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INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

Ab initio Effective Potentials for NA Scattering based on NCSM Nonlocal One-Body Densities

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NERSC National Energy Research Scientific Computing Center

TRIUMF 2019 Workshop
Progress in *ab initio* Techniques in Nucear Physics

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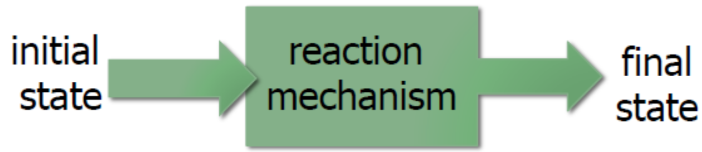
Exotic Nuclei are usually short lived:

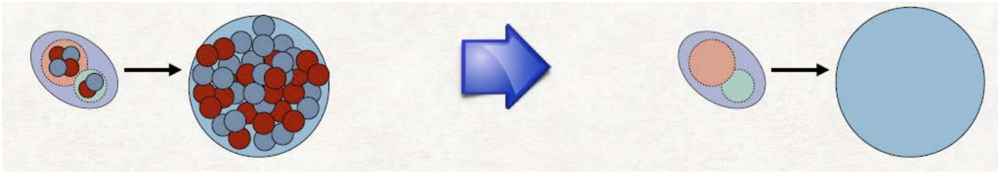
Have to be studied with reactions in inverse kinematics

e.g. direct reaction:



Challenge:

- **In the continuum, theory can solve the few-body problem exactly.**



Many-body problem

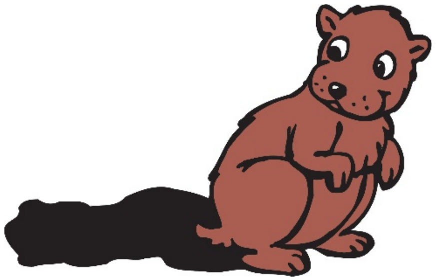
Few-body problem

Example (d,p) Reactions:

Reduce Many-Body to Few-Body Problem

Solve few-body problem

“Shadow” ?



Challenges & Opportunities

Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

- **Nucleon-nucleon interaction believed to be well known:**
  today: chiral interactions

- **Effective proton (neutron) interactions:**
  - purely phenomenological optical potentials fitted to data
  - optical potentials with theoretical guidance
  - microscopic optical potentials

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Isolate relevant degrees of freedom

Formally: separate Hilbert space into P and Q space, and calculate in P space

Projection on P space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly (Feshbach, Annals Phys. 5 (1958) 357-390)

Effective Interactions: non-local and energy dependent

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Effective Interactions: non-local and energy dependent

History: Phenomenological optical potentials

Either fitted to a large global data set OR to a restricted data set

Most general form of optical potential

- Σ_i [$V_{A,Z,N,E}(r)$ + i $W_{A,Z,N,E}(r)$] $Operator_{(i)}$
- **Functions are of Woods-Saxon type**

Have central and spin orbit term

Fit cross sections, angular distributions polarizations, for a set of nuclei (lightest usually ^{12}C).

No connection to microscopic theory

Today's Goal: effective interaction from *ab initio* methods

Start from many-body Hamiltonian with 2 (and 3) body forces

Theoretical foundations laid by Feshbach and Watson in the 1950s

Feshbach:

→ **effective nA interaction via Green's function from solution of many body problem using basis function expansion, e.g. SCGF, CCGF (current truncation to singles and doubles)** energy ~ 10 MeV

Rotureau, Danielewicz, Hagen, Jansen, Nunes
PRC 95, 024315 (2017)

Idini, Barbieri, Navratil
J.Phys.Conf. 981. 012005 (2018)
Acta Phys. Polon. B48, 273 (2017))

Watson:

→ **Multiple scattering expansion, e.g. spectator expansion (current truncation to 2 active particles)**

Spectator Expansion:

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Expansion in:
- particles active in the reaction
- antisymmetrized in active particles

"fast reaction", i.e. ≥ 100 MeV

Elastic Scattering (Watson approach)

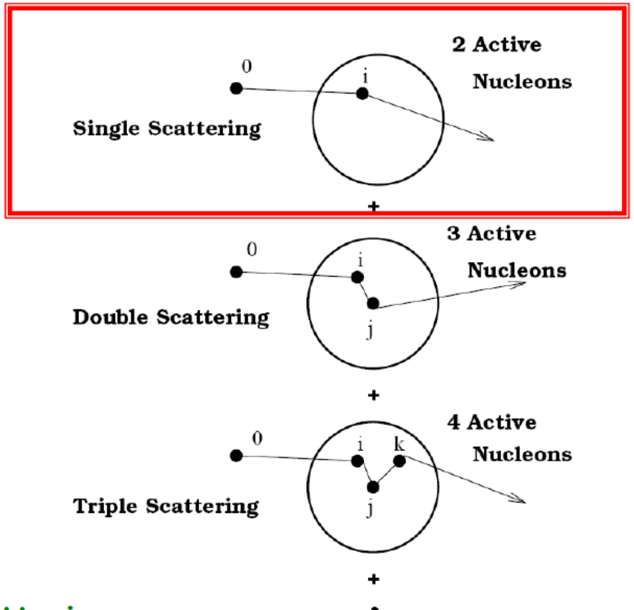
- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $\mathbf{P} = |\Phi_0\rangle\langle\Phi_0|$
  - With $\mathbf{1=P+Q}$ and $\mathbf{[P,G_0]=0}$
- For elastic scattering one needs: $\mathbf{P\,T\,P = P\,U\,P + P\,U\,P\,G_0(E)\,P\,T\,P}$

$$\mathbf{T = U + U\,G_0(E)\,P\,T}$$

$$\mathbf{U = V + V\,G_0(E)\,Q\,U} \quad \Leftarrow \text{ effective (optical) potential}$$

Up to here exact

Spectator Expansion of U :



1st order: single scattering: $\mathbf{U^{(1)} \approx \Sigma^{A}_{i=0}\,\tau_{0i}}$

Chinn, Elster, Thaler, PRC 47, 2242 (1993)

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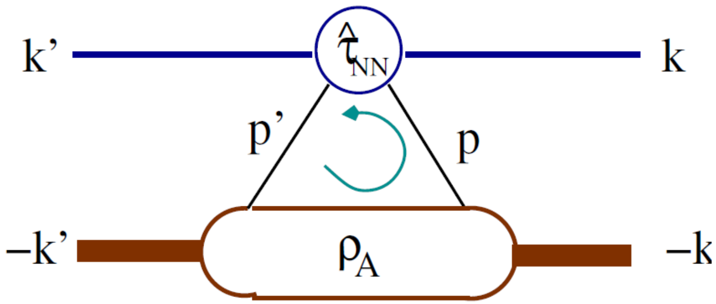
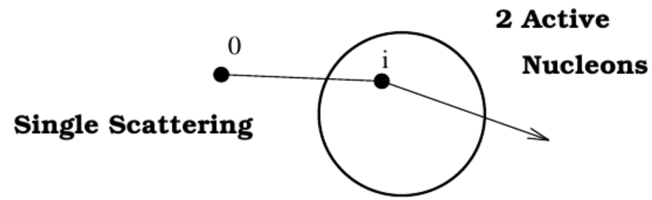
Computing the first order folding potential $U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$





