Evgeny Epelbaum, RUB

Progress in Ab-Initio Techniques in Nuclear Physics
March 3-6, 2020, TRIUMF, Vancouver, Canada

Isospin-violating two-nucleon interactions

in collaboration with Patrick Reinert and Hermann Krebs

Detailed understanding of isospin-breaking nuclear interactions is needed for precision nuclear physics:

— BEs of mirror nuclei, low-energy 3N scattering, nd scattering length, beta-decay, …

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft

SMS NN forces up to N^{4}LO$^+$ [Reinert, Krebs, EE, EPJA 54 (18)]

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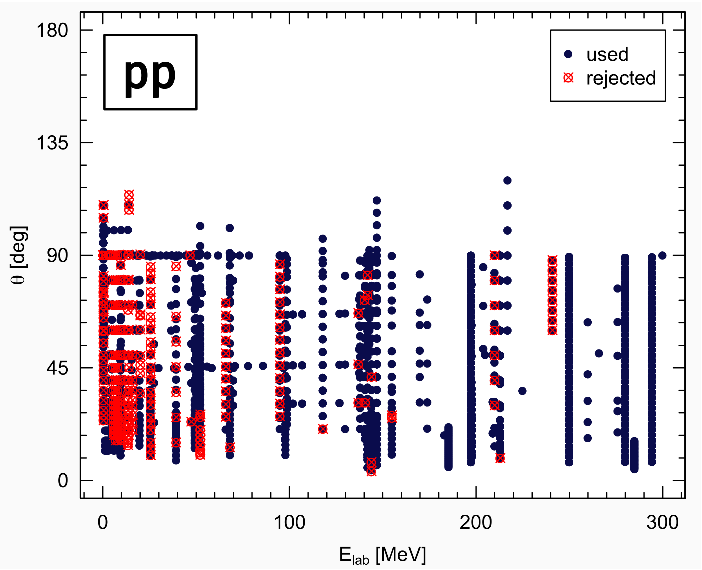
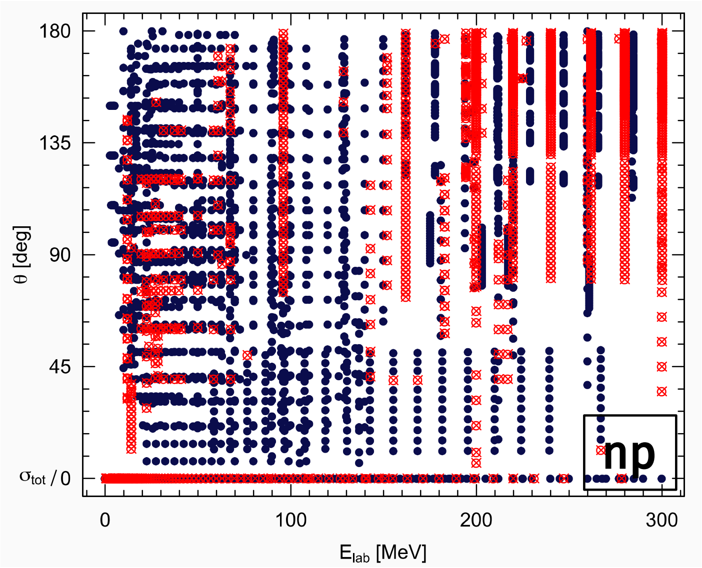
Experimental data:

- The coherent np scattering length b_{np} = 3.7405(9) fm, deuteron BE B_d = 2.224575(9) MeV and some deuteron properties
- About 8000 published np and pp scattering data below E_{lab} = 350 MeV. Granada 2013 database of mutually compatible data: 2996 pp + 3717 np data.

  [Navarro-Perez, Amaro, Ruiz Arriola '13]



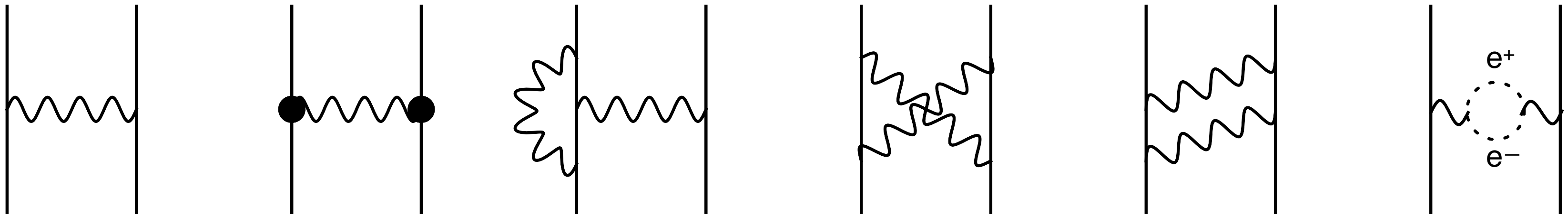
pp rejection rate ~ 11%



np rejection rate ~ 31%

Long-range electromagnetic NN interactions

Dominant long-range electromagnetic interactions are taken into account in all PWAs of NN scattering



— The Coulomb potential

— Magnetic moment interaction (Schwinger-Mott) **[Stoks, de Swart 90]**

— Two-photon exchange (the modified Coulomb potential) **[Austin, de Swart 83]**

— Vacuum polarization **[Durand III 57]**

Notice:

- Short-range parts of the e.m. potentials (FFs) are shifted to the strong NN force
- E.m. scattering amplitudes are either known in a closed form or have to be evaluated numerically by summing of > 1000 partial waves…

Chiral expansion of the NN amplitude

How far in the χ expansion does one need to go to obtain a precise description of data?

