Self-Consistent Green's Functions Calculations of Charge Radii and Distribution up to A=140

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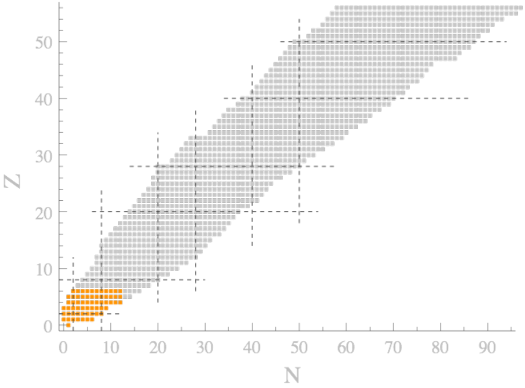
Progress in *Ab Initio* Techniques in Nuclear Physics
TRIUMF, Vancouver, BC – 4 March 2020

Ab initio methods for the heavy sector

Reach of *ab initio* methods



Courtesy of V. Somà, T. Duguet

"Exact" methods (80's)

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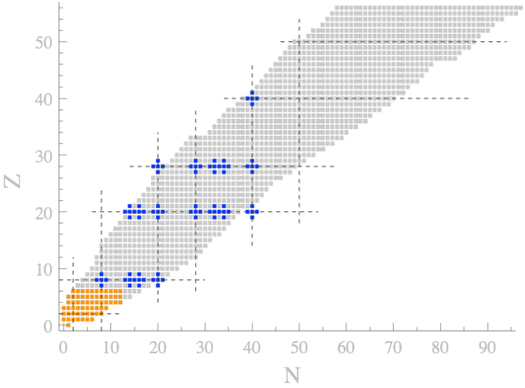

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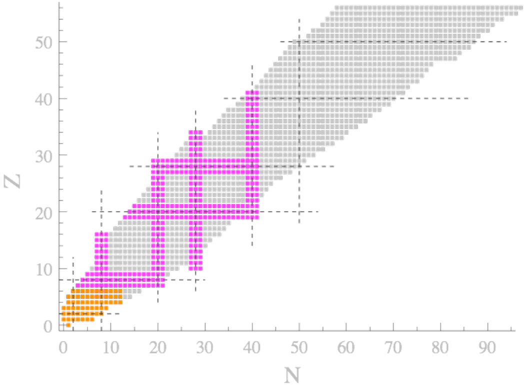

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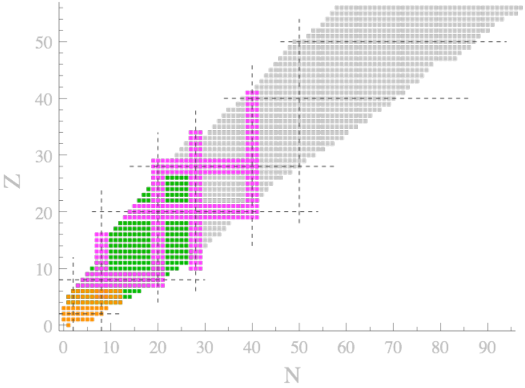

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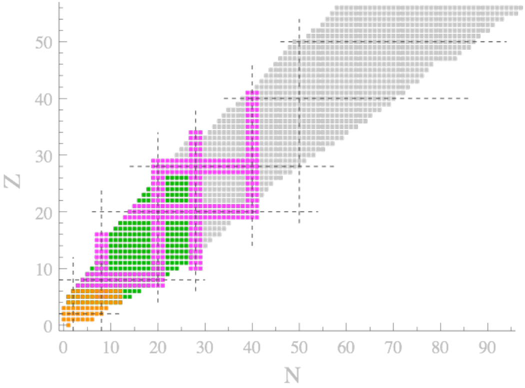

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What does prevent us from going further up in mass?

Nowadays limitations

Nuclear interaction accuracy

- Chiral potentials tend to largely overbind in mid-mass nuclei
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How to circumvent it

- New interactions: NNLO_{sat}, NN+3N_{lnl}, 1.8/2.0(EM), ΔNNLO_{GO}...
- Work on storage cost: CPD, THC, IT, alternative NO2B...
- Promising but requires to adapt the existing set-ups, takes time

How to move forward with today's tools?

***Ab initio* method**

- Limit the computational cost: single-reference method
- Fewer and fewer closed-shell nuclei: symmetry-breaking method
- Access to various observables

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NNLOsat

Self-consistent Green's functions in a nutshell

Concept behind SCGFs

Recast the solution of the SE as a set of one- to A-body Green's functions

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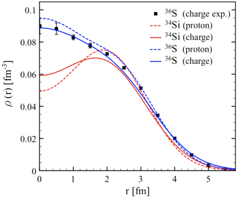
- Ground-state energy
- One-body observables (radii, densities...)
- Spectroscopy of the $A \pm 1$-body systems
- Elastic nucleon-nucleus scattering



[Duguet *et al.*, 2017]

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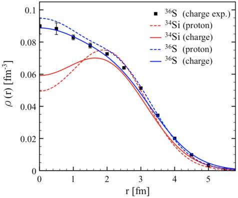

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See Mehdi Drissi's talk tomorrow for neutron matter SCGF!

