Electromagnetic Strength Distributions from the (In-Medium) NCSM

Laura Mertes, Christina Stumpf and Robert Roth

TECHNISCHE UNIVERSITÄT DARMSTADT

TRIUMF Workshop 2019

Motivation

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- strength distributions

  → provide information about nucleus

  → are accessible in **experiments**

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  → provide information about nucleus

  → are accessible in **experiments**

- conventional approximate methods as **RPA** are used for description of **collective excitations**

- combination of **(IT-)NCSM** and **Lanczos strength functions**

  → arbitrary nuclei up to sd-shell from **low-lying** excitations to giant-resonance region

Lanczos Strength Functions

- **idea**: construct basis in which Hamilton matrix is **tridiagonal**
  → diagonalize T: **fast-converging** approximations of EV of H

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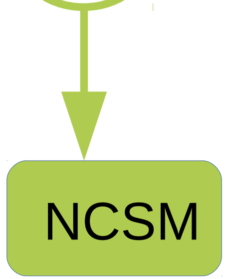

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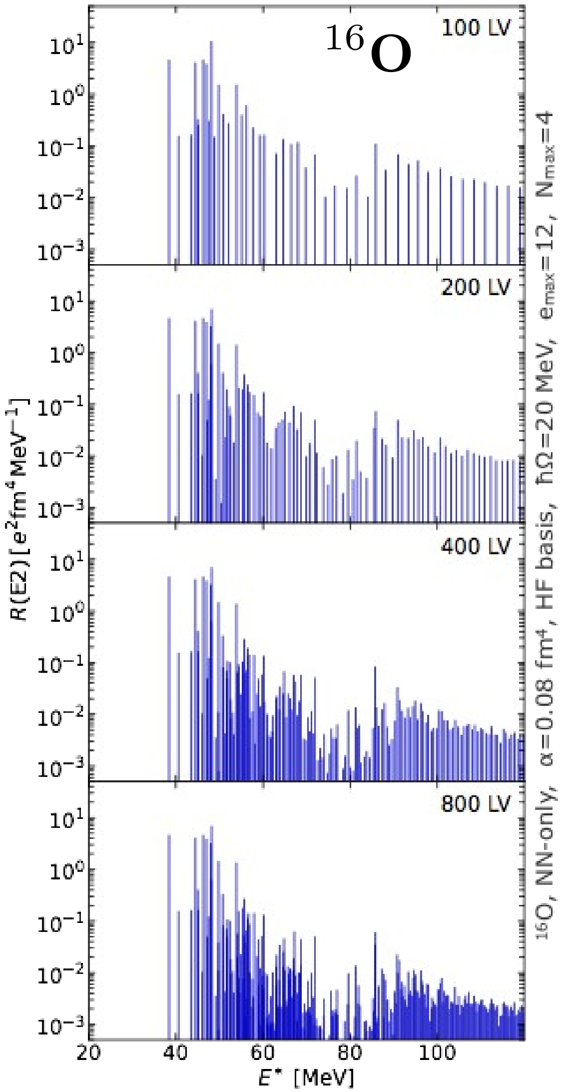
- **discrete strength** distribution

$$\mathrm{R(E^*)} = \sum_n |\,\langle E_0||\mathbf{O}||E_n\rangle\,|^2\delta(E^* - (E_n - E_0))$$

