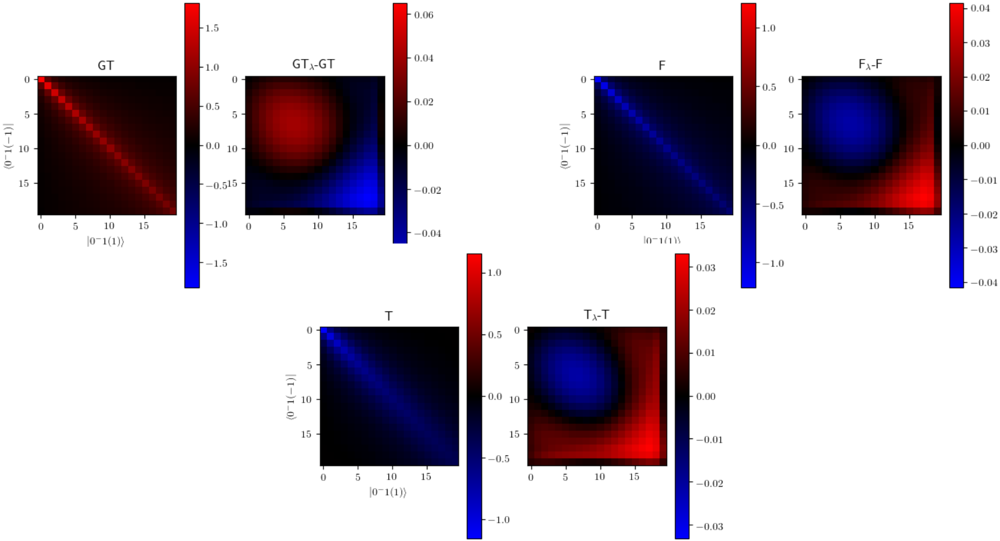
$$\hat{O}_{0\nu\beta\beta} = \hat{O}_{GT} + \hat{O}_F + \hat{O}_T \quad \text{from J. Engel}$$

$^1S_0$:



Two-body SRG: $\lambda = 2\ \text{fm}^{-1}$, chiral NN @ N^{3}LO

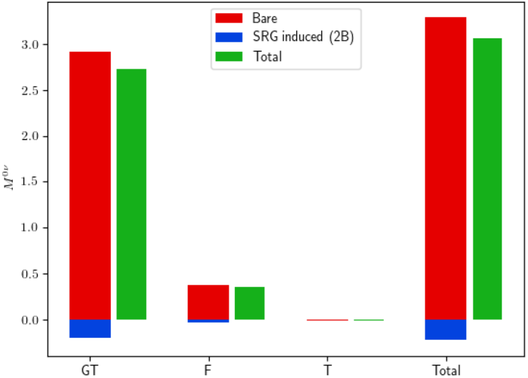
$^3P_0$:



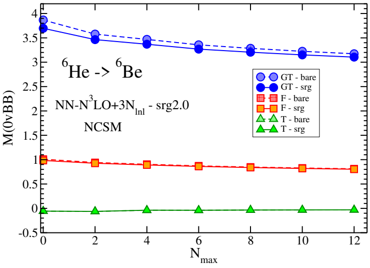
Two-body SRG: $\lambda = 2\ \text{fm}^{-1}$, chiral NN @ N^{3}LO

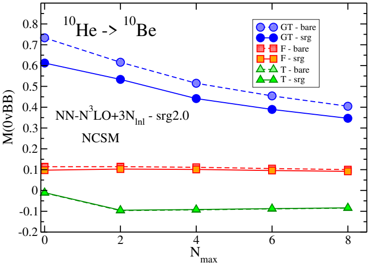
Preliminary Results: $0\nu\beta\beta_{\lambda,2b}$

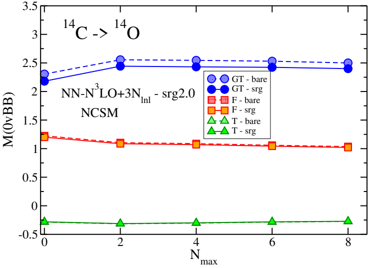
- Shell-model (J. Engel, M. Horoi)
- ^{76}Ge
- JUN45 interaction, $\hbar\omega = 9.23$ MeV
- $\sim 7\%$ effect, $\lambda = 2\ \text{fm}^{-1}$

