Consistently regularized Nuclear Forces and Currents in Chiral EFT

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Progress in Ab Initio Techniques in Nuclear Physics
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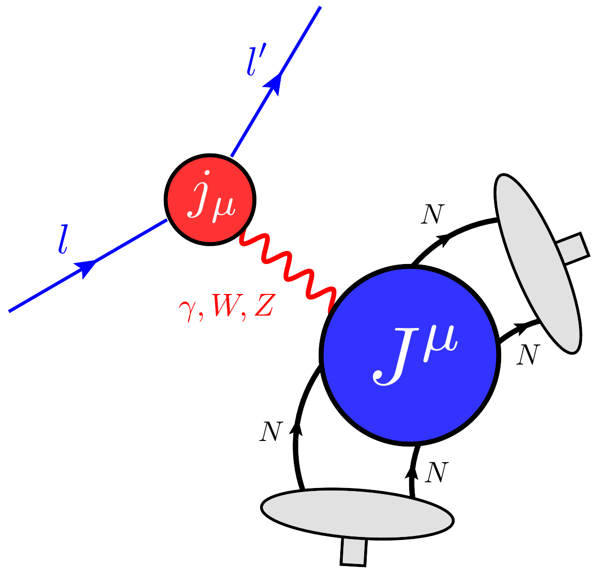
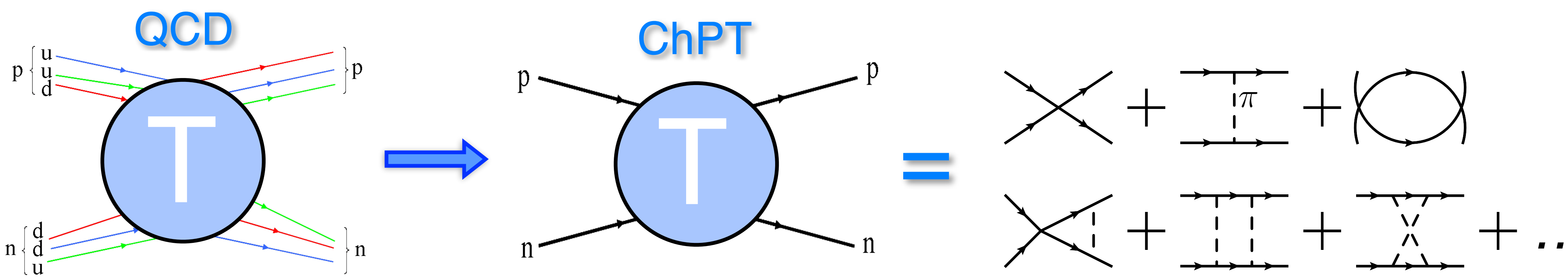
RUB

Outline

- Nuclear forces in chiral EFT
- Nuclear currents in chiral EFT up to N^3LO
- Symmetry preserving regularization
- Application to em deuteron form factor



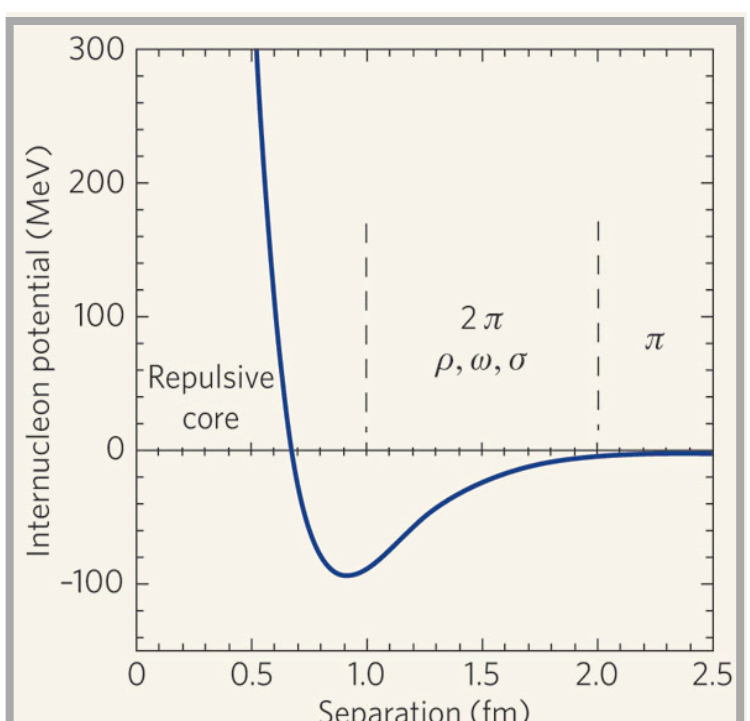
From QCD to nuclear physics

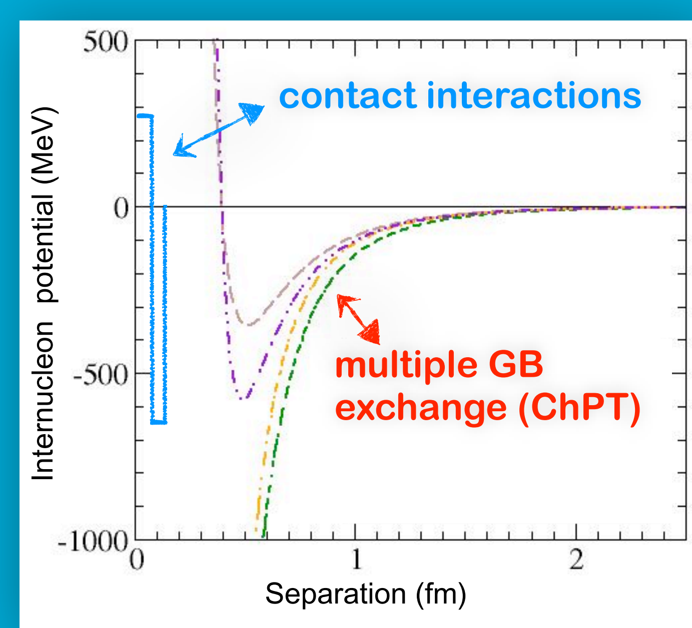


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⇨ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

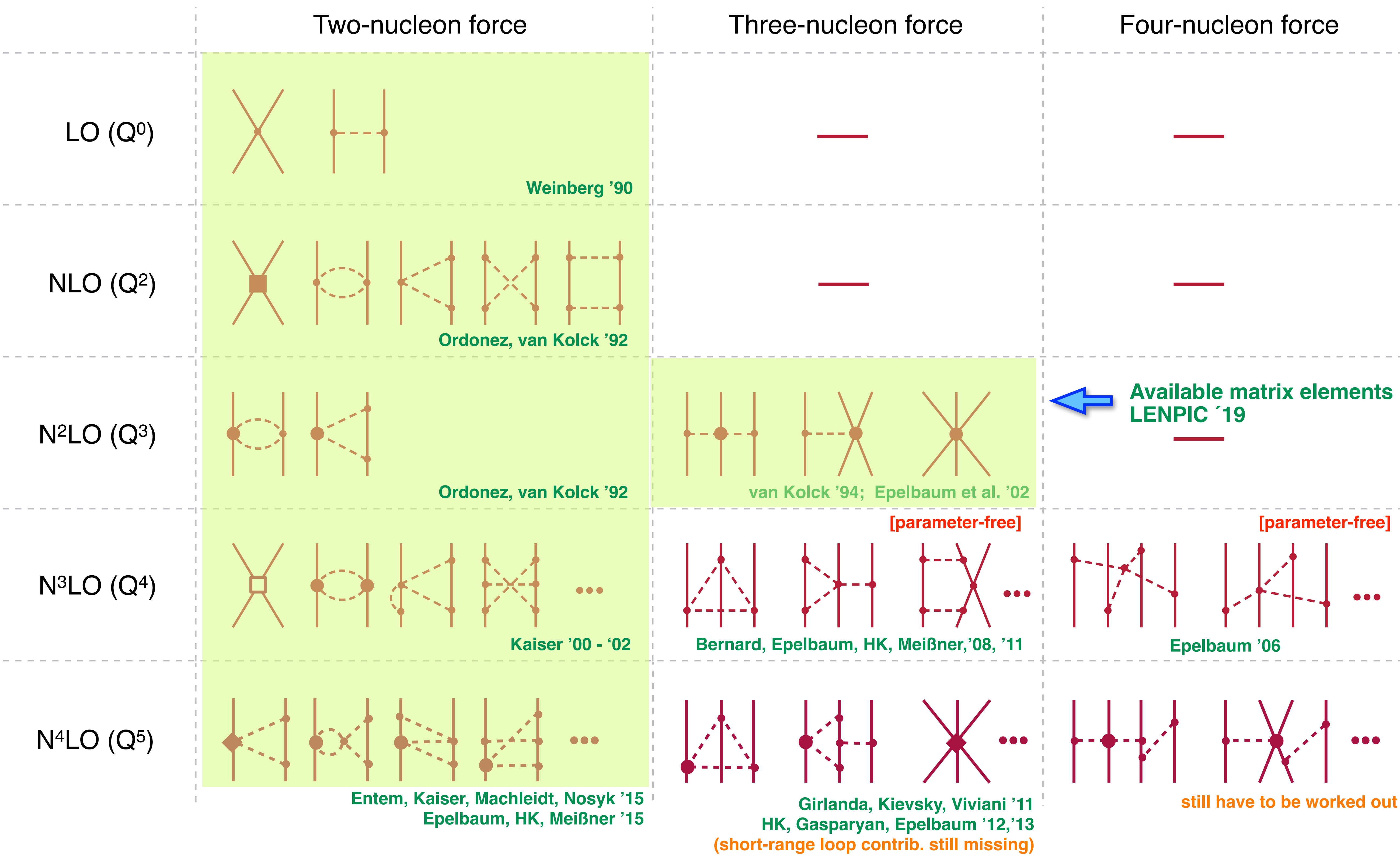
Weinberg '91





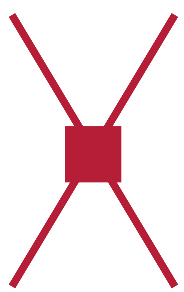
- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

Chiral Expansion of the Nuclear Forces



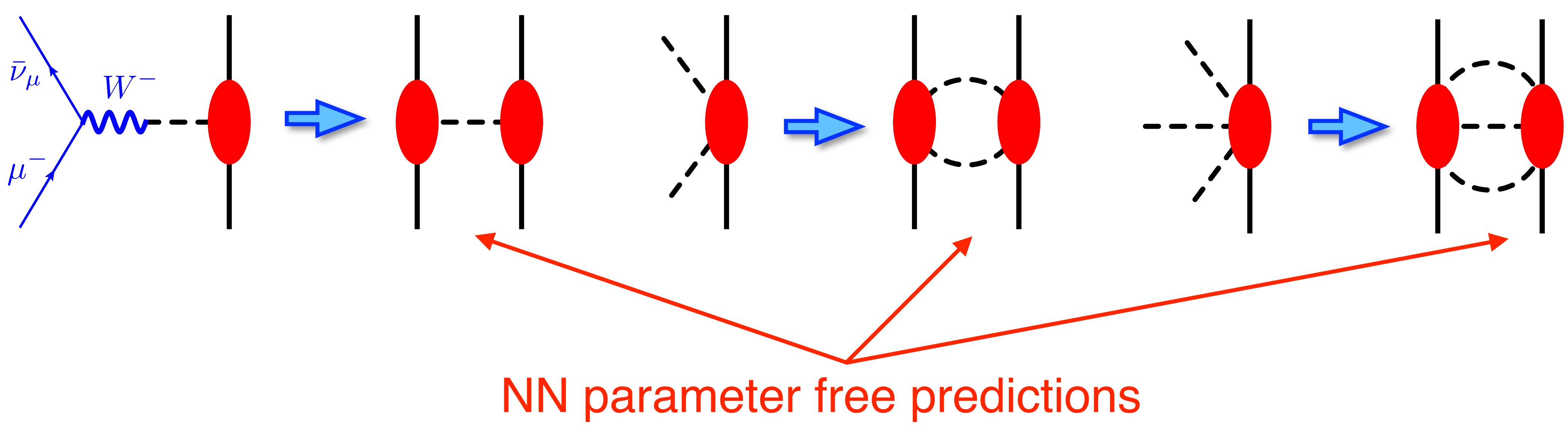
Long and Short Range Interactions

- Couplings of short-range interactions are fixed from NN - data.
  In the isospin limit we have:



