Applying new power counting of chiral EFT in ab-initio calculations

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Triumf workshop: Progress in Ab-initio Techniques in

Nuclear Physics

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EFT-like approaches

Select a model space

(since we cannot do calculations in (or even actually know the structure of) the entire Hilbert space)

Build/Derive the interactions in that model space

Solve for any observable desired

Compare to data (check your assumptions)

Conventional power counting

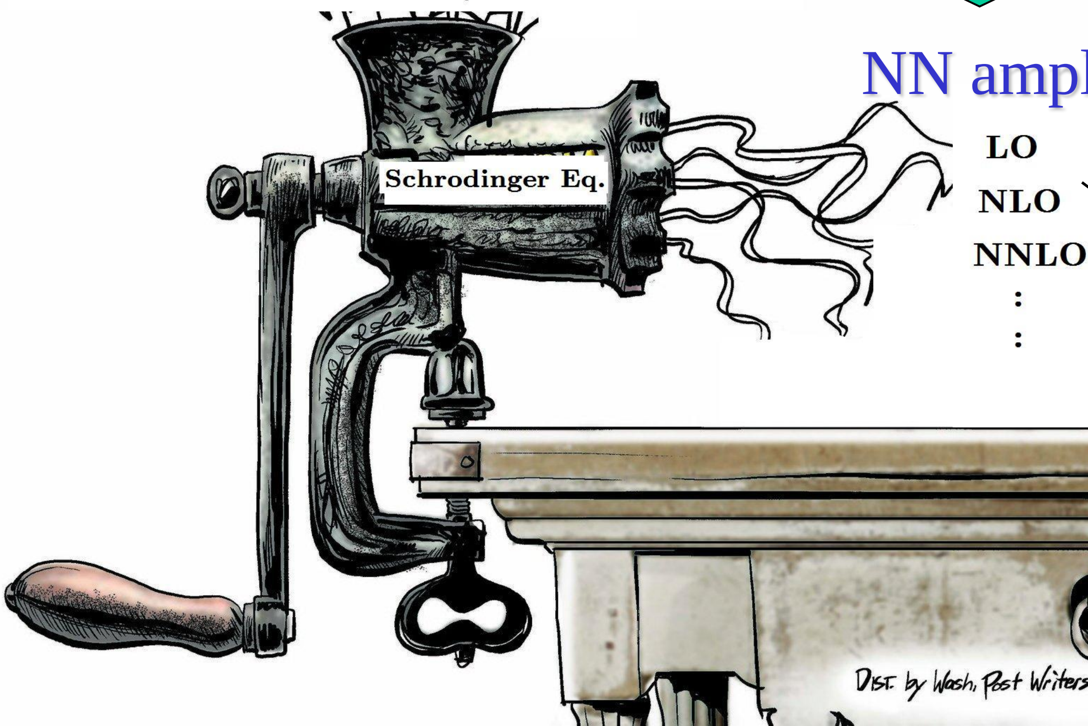
Epelbaum, Entem, Machleidt, Kaiser, Meissner, … etc., ~**90% of the people**

Hope:

V up to LO, NLO, NNLO, N3LO, N4LO, :

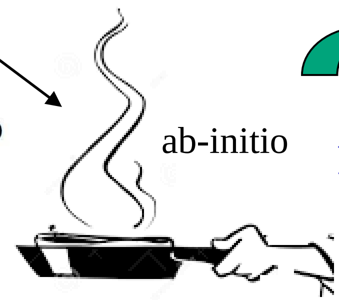
(one to one correspondence) ← assumption

(Under Λ≤550 MeV)



NN amplitude

LO
NLO
NNLO
:
:

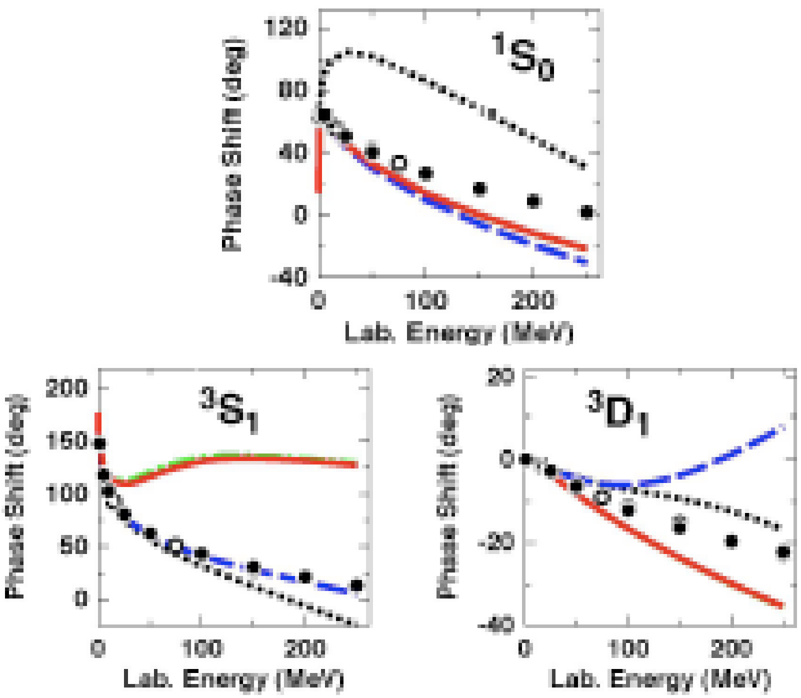


ab-initio

Properties of nuclei

Problems in RG

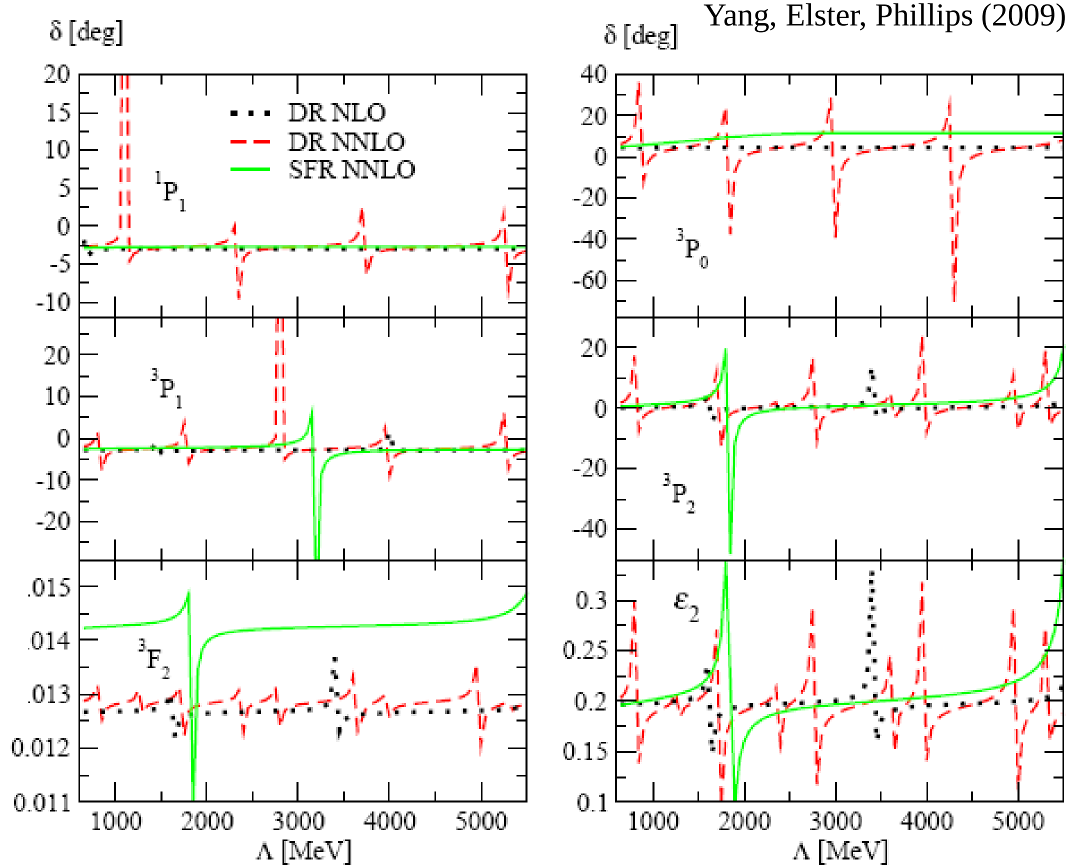
- Singular attractive potentials demand contact terms. (Nogga, Timmermans, van Kolck (2005))
- Beyond LO: Has RG problem at Λ>1 GeV (due to iterate to all order)