NN scattering amplitudes

Nuclear one-body density

$$U_{el}(\mathbf{q},\mathbf{K}) = \sum_{i=n,p} \int d\mathbf{P}\; \eta(\mathbf{q},\mathbf{K},\mathbf{P})\; \hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}\left(\frac{A+1}{A}\mathbf{K} - \mathbf{P}\right), \mathcal{E}\right) \rho_i\left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right)$$

$$\mathbf{K} \equiv \frac{\mathbf{k}+\mathbf{k}'}{2} \qquad \mathbf{P} \equiv \frac{\mathbf{k}_i + \mathbf{k}'_i}{2} + \frac{\mathbf{K}}{A}$$

$$\mathrm{q} \equiv \mathrm{k}' - \mathrm{k}$$

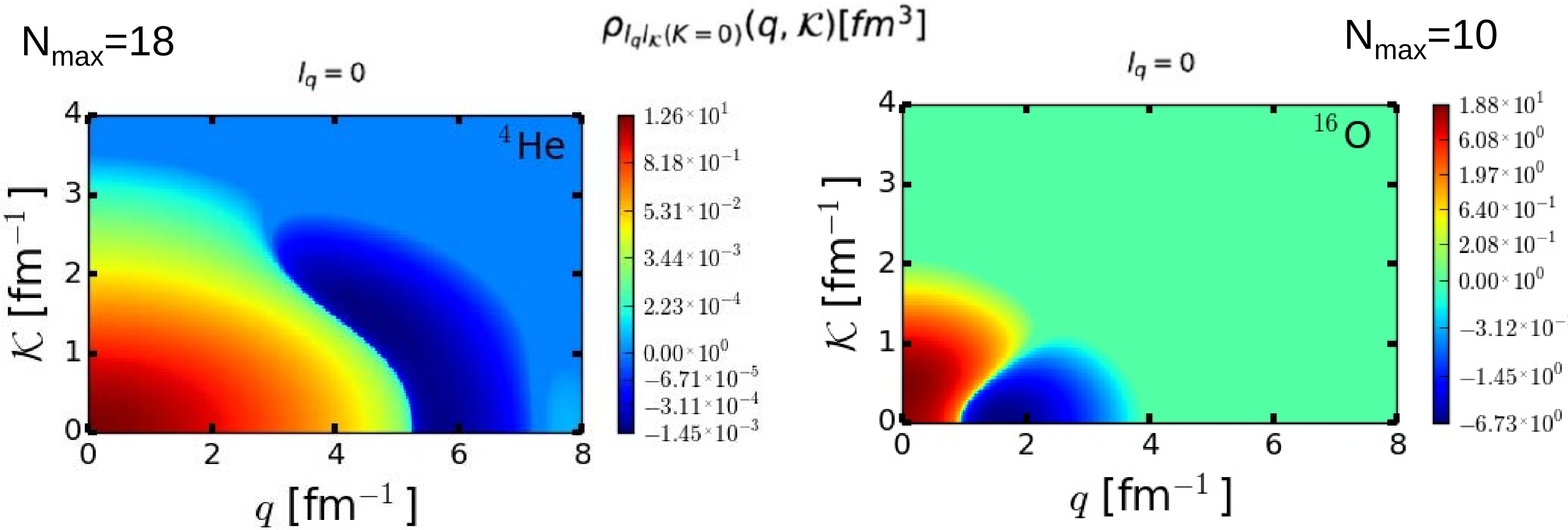
Same NN Interaction can now be used for NN t-matrix and one-body density matrix

Effective Potential is non-local and energy dependent

Details of implementation designed for energies ≥ 100 MeV

Nonlocal one-body densities from NCSM (or SA-NCSM)

translationally invariant ($NNLO_{opt}$, proton distribution, ħω=20 MeV)



$$\vec{q} = \vec{p'} - \vec{p}$$
$$\vec{\mathcal{K}} = \frac{1}{2}(\vec{p'} + \vec{p})$$

start from:

$$\rho_{sf}(\vec{r},\vec{r'}) = \langle \Psi' | \sum_{i=1}^{A} \delta^3(\vec{r}_i - \vec{r})\delta^3(\vec{r}_i' - \vec{r'}) | \Psi \rangle$$

remove CoM:

Burrows, Elster, Popa, Launey, Nogga, Maris, PRC 97, 024325 (2018)

and obtain $\rho_{ti}(\vec{q},\vec{\mathcal{K}})$ ≡ scalar function of 2 vector momenta

NN amplitude: $f_{NN}(k'k;E) = C \langle k' | t_{NN}(E) | k \rangle$

Variables (E,k',k,φ) ⇒ (E, q, K, θ)

with $q = k' - k$
$K = ½ (k' + k)$

NN t-matrix in Wolfenstein representation:

Projectile "*0*" : plane wave basis
Struck nucleon "*i*" : target basis

Usual assumption: Spin saturated ground state

$$\overline{\mathrm{M}}(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) = A(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,\mathbf{1} + i\cdot C(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,(\boldsymbol{\sigma}^{(0)}\otimes\mathbf{1} + \mathbf{1}\otimes\boldsymbol{\sigma}^{(i)})\cdot\hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{n}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{n}}_{NN})$$
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- -

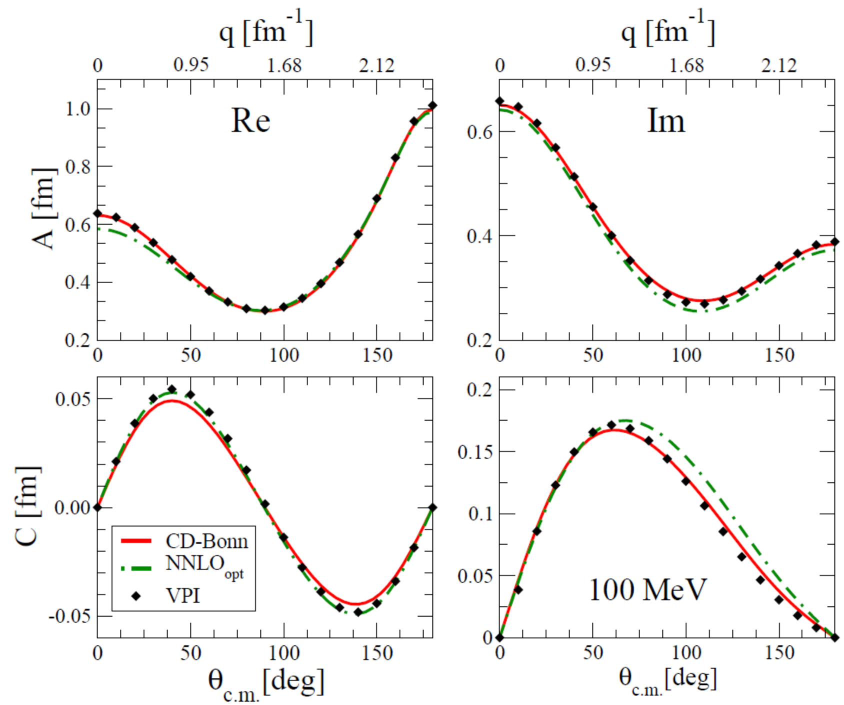
$$+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})\right]$$ Off-shell