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$\chi^2/\text{datum}\,(np,\ 0-300\text{ MeV})$	75	14	4.1	2.01	1.06
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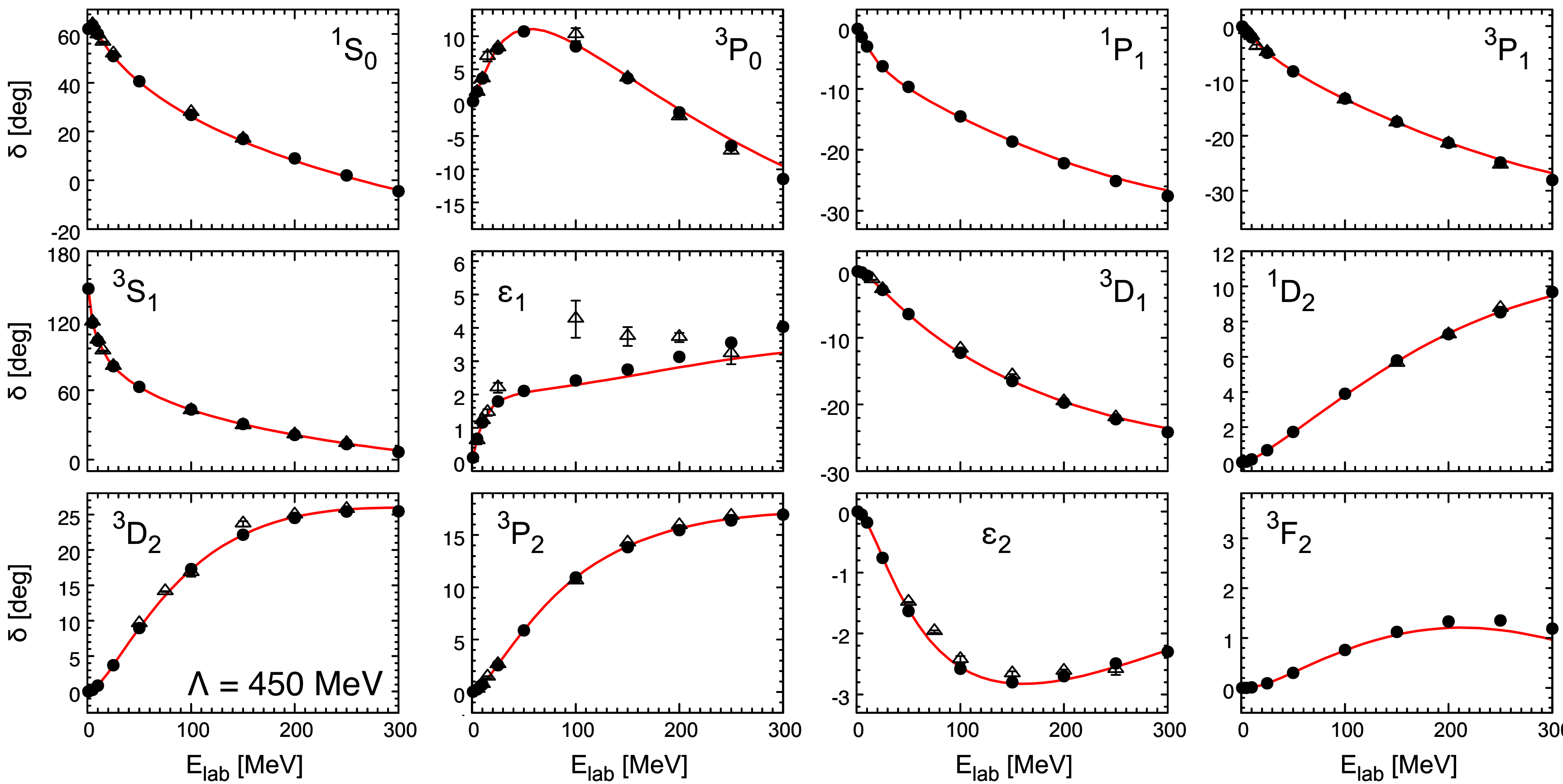
P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



