# Nuclear Electroweak Processes in a Multipole Decomposition Framework

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Progress in ab initio techniques in nuclear physics
TRIUMF, Vancouver, BC, Canada

March 6, 2020

# Nuclear electroweak currents for spherical basis calculations

- Recently extended to fourth order in $\chi$EFT [Krebs et al. (2019), Krebs et al. (2017), Baroni et al. (2016)].
- Contain scalar and vector products of $\boldsymbol{\sigma}$, nucleon momenta $\mathbf{p}$, momentum transfer $\mathbf{q}$, etc.
- Many ab initio methods use spherical basis expansions and can benefit from a multipole decomposition of the current operators.

$$\rho = \sqrt{4\pi} \sum_{\Lambda} i^{\Lambda} \sqrt{2\Lambda + 1}\, \mathcal{C}_{\Lambda}^{0}(q)\,,$$

$$j_{\lambda} = -\sqrt{2\pi(1 + \delta_{0\lambda})} \sum_{\Lambda} i^{\Lambda} \sqrt{2\Lambda + 1} \left( \mathcal{L}_{\Lambda}^{0}(q)\, \delta_{0\lambda} + [\lambda \mathcal{M}_{\Lambda}^{\pm 1}(q) + \mathcal{E}_{\Lambda}^{\pm 1}(q)]\, \delta_{\pm 1\lambda} \right).$$

- Calculated matrix elements of $\chi$EFT currents up to third chiral order.

# The electromagnetic response functions

The electrodissociation of a nucleus by a lepton can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\mathrm{d}\omega} = \sigma_{\mathrm{Mott}} \left( \left[ \frac{q^\mu q_\mu}{\mathbf{q}^2} \right]^2 R_L - \left[ \frac{q^\mu q_\mu}{2\mathbf{q}^2} - \tan^2(\theta/2) \right] R_T \right)$$

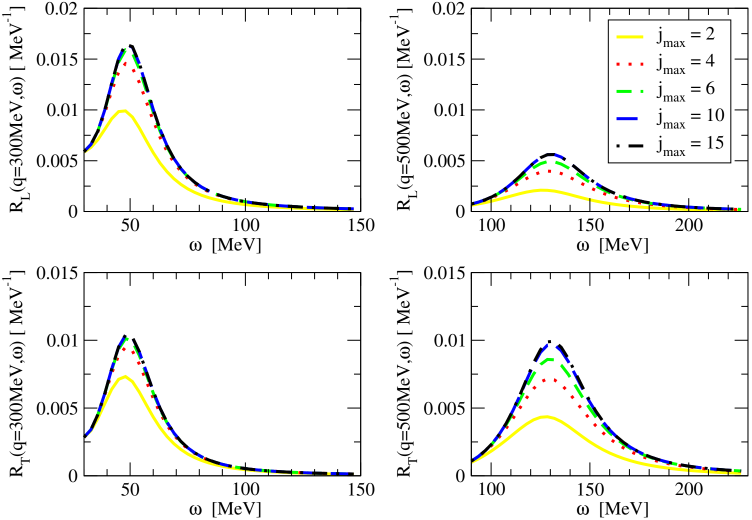
in terms of the longitudinal inelastic response function,

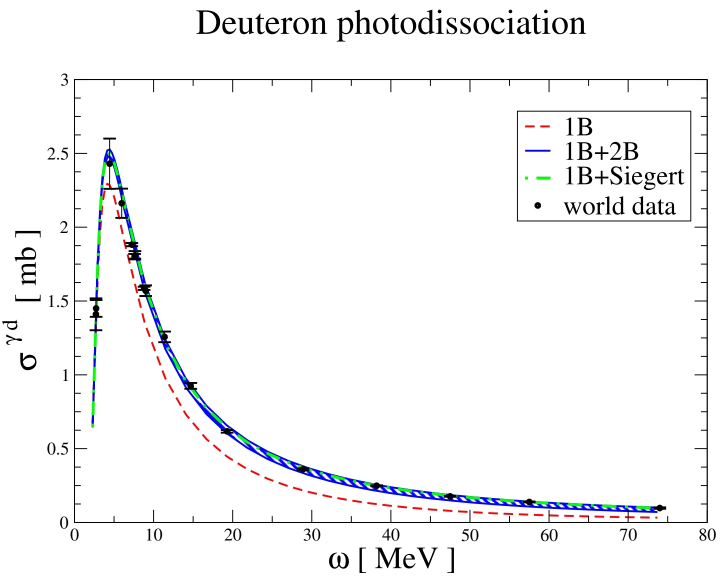
$$R_L = \sum_f |\langle \psi_f | \rho(q^\mu) | \psi_i \rangle|^2 \, \delta(\omega + m_d - E_f)$$

and the transverse one,

$$R_T = \sum_f |\langle \psi_f | \mathbf{j}_\perp(q^\mu) | \psi_i \rangle|^2 \, \delta(\omega + m_d - E_f)$$