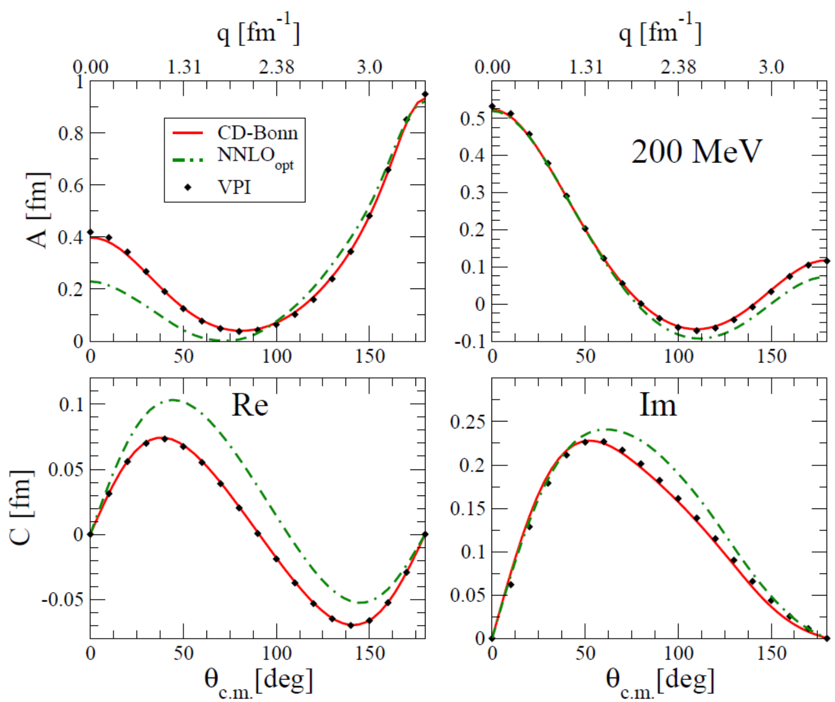
Remark: Spin dependence is not explicit in usual definition of one-body density matrix

Wolfenstein Amplitudes A and C

$NNLO_{opt}$ fitted to E_{lab}=125 MeV

→ max. momentum transfer ≈ 2.45 fm^{-1}



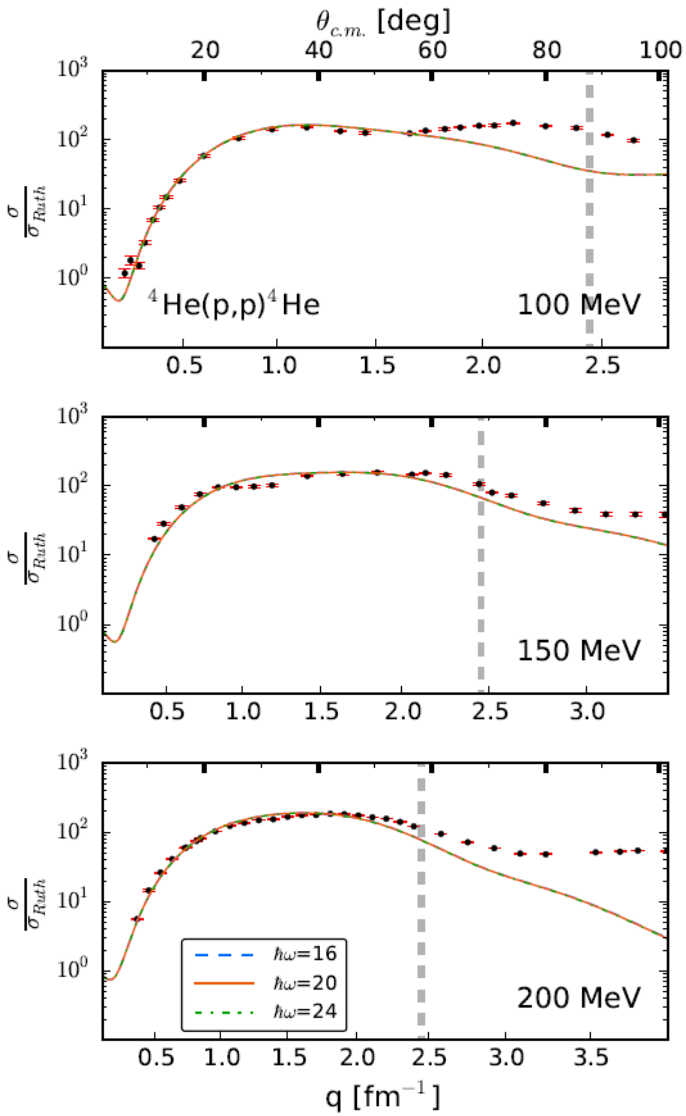


$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

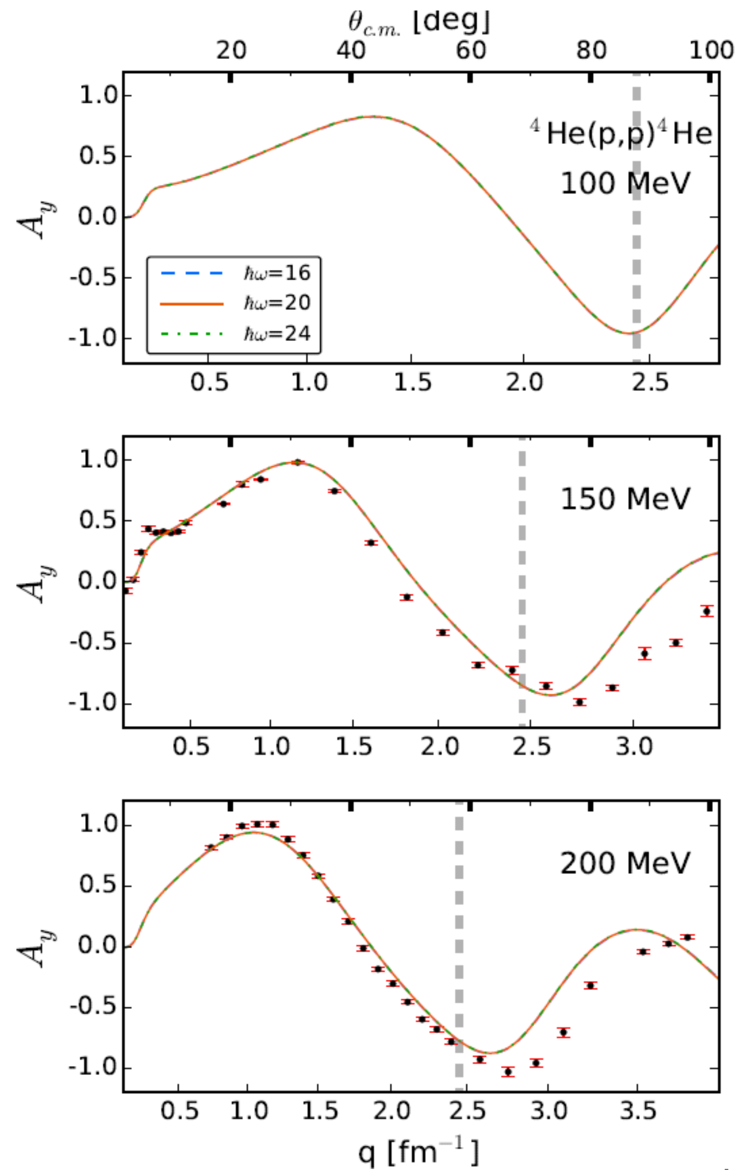
$$q \approx 480 \text{ MeV} = 2.45 \text{ fm}^{-1}$$