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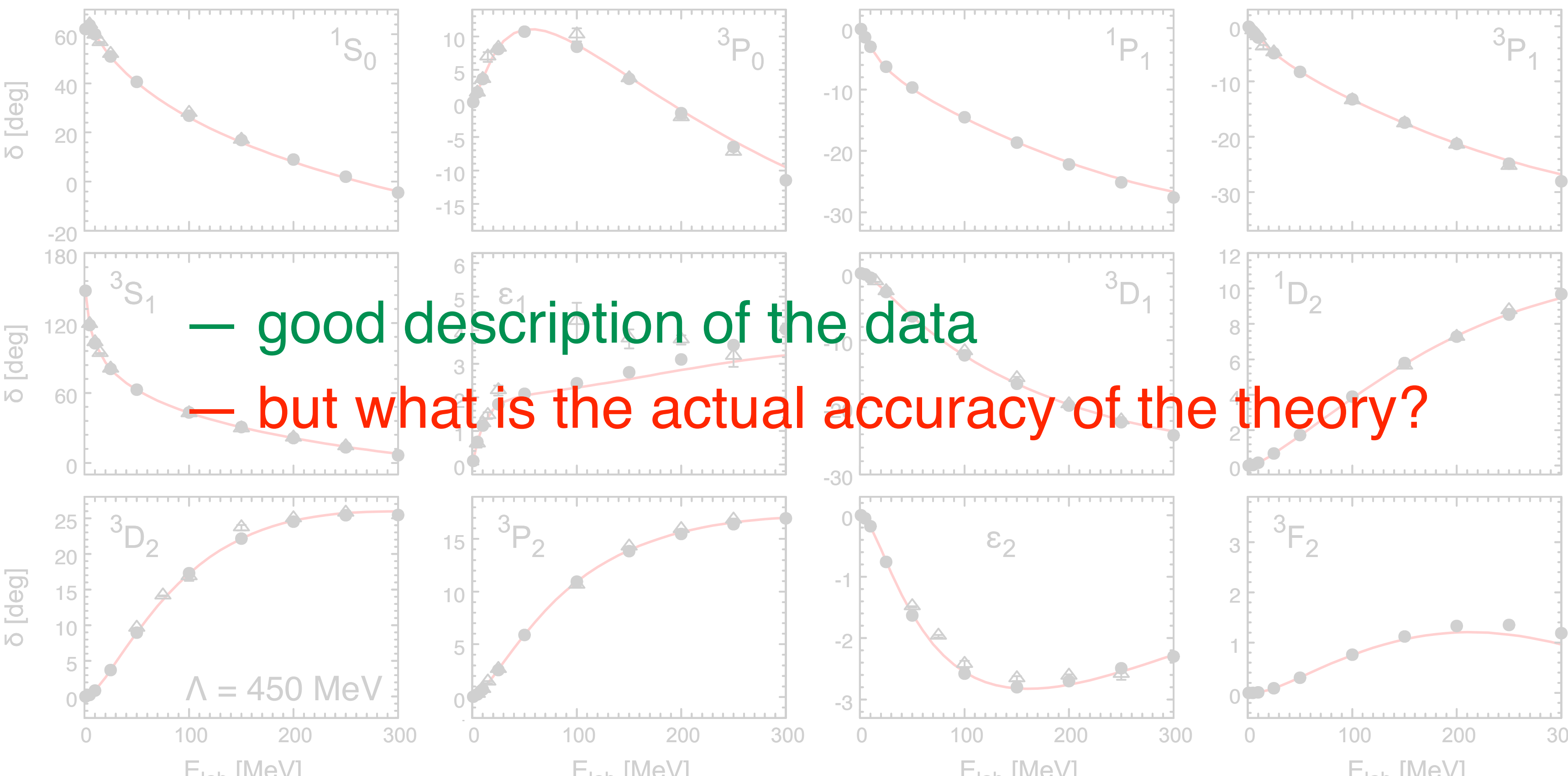

— good description of the data

— but what is the actual accuracy of the theory?

Uncertainty quantification

In most cases, the uncertainty is dominated by truncation errors. Consider an observable X(p):

$$X(p) = \underbrace{X^{(0)} + \Delta X^{(2)} + \Delta X^{(3)} + \ldots + \Delta X^{(k)}}_{\text{known from explicit calculations}} + \underbrace{\Delta X^{(k+1)} + \ldots}_{\text{truncation error } \delta X^{(k)}} \quad \text{where} \quad \Delta X^{(n)} = \mathcal{O}(\overbrace{Q}^{\max\left(\frac{p}{\Lambda_b}, \frac{M_\pi^{\text{eff}}}{\Lambda_b}\right)}{}^{n} X^{(0)})$$

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- In [EE, Krebs, Meißner, EPJA 51 (15) 53], a simple algorithm was proposed (1-st term dominance):

  $$\delta X^{(k)} = \max\left(Q^{k+1}|X^{(0)}|,\; Q^{k+1-j}|\Delta X^{(j)}|\right) \quad \wedge \quad \delta X^{(i)} \geq \max_{j,k}\left(|X^{(j\geq i)} - X^{(k\geq i)}|\right)$$

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- Bayesian approach by the BUQEYE Collaboration Furnstahl et al. '15, '17

— rescale X to make the coefficients dim-less: $X =: X_{\text{ref}}\,(c_0 + c_2 Q^2 + c_3 Q^3 + \ldots)$

— assume that c_i's are distributed according to some common pdf $\text{pr}(c_i|\bar{c})$ (≡ prior)

— calculate pdf for the dimension-less residual to take the value $\sum_{n=k+1}^{k+h} c_n Q^n = \Delta$:

$$\text{pr}_h(\Delta|\bar{c}) \equiv \left[\prod_{i=k+1}^{k+h} \int_{-\infty}^{\infty} dc_i\; \text{pr}(c_i|\bar{c})\right] \delta\Big(\Delta - \sum_{j=k+1}^{k+h} c_j Q^j\Big)$$

— assuming some pdf $\text{pr}(\bar{c})$, typically chosen uniform, marginalize over $\bar{c}$:

$$\text{pr}_h(\Delta|\{c_{i\leq k}\}) = \frac{\int_0^\infty d\bar{c}\; \text{pr}_h(\Delta|\bar{c})\; \text{pr}(\bar{c}) \prod_{i\in A} \text{pr}(c_i|\bar{c})}{\int_0^\infty d\bar{c}\; \text{pr}(\bar{c}) \prod_{i\in A} \text{pr}(c_i|\bar{c})}$$