LO [Q^0]: 2 operators (S-waves)
NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)
N^{2}LO [Q^3]: no new terms
N^{3}LO [Q^4]: + 12 operators (S-, P-, D-waves and ε_1, ε_2)
N^{4}LO [Q^5]: no new terms

- Long range part of the nuclear forces are predictions (chiral symmetry of QCD) once couplings from single-nucleon subprocess are determined



Short-Range LECs

- Slow convergence of fits to data at N3LO & beyond → Redundancy of LECs

Hammer, Furnstahl '00, Beane, Savage '01, Wesolowski et al.'16

Short-range LECs at N³LO: $V_{\rm cont}^{(Q^4)} = D_1 q^4 + D_2 k^4 + D_3 q^2 k^2 + \cdots + D_{15} \vec{\sigma}_1 \cdot (\vec{q} \times \vec{k}) \vec{\sigma}_2 \cdot (\vec{q} \times \vec{k})$

$U = e^{\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3}$ with $T_1 = \frac{m_N}{2\Lambda_b^4} \vec{k} \cdot \vec{q}, \quad T_2 = \frac{m_N}{2\Lambda_b^4} \vec{k} \cdot \vec{q}\, \vec{\sigma}_1 \cdot \vec{\sigma}_2, \quad T_3 = \frac{m_N}{2\Lambda_b^4} \Big(\vec{\sigma}_1 \cdot \vec{k}\, \vec{\sigma}_2 \cdot \vec{q} + \vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{k} \Big)$

applied to kinetic energy operator generates short-range N³LO structures

$$U^\dagger H_0 U = H_0 + \frac{\gamma_1}{\Lambda_b^4} (\vec{k} \cdot \vec{q})^2 + \frac{\gamma_2}{\Lambda_b^4} (\vec{k} \cdot \vec{q})^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \frac{\gamma_3}{\Lambda_b^4} \vec{k} \cdot \vec{q} \Big(\vec{\sigma}_1 \cdot \vec{k}\, \vec{\sigma}_2 \cdot \vec{q} + \vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{k} \Big) + \mathcal{O}(Q^5)$$

Conventional choice of $\gamma_{1,2,3}$

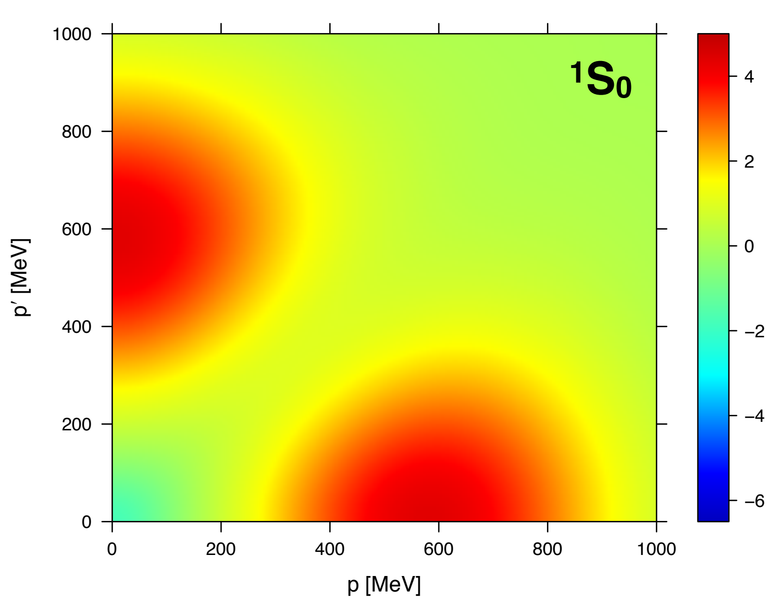
$D_{1S0}^{\rm off} = D_{3S1}^{\rm off} = D_{\epsilon 1}^{\rm off} = 0$

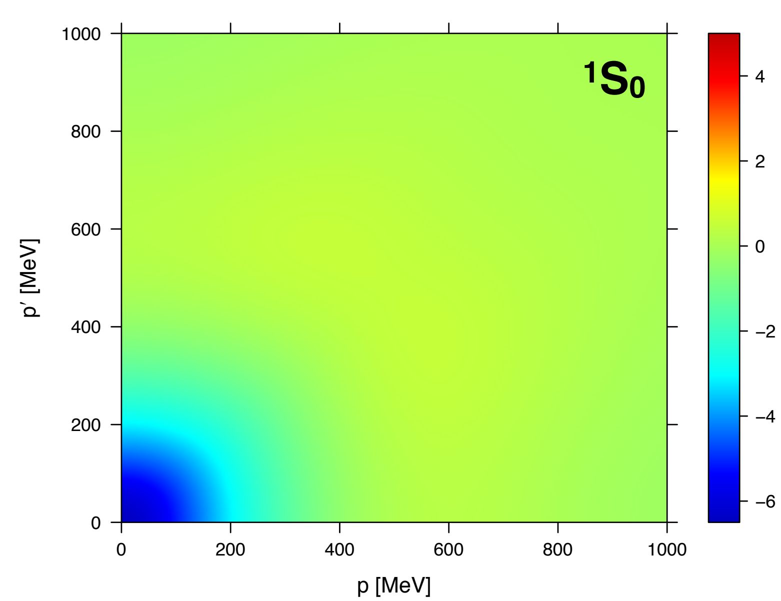
leads to softer NN interactions

$$\langle {}^1S_0, p' | V_{\rm cont} | {}^1S_0, p \rangle = \tilde{C}_{1S0} + C_{1S0}(p^2 + p'^2) + D_{1S0}\, p^2 p'^2 + D_{1S0}^{\rm off}\, (p^2 - p'^2)^2$$

$$\langle {}^3S_1, p' | V_{\rm cont} | {}^3S_1, p \rangle = \tilde{C}_{3S1} + C_{3S1}(p^2 + p'^2) + D_{3S1}\, p^2 p'^2 + D_{3S1}^{\rm off}\, (p^2 - p'^2)^2$$

$$\langle {}^3S_1, p' | V_{\rm cont} | {}^3D_1, p \rangle = C_{\epsilon 1} p^2 + D_{\epsilon 1}\, p^2 p'^2 + D_{\epsilon 1}^{\rm off}\, p^2 (p^2 - p'^2)$$

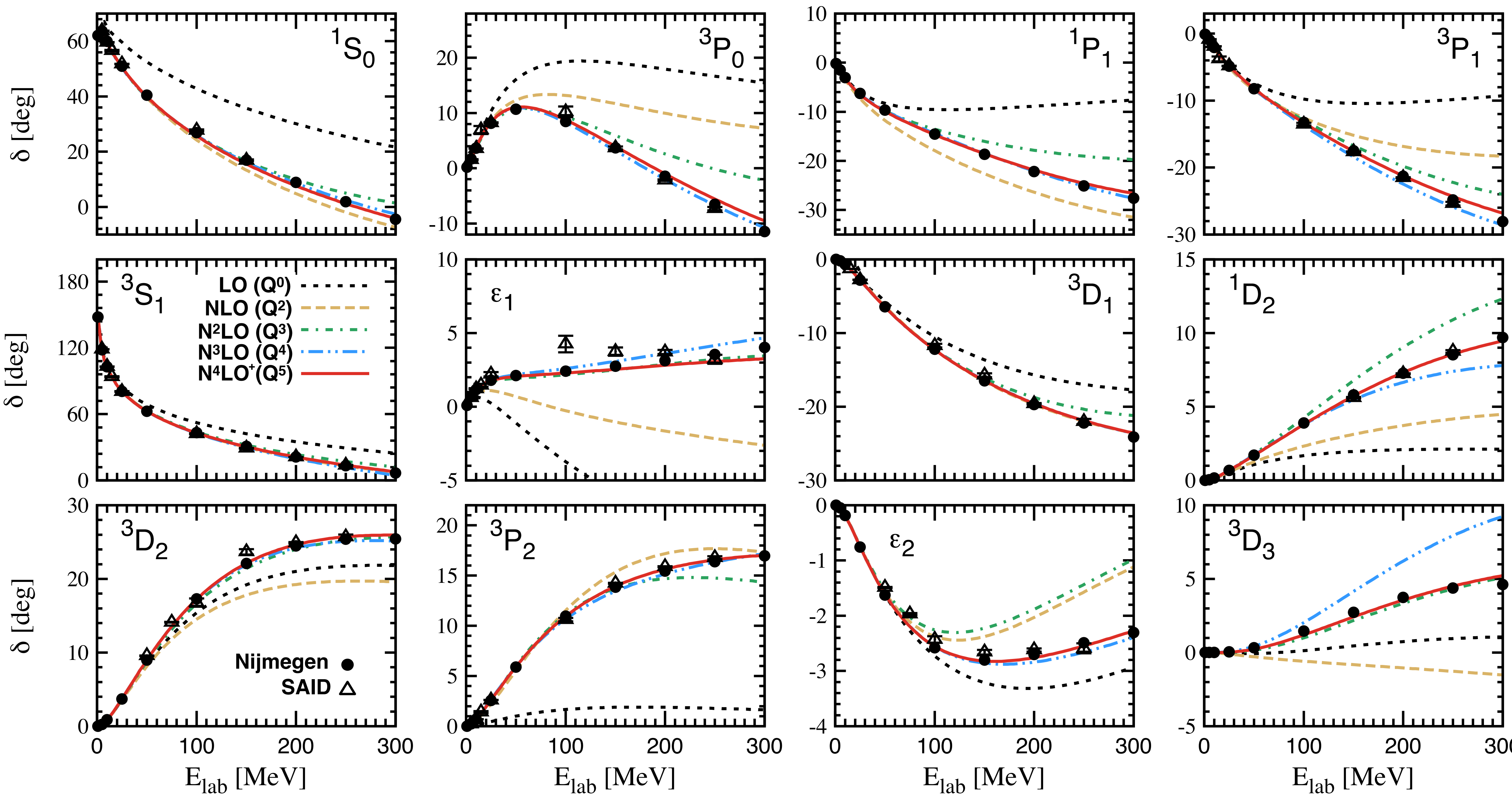




Softness of NN interaction for made choice of $\gamma_{1,2,3}$ confirmed by Weinberg eigenvalue analysis