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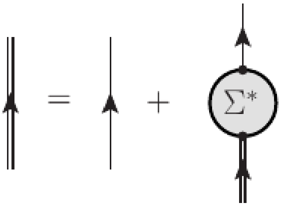
How to proceed

Solve the Dyson equation self-consistently: infinite diagram series resummation



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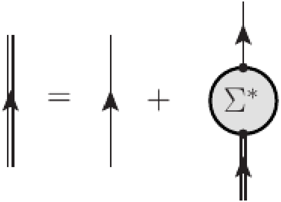

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Algebraic Diagrammatic Construct at order (n) [ADC(n)]

- Perturbative order + necessary diagrams to preserve analytical properties
- ADC(∞) gives exact result
- In practice numerically: ADC(3) for Dyson, ADC(2) for Gorkov

Our case study:
Xenon and Tin isotopes

[Arthuis, Barbieri, Vorabbi, Finelli, arXiv:2002.02214 (2020)]

Why and where to go above the Sn isotopic chain?

What are the present-day limits?

- ^{100}Sn, ^{132}Sn with CC: proof-of-principle calculations [Binder *et al.*, 2013]
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Our goals

- ^{100}Sn, ^{132}Sn: doubly-closed shell milestones, convergence study
- ^{132}Xe, ^{136}Xe and ^{138}Xe: open-shell nuclei of experimental interest
- Use interactions with good accuracy in the mid-mass area
- Look at both ground-state energy and radii

Computational details

Model space

- Harmonic oscillator basis
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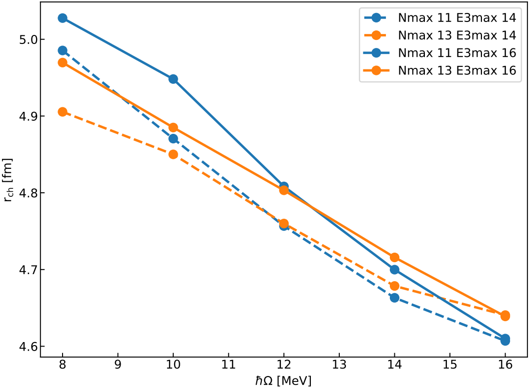
First exploratory calculations

- Variational minimum for $\hbar\Omega$ between 10 and 16 MeV
- Ground-state energy not converged w.r.t. model space size
- Can we say something about the radius?

Model space convergence on ^{132}Xe



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- Low influence of the E3max truncation on the expected radius value
- ADC(3) included in the error bars for Sn isotopes

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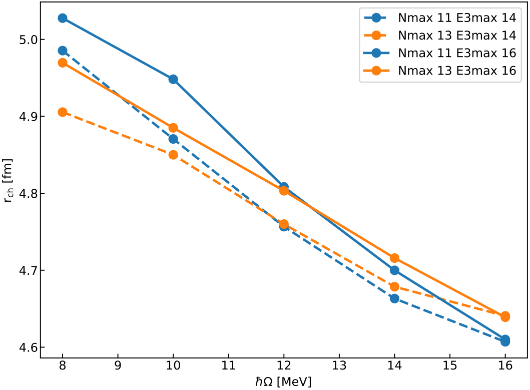

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Possible to extract radii with NNLOsat despite small model space

Charge radii values

	SCGF	Exp.
^{100}Sn	4.525 – 4.707	
^{132}Sn	4.725 – 4.956	4.7093
^{132}Xe	4.700 – 4.948	4.7859
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Table: Charge radii in fm obtained from SCGF calculations.

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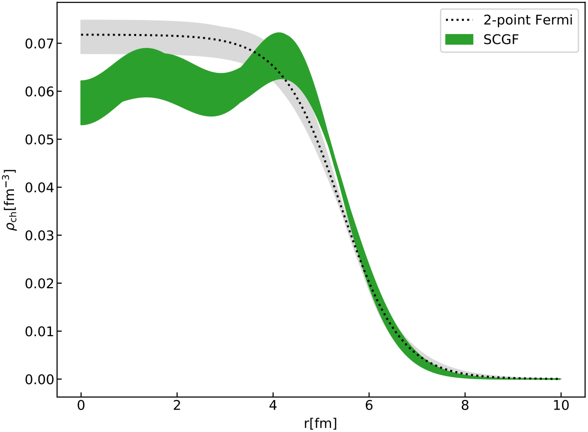
NNLOsat confirmed to work in the heavy mass regime