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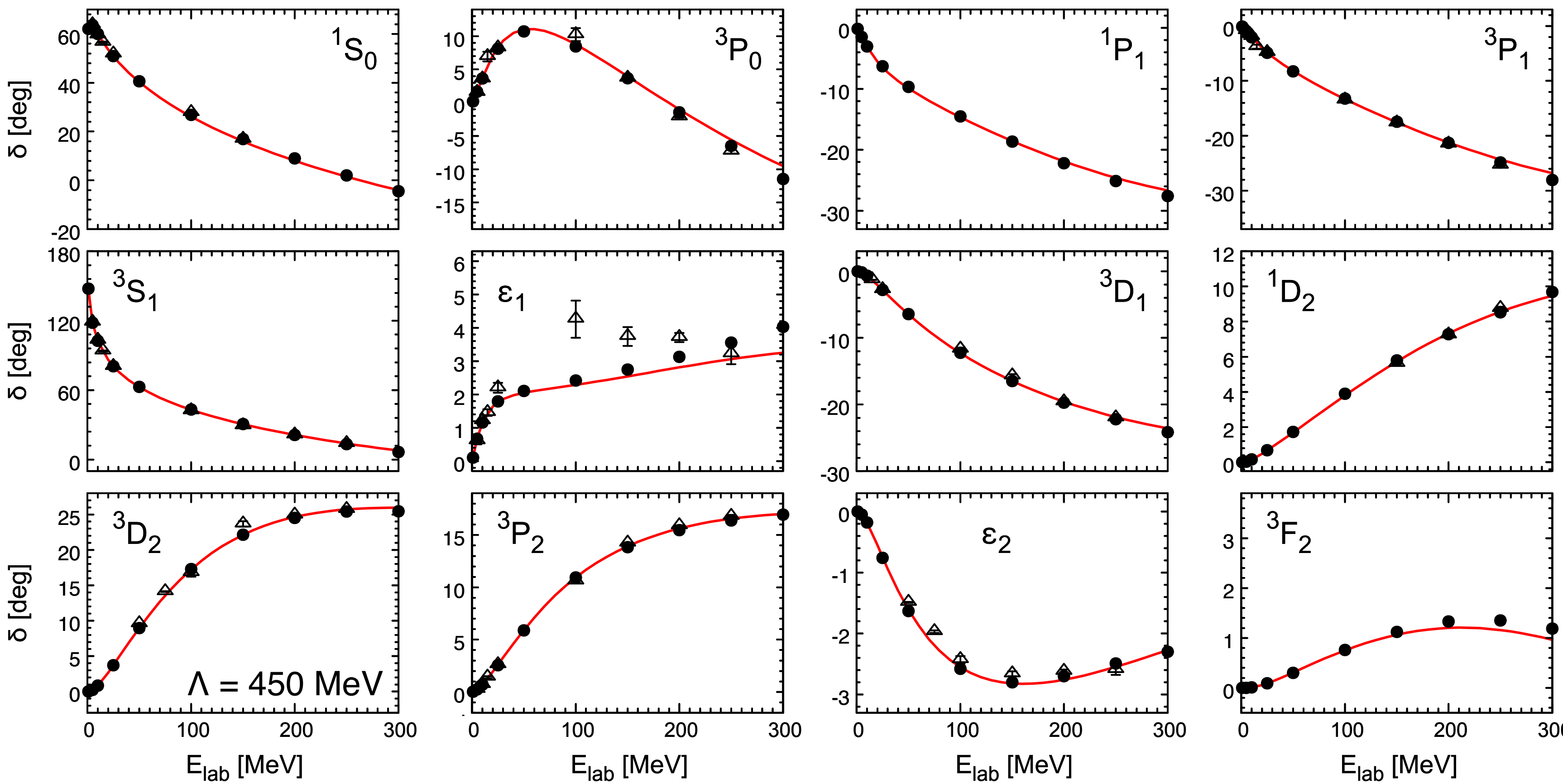

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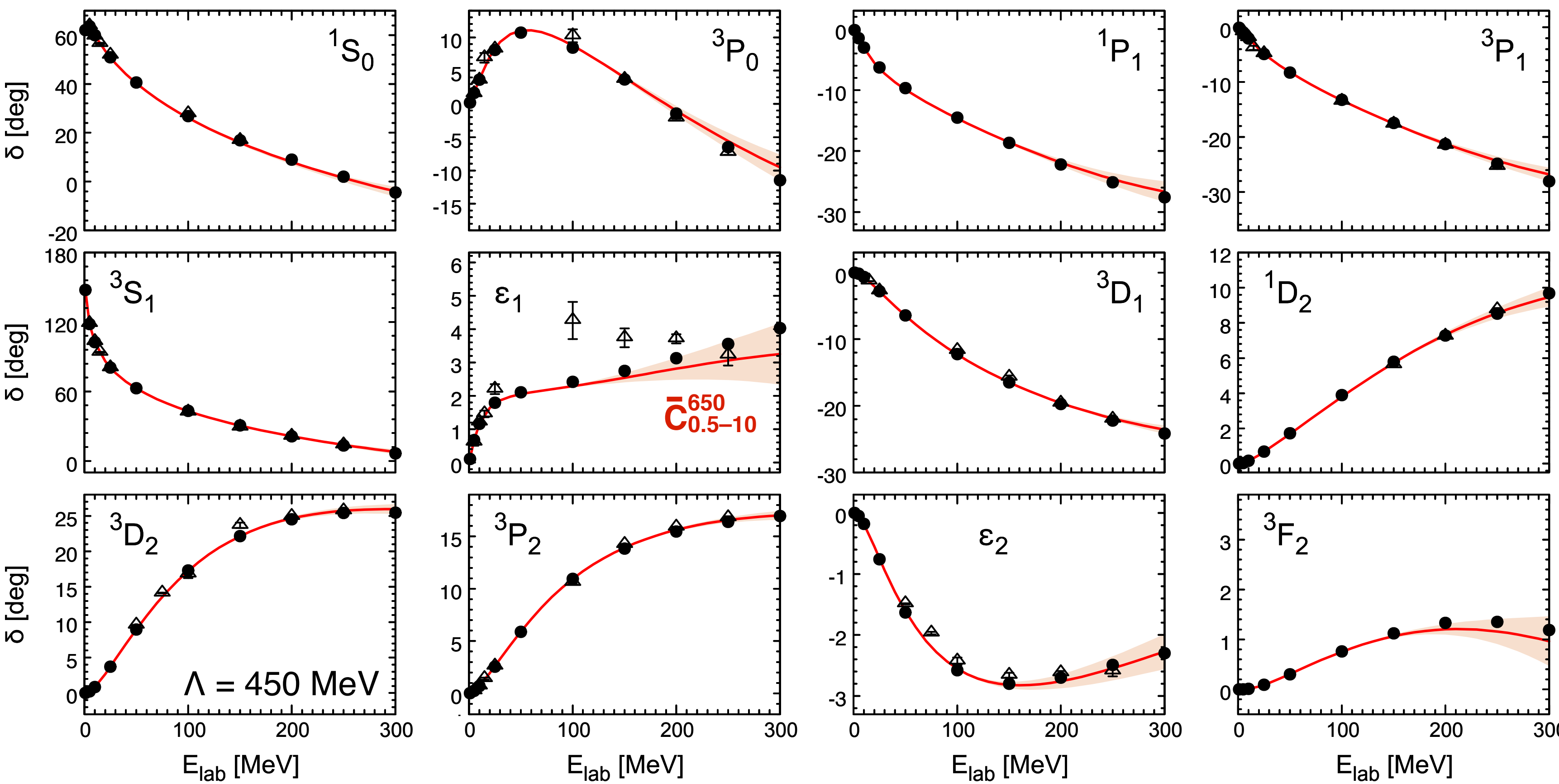