Nmax=18

^{4}He



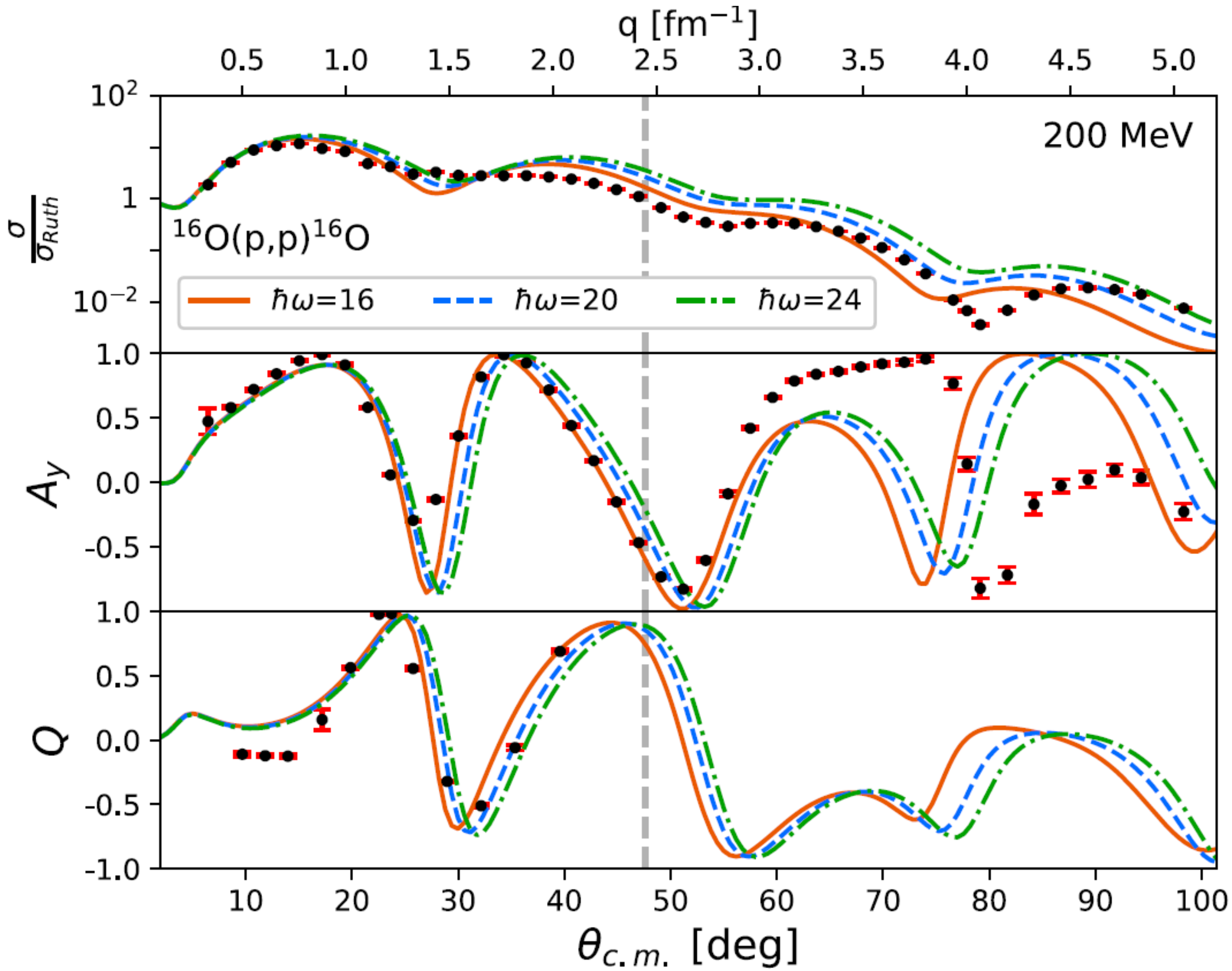
NNLO$_{opt}$ fitted up to Elab=125 MeV



Burrows, Elster, Weppner, Launey, Maris, Nogga, Popa
arXiv:1810.06442

^{16}O

N_{max}=10



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Burrows, Elster, Weppner, Launey,
Maris, Nogga, Popa
arXiv:1810.06442

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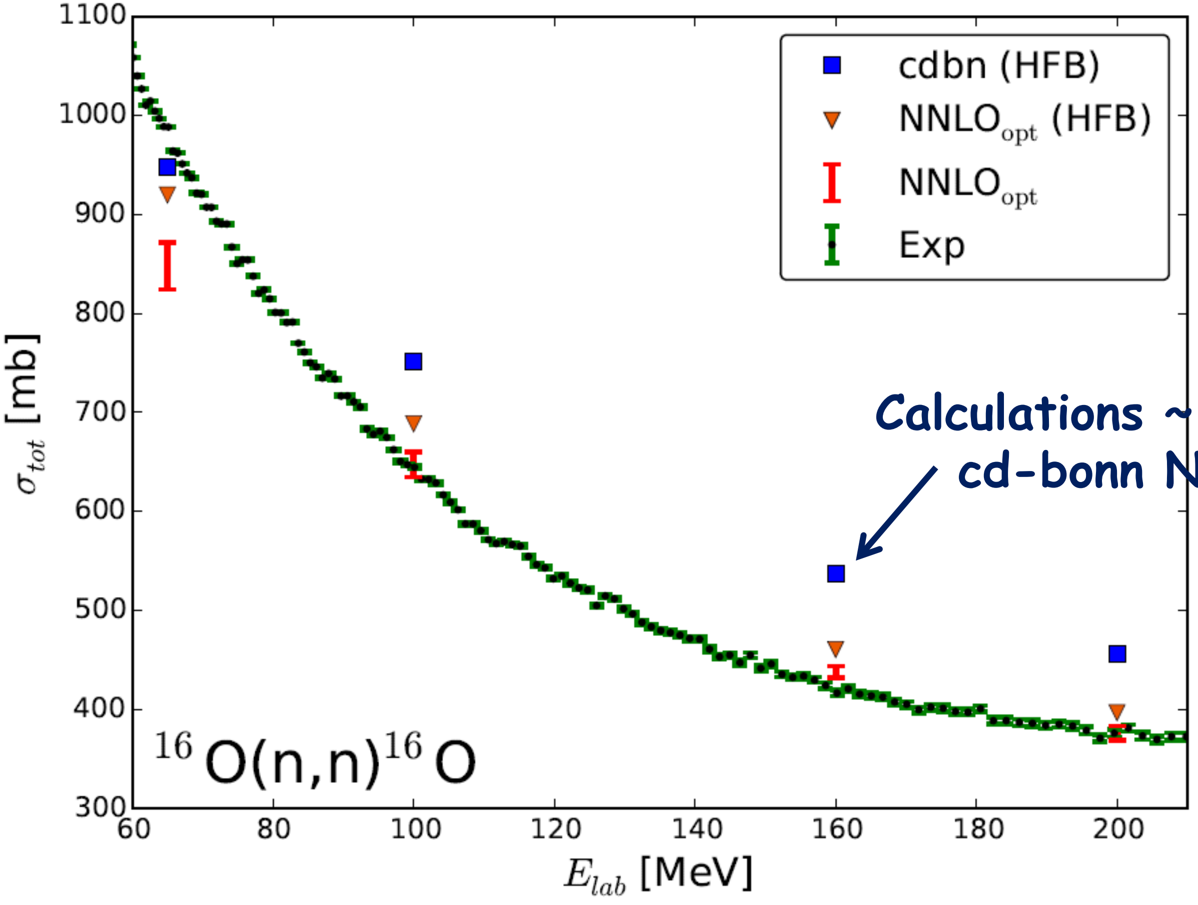
Total cross section for neutron scattering