Lanczos Strength Functions

C. Stumpf, T. Wolfgruber, R. Roth, arXiv:1709.06840



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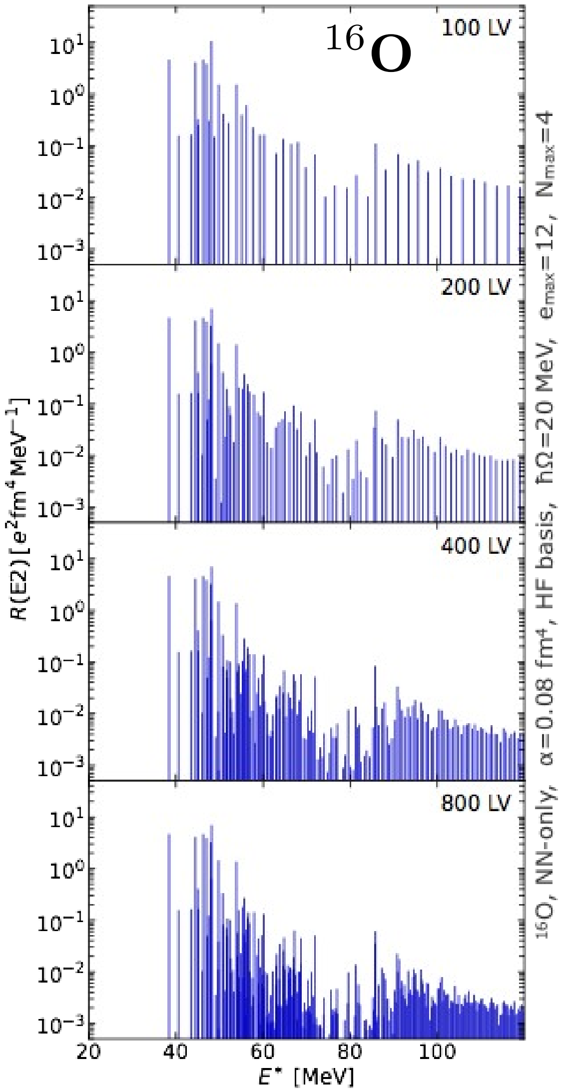

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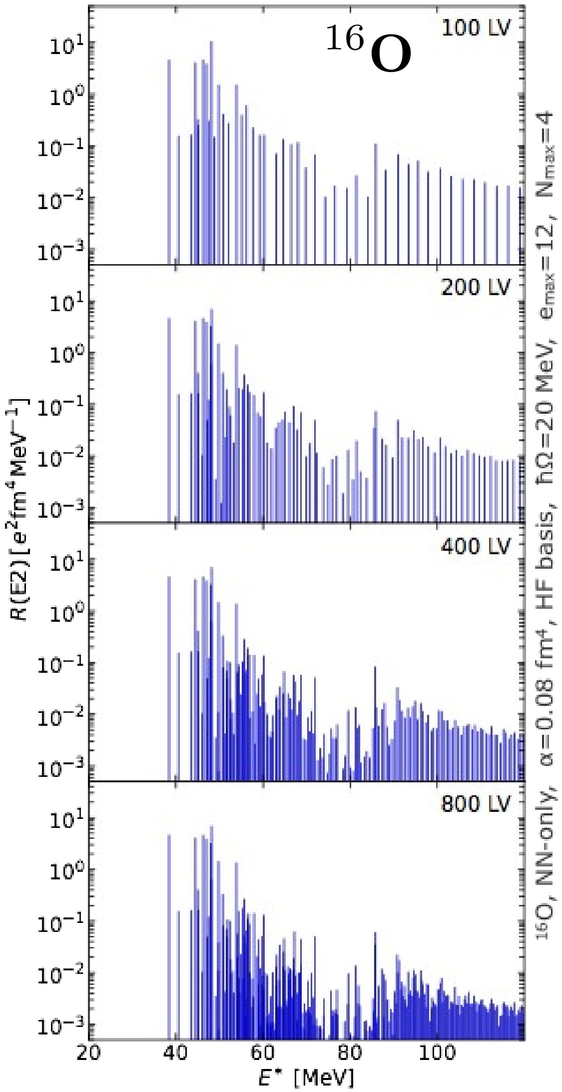