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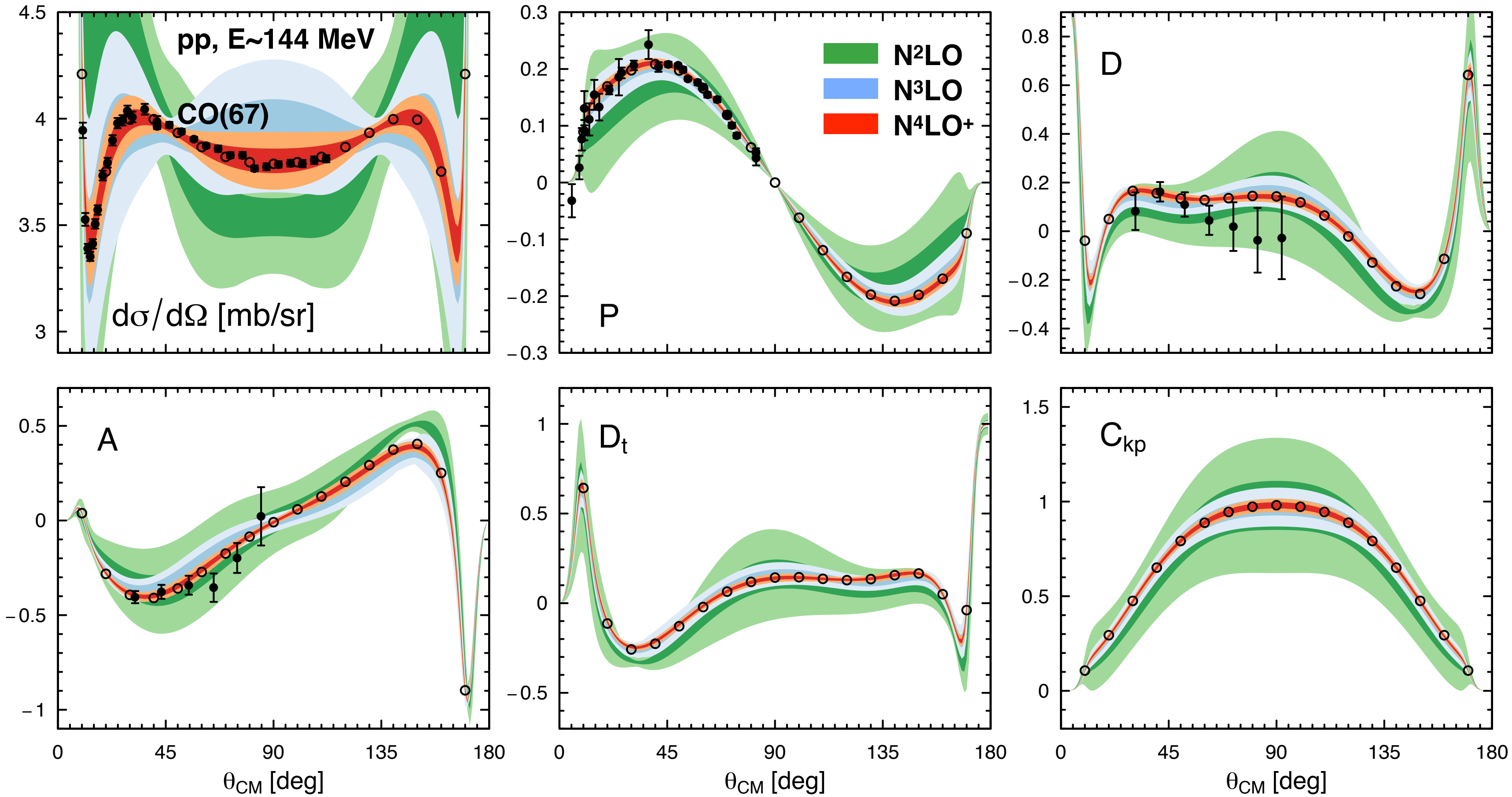

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Are we done in the 2N sector?