Calculations ~1995 using cd-bonn NN t-matrix

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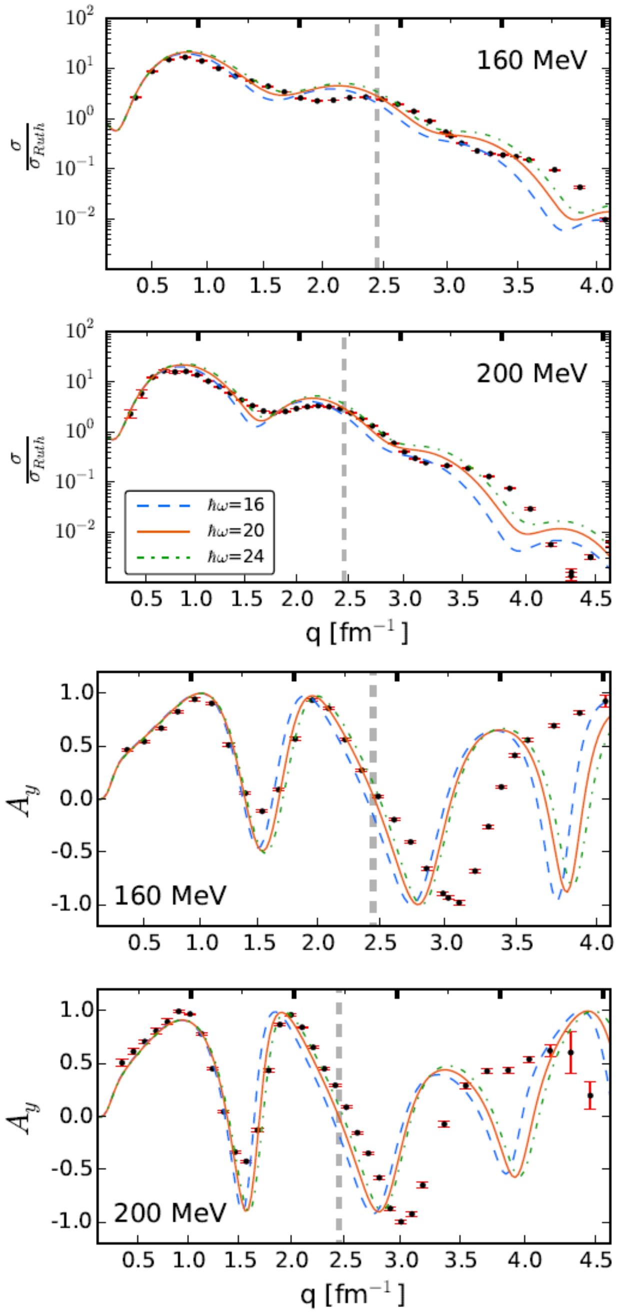
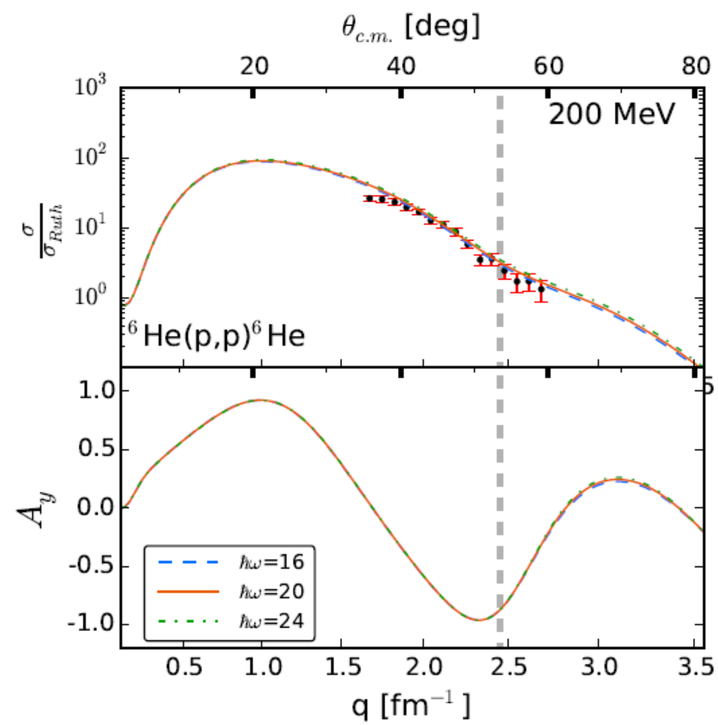
$^{12}C(p,p)^{12}C$



Note:

Implementation of first order term
(past, present, all groups)

only exact for spin saturated ground states
(≡ spin-flip of struck target nucleon neglected)



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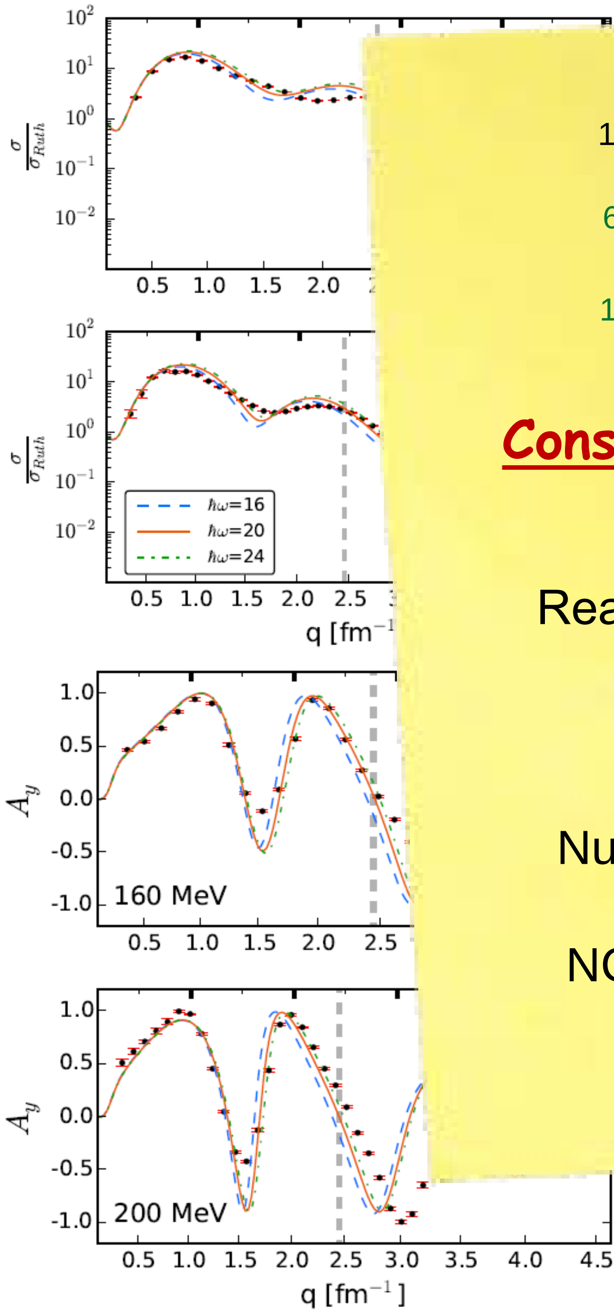
$^{12}C(p,p)^{12}C$



^{16}O: spin-0 contribution ~95%

^{6}He: spin-0 contribution ~ 80-85%

^{12}C: spin-0 contribution ~60%

Consider that calculations contain:

Reaction Theory ≈ 1992 ± 2

Crespo, Johnson, Tostevin
Chinn, Elster, Thaler, Weppner
Arellano, Brieva, Love

Nuclear force employed ≈ 2013

NCSM calculation matured ≈ 2000 +

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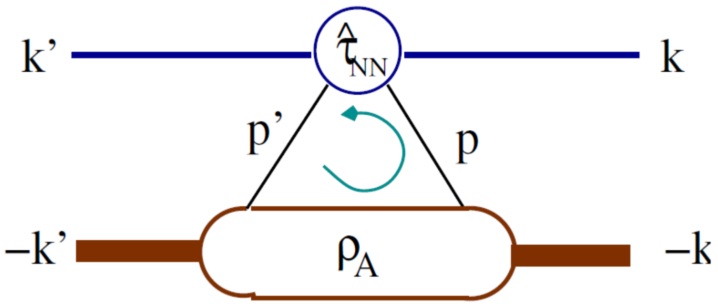
Improve on reaction theory (first order term)