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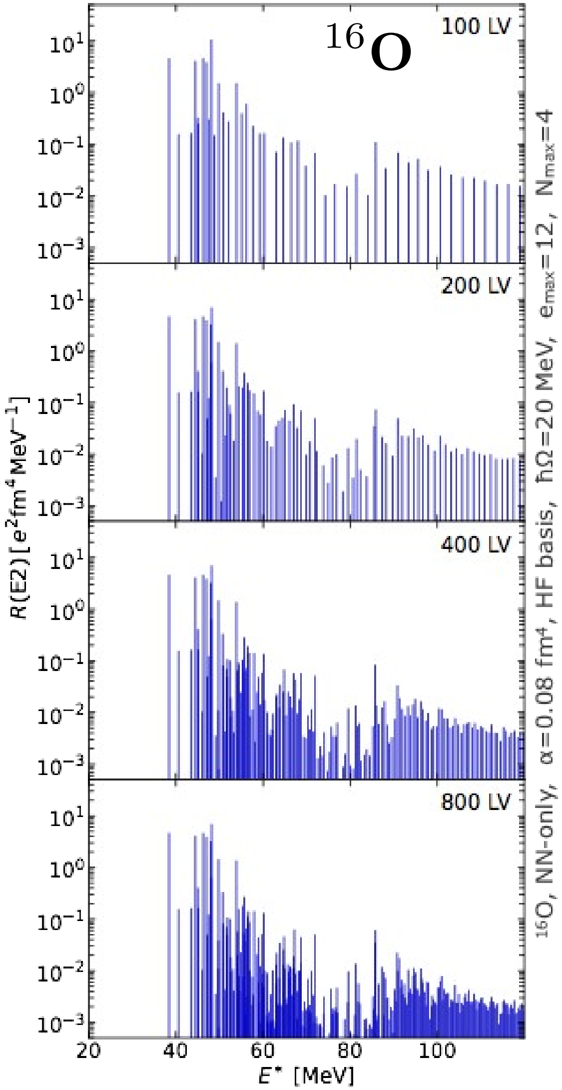
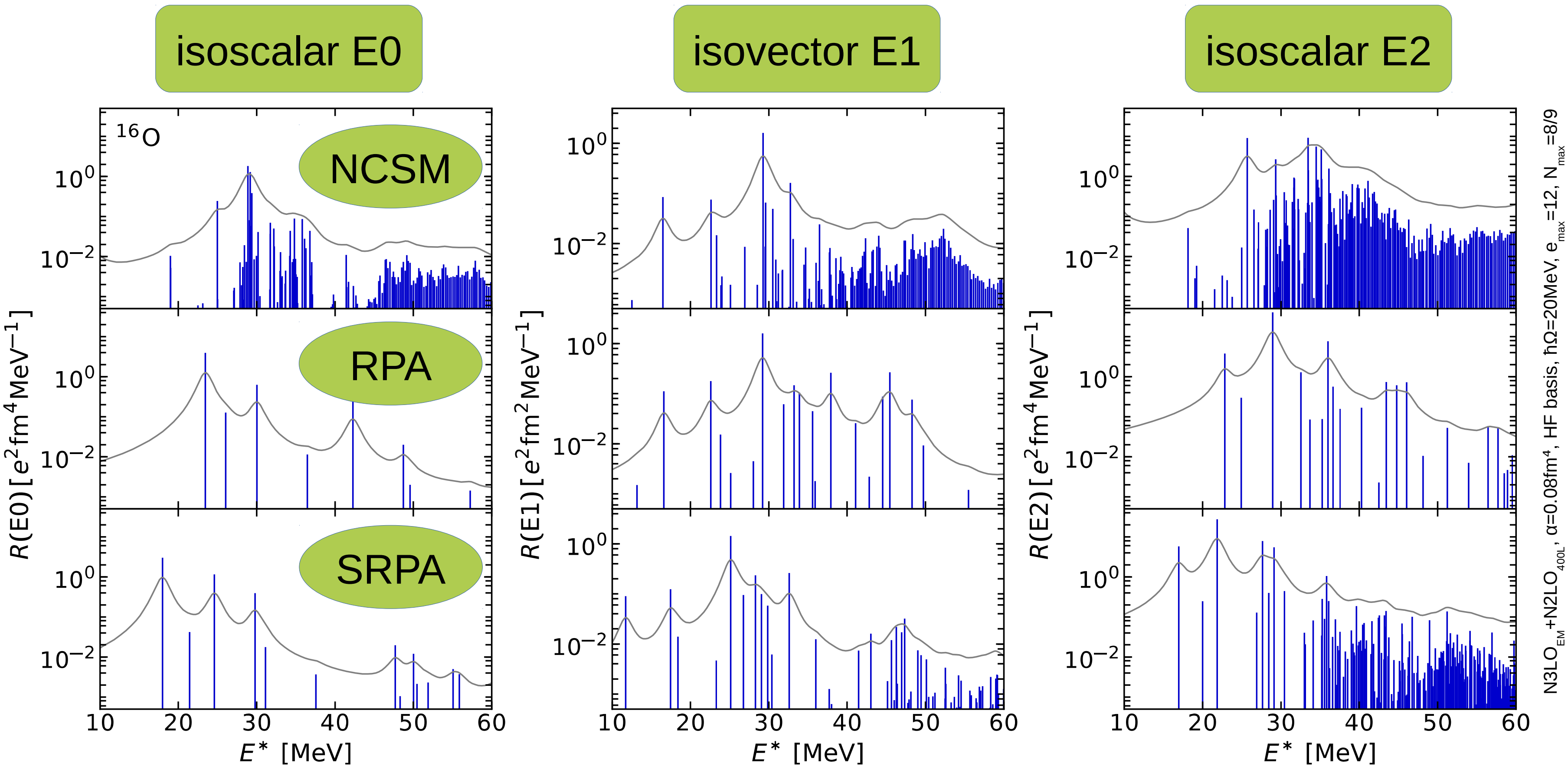