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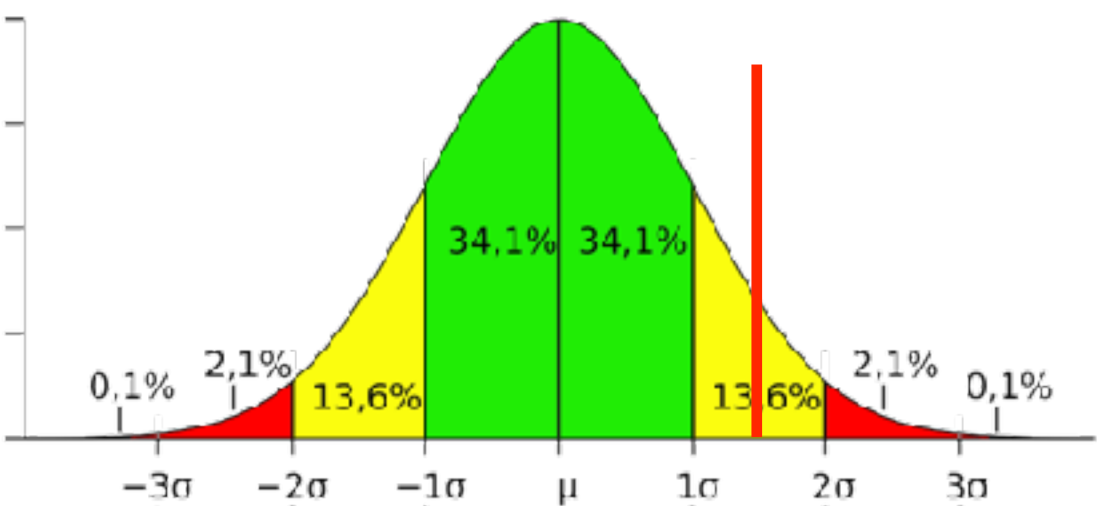
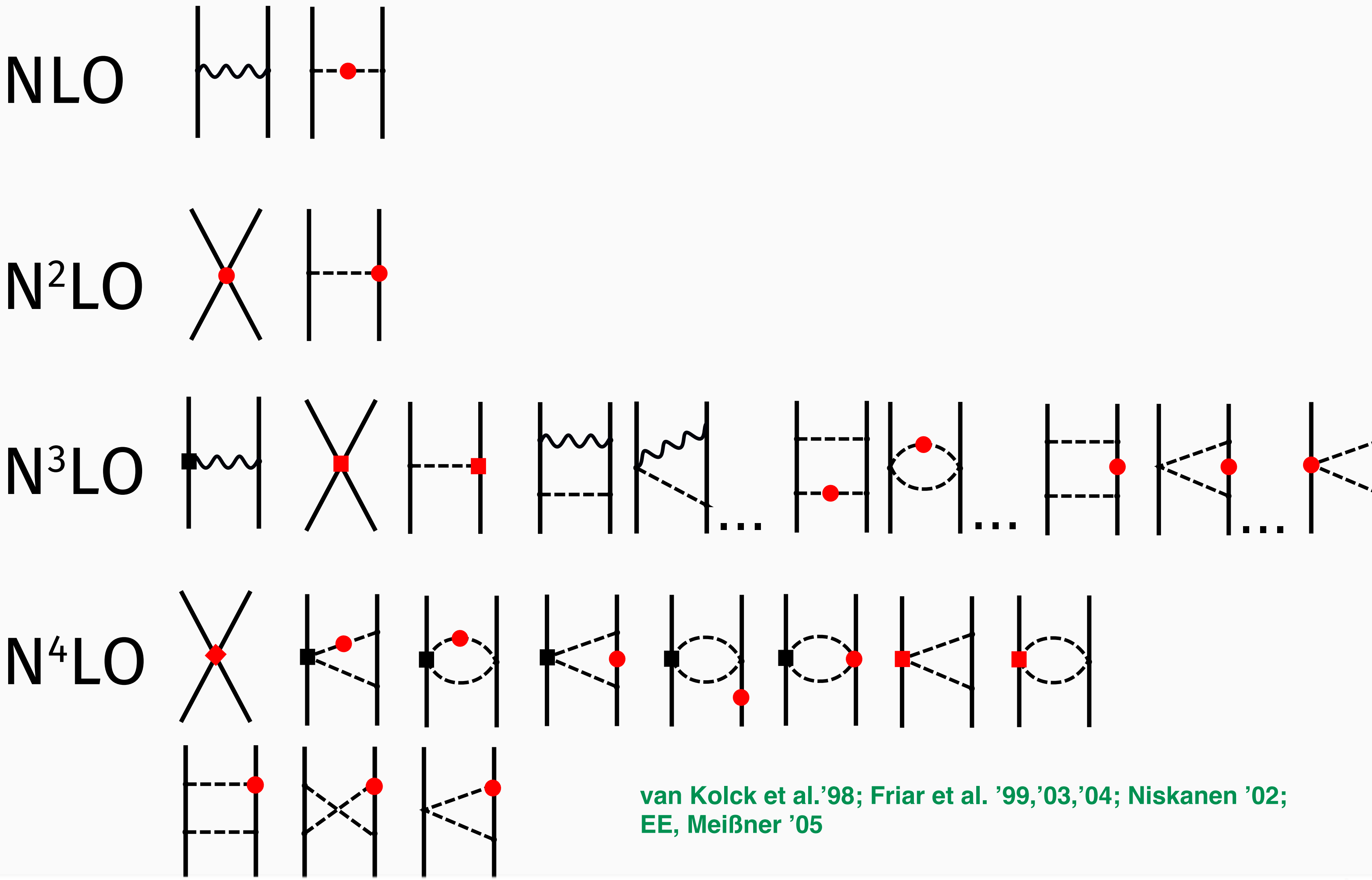
Are we done in the 2N sector?

Almost… In the fitted range of E_{lab} = 0-280 MeV, we obtain χ^2 = 5003 for 4895 np + pp scattering data, which leads to $\chi^2 / N_{dat} = 1.022$ or $\chi^2 / (N_{dat} - N_{LEC}) = 1.028$

Can this be traced back to the incomplete treatment of IB corrections?