^{132}Xe charge density distribution



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Hamiltonian

- Chiral EFT
  Bare NNLOsat
- Effective 3NF

Basis parameters

- $\hbar\Omega = 10$ to 14 MeV
- $N_{max} = 11 - 13$
- $E_{3max} = 16$

- Radius very close to the experiment: 4.82 ± 0.12 fm vs. $4.79^{+0.11}_{-0.08}$ fm
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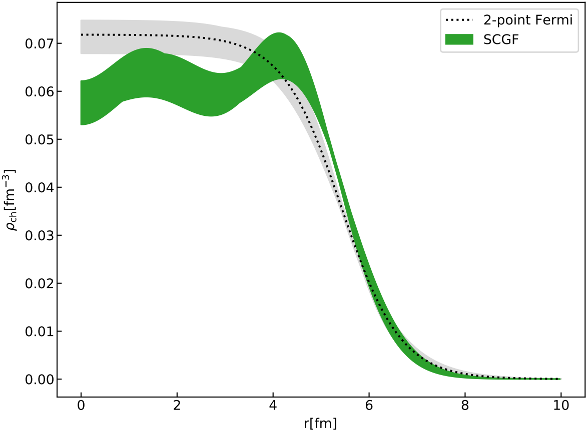

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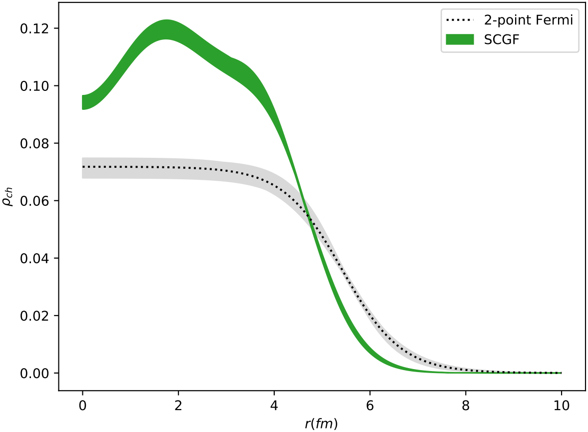
Need more precise experiment to determine the charge distribution

^{132}Xe charge density distribution... with NN+3nlnl



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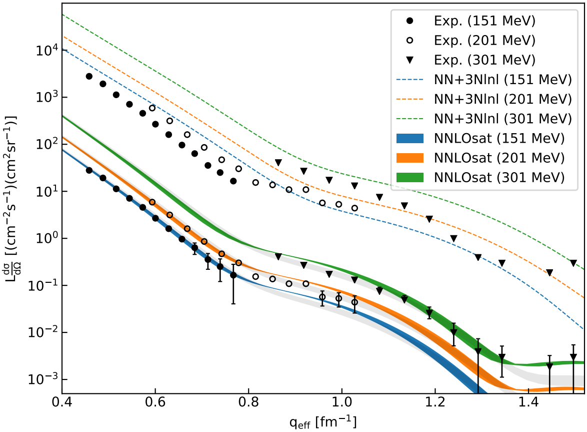
- Radius largely underestimated: 4.070 ± 0.045 fm
- Inability of NN+3NInl to reproduce the experimental properties

Reproducing the SCRIT experiment data



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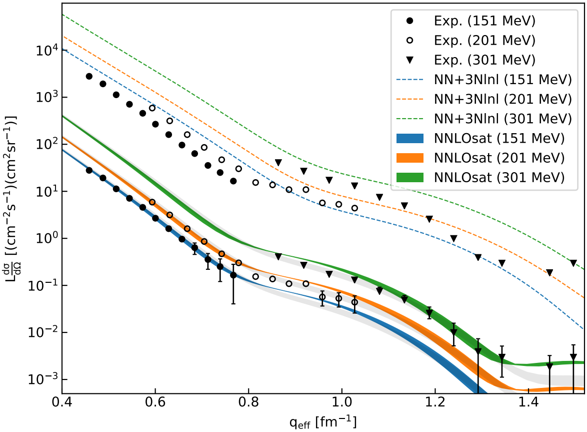

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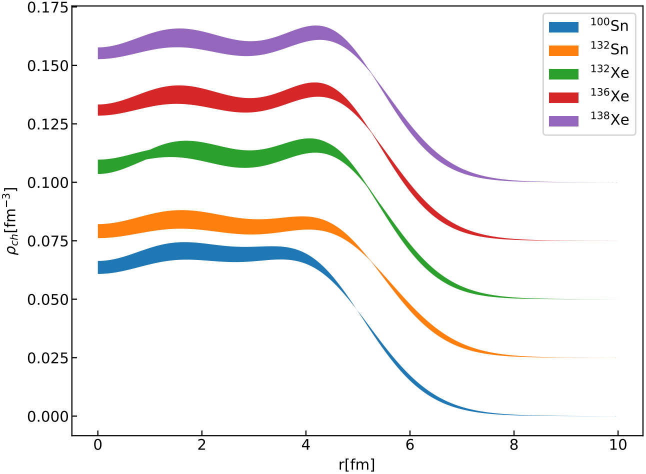
Theory and experiment can be compared in the heavy mass sector

Heavy nuclei charge density distributions



- Charge distributions consistent over the different isotopes
- Paves the way to density distribution studies of heavy nuclei

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- Mid-mass area as a good test bed for heavy-nuclei accuracy
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The future of *ab initio* methods

- Experiments can already be constrained by *ab initio* calculations
- Every improvement in ME storage is a move forward in mass

Group and collaborators

C. Barbieri
M. Drissi
J. Keeble
A. Rios

M. Vorabbi

P. Finelli

Thank you for your attention!