- strength distributions converge fast w.r.t. size of Lanczos basis
- same limitations as NCSM
- use simple Lanczos algorithm
  → store only three Lanczos vectors
- description of fine structure and fragmentation

Oxygen Isotopes



- positions of resonances of NCSM **compatible** with RPA

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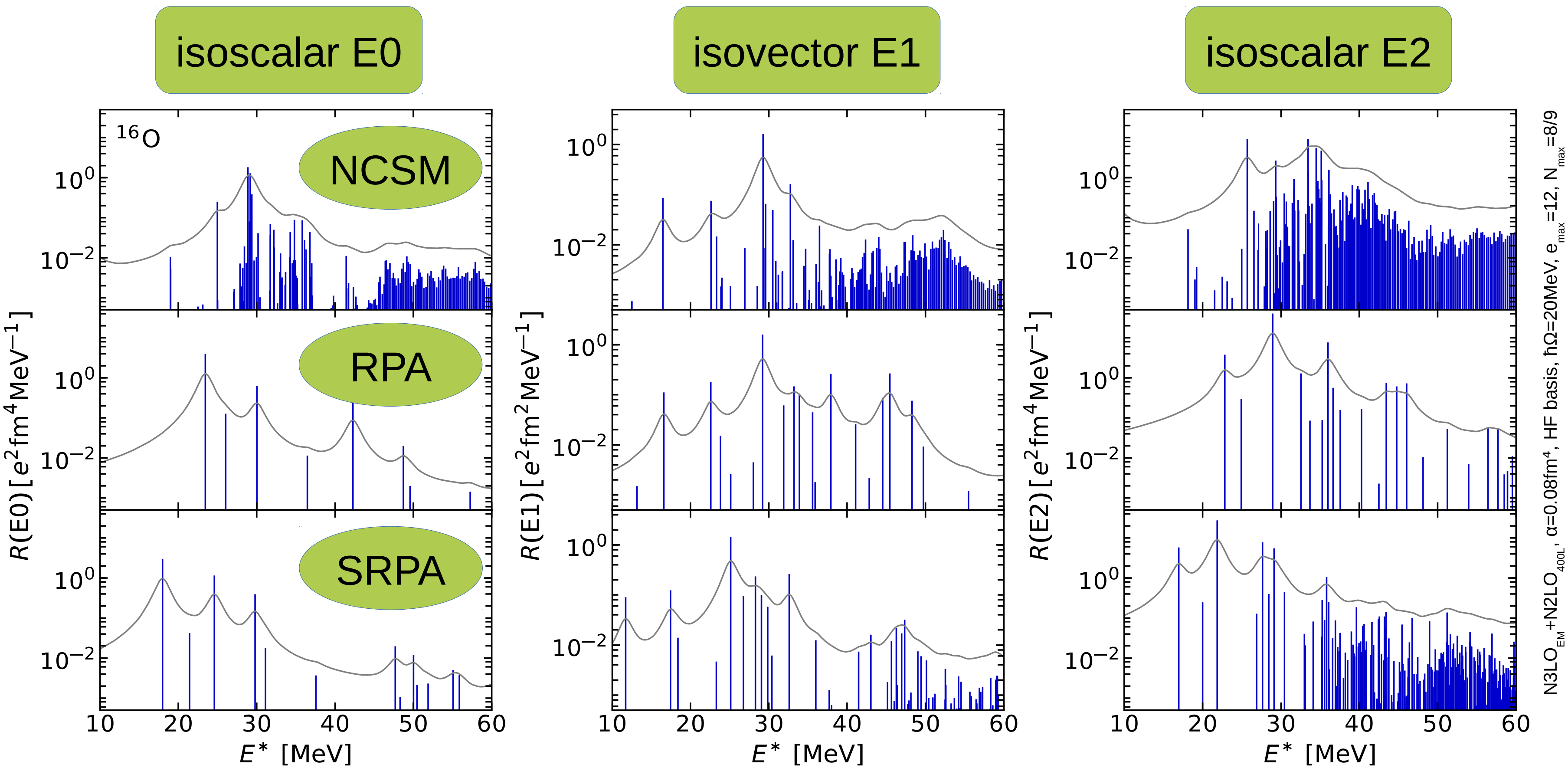

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- SRPA **shifts** strength to lower energies