N^{3}LO(Q^4)

Ch. Zeoli R. Machleidt D. R. Entem (2012)

Yang, Elster, Phillips (2009)



Renormalization group (RG)

Select a model space

(since we cannot do calculations in (or even actually know the structure of) the entire Hilbert space)

Build/Derive the interactions in that model space

Solve for any observable desired

Vary your model space (mostly enlarge→ more shouldn't hurt)

Compare to data (check your assumptions)

Minimum requirement of **EFT©**

Select a model space

(since we cannot do calculations in (or even actually know the structure of) the entire Hilbert space)

Renormalize **the interactions in that model space**

Solve for any observable desired

Vary your model space (mostly enlarge→ more shouldn't hurt)

↓

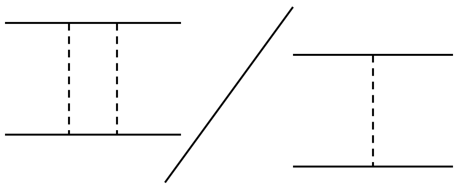
Compare to data (check your assumptions)

New power counting

Long & Yang, (2010-2012)

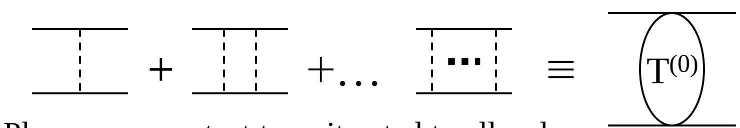
LO: Still iterate to all order (at least for most l<2).

Reason: van Kolck, Bedaque,… etc.



$$\sim \frac{g_A^2 M}{8\pi f_\pi^2} Q$$

Thus, at LO:



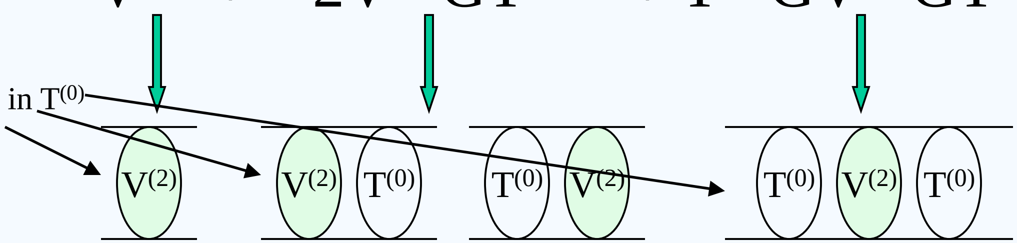
Plus proper contact term iterated to all order

Start at NLO, do perturbation. $(T = T^{(0)}+T^{(1)}+T^{(2)}+T^{(3)}+\ldots)$

If $V^{(1)}$ is absent:

$$T^{(2)} = V^{(2)} + 2V^{(2)}GT^{(0)} + T^{(0)}GV^{(2)}GT^{(0)}.$$

One insertion of $V^{(2)}$ in $T^{(0)}$



$$G \equiv \frac{2M_N}{\pi}\int_0^\Lambda \frac{p^2 dp}{p_0^2 - p^2 + i\varepsilon}$$

$$T^{(3)} = V^{(3)} + 2V^{(3)}GT^{(0)} + T^{(0)}GV^{(3)}GT^{(0)}.$$

RG-invariant power counting (call it: MWPC)

- Only LO (contain: S-waves & some P-waves) interaction is treated non-perturbatively.

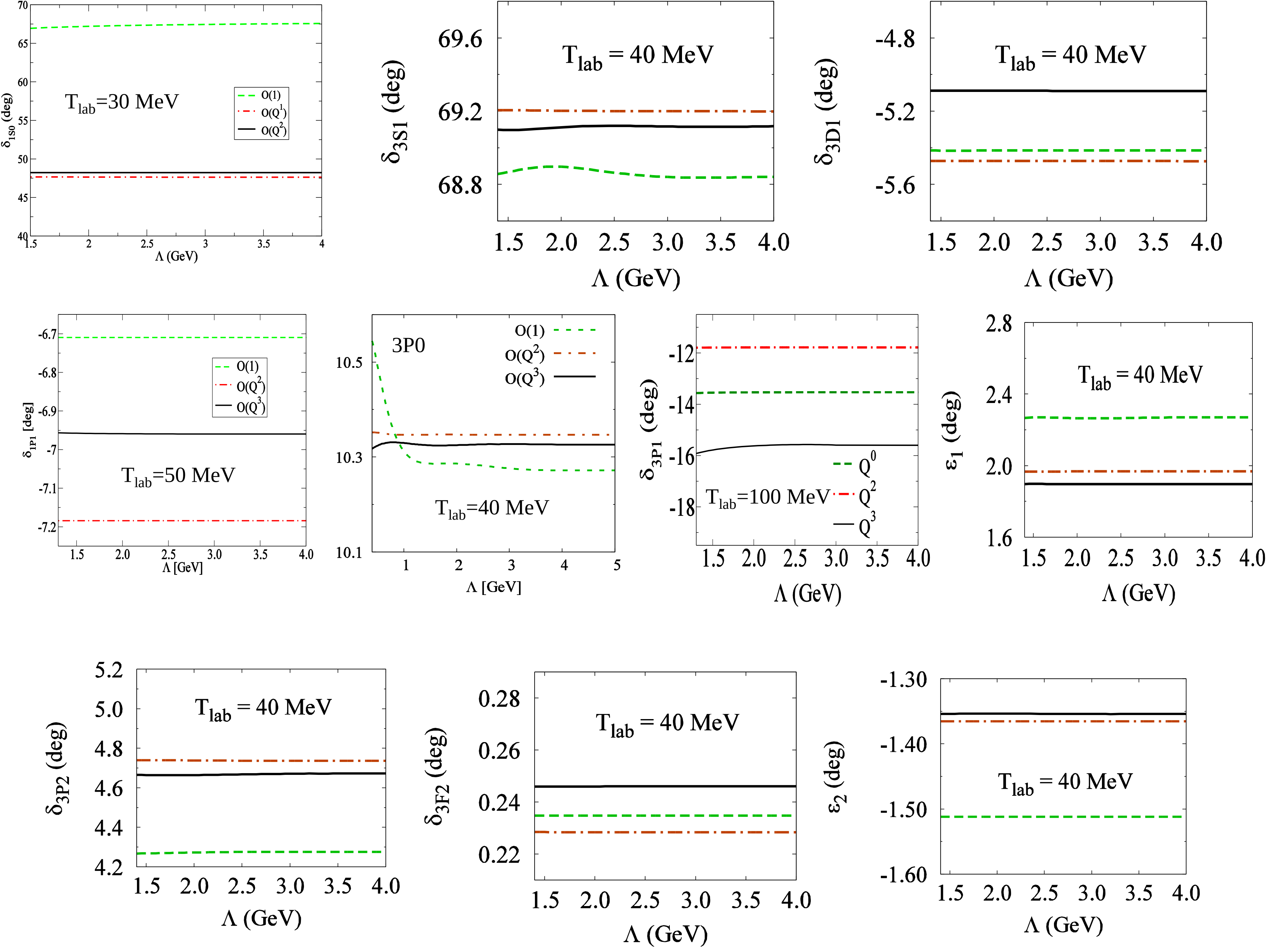
Same as WPC, except $^3P_0$ and $^3P_2$-$^3F_2$ →has 1 contact term to achieve RG.

- All other corrections need to be treated in perturbation theory.

To avoid the Wigner-bound-like effect (which destroy RG).

- Observables at LO need to be not "crazily far" from data. Otherwise it would be difficult to correct it.

However, this applies generally for any EFT.