Chiral Expansion of np Phase Shifts

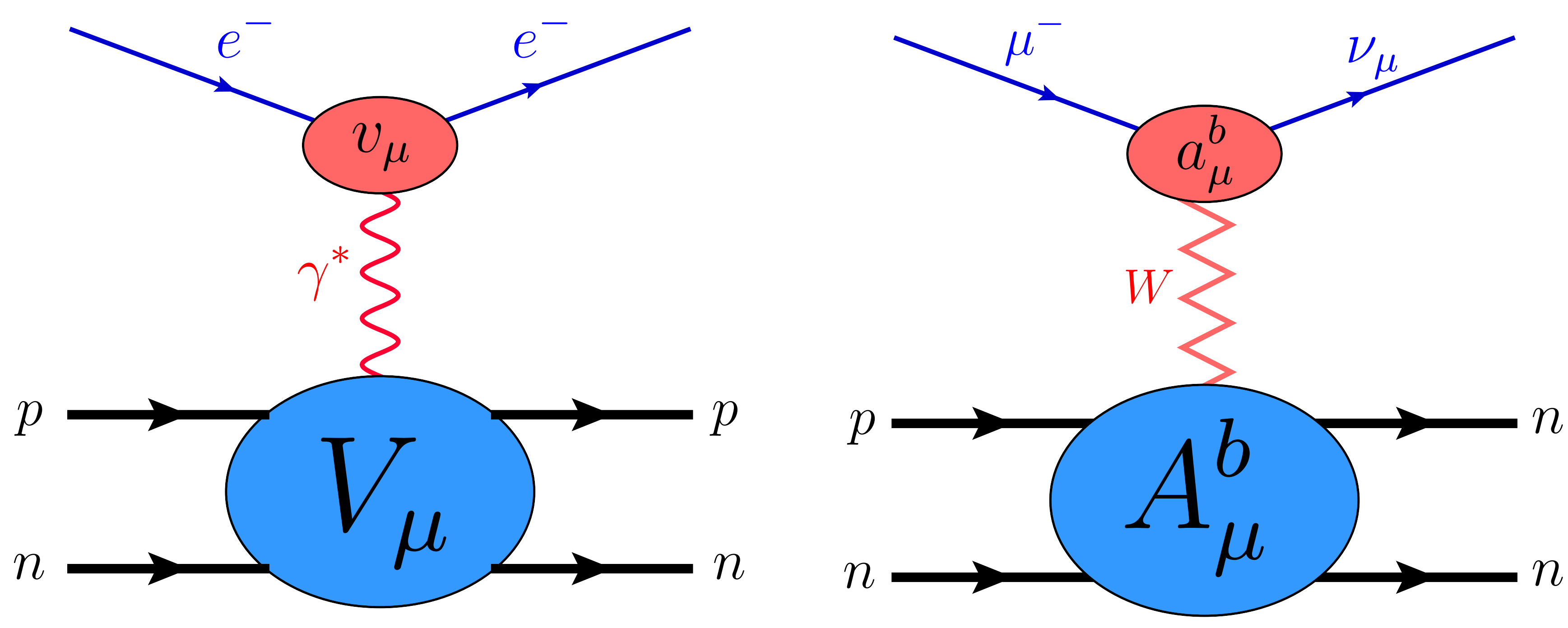
Reinert, HK, Epelbaum '17



- Good convergence of chiral expansion & excellent agreement with NPWA data
- Chiral potential match in precision phenomenological potentials (CD Bonn, Av18,…) with around 40% less parameter

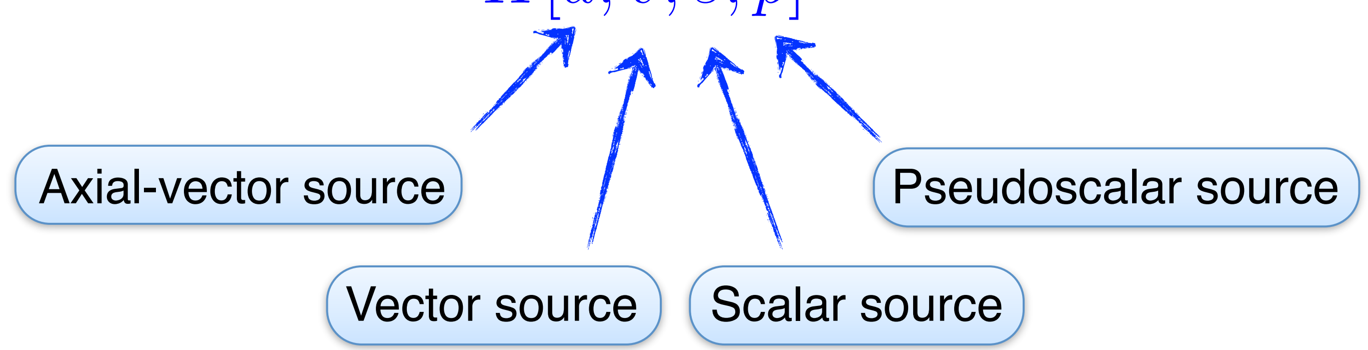
Nuclear currents in chiral EFT

Electroweak probes on nucleons and nuclei can be described by current formalism



Chiral EFT Hamiltonian depends on external sources

$$H[a, v, s, p]$$



Historical remarks

- Meson-exchange theory, Skyrme model, phenomenology, …
  Brown, Adam, Mosconi, Ricci, Truhlik, Nakamura, Sato, Ando, Kubodera, Riska, Sauer, Friar, Gari …

- First derivation within chiral EFT to leading 1-loop order using TOPT
  Park, Min, Rho Phys. Rept. 233 (1993) 341; NPA 596 (1996) 515;
  Park et al., Phys. Rev. C67 (2003) 055206
  – only for the threshold kinematics
  – pion-pole diagrams ignored
  – box-type diagrams neglected
  – renormalization incomplete

- Leading one-loop expressions using TOPT for general kinematics (still incomplete, e.g. no 1/m corrections)
  Pastore, Girlanda, Schiavilla, Goity, Viviani, Wiringa;
  PRC78 (2008) 064002; PRC80 (2009) 034004; PRC84 (2011) 024001 ← Vector current

  Baroni, Girlanda, Pastore, Schiavilla, Viviani;
  PRC93 (2016) 015501, Erratum: PRC 93 (2016) 049902 ← Axial vector current

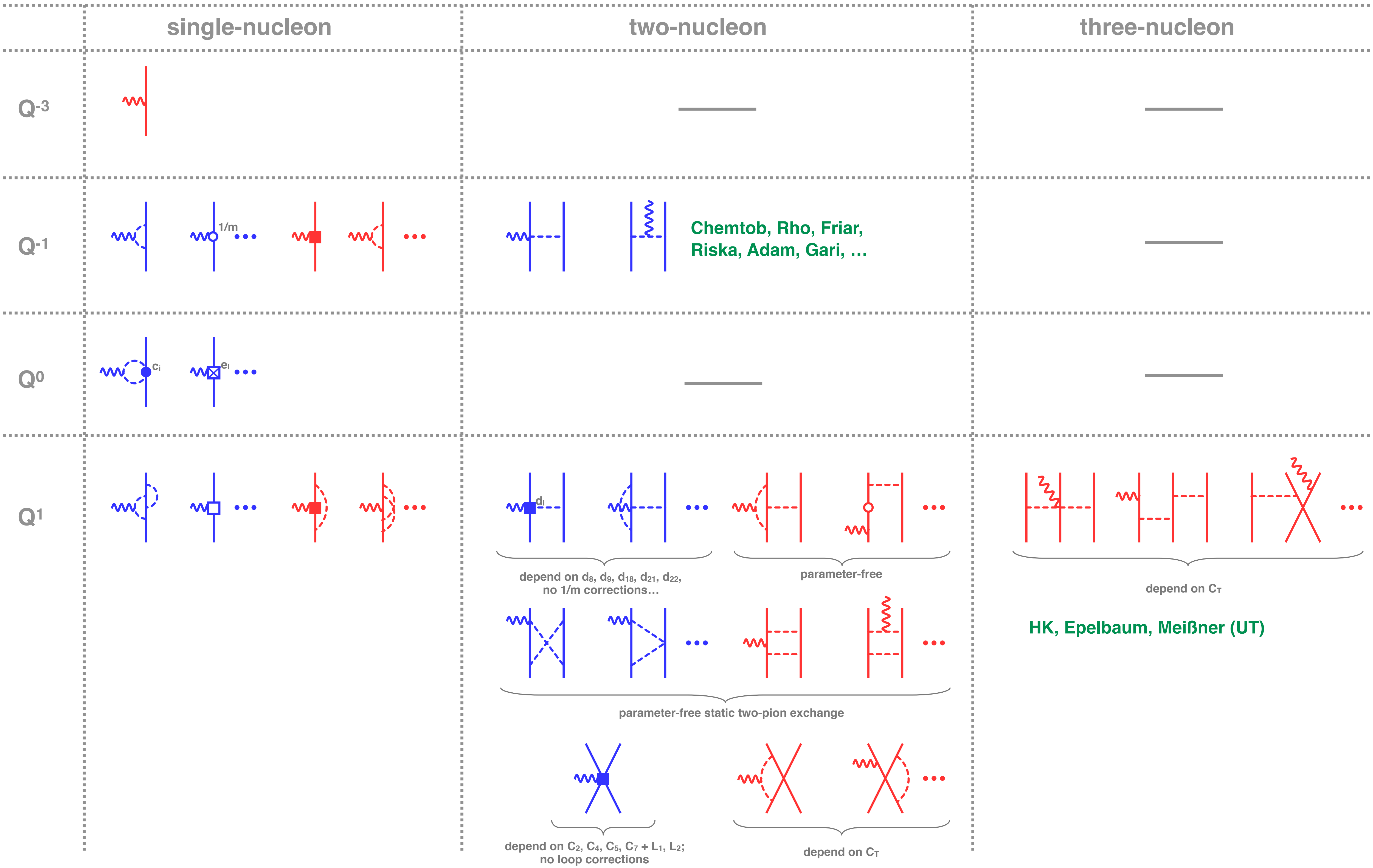
Complete derivation to leading one-loop order using the method of UT

Kölling, Epelbaum, HK, Meißner;
PRC80 (2009) 045502; PRC84 (2011) 054008; FBS 60 (2019) 31 ← Vector current