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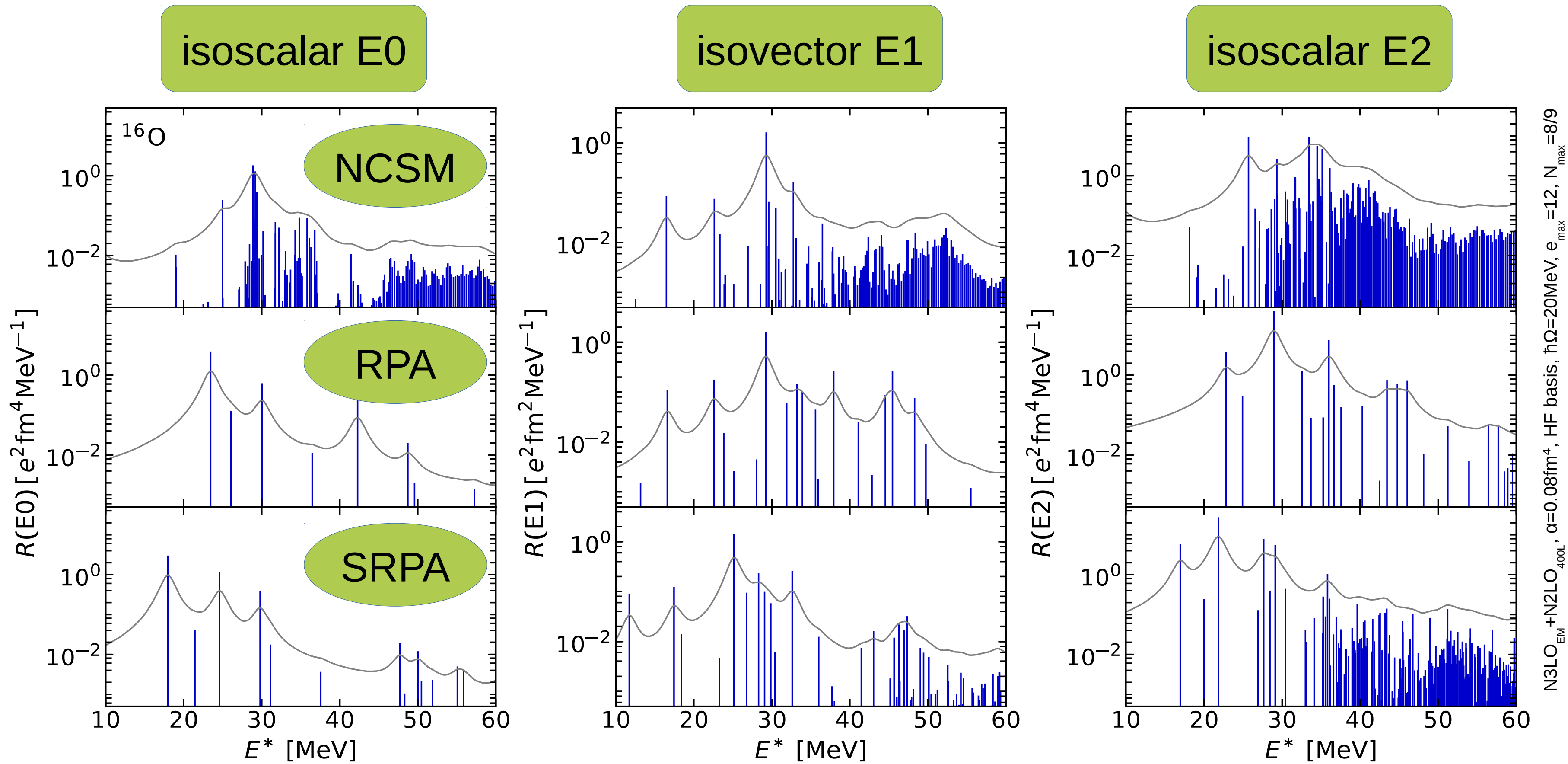
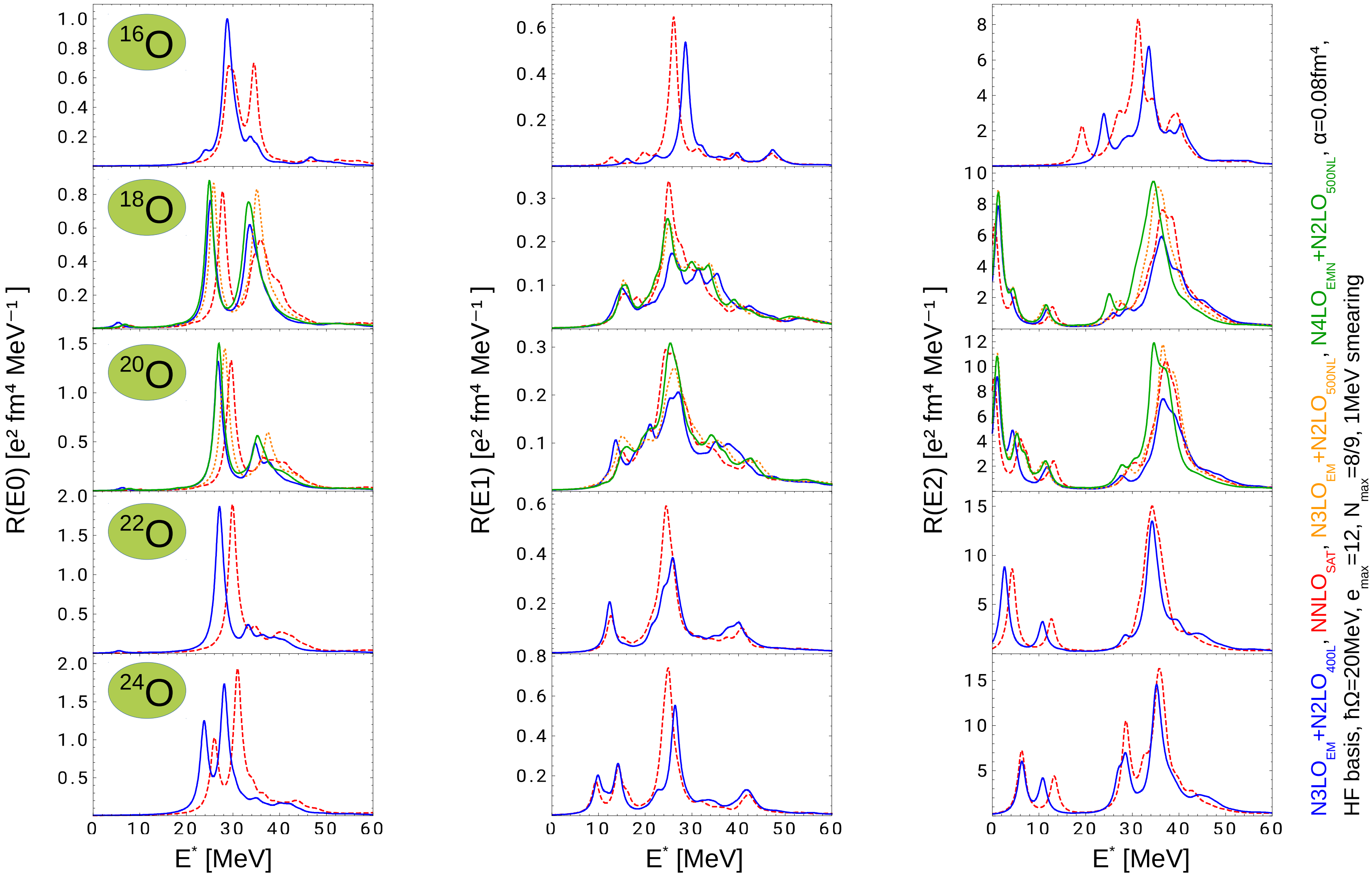