T_lab=30 MeV
O(1)
O(Q^1)
O(Q^2)
δ_1S0 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3S1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3D1 (deg)
Λ (GeV)
T_lab=50 MeV
O(1)
O(Q^2)
O(Q^3)
δ_1P1 [deg]
Λ [GeV]
3P0
O(1)
O(Q^2)
O(Q^3)
T_lab=40 MeV
Λ [GeV]
δ_3P1 (deg)
T_lab=100 MeV
Q^0
Q^2
Q^3
Λ (GeV)
T_lab = 40 MeV
ε_1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3P2 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3F2 (deg)
Λ (GeV)
T_lab = 40 MeV
ε_2 (deg)
Λ (GeV)

Ab-initio calculations

- For ^3H, ^3He, ^4He, use NCSM.
- For ^{16}O, use CC.

regulator

$$V_{ll'}(p,k;\Lambda) = [V_{short} + V_{long}] \cdot exp(-\frac{p^4 + k^4}{\Lambda^4}),$$

$$< V_{ll'} >_{nm} = \frac{2}{\pi}\int p^2 dp \int k^2 dk \;\; \phi_{nl}(p) V_{ll'}(p,k;\Lambda)\phi_{ml'}(k).$$

↓

Input for NCSM & CC

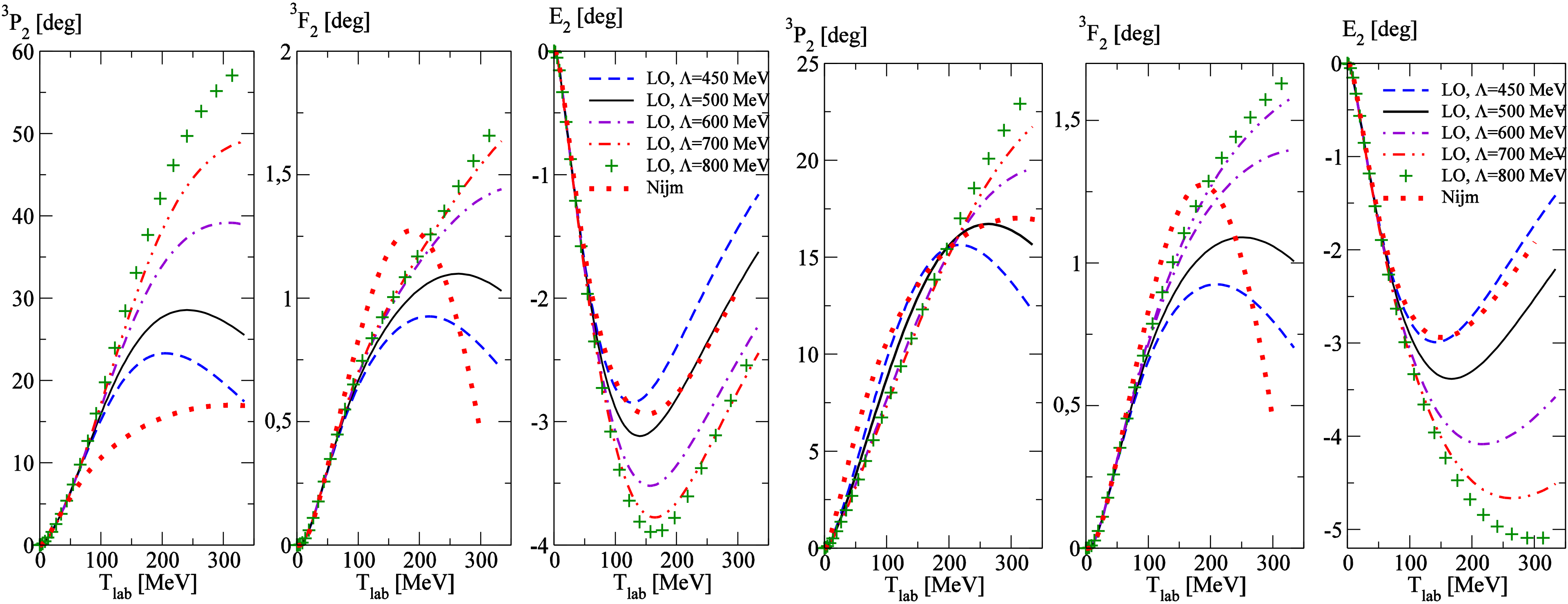
- Deep bound states ($^3P_0$ from $\Lambda \geq 750$ MeV and $^3S_1$-$^3D_1$ from $\Lambda \geq 1050$ MeV) are removed.

Input & fitting

Channel	LEC fit up to
$^1S_0$	a_0=-23.7 fm
$^3S_1$-$^3D_1$	E_b=-2.225 MeV
$^1P_1$	N/A
$^3P_0$	k_{cm}=140 MeV
$^3P_1$	N/A
$^3P_2$-$^3F_2$	Set I: T_{lab}=40 MeV Set II: T_{lab}=200 MeV

All other channels set to zero.

$^3P_2$-$^3F_2$ (fit up to m_π or higher?)