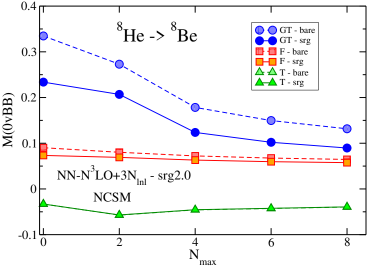


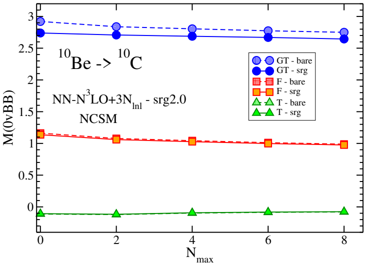
Two-body SRG in Light Nuclei



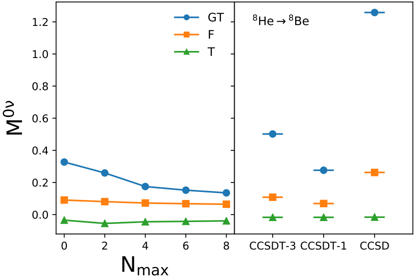


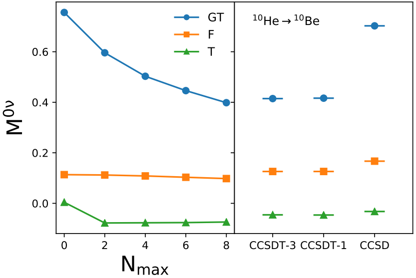




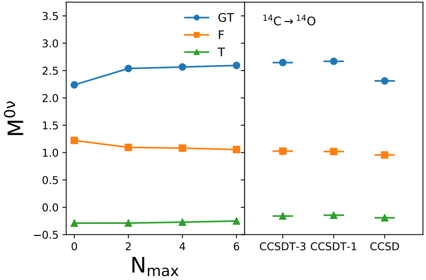


Benchmarks with Coupled Cluster





(Figures from Sam Novario)
EM 1.8/2.0



Also seems consistent with GFMC
(Pastore **PRC97** 014606 2018)

Summary and Outlook

- Operators must be SRG evolved to converge to the correct result
- Method implemented in 2B and 3B for arbitrary operators
- So far: $\sigma\tau$, axial MEC, $0\nu\beta\beta$, radius, E2
- Available in single-particle coordinates
- Results for $0\nu\beta\beta_{\lambda,2b}$: ^{8}H$\rightarrow^{8}$Be, ^{14}C$\rightarrow^{14}$O, etc
- In progress:
  - Application of $0\nu\beta\beta_{\lambda,3b}$ matrix elements in many-body methods
  - Quantification of 2- and 3-body evolution effects

Induced many-body terms

SRG transformations introduce higher-body terms in operators:

$$U_\alpha \hat{O} U_\alpha^\dagger = \hat{O}_\alpha^{(1)} + \hat{O}_\alpha^{(2)} + \hat{O}_\alpha^{(3)} + \dots$$

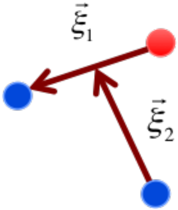
Each term, $\hat{O}_\alpha^{(a)}$, must be determined in the appropriate a-body system ($a \leq A$).
E.g. if $O = O^{(2)}$:

$$O_\alpha^{(2)} = U_\alpha^{(2)} O^{(2)} U_\alpha^{\dagger(2)} \qquad U_\alpha^{(2)} = \sum \left|\psi_{\alpha,a=2}\right\rangle \left\langle\psi_{a=2}\right|$$

$$O_\alpha^{(3)} = U_\alpha^{(3)} \left\langle O^{(2)} \right\rangle^{(3)} U_\alpha^{\dagger(3)} - \left\langle O_\alpha^{(2)} \right\rangle^{(3)} \qquad U_\alpha^{(3)} = \sum \left|\psi_{\alpha,a=3}\right\rangle \left\langle\psi_{a=3}\right|$$

For $|\psi_k\rangle = |kJ^\rho T\rangle = \sum c^k_{Ni} |NiJ^\rho T\rangle$
$|NiJ^\rho T\rangle = \sum \mathcal{C}^{NiJT}_{n\ell sjt;\mathcal{N}\mathcal{L}\mathcal{J}} |(n\ell sjt;\mathcal{N}\mathcal{L}\mathcal{J})JT\rangle \quad (N = 2n + \ell + 2\mathcal{N} + \mathcal{L})$

$$U^{J^\rho T}_\alpha = \sum_k |kJ^\rho T, \alpha\rangle \langle kJ^\rho T|$$



Non-scalar operators may connect states with $J^\rho T(T_z)$, e.g.

$$\langle f, J_f| O_\alpha |i, J_i\rangle = \langle f, J_f| U^{J_f}_\alpha O U^{J_i\dagger}_\alpha |i, J_i\rangle$$

Converting to single-particle basis:

$$\langle a'b'J_{a'b'}c'J_{a'b'c'}| O_\alpha |abJ_{ab}cJ_{abc}\rangle \qquad a = n_a, \ell_a, j_a$$
$$= \sum_{if} c^{if}_{a'b'c'abc} \langle f, J_f| O_\alpha |i, J_i\rangle$$