# EM responses of the deuteron as mulipole expansions





Deuteron photodissociation
1B
1B+2B
1B+Siegert
world data
σ γd [ mb ]
ω [ MeV ]
0
0.5
1
1.5
2
2.5
3
0
10
20
30
40
50
60
70
80

# Application: Lamb shift of muonic deuterium

$\mathcal{O}(\alpha^5)$ nuclear+hadronic correction:

$$\Delta E_{nl} = \frac{8\alpha^2 m}{i\pi} |\phi_{nl}(0)|^2 \int d^4Q \frac{(Q^2 - 2\nu^2)T_1(\nu, Q^2) - (Q^2 + \nu^2)T_2(\nu, Q^2)}{Q^4(Q^4 - 4m^2\nu^2)}$$

Scalar VVCS amplitudes $T_{1,2}$:

- Consist of elastic and finite size and polarizability contributions.
- Imaginary parts are related to deuteron structure functions $F_{1,2}$ (or $R_{L,T}$) by optical theorem.
- Real parts are given by the dispersion relations:

$$\Re T_1(\nu, Q^2) = \widetilde{T}_1(0, Q^2) + \frac{\nu^2}{2\pi M_d}\mathcal{P}\int_{\nu_0}^{\infty} \frac{\mathrm{d}\omega}{\omega}\frac{F_1(\omega, Q^2)}{\omega^2 - \nu^2},$$

$$\Re T_2(\nu, Q^2) = \frac{1}{2\pi}\int_{\nu_0}^{\infty} \mathrm{d}\omega\, \frac{F_2(\omega, Q^2)}{\omega^2 - \nu^2},$$

where full $Q^2$ dependence of the subtraction constant $\widetilde{T}_1(0, Q^2)$ can be determined using a newly discovered sum rule [Gorchtein (2015)].

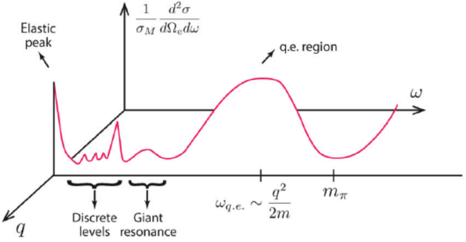
# Application: Lamb shift of muonic deuterium

| | 1B (pt. nucl.) | 1B | 1B+Siegert | 1B+2B | |
|---|---|---|---|---|---|
| Longitudinal | -1.4931 | -1.4610 | -1.5137 | -1.4968 | |
| Transverse | -0.0024 | -0.0031 | -0.0062 | -0.0046 | |
| Total | -1.4955 | -1.4641 | -1.5199 | -1.5014 | |
| Hernandez et al. (2019) | | | -1.531(0.012) | | |
| Carlson et al. (2014) | | | | | -1.589(740) |

Table: Nuclear polarizability correction (meV) with E&M-500 interactions.

- Hadronic contribution is much smaller [0.028(2) meV].
- Expect an uncertainty of $\mathcal{O}(1\%)$ on the above results.
- Sensitive to longitudinal response at low virtualities ($Q^2 < 10^4\,\mathrm{MeV}^2$).

# Neutrino nucleus scattering



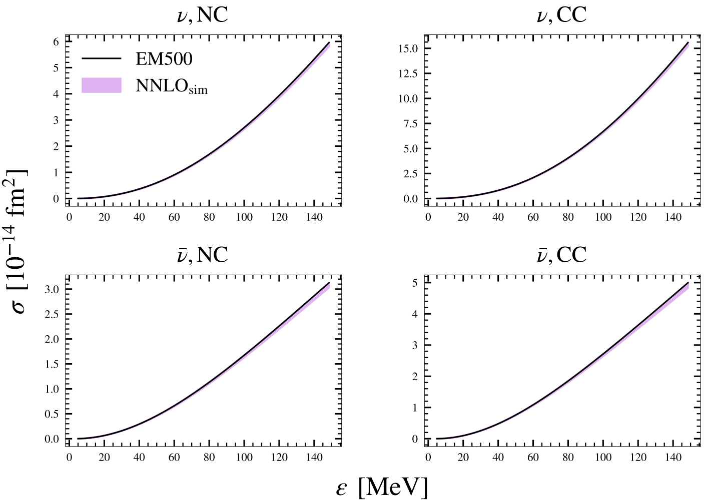

- Much smaller statistical uncertainties in next-generation $\nu$ experiments warrant better control over systematics. $\nu$-$A$ cross section is the major contributor.
- *Ab initio* $\chi$EFT calculations can provide important benchmarks for modeling the quasielastic peak.
- Rigorous accounting of both FSI and 2B currents hasn't been done yet.
- $^{16}O$ and $^{40}Ar$ are both amenable to LIT-CC treatment.