- positions of resonances of NCSM **compatible** with RPA
- SRPA **shifts** strength to lower energies
- NCSM strength: more **fragmentation** and **fine structure**

Oxygen Isotopes



Electromagnetic Strength Distributions from the (In-Medium) NCSM

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Motivation

- electromagnetic transitions provide information about nucleus and depend on detailed form of nuclear wave function
- strength distributions are accessible in experiments and allow for benchmark and improvement of theoretical models
- conventional approximate methods as (S)RPA are used for description of collective excitations
- combination of (IT-)NCSM and Lanczos strength functions yields strength distribution of arbitrary nuclei up to sd-shell from low-lying excitations to giant-resonance region

Simple Lanczos Method

- iterative method for calculation of extreme eigenvalues of Hermitian matrix A
- orthogonal projection method onto Krylov subspaces
- Lanczos algorithm constructs basis → input matrix A is tridiagonal: T_m
- diagonalize T_m → fast-converging approximations for eigenpairs of A

start: pivot vector $\vec{v}_1, ||\vec{v}_1|| = 1$

$\beta_0 = 0, \vec{v}_0 = \vec{0}$

iterate: for $j = 1, 2, ..., m$ do

$\vec{w}_j \leftarrow A\vec{v}_j - \beta_{j-1}\vec{v}_{j-1}$

$\alpha_j \leftarrow (\vec{w}_j, \vec{v}_j)$

$\vec{w}_j \leftarrow \vec{w}_j - \alpha_j\vec{v}_j$

$\beta_j \leftarrow ||\vec{w}_j||$

$\vec{v}_{j+1} \leftarrow \vec{w}_j/\beta_j$

$$\Rightarrow T_m = \begin{pmatrix} \alpha_1 & \beta_1 & & & \\ \beta_1 & \alpha_2 & \beta_2 & & \\ & \ddots & \ddots & \ddots & \\ & & & & \beta_{m-1} \\ & & & \beta_{m-1} & \alpha_m \end{pmatrix}$$

Lanczos Strength Functions

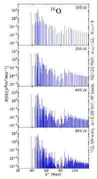
- define pivot state:

$$|v_1\rangle = \frac{1}{\sqrt{\tilde{S}}}\mathbf{O}\,|E_0\rangle \ , \quad \tilde{S} = \langle E_0|\,\mathbf{O}^\dagger\mathbf{O}\,|E_0\rangle$$

- initial state $|E_0\rangle$ from (IT- or IM-)NCSM calculation
- normalization factor $\tilde{S}$ corresponds to total transition strength from GS to any excited state
- use simple Lanczos algorithm (m iterations) with nuclear Hamiltonian and $|v_1\rangle$ as initial vector → tridiagonal matrix T_m
- diagonalization of T_m with standard methods:
  → eigenvalues E_n and eigenvectors $|E_n\rangle = \sum_{i=1}^m C_i^{(n)}\,|v_i\rangle$ as superposition of Lanczos vectors
- with first coefficient $C_1^{(n)} = \langle v_1|E_n\rangle$
  → obtain reduced transition matrix element
- discrete strength distribution:

$$R(E^*) = \sum_n |\,\langle E_0||\mathbf{O}||E_n\rangle\,|^2\delta(E^* - (E_n - E_0))$$

- strength distributions converge fast w.r.t. size of Lanczos basis
- fold discrete strength distribution with Lorentz curve to mimic resolution/continuum effects



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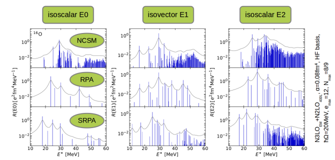
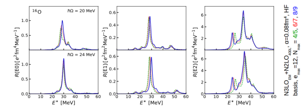
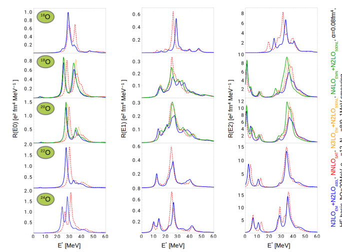


- comparison of Lanczos strength functions with conventional RPA and SRPA
- positions of resonances of NCSM compatible with RPA
- SRPA artificially shifts strength to lower energies
- NCSM strength: more fragmentation and fine structure



- convergence of peak positions with N_{max} and $\hbar\Omega$



- NCSM calculation for oxygen isotopic chain
- structure of strength distributions similar for different up-to-date interactions
- E0: energy shift of a few MeV with increasing mass
- E1: broadening till mid-shell, narrowing till closed neutron shell, strength @ low-energy emerge → Pygmy dipole excitations
- E2: strong contributions of low-lying strength → Pygmy quadrupole resonance

Outlook: In-Medium NCSM

- use simple Lanczos algorithm with IM-SRG-evolved operators and intial state from IM-NCSM calculation → extension to higher mass regime

Technische Universität Darmstadt, Germany

[1] C. Stumpf, T. Wolfgruber, R. Roth, arXiv:1709.06840
[2] E. Gebrerufael, K. Vobig, H. Hergert, R. Roth, Phys. Rev. Lett. 118, 152503 (2017)

Thank you for your attention!

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COMPUTING TIME