Isospin-breaking NN forces



van Kolck et al.'98; Friar et al. '99,'03,'04; Niskanen '02;
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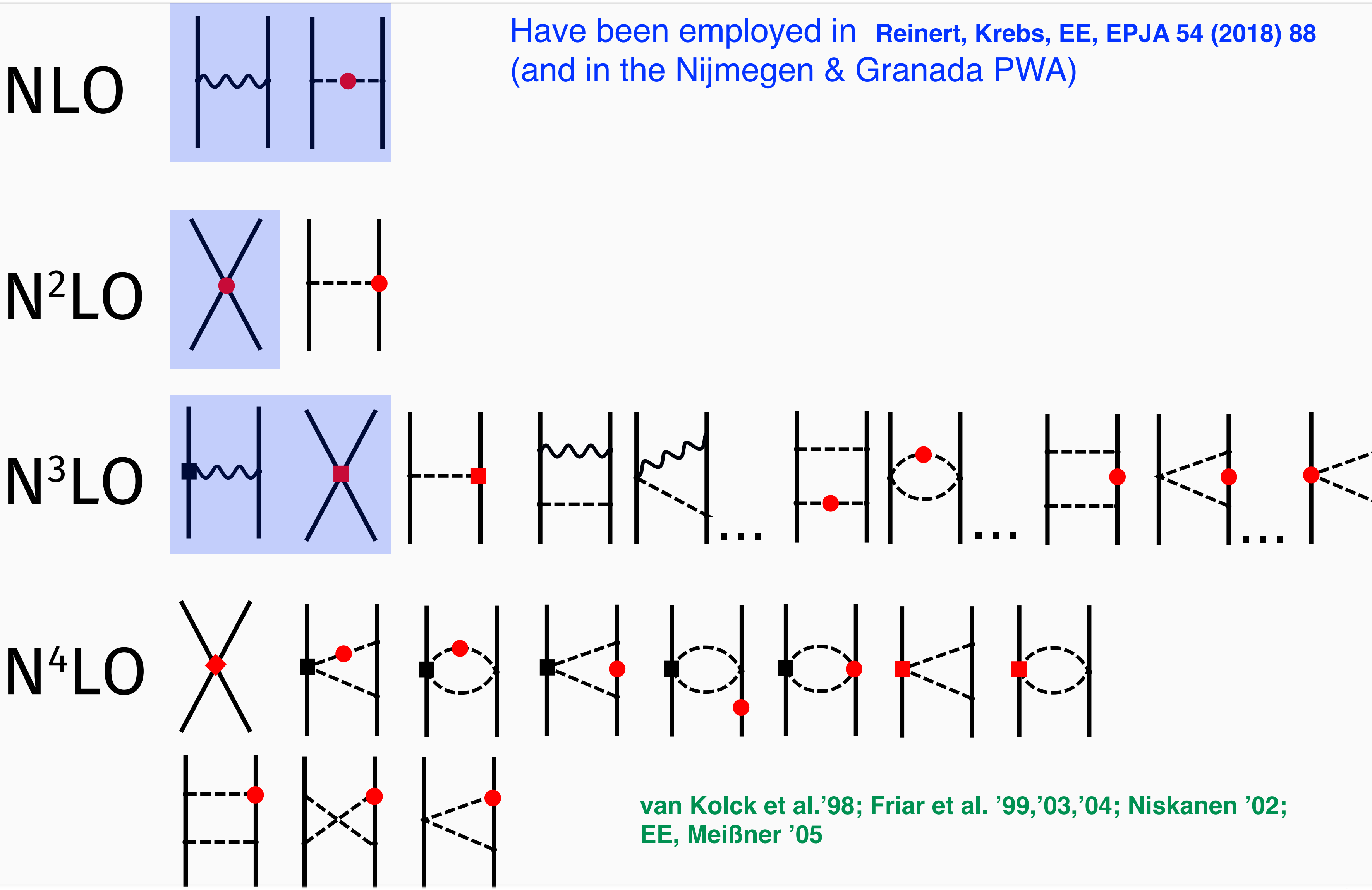

Have been employed in **Reinert, Krebs, EE, EPJA 54 (2018) 88**
(and in the Nijmegen & Granada PWA)

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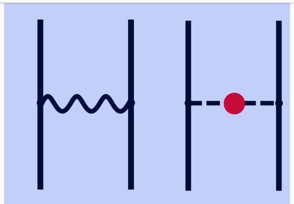
Isospin-breaking NN forces

NLO



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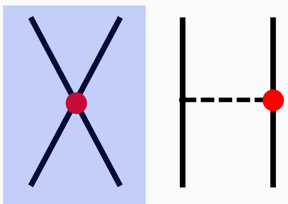
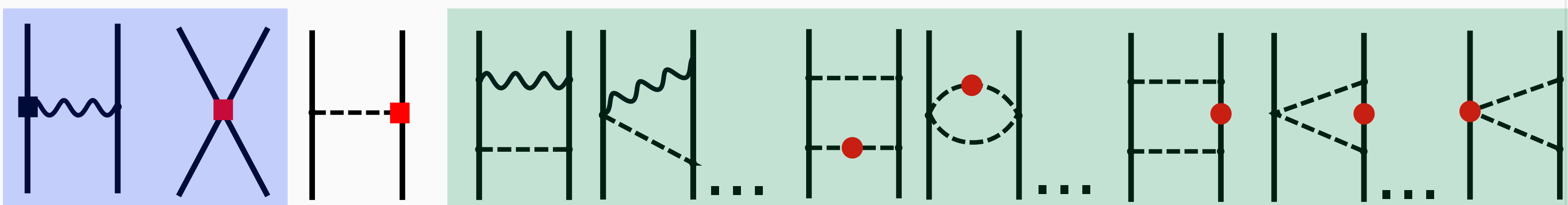
Parameter-free: depend on δM_π, $\delta m = 1.29$ MeV and $(\delta m)^{QCD} = 2.05(30)$ MeV **[Gasser, Leutwyler '75]**