HK, Epelbaum, Meißner, Ann. Phys. 378 (2017) 317 ← Axial vector current

Vector currents in chiral EFT

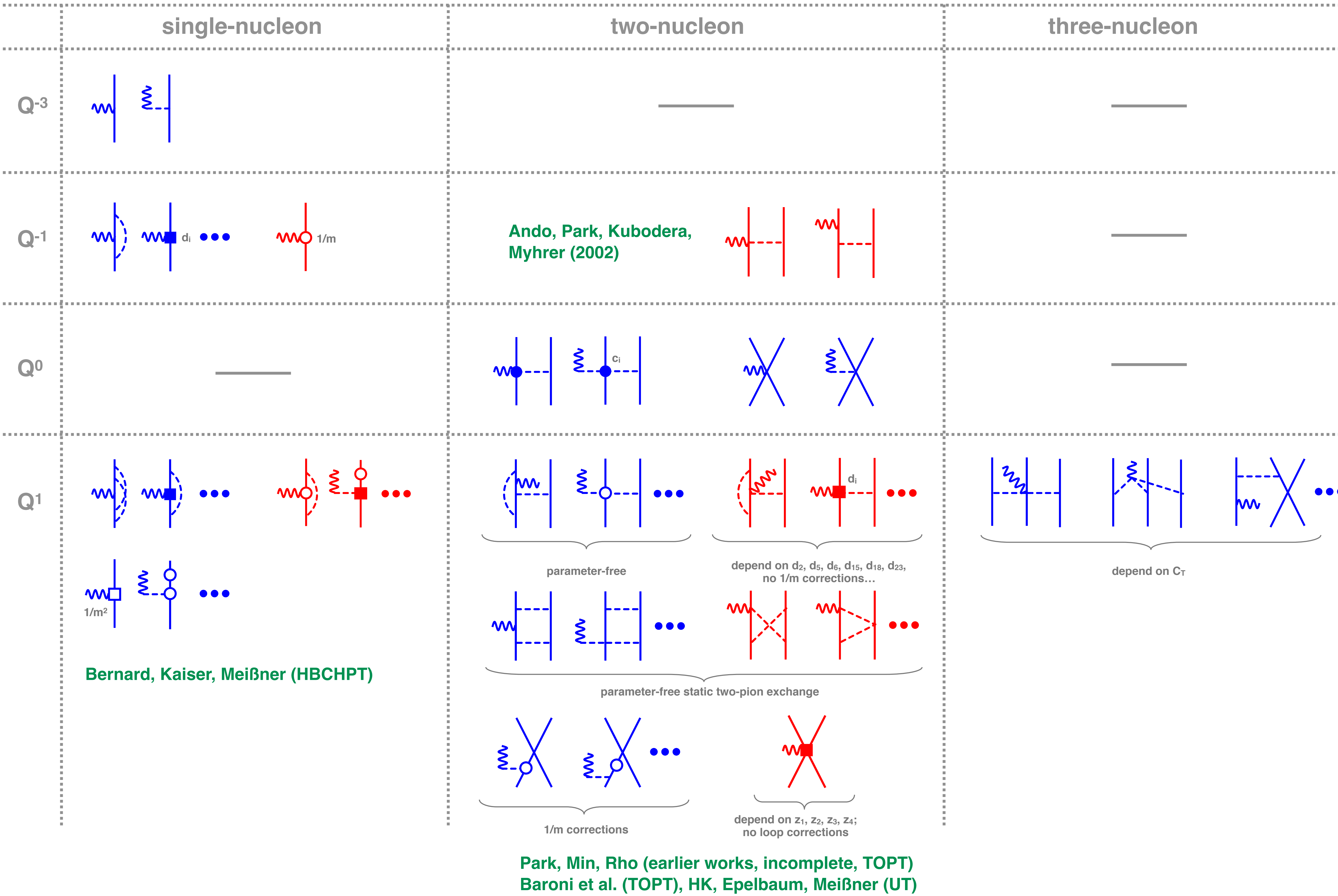
Chiral expansion of the electromagnetic **current** and **charge** operators



Park, Min, Rho, Kubodera, Song, Lazauskas (earlier works, incomplete, TOPT)
Pastore, Schiavilla et al. (TOPT), Kölling, Epelbaum, HK, Meißner (UT)

Axial vector operators in chiral EFT

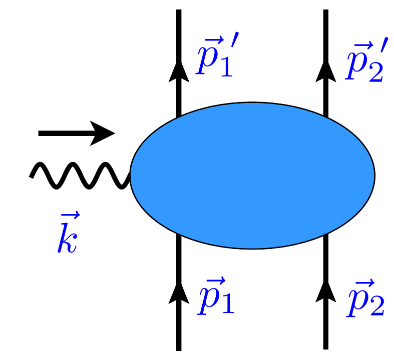
Chiral expansion of the axial vector **current** and **charge** operators



Compare of Vector Currents

- One - pion - exchange current at order Q **Kölling et al. PRC84 (2011) 054008 - parametrization**

$$\vec{V}^{(Q)}_{2\text{N}:1\pi,\,\text{static}} = i\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\vec{q}_1\times\vec{q}_2\,\left[[\tau_2]^3\,f_1(k)+\vec{\tau}_1\cdot\vec{\tau}_2\,f_2(k)\right]+i\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\Big\{\vec{k}\times[\vec{q}_2\times\vec{\sigma}_1]\,f_3(k)$$
$$+\ \vec{k}\times[\vec{q}_1\times\vec{\sigma}_1]\,f_4(k)+\vec{\sigma}_1\cdot\vec{q}_1\left(\frac{\vec{k}}{k^2}-\frac{\vec{q}_1}{q_1^2+M_\pi^2}\right)f_5(k)+\left[\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\vec{q}_1-\vec{\sigma}_1\right]f_6(k)\Big\}+1\leftrightarrow 2$$



$$V^{0,(Q)}_{2\text{N}:1\pi,\,\text{static}} = \frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}[\tau_2]^3\left[\vec{\sigma}_1\cdot\vec{k}\,\vec{q}_2\cdot\vec{k}\,f_7(k)+\vec{\sigma}_1\cdot\vec{q}_2\,f_8(k)\right]+1\leftrightarrow 2$$

$$\vec{q}_i = \vec{p}_i{}'-\vec{p}_i$$

$$f_1^{\text{Pisa/J-Lab}}(k)=e\frac{g_A}{F_\pi^2}\frac{G_{\gamma N\Delta}(k^2)}{\mu_{\gamma N\Delta}}d_8' \qquad f_2^{\text{Pisa/J-Lab}}(k)=-e\frac{g_A}{F_\pi^2}G_{\gamma N\rho}(k^2)d_9' \qquad f_3^{\text{Pisa/J-Lab}}(k)=-e\frac{g_A}{F_\pi^2}\frac{G_{\gamma N\Delta}(k^2)}{\mu_{\gamma N\Delta}}d_{21}'$$

$f_{4-8}^{\text{Pisa/J-Lab}}(k)=0$ $\longleftarrow$ restricted spin-isospin structure of OPE in Pisa/J-Lab current

↖

No static OPE charge contribution at order Q in most recent Pisa/J-Lab current

Identical magnetic moment operators from UT and Pisa/J-Lab currents

Piarulli et al. PRC87 (2013) 014006

- Two - pion - exchange current at order Q

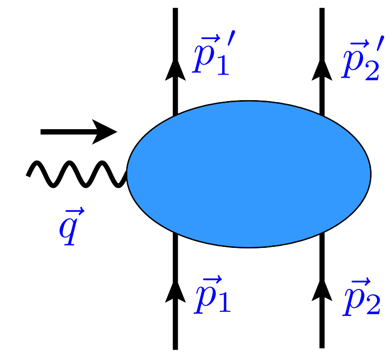
TPE UT and Pisa/J-Lab current contributions are unitary equivalent

Pastore et al. PRC84 (2011) 024001

Compare of Axial Vector Currents

Baroni et al. PRC94 (2016) 2, 024003; Erratum PRC95 (2017) 5, 059902;
PRC93 (2016) 1, 015501; Erratum PRC93 (2016) 4, 049902; Erratum PRC95 (2017) 5, 059901

At momentum transfer $\vec{q} = 0$ the result of Baroni et al. is



$\vec{k} = \vec{p}_2' - \vec{p}_2 = \vec{p}_1 - \vec{p}_1'$

$$\mathbf{j}_{\pm}^{\rm N4LO}({\rm OPE};\mathbf{k}) = \frac{g_A^5\, m_\pi}{256\,\pi\, f_\pi^4}\left[18\,\tau_{2,\pm}\,\mathbf{k} - (\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\,\boldsymbol{\sigma}_1\times\mathbf{k}\right]\boldsymbol{\sigma}_2\cdot\mathbf{k}\,\frac{1}{\omega_k^2} + (1\rightleftharpoons 2)\ , \qquad (5)$$

$$\mathbf{j}_{\pm}^{\rm N4LO}({\rm MPE};\mathbf{k}) = \frac{g_A^3}{32\,\pi f_\pi^4}\,\tau_{2,\pm}\left[W_1(k)\,\boldsymbol{\sigma}_1 + W_2(k)\,\mathbf{k}\;\boldsymbol{\sigma}_1\cdot\mathbf{k} + Z_1(k)\left(2\,\mathbf{k}\;\boldsymbol{\sigma}_2\cdot\mathbf{k}\frac{1}{\omega_k^2} - \boldsymbol{\sigma}_2\right)\right] + \frac{g_A^5}{32\,\pi f_\pi^4}\,\tau_{1,\pm}\,W_3(k)\,(\boldsymbol{\sigma}_2\times\mathbf{k})\times\mathbf{k} - \frac{g_A^3}{32\,\pi f_\pi^4}\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\,Z_3(k)\,\boldsymbol{\sigma}_1\times\mathbf{k} \times\boldsymbol{\sigma}_2\cdot\mathbf{k}\,\frac{1}{\omega_k^2} + (1\rightleftharpoons 2)\ , \qquad (6)$$

$$W_1(k) = \frac{M_\pi}{2}\left(1 + g_A^2\left(-9 + \frac{4M_\pi^2}{k^2+4M_\pi^2}\right)\right) + \frac{1}{2}\left((1-5g_A^2)k^2 + 4(1-2g_A^2)M_\pi^2\right)A(k),$$

$$W_2(k) = \frac{M_\pi}{2k^2(k^2+4M_\pi^2)}\left(\left(1+3g_A^2\right)k^2 + 4\left(1+2g_A^2\right)M_\pi^2\right) - \frac{1}{2k^2}\left((-1+g_A^2)k^2 + 4(1+2g_A^2)M_\pi^2\right)A(k)$$

$$W_3(k) = -2\,A(k)$$

$$Z_1(k) = 2M_\pi + 2(k^2+2M_\pi^2)A(k),$$

$$Z_3(k) = \frac{M_\pi}{2} + \frac{1}{2}(k^2+4M_\pi^2)A(k).$$

with loop function $A(k) = \frac{1}{2k}\arctan\frac{k}{2M_\pi}$

$$\vec{j}_a^{\,\rm N4LO}\left({\rm MPE},\vec{k}\right) - \vec{A}_{\rm NN:2\pi}^{a\,(Q)} - \vec{A}_{\rm NN:1\pi}^{a\,(Q)} = -\vec{k}\,\frac{g_A^5\,\left(4M_\pi^2+k^2\right)\vec{k}\cdot\vec{\sigma}_2\tau_1^a}{32\,\pi F_\pi^4\,k^2}A(k) + \text{rational function in } \vec{k} \,+\, 1 \leftrightarrow 2$$

Contributions of the difference to GT: Baroni et al. PRC98 (2018) 4, 044003

Subtraction Method within TOPT

- Nuclear force and current operators from inversion of off-shell *T*-matrix

  Method of Baroni et al. PRC98 (2018) 4, 044003

  - Calculate T-matrix within TOPT
  - Invert LS equation

$$T = V + VGT \longrightarrow V = T(1+GT)^{-1}$$

Off-shell change of the *T*-matrix $\longleftrightarrow$ Similarity transformation of *V*

- Compare of UT and TOPT methods HK, Epelbaum, Meißner arXiv:2001.03904

  On the Fock-space level UT and TOPT currents are unitary equivalent ✓

$$v_{5\mathrm{b,\,np}}^{(1)} = \alpha_1\left(\eta A_N \eta V \frac{\lambda^1}{\omega} V\eta V \frac{\lambda^1}{\omega^3} V\eta - \eta A_N \eta V \frac{\lambda^1}{\omega^3} V\eta V \frac{\lambda^1}{\omega} V\eta\right) + \alpha_2\left(\eta A_N \eta V \frac{\lambda^1}{\omega} V \frac{\lambda^2}{\omega} V \frac{\lambda^1}{\omega^2} V\eta - \eta A_N \eta V \frac{\lambda^1}{\omega^2} V \frac{\lambda^2}{\omega} V \frac{\lambda^1}{\omega} V\eta\right)$$
$$+ \frac{1}{2}\eta A_N \eta V \frac{\lambda^1}{\omega^3} V\eta V \frac{\lambda^1}{\omega} V\eta + \frac{3}{8}\eta A_N \eta V \frac{\lambda^1}{\omega^2} V\eta V \frac{\lambda^1}{\omega^2} V\eta - \frac{1}{2}\eta A_N \eta V \frac{\lambda^1}{\omega^2} V \frac{\lambda^2}{\omega} V \frac{\lambda^1}{\omega} V\eta + \frac{1}{2}\eta A_N \eta V \frac{\lambda^1}{\omega} V\eta V \frac{\lambda^1}{\omega^3} V\eta + \ldots$$

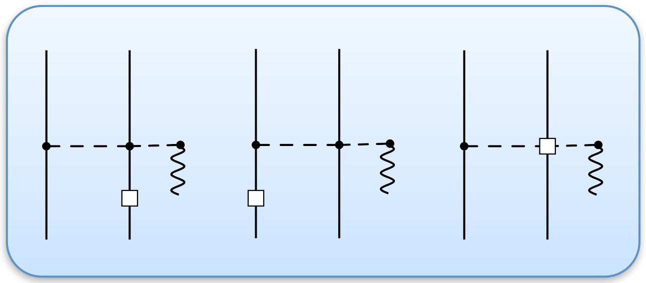
  Final results disagree even with properly adjusted unitary phases ✖

  Baroni et al. do not give explicit expression for $v_5^{(1)}$ on the Fock-space level.