# Deuteron neutrinodissociation

| | $\nu$,NC | $\bar{\nu}$,NC | $\nu$,CC | $\bar{\nu}$,CC |
|---|---|---|---|---|
| $(Q/\Lambda_b)^{-3}$ | 2.08 | 4.38 | 2.08 | 4.02 |
| $(Q/\Lambda_b)^{-2}$ | 2.58 | 6.46 | 1.61 | 2.61 |
| Shen et al. (2012) [1B, AV-18] | 2.58 | 6.42 | 1.60 | 2.57 |
| $(Q/\Lambda_b)^{-1}$ | 2.62 | 6.53 | 1.60 | 2.60 |
| $(Q/\Lambda_b)^{0}$ | 2.71 | 6.67 | 1.69 | 2.71 |
| Baroni et al. (2017) [$(Q/\Lambda_b)^{1}$] | 2.74 | 6.85 | 1.68 | 2.68 |

Table: Convergence of deuteron $\bar{\nu}/\nu$ cross sections (in $10^{-14}\,\mathrm{fm}^2$) at 100 MeV with wave functions obtained from E&M-500 N3LO interactions, with dipole form factors, and counterterm $d_R$ from $c_{D,E}$ fit to tritium binding energy and beta decay half life.

# The NNLO$_{\rm sim}$ uncertainties



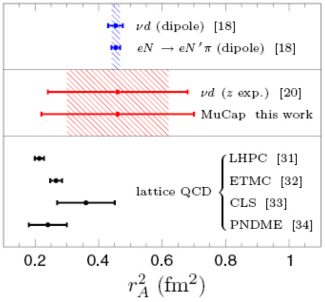



Figure: Compilation of nucleon axial radius values [Hill et al. (2018)].

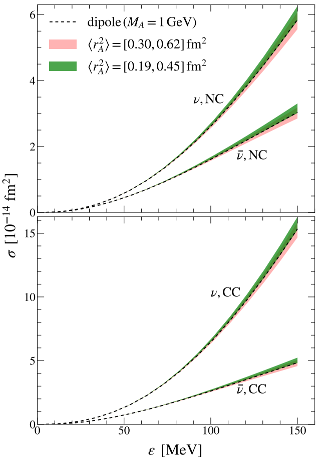
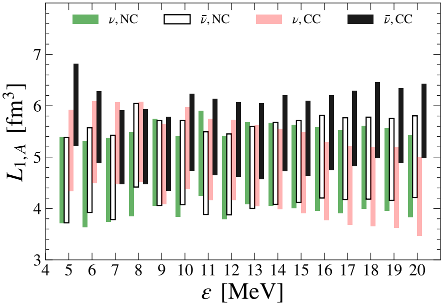



Figure: Nucleon-structure uncertainties in the 1B cross sections. Bands are due to the span of axial radius values shown on the left.

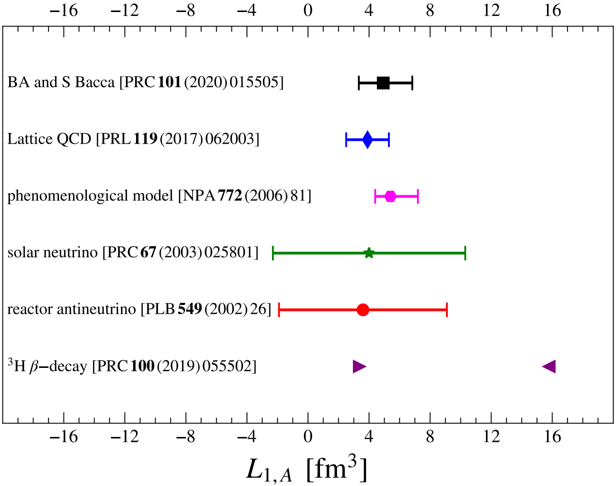
# $L_{1,A}$ constraints from $\chi$EFT

The Pionless EFT counterterm $L_{1,A}$:

- Occurs at NLO in the isovector axial current.
- Major source of uncertainty in Pionless EFT calculation of *pp* fusion.
- Normally quoted at Pionless EFT renormalization scale set to $m_\pi$.
- Constrained here by matching the NNLO$_{\rm sim}$ results with the (updated) NNLO Pionless EFT calculation [Butler et al. (2001)].



# $L_{1,A}$ constraints from $\chi$EFT



# Summary

- Photo-, electro and neutrino-dissociation of the deuteron were calculated with multipole-decomposed chiral electroweak currents and validated by comparing to experiments and other calculations.
- Nuclear-structure uncertainty in $\bar{\nu}/\nu$-$d$ cross sections constrained better than nucleon-structure uncertainty.
- Matching low-energy chiral EFT results with and Pionless EFT yields better constraints than existing experiments.

*Thank you!*

# References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