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Define one-body density such that information on spin is contained explicitly

$$\rho_{q_s}^{K_s}(\vec{r}_s,\vec{r}_s') = \left\langle \Phi' \left| \sum_{i=1}^{A} \delta^3(r_i - r_s)\delta^3(r_i' - r_s')\,\widehat{\tau_{(i)}}_{q_s}^{K_s} \right| \Phi \right\rangle$$

Scalar:

$$K_s = 0 \quad : \quad \widehat{\tau_{(i)}}_0^0 = 1$$

Spin-dependent

$$\begin{aligned} K_s = 1 \quad &: \quad \widehat{\tau_{(i)}}_0^1 = 2\hat{\sigma}_z \\ &: \quad \widehat{\tau_{(i)}}_{-1}^1 = \frac{1}{\sqrt{2}}(\hat{\sigma}_x - i\hat{\sigma}_y) \\ &: \quad \widehat{\tau_{(i)}}_1^1 = -\frac{1}{\sqrt{2}}(\hat{\sigma}_x + i\hat{\sigma}_y) \end{aligned}$$

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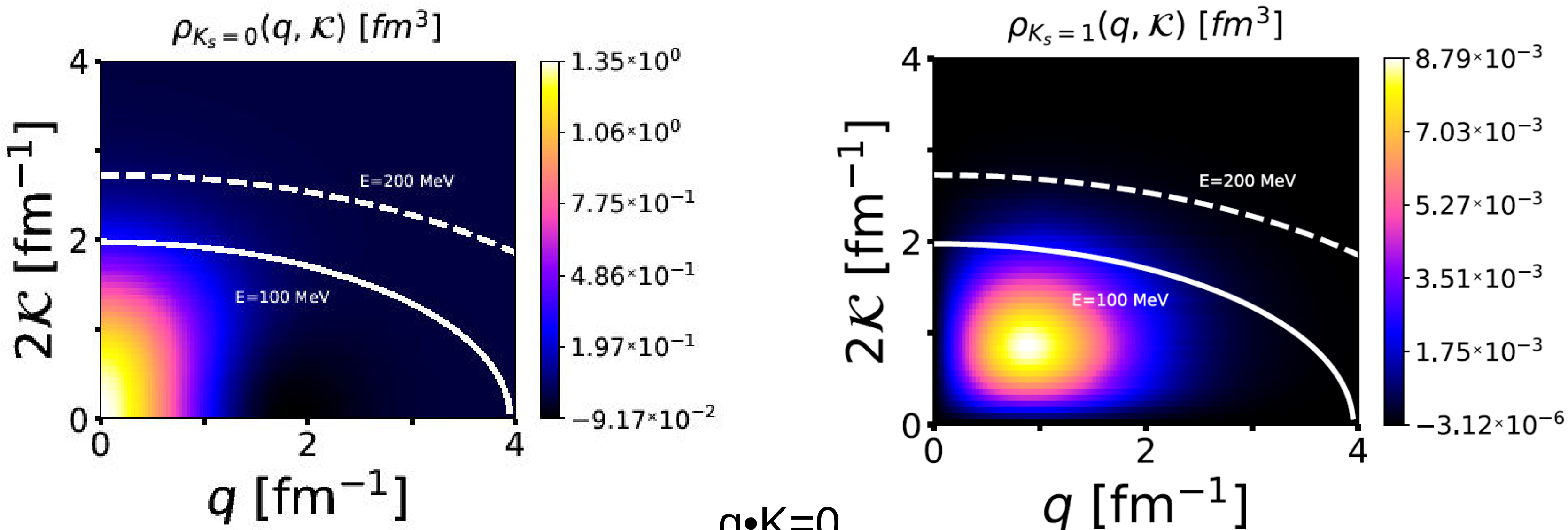
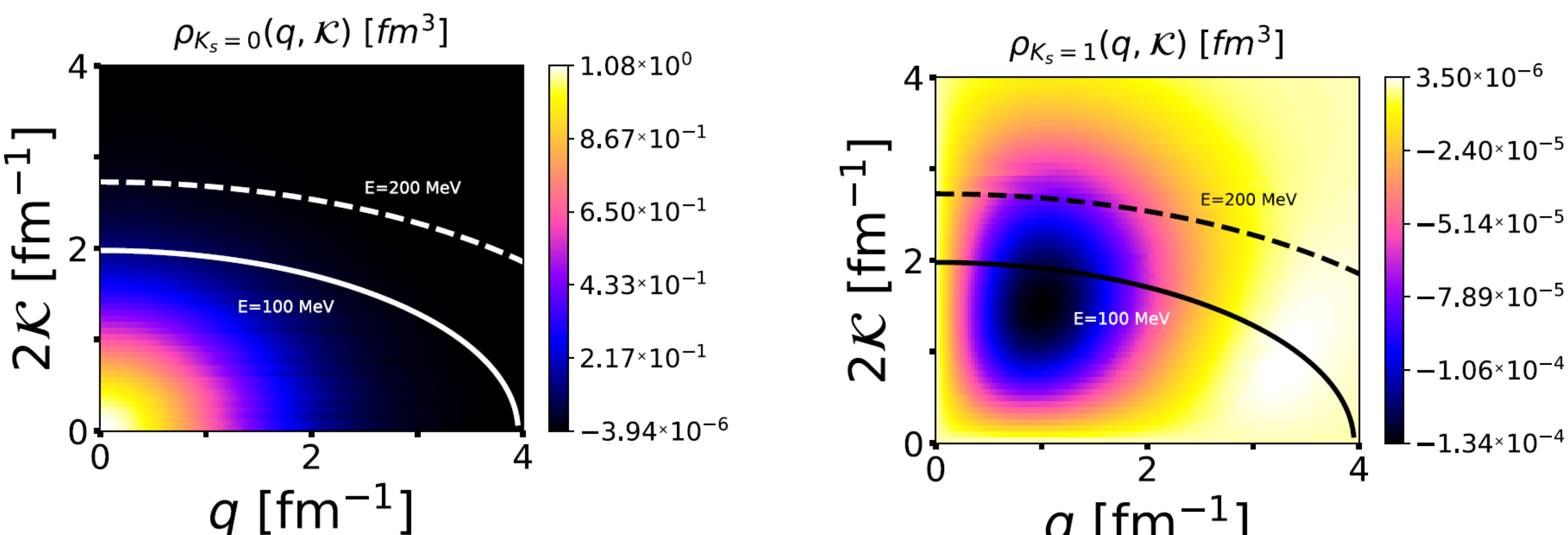
^{6}He : neutron density matrix (N$_{max}$=6 ħω=20 MeV) [CoM removed]



q•K=0

Proton density matrix



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$$+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathbf{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathbf{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathbf{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathbf{q}})\right]$$ Off-shell

Calculate matrix elements:

$$v_n(\vec{r}_s, \vec{r'}_s) = \left\langle \Phi' \left| \sum_{i=1}^{A} \delta^3(r_i - r_s)\delta^3(r'_i - r'_s) \left[\widehat{\mathcal{T}_{(i)}}^{K_s=1} \cdot \widehat{\mathbf{n}_{\mathrm{t.i.}}}^{1} \right]_0^0 \right| \Phi \right\rangle$$

Result: matrix elements containing σ•q and σ•K give zero net contribution

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NN amplitude: $f_{NN}(k'k;E) = C \langle k' | t_{NN}(E) | k \rangle$