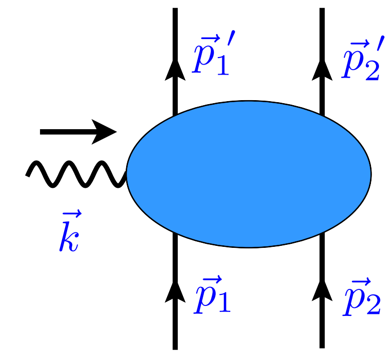
  This expression might clarify the disagreement

Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.



1/m - corrections to pion-pole OPE current proportional to g_A



$$\vec{A}^{a,(Q:\,g_A)}_{2\mathrm{N}:\,1\pi,1/m} = i\,[\tau_1\times\tau_2]^a\,\frac{g_A}{8F_\pi^2 m}\,\frac{\vec{k}}{(k^2+M_\pi^2)(q_1^2+M_\pi^2)}\left(\vec{k}_2\cdot(\vec{k}+\vec{q}_1)-\vec{k}_1\cdot\vec{q}_1+i\,\vec{k}\cdot(\vec{q}_1\times\vec{\sigma}_2)\right)+1\leftrightarrow 2$$

$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i$$

$$\vec{k}_i = \frac{1}{2}\,(\vec{p}_i{}' + \vec{p}_i)$$

Naive local cut-off regularization of the current and potential

$$\vec{A}^{a,(Q:\,g_A,\Lambda)}_{2\mathrm{N}:\,1\pi,1/m} = \vec{A}^{a,(Q:\,g_A)}_{2\mathrm{N}:\,1\pi,1/m}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V^{Q^0,\Lambda}_{1\pi} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

First iteration with OPE NN potential

$$\vec{A}^{a,(Q:\,g_A,\Lambda)}_{2\mathrm{N}:\,1\pi,1/m}\frac{1}{E-H_0+i\epsilon}V^{O^0,\Lambda}_{1\pi}+V^{O^0,\Lambda}_{1\pi}\frac{1}{E-H_0+i\epsilon}\vec{A}^{a,(Q:\,g_A,\Lambda)}_{2\mathrm{N}:\,1\pi,1/m} = \Lambda\frac{g_A^3}{32\sqrt{2}\pi^{3/2}F_\pi^4}([\tau_1]^a-[\tau_2]^a)\frac{\vec{k}}{k^2+M_\pi^2}\vec{q}_1\cdot\vec{\sigma}_1+\ldots$$

No such counter term in chiral Lagrangian

To be compensated by two-pion-exchange current $\vec{A}^{a\,(Q)}_{2\mathrm{N}:\,2\pi}$ if calculated via cutoff regularization

In dim. reg. $\vec{A}^{a\,(Q)}_{2\mathrm{N}:\,2\pi}$ is finite

Higher Derivative Regularization

Based on ideas: Slavnov, NPB31 (1971) 301;
Djukanovic et al. PRD72 (2005) 045002; Long and Mei PRC93 (2016) 044003

- Change leading order pion - Lagrangian (modify free part)

$$S_\pi^{(2)} = \int d^4x \frac{1}{2}\vec{\pi}(x)\left(-\partial^2 - M_\pi^2\right)\vec{\pi}(x) \to S_{\pi,\Lambda}^{(2)} = \int d^4x \frac{1}{2}\vec{\pi}(x)\left(-\partial^2 - M_\pi^2\right)\exp\left(\frac{\partial^2 + M_\pi^2}{\Lambda^2}\right)\vec{\pi}(x)$$

$$\frac{1}{q^2 + M_\pi^2} \to \frac{\exp\left(-\frac{q^2+M_\pi^2}{\Lambda^2}\right)}{q^2 + M_\pi^2}$$

$\mathcal{L}_{\pi,\Lambda}^{(2)}$ has to be invariant under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_V}$

- Every derivative should be covariant one
- Lagrangian $\mathcal{L}_{\pi,\Lambda}^{(2)}$ should be formulated in terms of $U(\vec{\pi}(x)) \in \mathrm{SU(2)}$

Gasser, Leutwyler ´84, ´85; Bernard, Kaiser, Meißner ´95

Building blocks $\chi = 2B(s + ip)$

$$\nabla_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)U$$

Higher Derivative Regularization

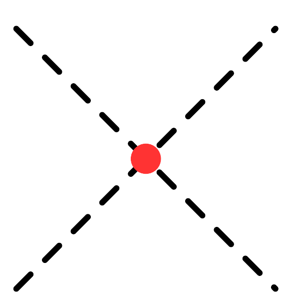
- Regularization of pion - Lagrangian will not affect nucleon Green function
  - Schrödinger or LS-equations get not modified
  - Only nuclear forces get affected

  We are not going to change pion-nucleon Lagrangian

- Not every chiral symmetric higher derivative extension of pion - Lagrangian leads to a regularized theory

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] \to \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U^\dagger \exp\left(-\vec{\partial}^2/\Lambda^2\right)\partial^\mu U\right]$$

$$= \frac{i}{q^2}\exp\left(-q^2/\Lambda^2\right) \checkmark$$

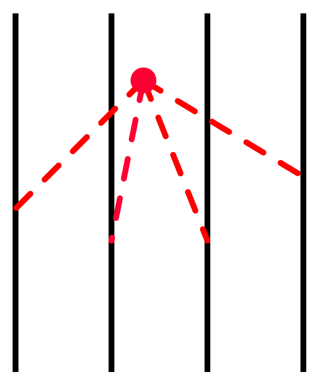


$$= \exp\left((\vec{q}_1 + \vec{q}_2)^2/\Lambda^2\right)\,\mathrm{Polynomial}(\mathrm{q_i's}) + \ldots \quad ✖$$

Unregularization of two propagators

Higher Derivative Regularization

Four-nucleon force as a regularization test



$$= \exp\left[\left(-\vec{q}_1^{\,2} - \vec{q}_2^{\,2} - \vec{q}_3^{\,2} - \vec{q}_4^{\,2} + (\vec{q}_1 + \vec{q}_2)^2\right)/\Lambda^2\right] \frac{1}{q_1^2 q_1^2 q_3^2 q_4^2} \cdots$$

$$= \exp\left[\left(-(\vec{q}_1 + \vec{q}_3)^2 - (\vec{q}_1 + \vec{q}_4)^2\right)/\Lambda^2\right] \frac{1}{q_1^2 q_2^2 q_3^2 q_4^2} \cdots$$

Only two linear combinations of momenta get regularized → Unregularized 4NF

Which additional constrain is needed to construct a regularized theory?

- All higher derivative terms of the non-linear sigma model Lagrangian in **Slavnov, NPB31 (1971) 301** are proportional to <u>equation of motion</u>

Generalize this idea to chiral EFT: all additional terms ~ EOM

$$\mathrm{EOM} = -\left[D_\mu, u^\mu\right] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}\left(\chi_-\right)$$

$\mathrm{EOM} = 0$ ← classical equation of motion for pions

Higher Derivative Lagrangian

- To construct a parity-conserving regulator it is convenient to work with building-blocks

$$u_\mu = i\, u^\dagger \nabla_\mu U u^\dagger, \quad D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u\, l_\mu u^\dagger$$

$$\chi_\pm = u^\dagger \chi\, u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B(s + i\,p), \quad u = \sqrt{U}, \quad \mathrm{ad}_A B = [A, B]$$

Possible ansatz for higher derivative pion Lagrangian

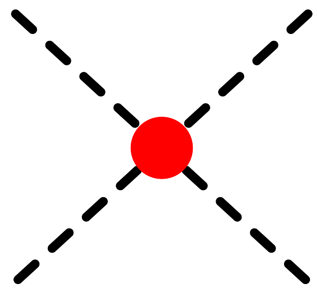
$$\mathcal{L}^{(2)}_{\pi,\Lambda} = \mathcal{L}^{(2)}_{\pi} + \frac{F^2}{4} \mathrm{Tr}\left[\mathrm{EOM} \frac{1 - \exp\left(\frac{\mathrm{ad}_{D_\mu}\mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+}{\Lambda^2}\right)}{\mathrm{ad}_{D_\mu}\mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+} \mathrm{EOM}\right]$$

$$\mathcal{L}^{(2)}_{\pi} = \frac{F^2}{4} \mathrm{Tr}\left[u_\mu u^\mu + \chi_+\right] \qquad \mathrm{EOM} = -\left[D_\mu, u^\mu\right] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}\left(\chi_-\right)$$

Use dimensional regularization on top of higher derivative one

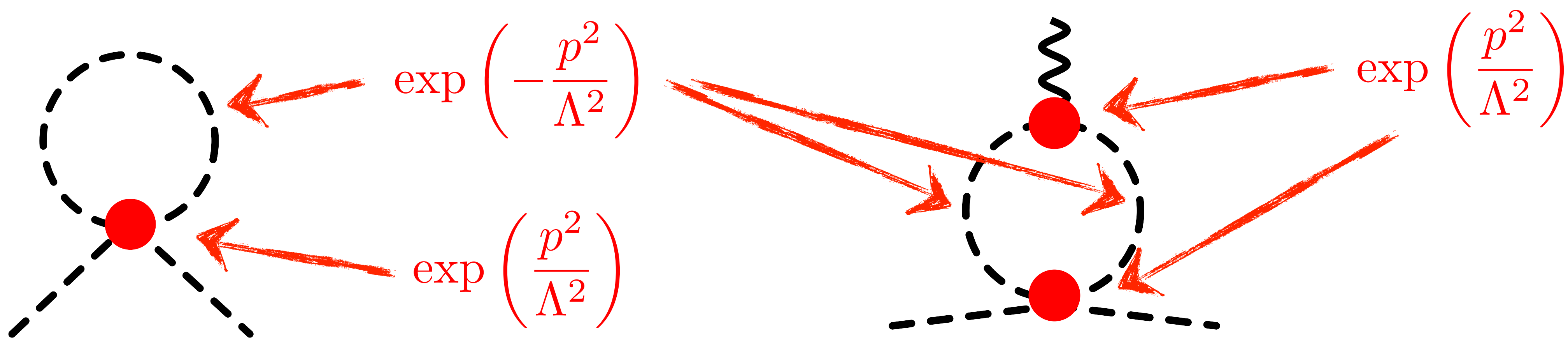
→ regularization of remaining divergencies in pion sector

Modified Vertices



- Enhanced by $\exp\left(\frac{p^2}{\Lambda^2}\right)$
- Every propagator is suppressed by $\exp\left(-\frac{p^2}{\Lambda^2}\right)$