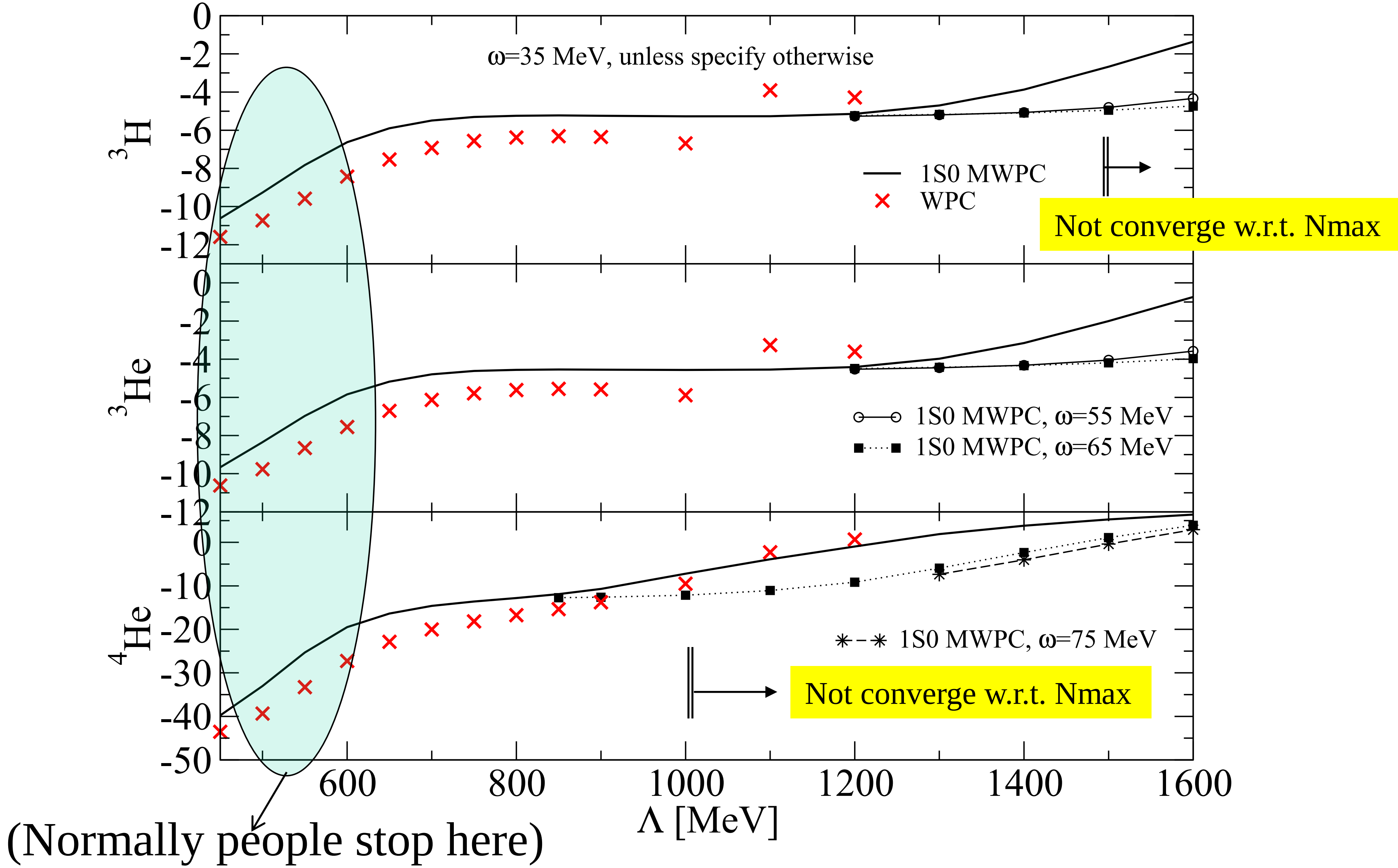
Fit up to $k=m_\pi$

Set I

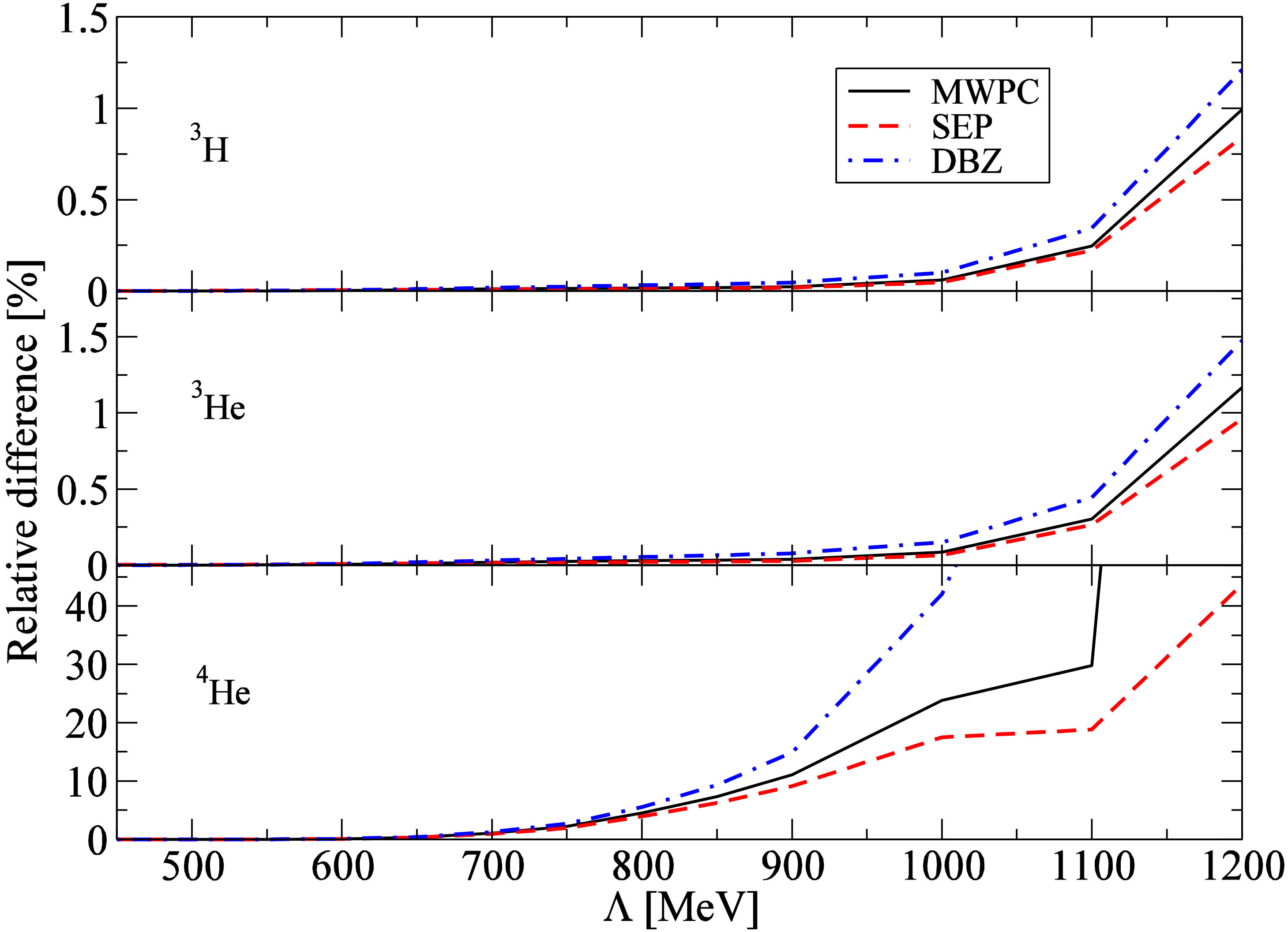
Fit up to T_{lab}=200 MeV

Set II

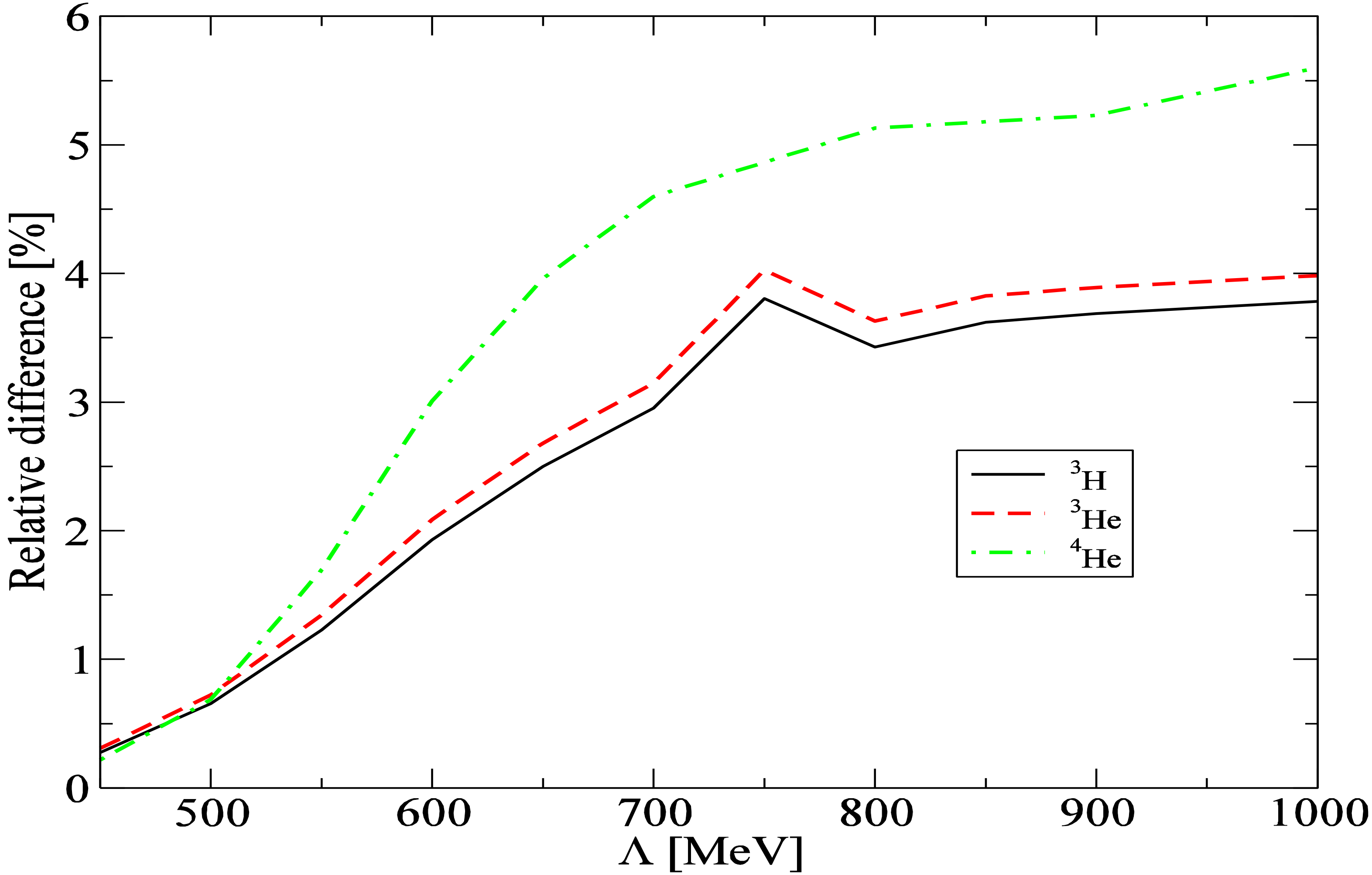
NCSM results: LO light nuclei



Relative difference between reuslts from $N_{max(A)}$ and $N_{max(A)} - 2$



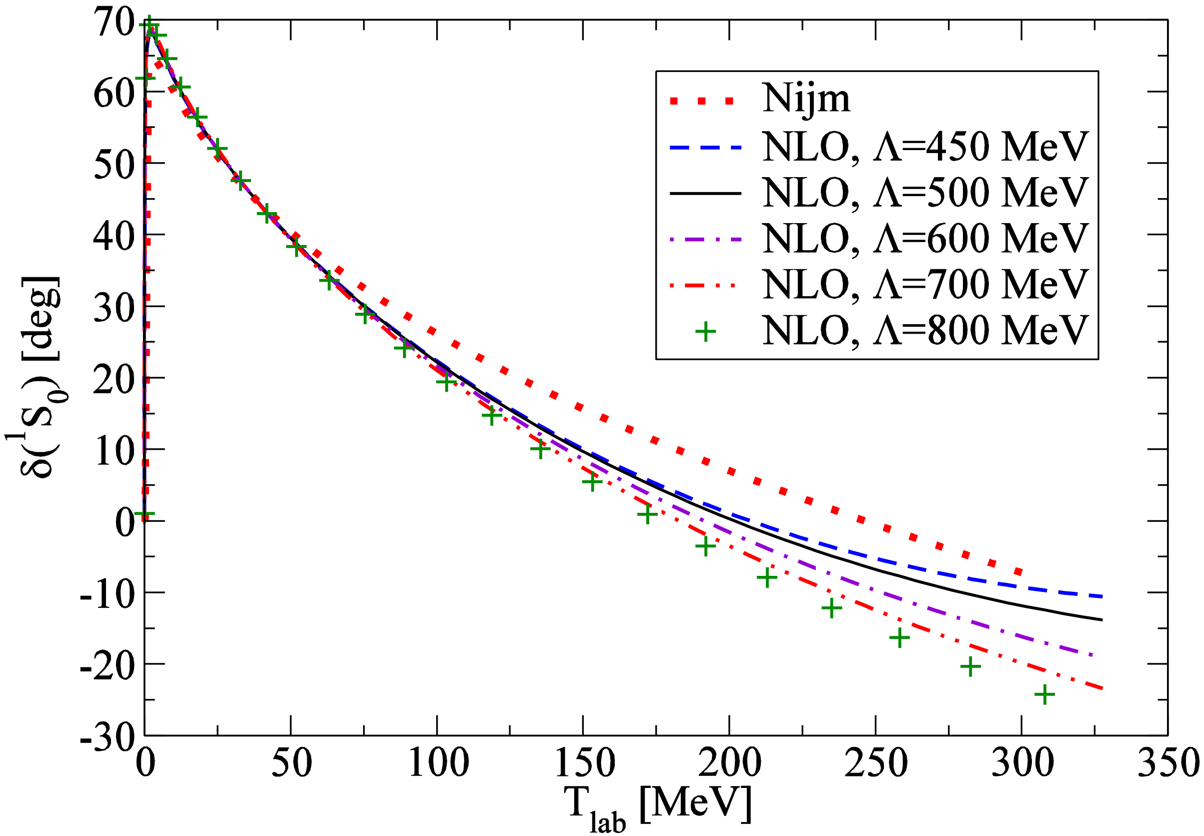
Relative difference between input Set I and Set II.



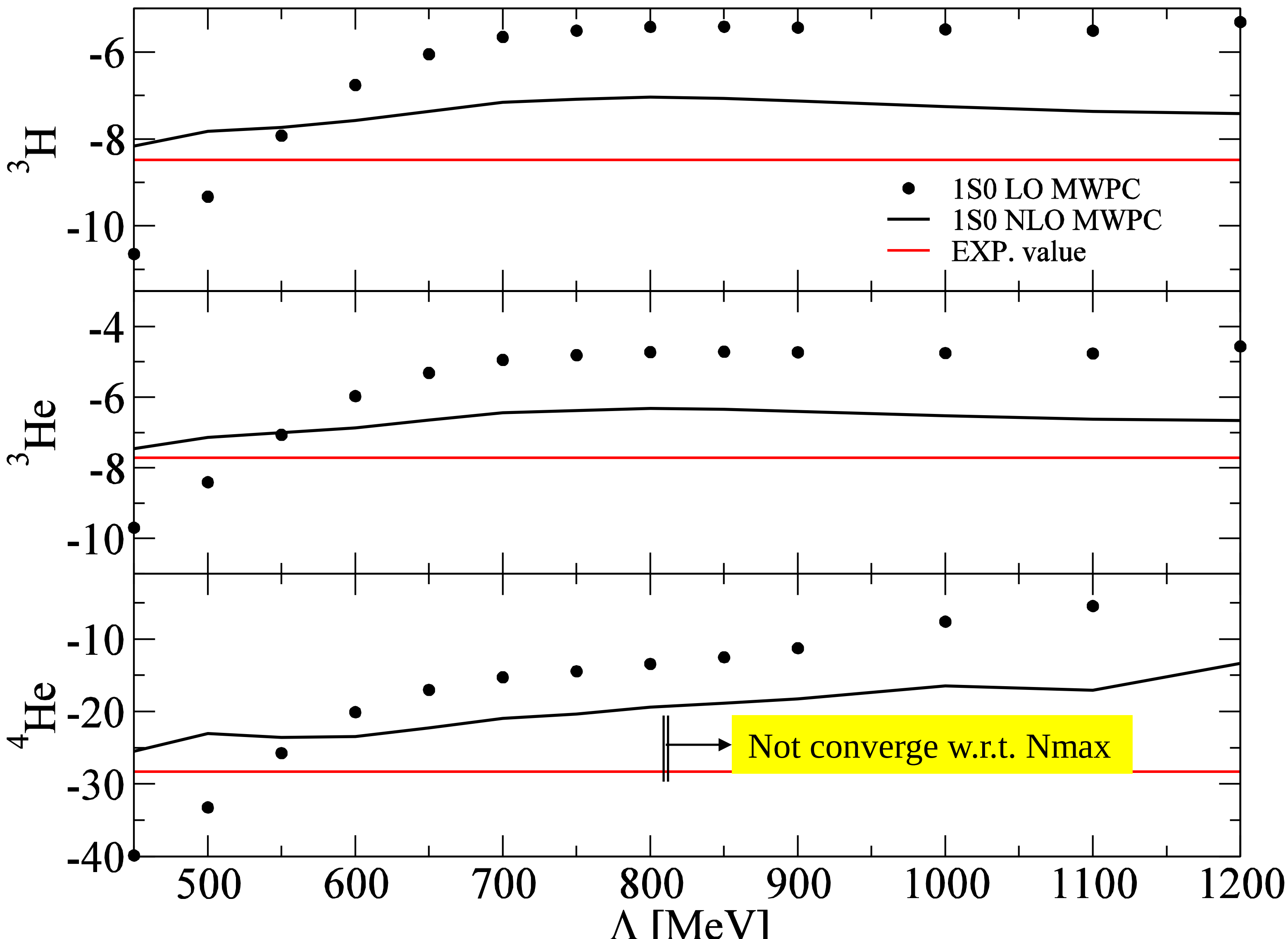
NLO

Just $^1S_0$ channel enters (perturbatively)

$$V_{nlo}(p,k) = [C + D(p^2 + k^2)]\exp(-\frac{p^4 + k^4}{\Lambda^4})$$



NLO for light nuclei



So far so good, but…

^{16}O results
(depend a lot on fitting details)

Cutoff (MeV)	^{16}O (MeV)		$4*^4He$ (MeV)	
	Fit m_π	Fit 200	Fit m_π	Fit 200
450	-269	-152	-159	-159
500	-225	-74.9	-133	-132

Huge effect!

Almost no Effect on ^4He

Higher cutoffs require more computational efforts.

^{16}O v.s. $4 \cdot {}^{4}He$

- Subleading corrections (enter perturbatively) can modify existing poles position, but cannot generate new pole.

- Thus, if we fail to get the "4-α"-like pole structure at LO, we are in trouble.

- This is the case if $^{3}P_{2}$-$^{3}F_{2}$ is fitted up to T_{lab}=200 MeV to get "good phase shifts".