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- -

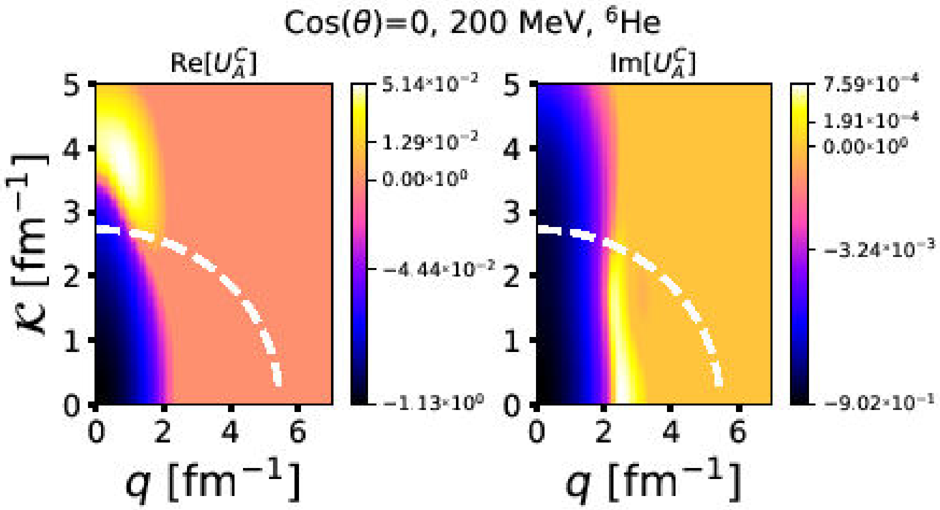
$$+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathbf{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathbf{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathbf{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathbf{q}})\right]$$ Off-shell

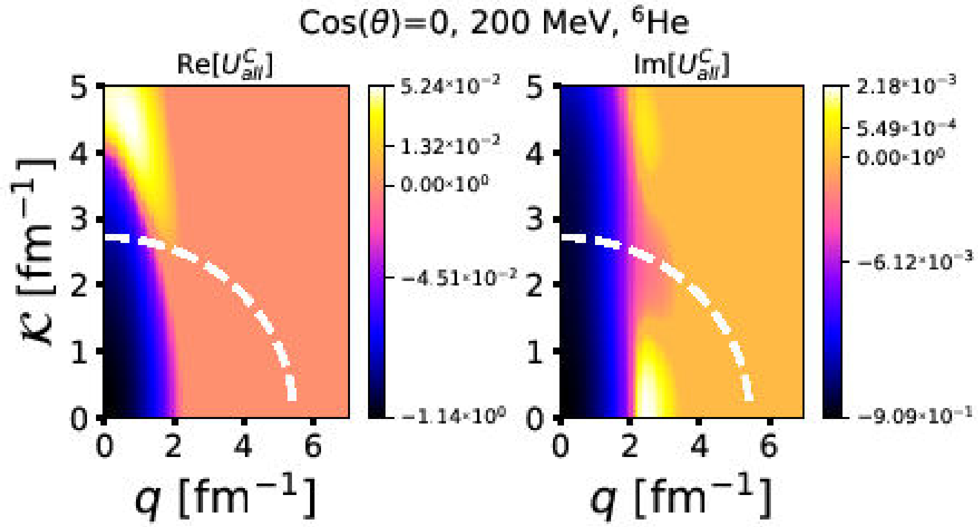
Contributions to an ab initio folding potential in first order for elastic p(n) scattering off spin-zero targets

Central part of effective potential (np)

On-shell condition:

$$\left(\frac{A-1}{2A}\right)^2 q^2 + K^2 = k_0^2$$





$$\mathbf{K} \equiv \frac{\mathbf{k}+\mathbf{k}'}{2}$$

$$\mathrm{q} \equiv \mathbf{k}' - \mathbf{k}$$

(N_{max}=6 ħω=20 MeV)

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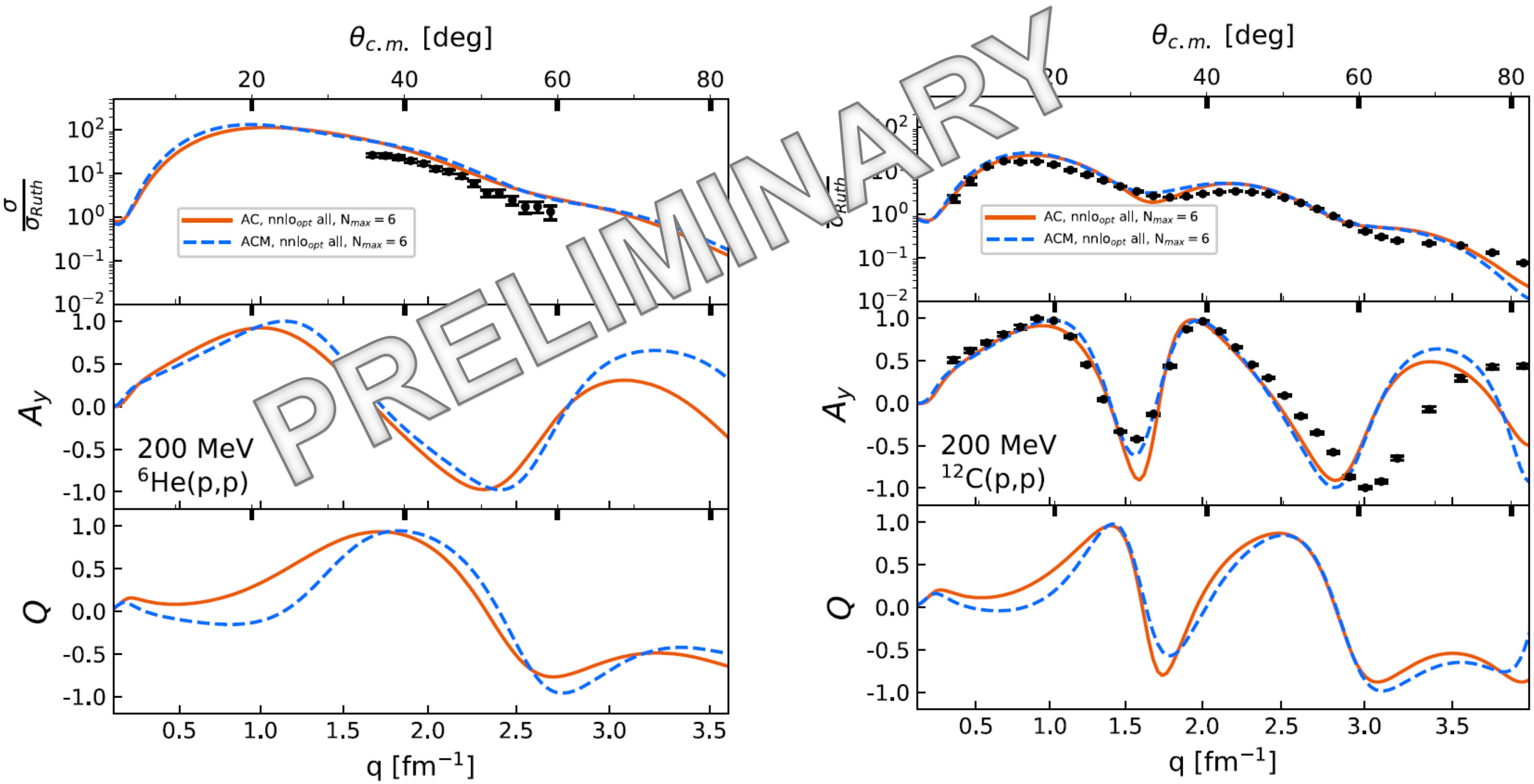
(N_{max}=6 ħω=20 MeV) -- only neutron-proton part with additional contributions



p+A and n+A effective interactions ('optical potentials')

- Renewed urgency in reaction theory community for microscopic input to e.g. (d,p) reaction models .
- Most likely complementary approaches needed for different energy regimes

Today: Consistent approach to p+A effective interaction becomes possible.