Pionic sector becomes unregularized



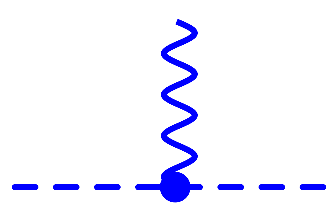
- Use dimensional on top of higher derivative regularization
- Dimensional regularization will not affect effective potential and Schrödinger or LS equations but will regularize pionic sector

Regularization of Vector Current

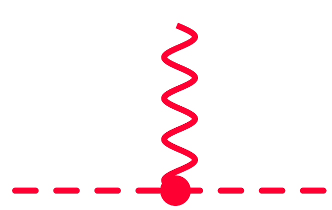
- Modify pion-propagators in a vector current

$$\text{- - - - - - - -} \quad = \quad \frac{1}{q^2+M^2} \to \frac{\exp\left(-\frac{q^2+M^2}{\Lambda^2}\right)}{q^2+M^2} \quad = \quad \text{- - - - - - - -}$$

- Modify two-pion-photon vertex



$$= \quad e\,\epsilon_\mu (q_2^\mu - q_1^\mu)\epsilon_{3,a_1,a_2}$$

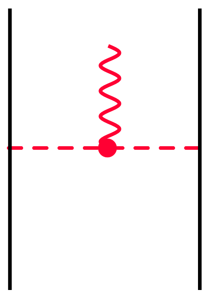


$$= \quad e\,\epsilon_\mu (q_2^\mu - q_1^\mu)\epsilon_{3,a_1,a_2} \quad \times \quad \frac{1}{q_1^2 - q_2^2}\left[(q_1^2+M^2)\exp\left(\frac{q_1^2+M^2}{\Lambda^2}\right) - (q_2^2+M^2)\exp\left(\frac{q_2^2+M^2}{\Lambda^2}\right)\right]$$

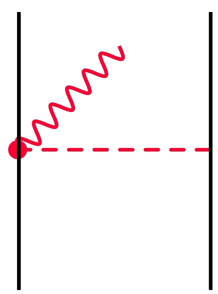
Modified two-pion-photon vertex leads to exponential increase in momenta

Regularization of Vector Current

Regularization of pion-exchange vector current



$$= \frac{i\, e\, g_A^2}{4F^2} \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_2 [\tau_1 \times \tau_2]_3 \frac{\vec{\epsilon} \cdot (\vec{q}_2 - \vec{q}_1)}{q_1^2 - q_2^2} \left[\frac{\exp\left(-\frac{q_2^2 + M^2}{\Lambda^2}\right)}{q_2^2 + M^2} - \frac{\exp\left(-\frac{q_1^2 + M^2}{\Lambda^2}\right)}{q_1^2 + M^2} \right]$$



$$= -\frac{i\, e\, g_A^2}{4F^2} \vec{\epsilon} \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_2 [\tau_1 \times \tau_2]_3 \frac{\exp\left(-\frac{q_1^2 + M^2}{\Lambda^2}\right)}{q_1^2 + M^2} + (1 \leftrightarrow 2)$$

Riska prescription: longitudinal part of the current can be derived from continuity equation

Riska, Prog. Part. Nucl. Phys. 11 (1984) 199

$$\left[H_{\text{strong}}, \boldsymbol{\rho} \right] = \vec{k} \cdot \vec{\boldsymbol{J}}$$

Higher orders → work in progress

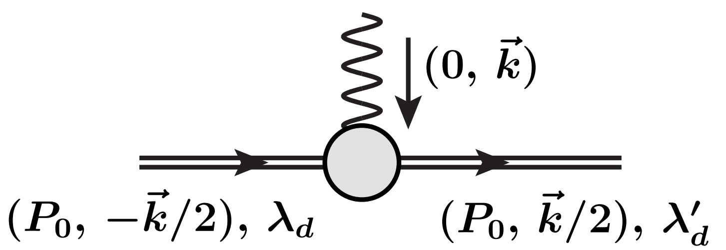
Deuteron Charge Formfactor

Deuteron FFs in the Breit frame ($\vec{k}^2 = Q^2,\ \eta \equiv \vec{k}^2/(4M_d^2)$):

$$G_C(Q^2) = \frac{1}{3|e|}\frac{1}{2P_0}\left(\langle P',1|J^0|P,1\rangle + \langle P',0|J^0|P,0\rangle + \langle P',-1|J^0|P,-1\rangle\right)$$

$$G_Q(Q^2) = \frac{1}{2|e|\eta}\frac{1}{2P_0}\left(\langle P',0|J^0|P,0\rangle - \langle P',1|J^0|P,1\rangle\right)$$

$$G_M(Q^2) = \frac{1}{\sqrt{\eta}|e|}\frac{1}{2P_0}\left\langle P',1\left|\frac{J^x + iJ^y}{\sqrt{2}}\right|P,0\right\rangle$$



Various contributions to the FFs ($i = \{C,\ Q\}$):

$$G_i(Q^2) = \underbrace{G_i^{\rm Main}(Q^2)}_{\rm LO} + \underbrace{G_i^{\rm DF}(Q^2) + G_i^{\rm SO}(Q^2) + G_i^{\rm Boost}(Q^2) + G_i^{1\pi}(Q^2)}_{\rm N^3LO} + \underbrace{G_i^{\rm Cont}(Q^2)}_{\rm N^4LO}$$

- Boost corrections **Friar '77; Schiavilla, Pandharipande '02**

$$\psi(\vec{p},\vec{v}) \simeq \left(1 - \frac{\vec{v}^2}{4}\right)\left[1 - \frac{1}{2}(\vec{v}\cdot\vec{p})(\vec{v}\cdot\vec{\nabla}_p) - \frac{i}{4m}\vec{v}\cdot(\vec{\sigma}_1 - \vec{\sigma}_2)\times\vec{p}\right]\psi(\vec{p},0)$$

- Deuteron wave functions: LO…N^3LO WFs based on the **RKE** potentials. For N^4LO, use the N^4LO^+ forces based on charge-dependent πN coupling constants **[RKE, in preparation]**

Radius and Quadrupole Moment

- Deuteron charge radius $r_d^2 = (-6)\frac{\partial G_C(Q^2)}{\partial Q^2}\Big|_{Q^2=0}$ can be decomposed as follows

$$r_d^2 = \underbrace{r_m^2}_{\text{matter radius (model-dependent)}} + r_p^2 + r_n^2 + r_{\rm DF}^2 + r_{\rm SO}^2 + r_{\rm Boost}^2 + r_{1\pi}^2 + r_{\rm Cont}^2$$

Deuteron structure radius is defined via

$$r_{str}^2 := r_d^2 - (r_p^2 + r_n^2 + r_{\rm DF}^2) = r_m^2 + r_{\rm SO}^2 + r_{\rm Boost}^2 + r_{1\pi}^2 + r_{\rm Cont}^2$$

$r_d^2 - r_p^2$ is precisely known from hydrogen-deuterium isotope shift measurements accompanied with theoretical QED $\mathcal{O}(\alpha^2)$ analysis

$$r_d^2 - r_p^2 = 3.82070(31)\ \text{fm}^2$$ [Pachucki et al. PRA97 (2018) 062511]

→ r_n^2 can be extracted from deuteron structure radius

- Deuteron quadrupole moment $Q_d = \frac{1}{M_d^2} G_Q(0)$ has a similar decomposition:

$$Q_d = Q_{\rm Main} + Q_{\rm DF} + Q_{\rm SO} + Q_{\rm Boost} + Q_{1\pi} + Q_{\rm Cont}$$

and is well known experimentally:

$$Q_d = 0.2860(15)\ \text{fm}^2$$ [Bishop, Cheung, Phys. Rev. A20 (1979) 381]

Deuteron Charge Formfactor

- OPE 2N charge operator at N^3LO (only relevant terms):

$$\rho_{2N}^{\rm N3LO} = \frac{e g_A^2}{16 F_\pi^2 m} (\vec{\tau}_1 \cdot \vec{\tau}_2) \frac{(\vec{\sigma}_2 \cdot \vec{q}_2)}{\vec{q}_2^{\,2} + M_\pi^2} \left[(1 - 2\bar{\beta}_9)(\vec{\sigma}_1 \cdot \vec{k}) + (2\bar{\beta}_8 - 1) \frac{(\vec{\sigma}_1 \cdot \vec{q}_2)(\vec{q}_2 \cdot \vec{k})}{\vec{q}_2^{\,2} + M_\pi^2} \right] + 1 \leftrightarrow 2$$

Same UTs affect relativistic corrections to the 2NF, 3NF
Minimal nonlocality (RKE): $\beta_8 = 1/4$, $\beta_9 = -1/4$

parameter-free

- Short-range 2N charge operator at N^4LO:

$$\rho_{2N}^{\rm N4LO} = \underbrace{e A\, \vec{k}^2 + B\, \vec{k}^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2)}_{\text{charge FFs, } S = 0,\, 1} + \underbrace{C\, \vec{k} \cdot \vec{\sigma}_1 \vec{k} \cdot \vec{\sigma}_2}_{\text{quadrupole FF}}$$

Chen, Rupak, Savage '99; Phillips '07

fixed from the structure radius and quadrupole moment

- $\rho_{2N}^{\rm N4LO}$ increases quadratically with increasing momentum transfer *k*

  → Finite momentum transfer behavior gets spoiled

- LECs *A*, *B*, *C* get shifts from unitary transformation $U = e^{\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3}$

$$\rho_{2N}^{\rm N4LO} \to \rho_{2N}^{\rm N4LO} + U \rho_{1N} U^\dagger - \rho_{1N} = \rho_{2N}^{\rm N4LO} + \left[\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3, \rho_{1N} \right]$$

Deuteron Charge Formfactor

- Consistent 1N operators expressible in terms of the nucleon FFs:

$$\rho_{1\mathrm{N}} = \underbrace{eG_E(k^2)}_{\textit{main}} \; - \; \underbrace{e\frac{k^2}{8m^2}G_E(k^2)}_{\textit{Darwin-Foldy}} \; + \; \underbrace{ie\frac{2G_M(k^2) - G_E(k^2)}{4m^2}\boldsymbol{\sigma}\cdot\boldsymbol{k}\times\boldsymbol{p}}_{\textit{spin-orbit}}$$

→ no need to rely on χ expansion of the 1N FFs known to converge slowly

empirical information on 1N FFs

Global analysis of experimental data **[Arrington, Hill, Lee, PLB777 (2018) 8]**
Dispersive approach **[Belushkin, Hammer, Meißner, PRC75 (2007) 035202]**

- Replace short-range NN charge by commutator of with 1N-form factor

$$\rho_{2\mathrm{N}}^{\mathrm{N4LO}} \to \tilde{\rho}_{2\mathrm{N}}^{\mathrm{N4LO}} \; = \; [\rho_{1\mathrm{N}}^{\mathrm{main}}, AT_1 + BT_2 + CT_3] \; = \; \rho_{2\mathrm{N}}^{\mathrm{N4LO}} + \text{higher order terms}$$