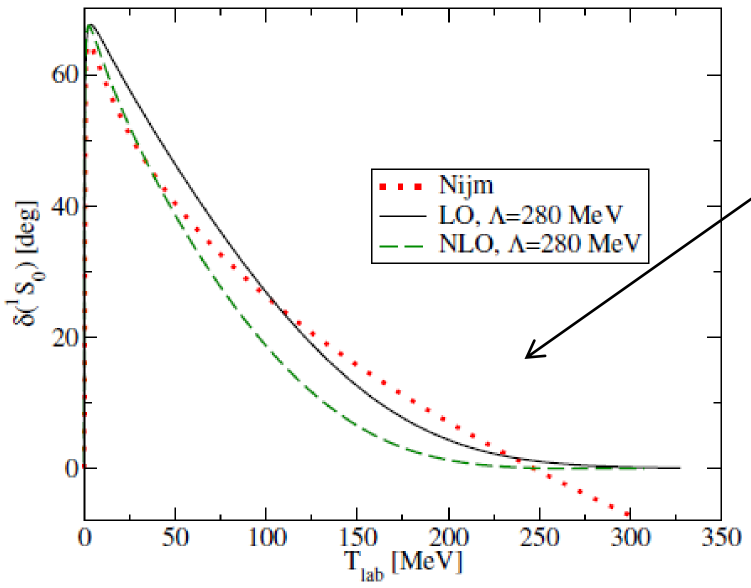
^{16}O results

Cutoff (MeV)	^{16}O (MeV) Fit m_π		^{16}O (MeV) Fit 200	
	LO	NLO	LO	NLO
450	-269	-76.6	-159	-43.5
500	-225	~0	-133	-105

Cannot be! Excluded now.

Already excluded by 4α

MWPC with low cutoffs works well
(if using Λ as a fitting parameter)



Λ=280 MeV for $^1S_0$
Λ=450 MeV for the rest.

Just LO, no 3-body force.

^{4}He: -29.5 MeV
^{16}O: -127 MeV

But this is not EFT!

To describe nuclei with an RG-invariant EFT requires more work!

Is the large NLO contribution to the $^1S_0$ phase shift a problem?

Further improvement in $^1S_0$

- At LO, although RG-invariant, the converged phase shift is far from data.
- Worrisome big change (>100%) from LO to NLO.

Re-thinking alternatives: Adopt dibaryon field

D.B. Kaplan, Nucl. Phys. B 494 (1997) 471.

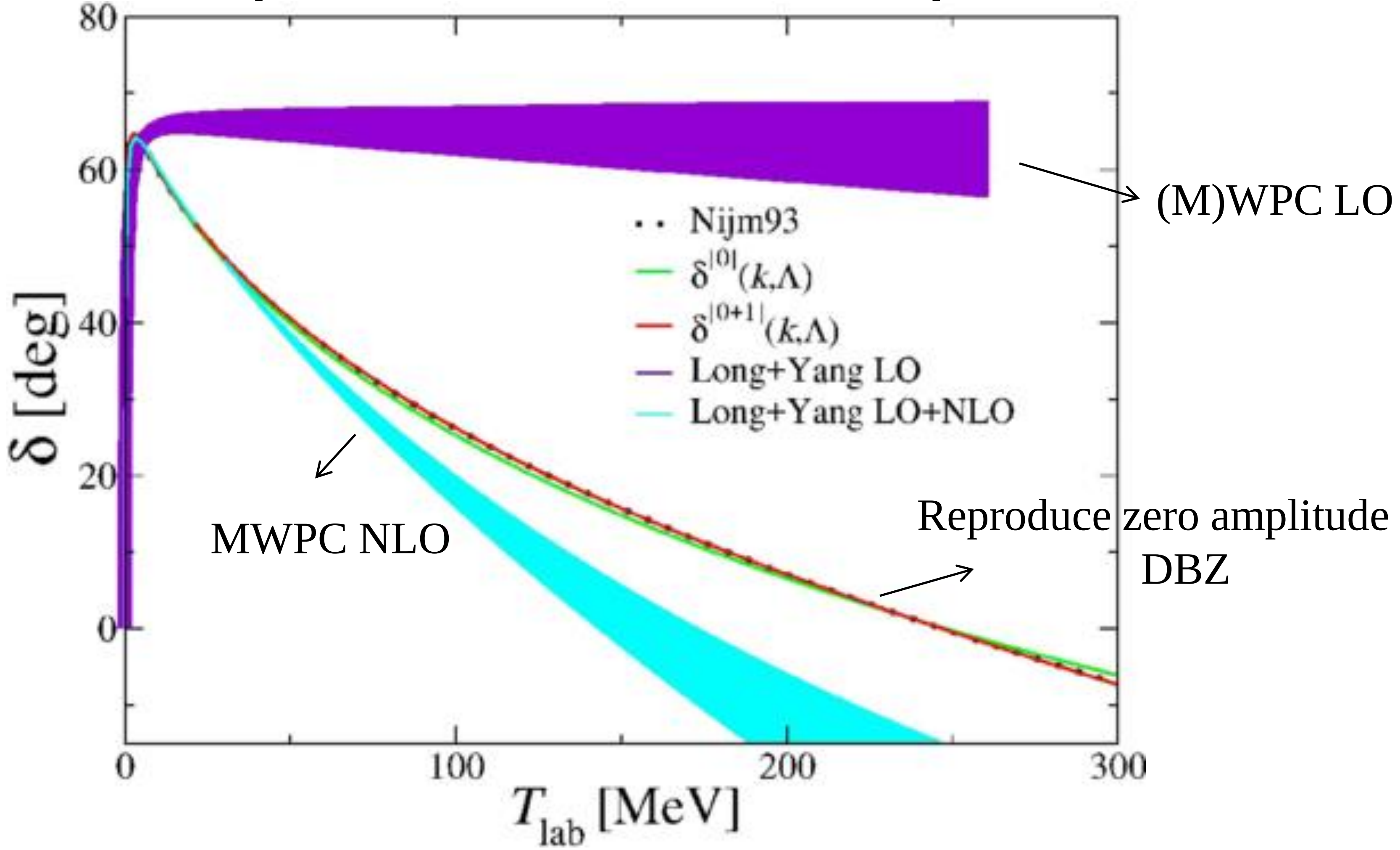
B. Long, Phys. Rev. C 88 (2013) 014002.

*M. S. Sánchez, C.-J. Yang, Bingwei Long, U. van Kolck, Phys.Rev. C97 (2018) no.2, 024001.

adopted here

All of them produce RG-invariant $^1S_0$ phase shifts.

A: 1.5 dibaryon field+OPE (denoted as DBZ)



Phys.Rev. C97 (2018) no.2, 024001

Dibaryon and phase equivalent transformation

$$V_{DB}(E)=\frac{1}{\Delta+cE},\left(E=\frac{k_0^2}{m_N}\right) \longrightarrow$$ E-dep! Cannot be used in many-body.

Full potential: $V_{DBZ}(p,k,E)=OPE(p,k)+C_0+V_{DB}(E)$.

Our treatment

$[H_0+V(E)]\psi=E\psi$. Solve iteratively to get for E_i.

$$1=\sum_{E_i}|\psi_{E_i}\rangle\langle\psi_{E_i}|,$$ (if $|\psi_{E_i}\rangle$ are orthgonal).

In fact it's not for E-dep V => re-orthgonalize them by Gram-Schmidt.

Then, $\langle \mathrm{p}|\mathrm{H}|\mathrm{p'}\rangle=\sum_{E_i}\sum_{E'_i}\langle \mathrm{p}|\psi_{E_i}\rangle\langle\psi_{E_i}|H|\psi_{E'_i}\rangle\langle\psi_{E'_i}|p'\rangle\approx\sum_{E_i}\langle p|\psi_{E_i}\rangle E_i\langle\psi_{E_i}|p'\rangle$

Fianlly, $V(p,p')=\langle \mathrm{p}|\mathrm{H}|\mathrm{p'}\rangle-\frac{p^2}{m_N}\delta_{pp'}$.