- **In the multiple scattering approach first order term non-spin-0 components in the ground state need to be included for an *ab initio* first order folding potential**

- **Different structure approaches need to be explored in this context: at present we use NCSM and SA-NCSM**

- Dependence on forces employed .....

- Revisit details of implementation of the reaction theory ...

- **Systematic approach to higher order corrections e.g. anti-symmetry, 3NFs**
  (hard but needs to be attempted)

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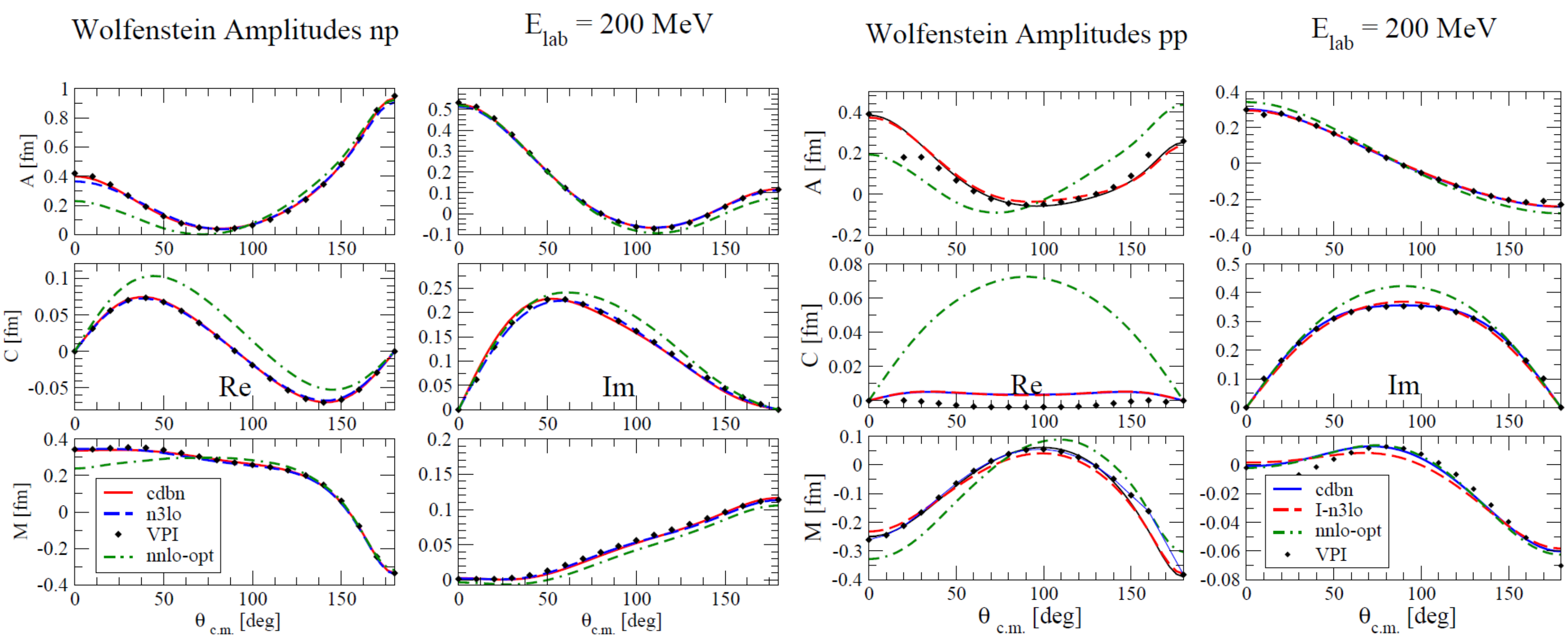
Wolfenstein Amplitudes np
E_lab = 200 MeV
Wolfenstein Amplitudes pp
E_lab = 200 MeV
A [fm]
C [fm]
M [fm]
Re
Im
Re
Im
cdbn
n3lo
VPI
nnlo-opt
cdbn
I-n3lo
nnlo-opt
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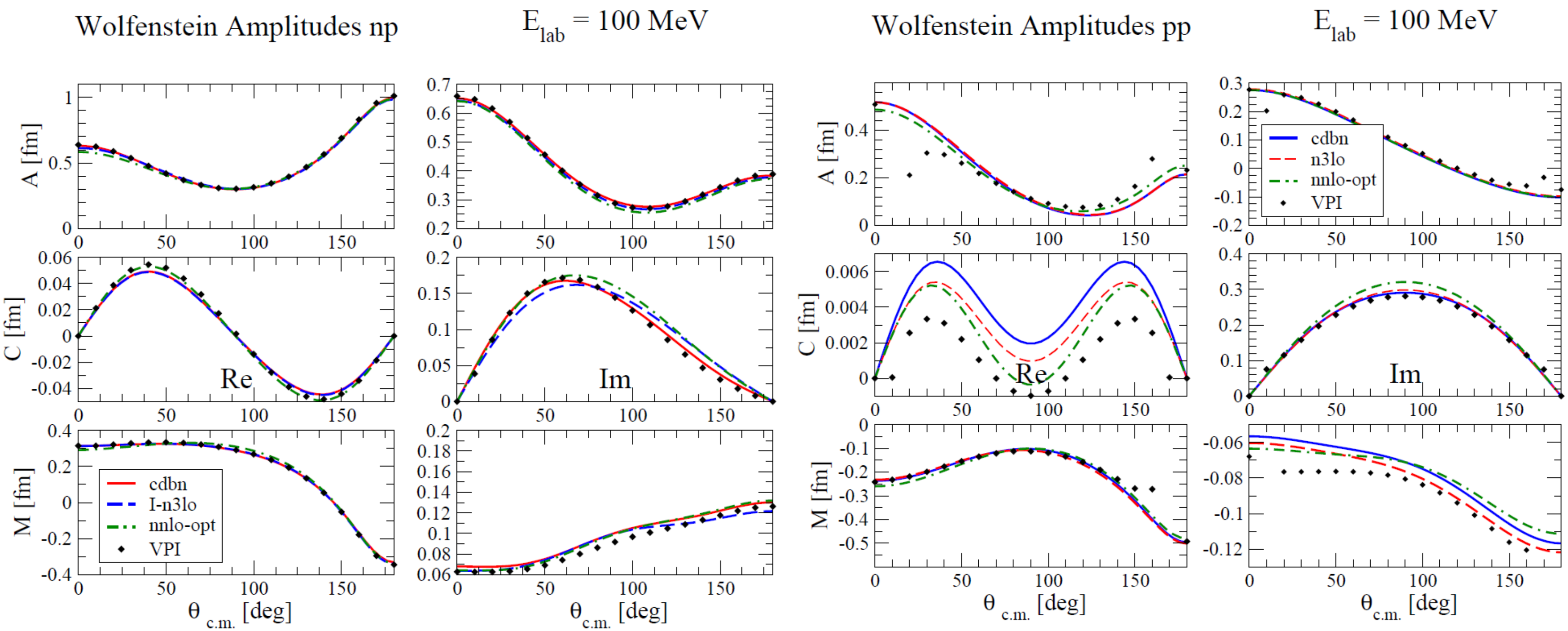
Wolfenstein Amplitudes np
E_lab = 100 MeV
Wolfenstein Amplitudes pp
E_lab = 100 MeV
A [fm]
C [fm]
M [fm]
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cdbn
I-n3lo
n3lo
nnlo-opt
VPI
θ c.m. [deg]