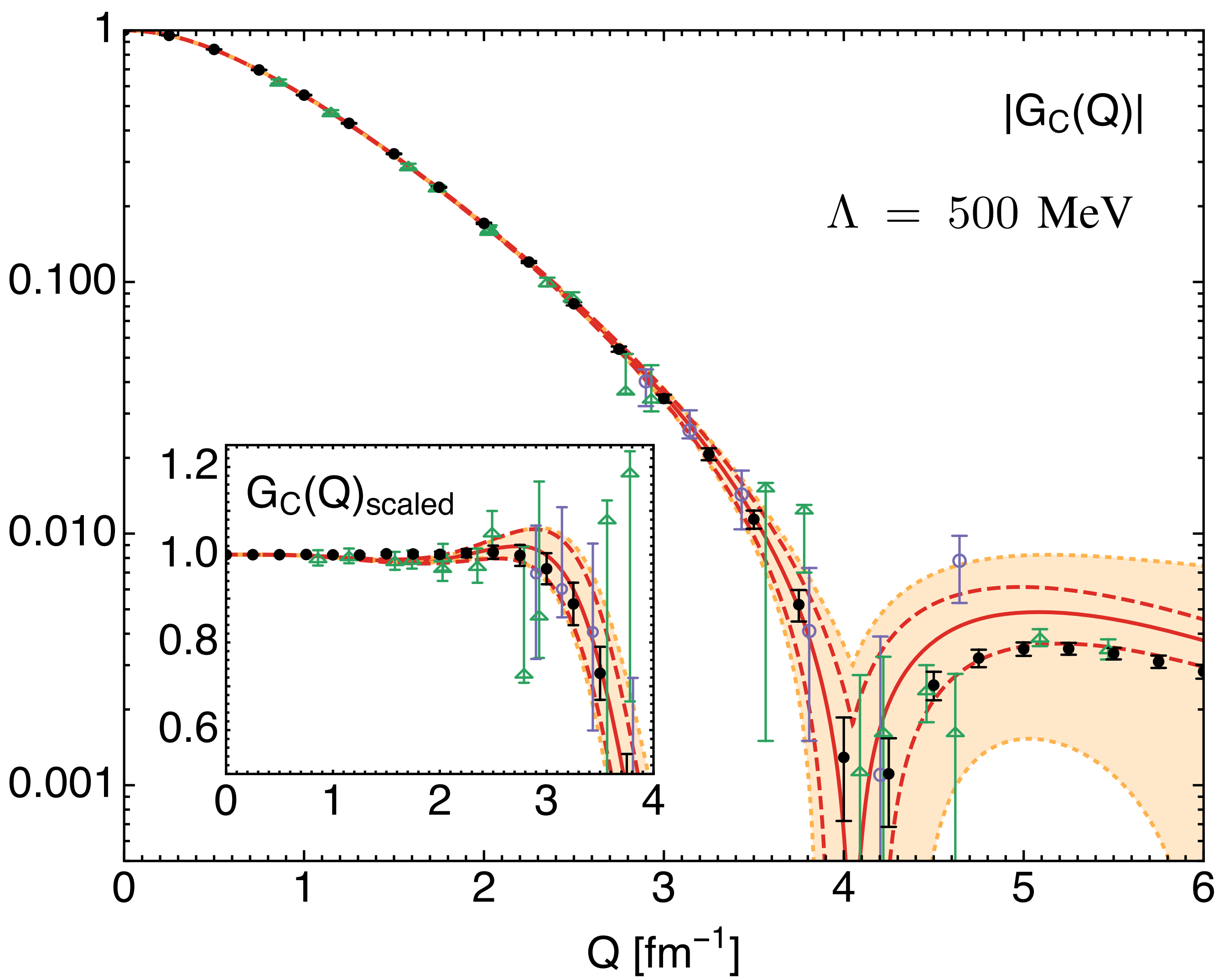
- Due to 1N FFs $\tilde{\rho}_{2\mathrm{N}}^{\mathrm{N4LO}}$ decreases with increasing momentum transfer *k*

→ proper finite *k* behavior of deuteron FFs

- Regularization of $\tilde{\rho}_{2\mathrm{N}}^{\mathrm{N4LO}}$ is automatically dictated by regularization of NN

Deuteron Charge Formfactor

Filin, Baru, Epelbaum, HK, Möller, Reinert, PRL124 (2020) 082501



- Best fit to data up to $Q = 4\,\text{fm}^{-1}$

  Dashed lines → 1σ - error in determination of short-range LECs

  Error band from Bayesian analysis: 68% DoB, $\Lambda_b = 600\,\text{MeV}$
  Furnstahl et al.´15; Epelbaum et al.´19

  Cutoff variation $\Lambda = 400 \ldots 550$ MeV yields results within error bands

- Accurate extraction of LECs
  → Determination of deuteron structure radius

$$r^2_{str} := r^2_d - (r^2_p + r^2_n + r^2_{\rm DF})$$

$$r^2_d - r^2_p = 3.82070(31)\,\text{fm}^2$$

[Pachucki et al. PRA97 (2018) 062511]

$r^2_{\rm str}$	truncation	$\rho_{2N}^{\rm cont}$	πN LECs	2N LECs	Q-range	total
3.8933	±0.0032	±0.0037	±0.0004	$^{+0.0010}_{-0.0047}$	±0.0017	$^{+0.0053}_{-0.0070}$

All numbers in fm²

→ $r^2_n = -0.106^{+0.007}_{-0.005}\,\text{fm}^2$ 1.7σ smaller than PDG value [Tanabashi et al. ´18]

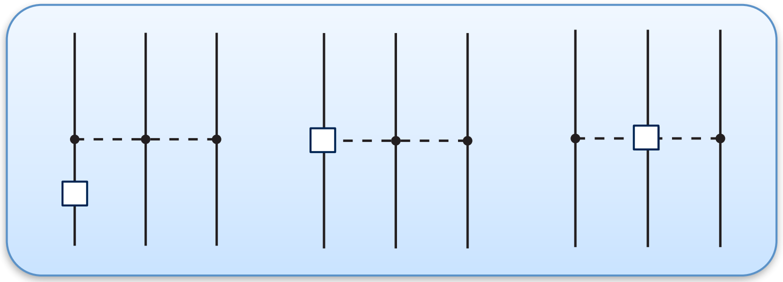
Summary

- Electroweak currents are analyzed up to order Q
- UT and TOPT currents are unitary equivalent on the Fock-space level
- Final results for UT and TOPT currents are *not* unitary equivalent ✖

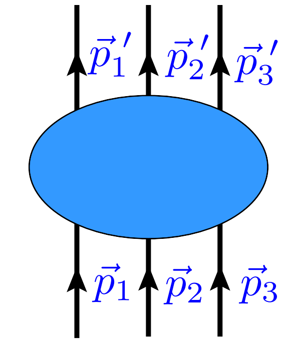
- Violation of chiral symmetry at one loop level if different regularizations for currents and forces are used
- Higher derivative regularization respects chiral/gauge symmetries
- Consistently regularized isoscalar part of em charge density operator is available
- Application to deuteron charge form factor: precise extraction of the neutron radius

Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.



1/m - corrections to pion-pole OPE current proportional to g_A



$$V_{2\pi,1/m}^{g_A^2} = i\frac{g_A^2}{32mF_\pi^4}\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_3\cdot\vec{q}_3}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)}\tau_1\cdot(\tau_2\times\tau_3)(2\vec{k}_1\cdot\vec{q}_3+4\vec{k}_3\cdot\vec{q}_3+i\,[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2)$$

$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i$$

$$\vec{k}_i = \frac{1}{2}(\vec{p}_i{}' + \vec{p}_i)$$

Naive local cut-off regularization of the current and potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} = V_{2\pi,1/m}^{g_A^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)\exp\left(-\frac{q_3^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V_{1\pi}^{Q^0,\Lambda} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

First iteration with OPE NN potential

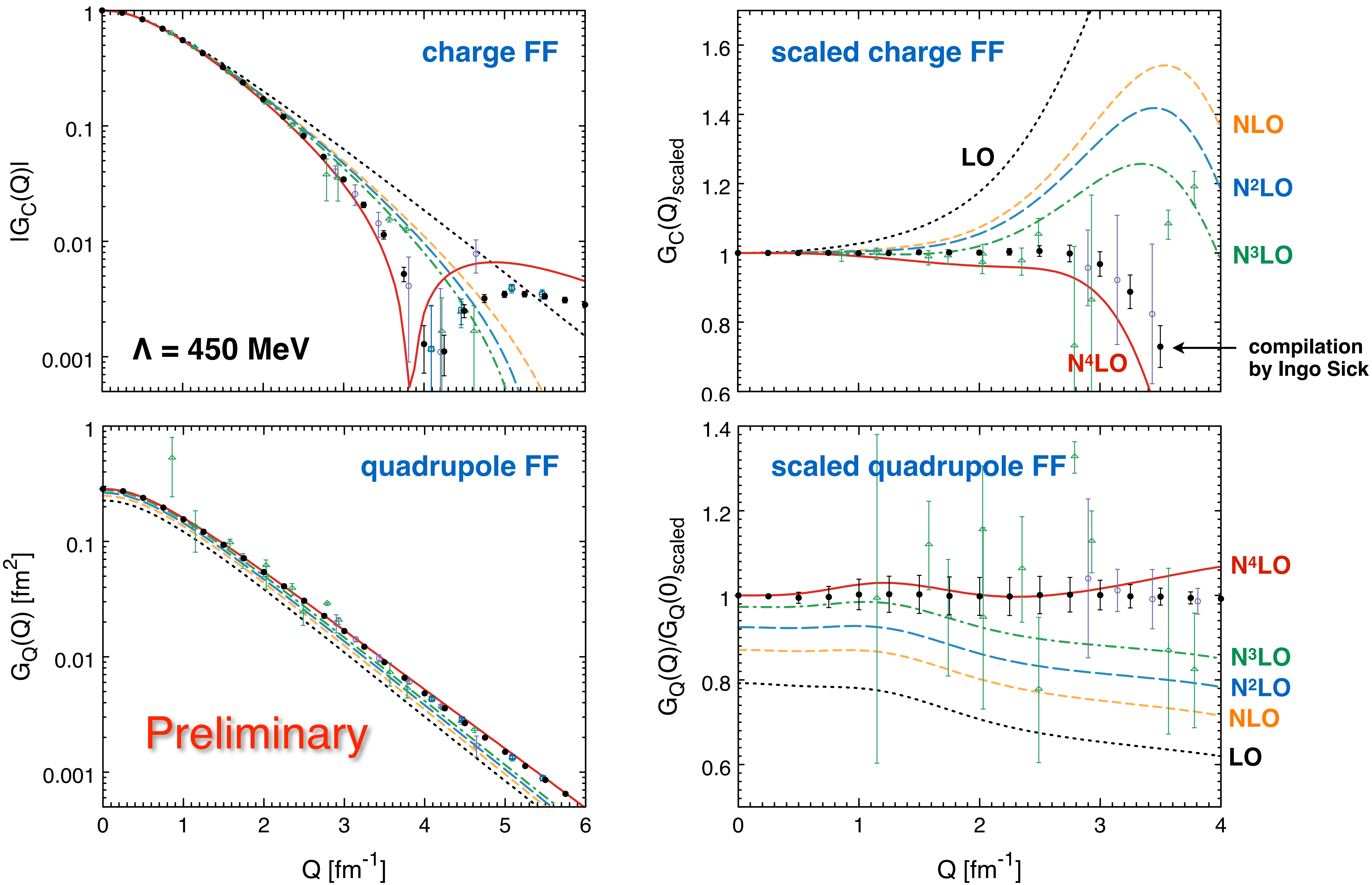
$$V_{2\pi,1/m}^{g_A^2,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{1\pi}^{Q^0,\Lambda} + V_{1\pi}^{Q^0,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{2\pi,1/m}^{g_A^2,\Lambda} = \Lambda\frac{g_A^4}{128\sqrt{2}\pi^{3/2}F_\pi^6}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)\frac{\vec{q}_2\cdot\vec{\sigma}_2\vec{q}_3\cdot\vec{\sigma}_3}{q_3^2+M_\pi^2}+\ldots$$