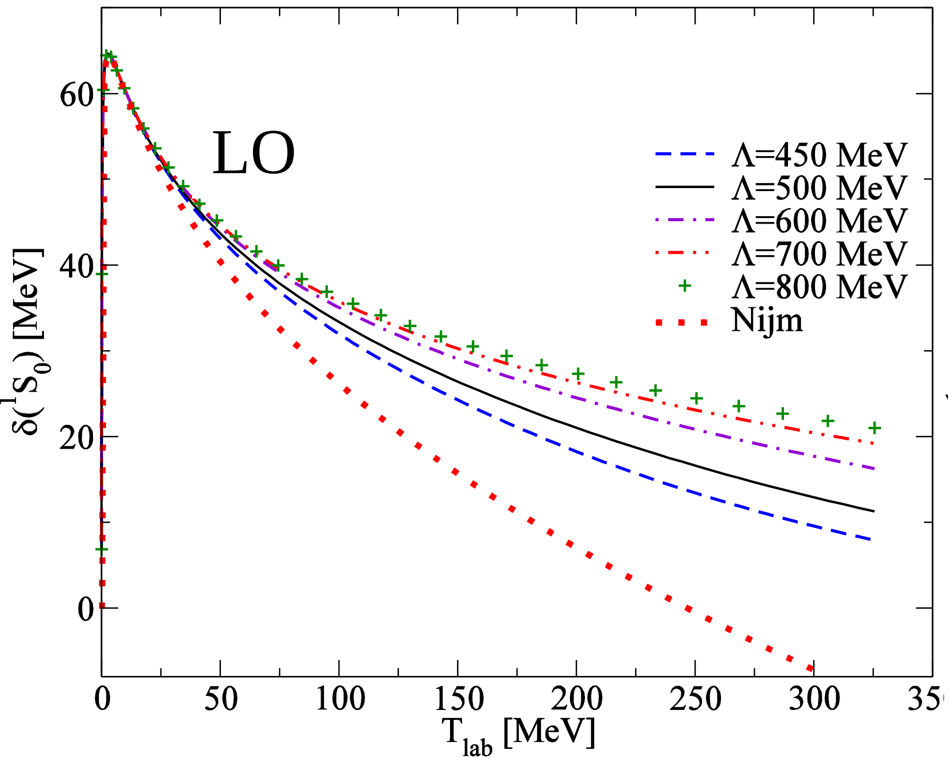
Introduce model dep., i.e., ordering of vectors in G.S. create ~15(20)% uncertainty on ^{3}H (^{4}He).

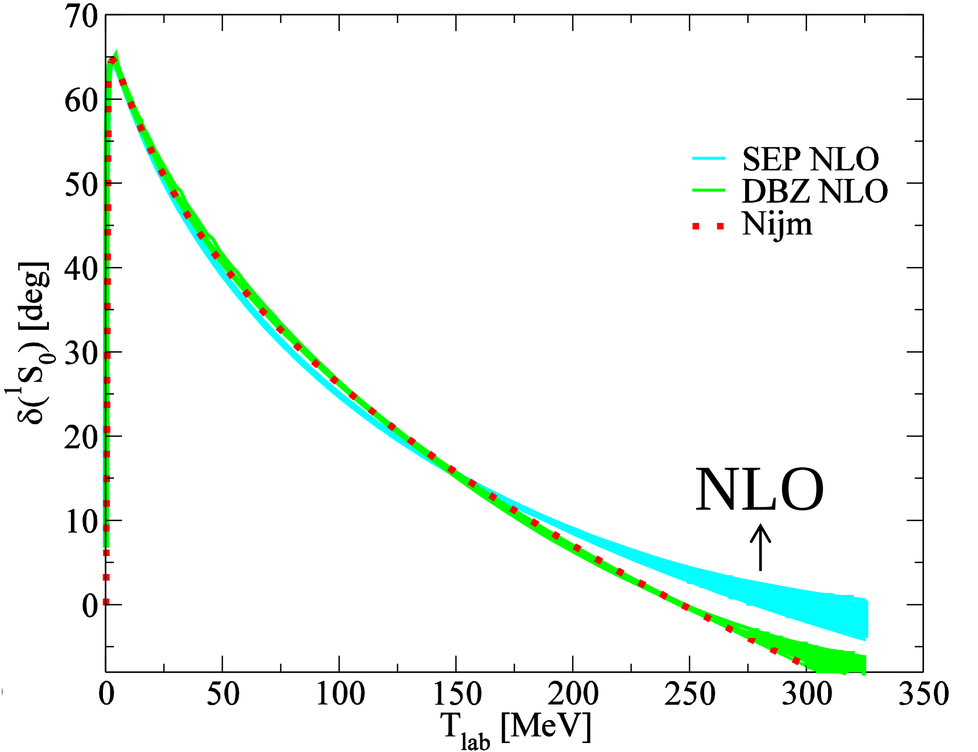
B: Separable potential

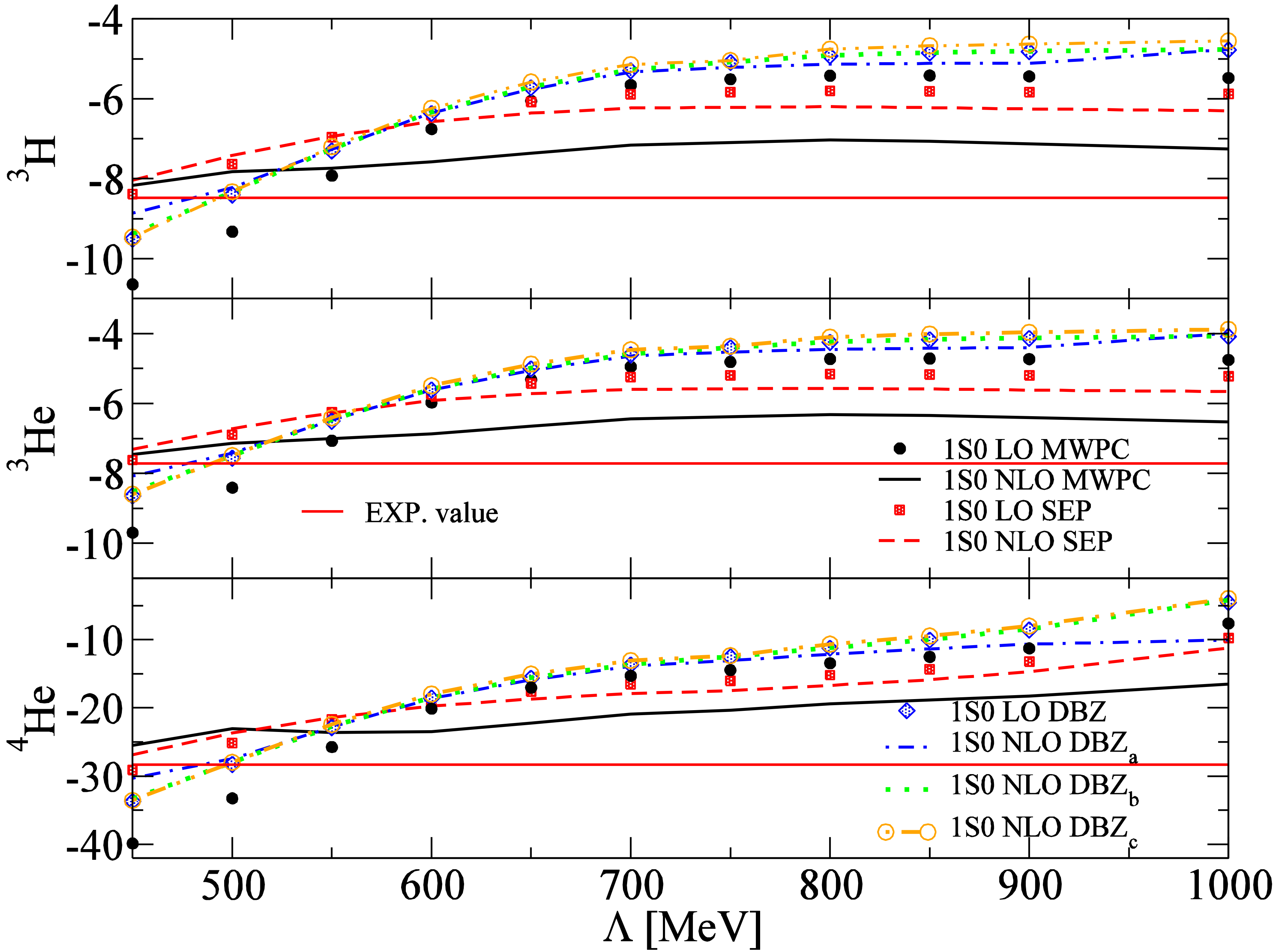
Shaowei Wu, Bingwei Long, coming soon.