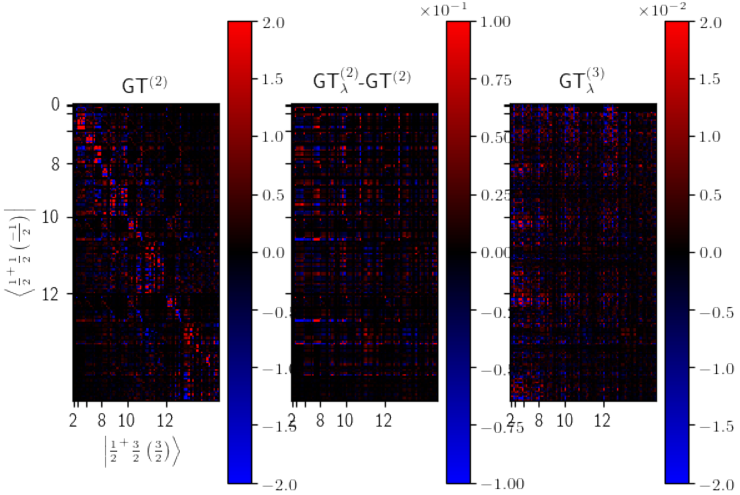
Three-body Implementation (cont.)

- Generalized code to calculate:

$$\langle f, J_f || \hat{O}^{(3)} || i, J_i \rangle = \frac{1}{36} \sum \langle \alpha\beta\gamma | \hat{O} | \delta\epsilon\omega \rangle \langle f, J_f || a^\dagger_\alpha a^\dagger_\beta a^\dagger_\gamma a_\omega a_\epsilon a_\delta || i, J_i \rangle$$

- Decouple $\langle abJ_{ab}cJ_{abc} | \hat{O} | deJ_{de}fJ_{def} \rangle \rightarrow \langle abc | \hat{O} | def \rangle$ on the fly
- Benchmarked general operator method with three-body interaction

Three-body SRG:
$\lambda = 2\ \text{fm}^{-1}$
NN@N^{3}LO



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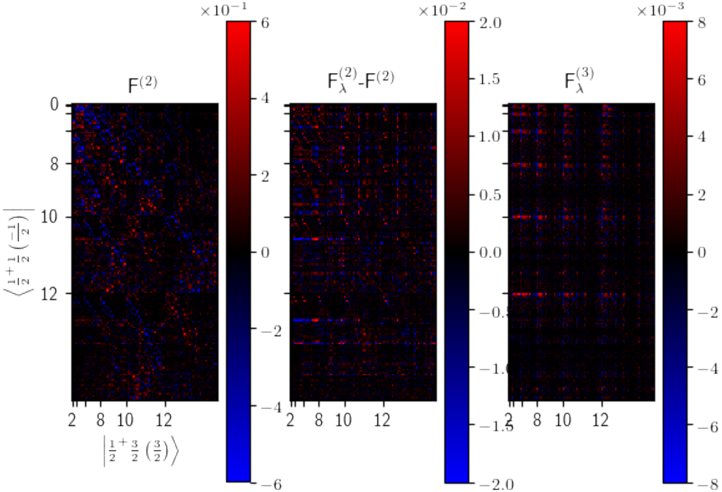

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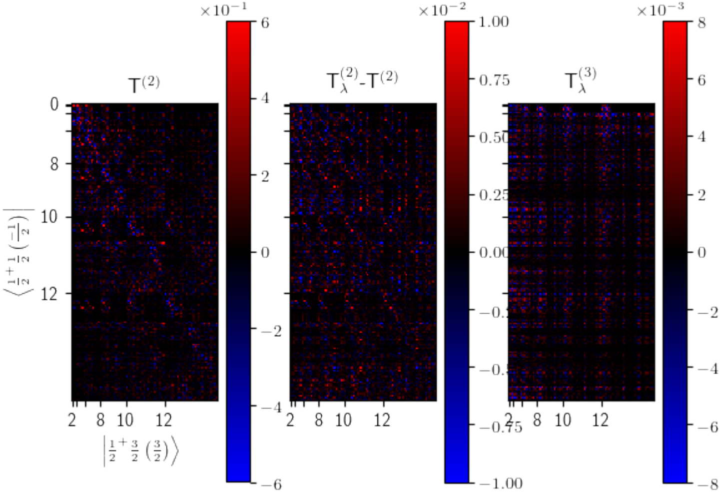

$0\nu\beta\beta$ Operators

$$\hat{O}_{0\nu\beta\beta} = \hat{O}_{GT} + \hat{O}_{F} + \hat{O}_{T}$$

$$O_\gamma = H_\gamma y_\gamma \tau_1^+ \tau_2^+$$

$$H_\gamma(r_{12}) = \frac{2R}{\pi} \int_0^\infty \mathrm{d}q \frac{q \cdot f_\gamma(q \cdot r_{12}) h_\gamma(q^2)}{q + E_0^{\text{cl}}}$$

$$y_\gamma = \begin{cases} 1 & \gamma = F \\ \sigma_1 \cdot \sigma_2 & \gamma = GT \\ \sqrt{\frac{24\pi}{5}} Y_2(\hat{r_{12}}) \left(3(\sigma_1 \cdot r_{12})(\sigma_2 \cdot r_{12}) - \sigma_1 \cdot \sigma_2\right) & \gamma = T \end{cases}$$