No such D-like term in chiral Lagrangian

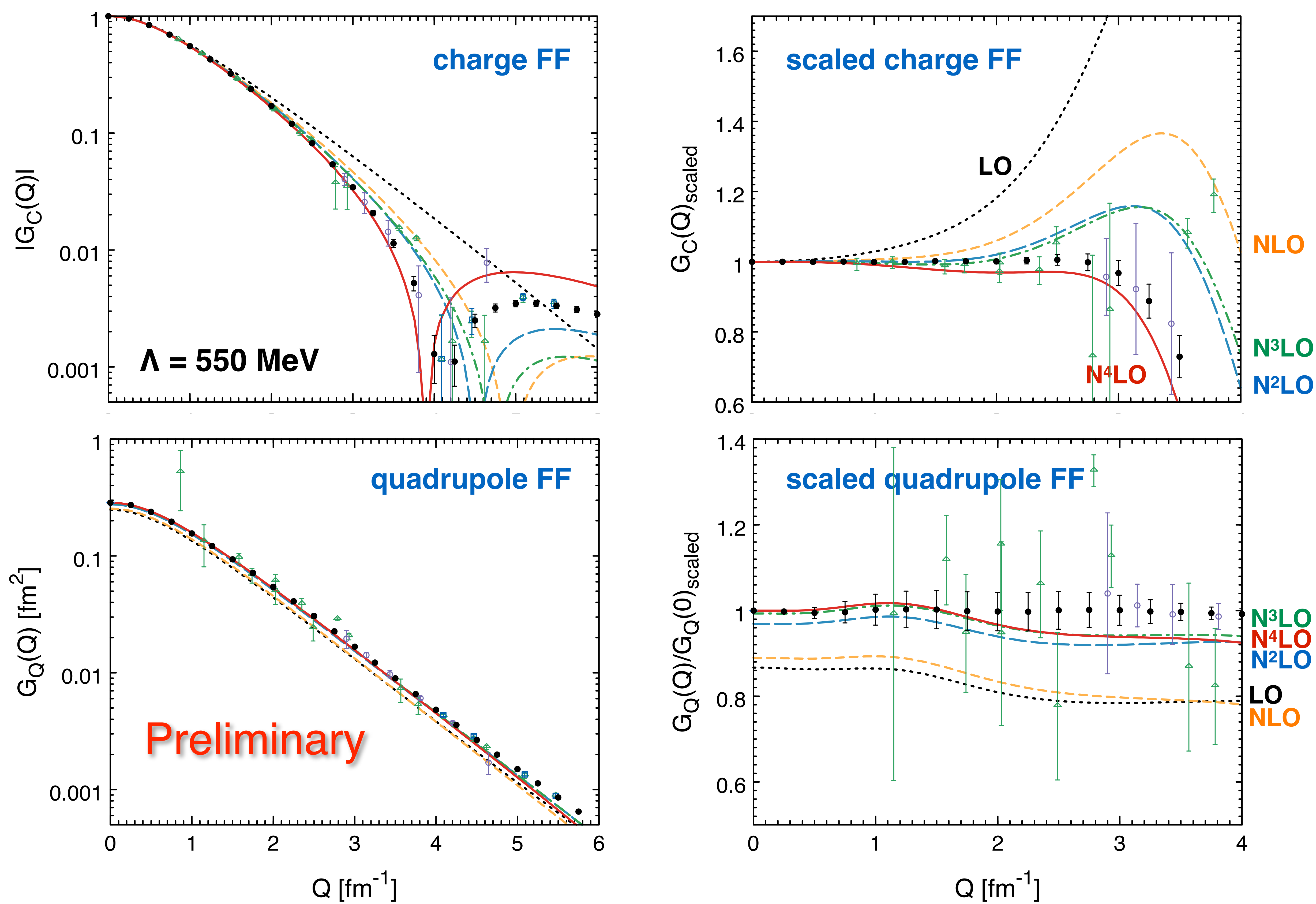
To be compensated by one-pion-two-pion-exchange $V_{2\pi-1\pi}$ if calculated via cutoff regularization

In dim. reg. $V_{2\pi-1\pi}$ is finite

Convergence of Chiral Expansion



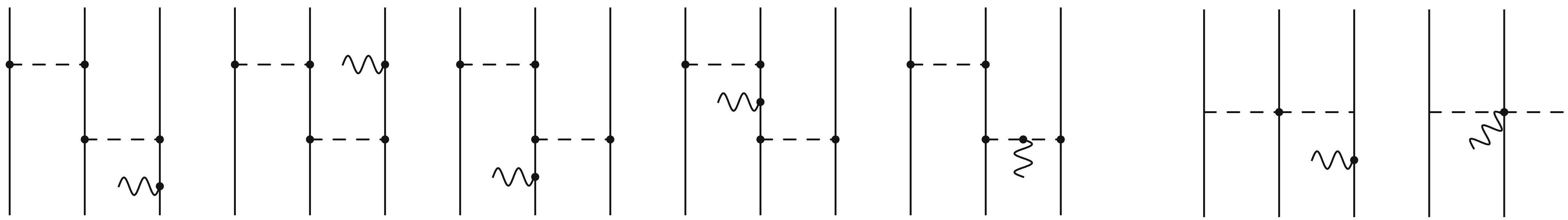
Convergence of Chiral Expansion



Three-Nucleon Charge Operator

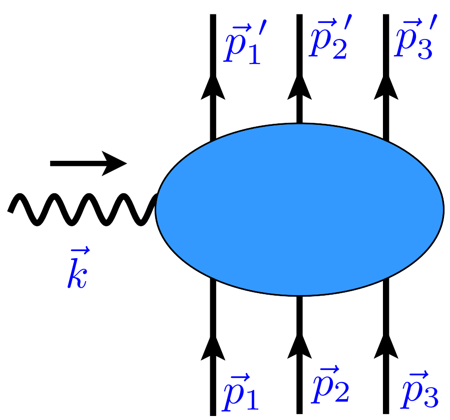
HK, Epelbaum, Meißner, FBS 60 (2019) 31

Long-range contributions to three-nucleon charge at order Q



$$V_{3\mathrm{N}:\pi}^{0\,(Q)} = -\frac{e\, g_A^4}{8F_\pi^4} \frac{\mathbf{q}_1 \cdot \boldsymbol{\sigma}^{(1)}}{(\mathbf{q}_1^2 + M_\pi^2)((\mathbf{q}_1+\mathbf{q}_2)^2 + M_\pi^2)} \left(\frac{(\mathbf{q}_1+\mathbf{q}_2)\cdot \boldsymbol{\sigma}^{(3)}}{(\mathbf{q}_1+\mathbf{q}_2)^2 + M_\pi^2} + \frac{\mathbf{q}_3 \cdot \boldsymbol{\sigma}^{(3)}}{\mathbf{q}_3^2 + M_\pi^2} \right)$$
$$\times \left(i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(3)}]_3 + \boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(3)} \tau_3^{(2)} - \boldsymbol{\tau}^{(2)} \cdot \boldsymbol{\tau}^{(3)} \tau_3^{(1)} \right) (\mathbf{q}_1^2 + \mathbf{q}_1 \cdot \mathbf{q}_2)$$
$$+ 5 \text{ permutations},$$

$$V_{3\mathrm{N}:\pi}^{0\,(Q)} = \frac{e\, g_A^2}{16F_\pi^4} \frac{\mathbf{q}_1 \cdot \boldsymbol{\sigma}^{(1)}}{\mathbf{q}_1^2 + M_\pi^2} \left(\frac{(\mathbf{q}_1+\mathbf{q}_2)\cdot \boldsymbol{\sigma}^{(3)}}{(\mathbf{q}_1+\mathbf{q}_2)^2 + M_\pi^2} + \frac{\mathbf{q}_3 \cdot \boldsymbol{\sigma}^{(3)}}{\mathbf{q}_3^2 + M_\pi^2} \right)$$
$$\times \left(\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(3)} \tau_3^{(2)} - \boldsymbol{\tau}^{(2)} \cdot \boldsymbol{\tau}^{(3)} \tau_3^{(1)} \right) + 5 \text{ permutations}.$$



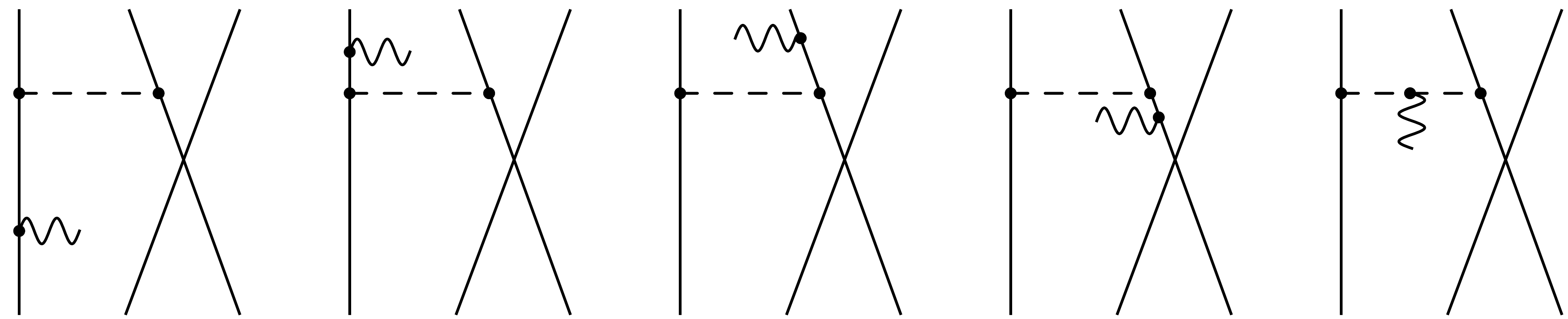
$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i$

Phenomenological impact still to be studied

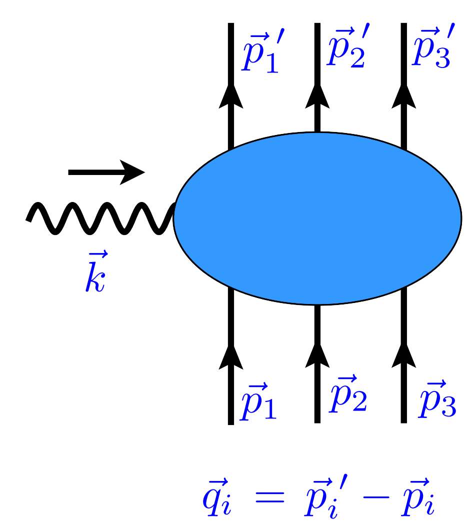
Three-Nucleon Charge Operator

HK, Epelbaum, Meißner, FBS 60 (2019) 31

Shorter-range contributions to three-nucleon charge at order *Q*



$$V^{0\,(Q)}_{3\mathrm{N:\,cont}} = -[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(3)}]_3 \frac{e\, g_A^2 C_T}{2F_\pi^2} \frac{(\mathbf{q}_2 + \mathbf{q}_3)\cdot(\boldsymbol{\sigma}^{(2)} \times \boldsymbol{\sigma}^{(3)})}{(\mathbf{q}_2+\mathbf{q}_3)^2 + M_\pi^2} \times \left(\frac{\mathbf{q}_1 \cdot \boldsymbol{\sigma}^{(1)}}{\mathbf{q}_1^2 + M_\pi^2} + \frac{(\mathbf{q}_2+\mathbf{q}_3)\cdot \boldsymbol{\sigma}^{(1)}}{(\mathbf{q}_2+\mathbf{q}_3)^2 + M_\pi^2} \right) + 5\ \text{permutations.}$$



From approximate SU(4) Wigner symmetry arguments are expected to be suppressed: $C_S >> C_T$