P-dep. version of 1 dibaryon field potential.

$$V_{sep}(p,k) = \frac{y m_N}{\sqrt{p^2 + m_N \Delta}\sqrt{k^2 + m_N \Delta}} + Yukawa$$







-4
-6
-8
-10
^{3}H
^{3}He
^{4}He
-10
-20
-30
-40
500
600
700
800
900
1000
Λ [MeV]
EXP. value
1S0 LO MWPC
1S0 NLO MWPC
1S0 LO SEP
1S0 NLO SEP
1S0 LO DBZ
1S0 NLO DBZ$_a$
1S0 NLO DBZ$_b$
1S0 NLO DBZ$_c$

^{16}O with DBZ (still not work)

		^{16}O	$4{*}^4$He
Set I	DBZ with $^3P_2 - ^3F_2$ up to T_{lab} = 40 MeV, Λ = 450 MeV	-173	-135
	DBZ with $^3P_2 - ^3F_2$ up to T_{lab} = 40 MeV, Λ = 500 MeV	-123	-113
	DBZ with $^3P_2 - ^3F_2$ up to T_{lab} = 40 MeV, Λ = 550 MeV	-70.5	-91.6
Set II	DBZ with $^3P_2 - ^3F_2$ up to T_{lab} = 200 MeV, Λ = 450 MeV	-103	-134
	DBZ with $^3P_2 - ^3F_2$ up to T_{lab} = 200 MeV, Λ = 500 MeV	-68.9	-112
	DBZ with $^3P_2 - ^3F_2$ up to T_{lab} = 200 MeV, Λ = 550 MeV	-44.7*	-91.2

^{16}O<4α!

^{16}O with SEP (still not work)

		^{16}O	$4{*}^4$He
Set I	SEP with $^3P_2-^3F_2$ up to T_{lab} = 40 MeV, Λ = 450 MeV	-146	-116
	SEP with $^3P_2-^3F_2$ up to T_{lab} = 40 MeV, Λ = 500 MeV	-103	-101
	SEP with $^3P_2-^3F_2$ up to T_{lab} = 40 MeV, Λ = 550 MeV	-73.5	-86.9
Set II	SEP with $^3P_2-^3F_2$ up to T_{lab} = 200 MeV, Λ = 450 MeV	-94.0	-115

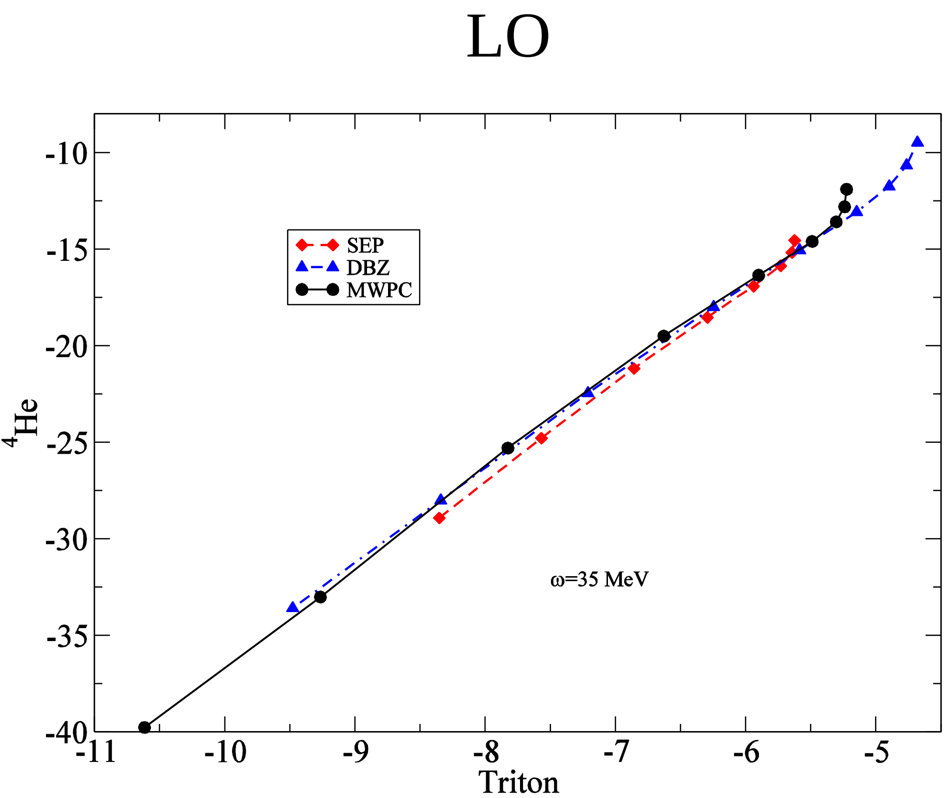
^{16}O<4α!

Conclusion

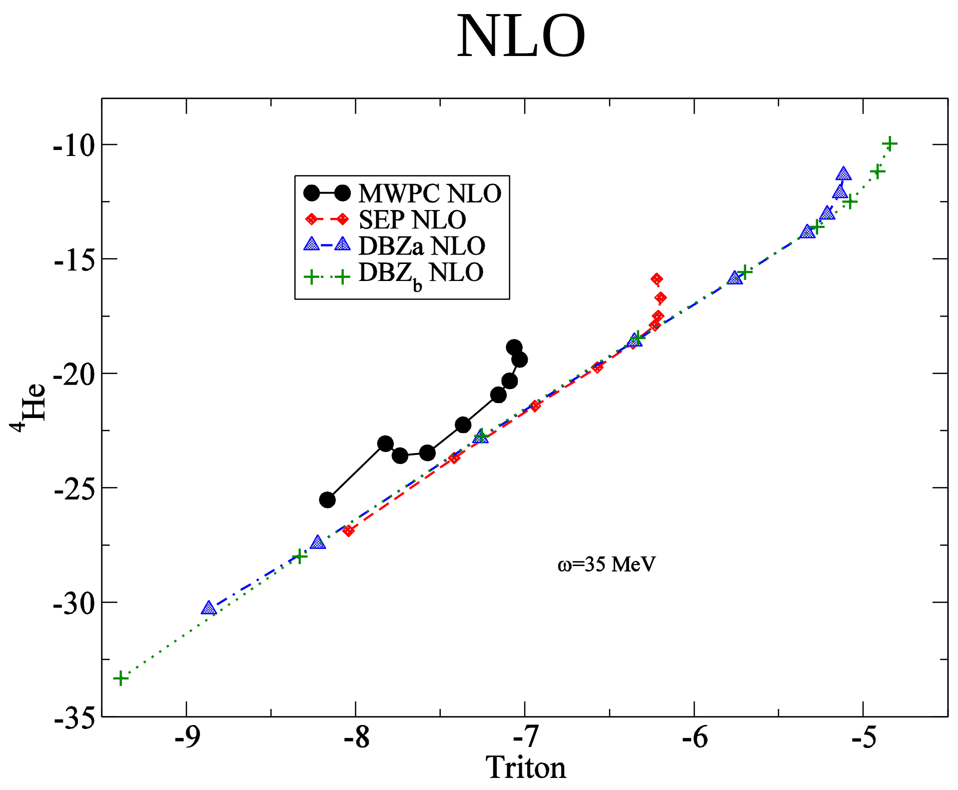
- To describe nuclei with an RG-invariant EFT requires more work!
- We have performed first ab-initio calculations of many-body systems (A>3) with a modified, RG-invariant power counting in chiral EFT.
- For ^{3}H, ^{3}He and ^{4}He, reasonable, RG-invariant results can be obtained.
- For ^{16}O, we observe:
  - LO results not a good starting point (4 alpha pole)
  - very large NLO contributions
  - results depend sensitively on fitting strategy
- It is quite possible that 3-body force needs to be promoted to LO for systems heavier than ^{4}He.

Thank you!

Tjon line



LO
SEP
DBZ
MWPC
ω=35 MeV
^{4}He
Triton
-10
-15
-20
-25
-30
-35
-40
-11
-10
-9
-8
-7
-6
-5



NLO
MWPC NLO
SEP NLO
DBZa NLO
DBZ$_b$ NLO
ω=35 MeV
^{4}He
Triton
-10
-15
-20
-25
-30
-35
-9
-8
-7
-6
-5

	LO	NLO
$^1S_0$ MWPC	Yukawa+1 LEC	2 LECs
$^1S_0$ DBZ	Yukawa+3 LECs	4 LECs
$^1S_0$ SEP	Yukawa+2 LECs	3 LECs
$^3S_1 - ^3D_1$	OPE+1 LEC	absent
$^1P_1$	OPE	absent
$^3P_0$	OPE+1 LEC	absent
$^3P_1$	OPE	absent
$^3P_2 - ^3F_2$	OPE+ 1 LEC	absent