N^{2}LO



N^{3}LO



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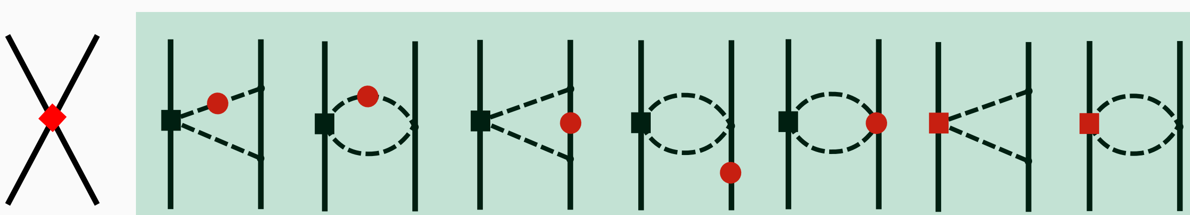

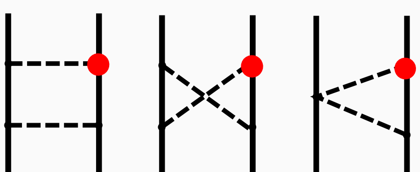

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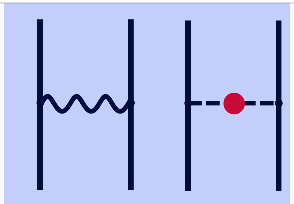

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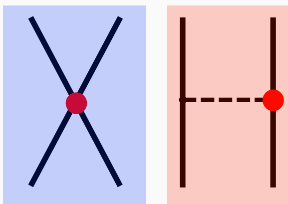
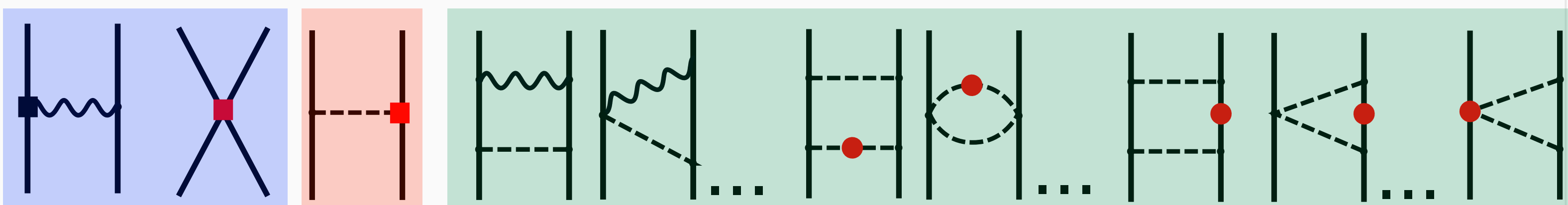


Depend on 3 πN coupling constants + 3 IB contact terms in p-waves

N^{3}LO



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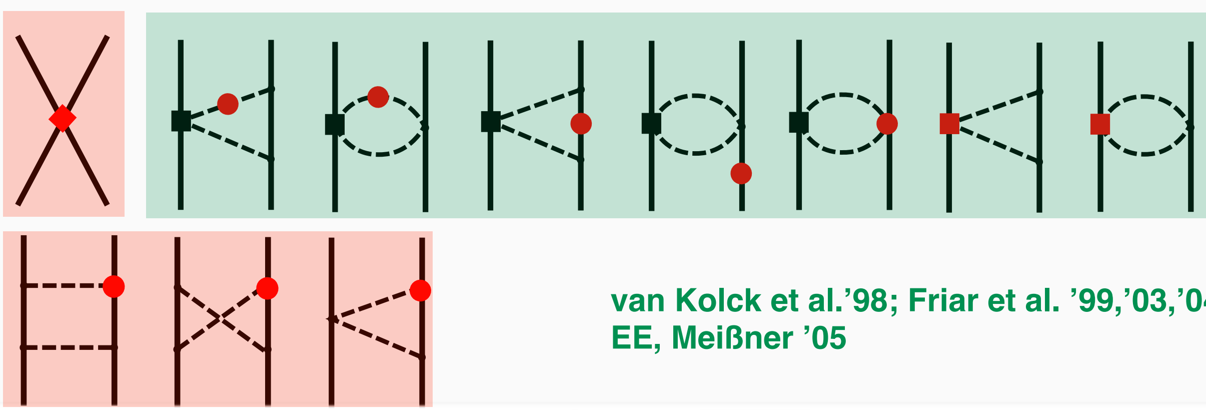

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Charge dependence of the pion-nucleon coupling constant

Patrick Reinert, EE, Hermann Krebs, PRELIMINARY

Assuming exact isospin symmetry:

$$\langle \mathrm{N}(p')|A_i^\mu(0)|\mathrm{N}(p)\rangle = \bar{u}(p')\left[\gamma^\mu G_A(p'-p) + \frac{(p'-p)^\mu}{2m_\mathrm{N}} G_P(p'-p)\right]\gamma_5 \frac{\tau_i}{2} u(p)$$

— axial charge of the nucleon: $g_A \equiv G_A(0) = 1.2724(23)$

$$g_{\pi\mathrm{NN}} = \frac{2\sqrt{4\pi}m_\mathrm{N}}{M_\pi} f_{\pi\mathrm{NN}}$$

— induced pseudoscalar FF at the pion pole: $G_P(q) = \frac{4m_\mathrm{N}F_\pi g_{\pi\mathrm{NN}}}{M_\pi^2 - q^2} + \text{ non-pole terms}$

— Goldberger-Treiman relation: $F_\pi g_{\pi\mathrm{NN}} = g_A m_\mathrm{N}(1 + \Delta_\mathrm{GT})$

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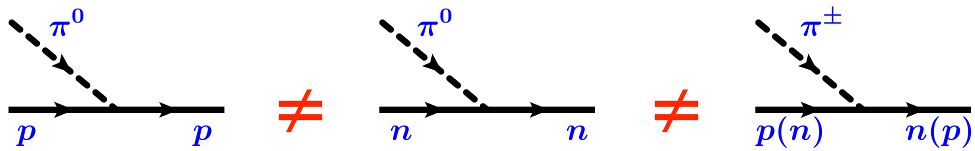
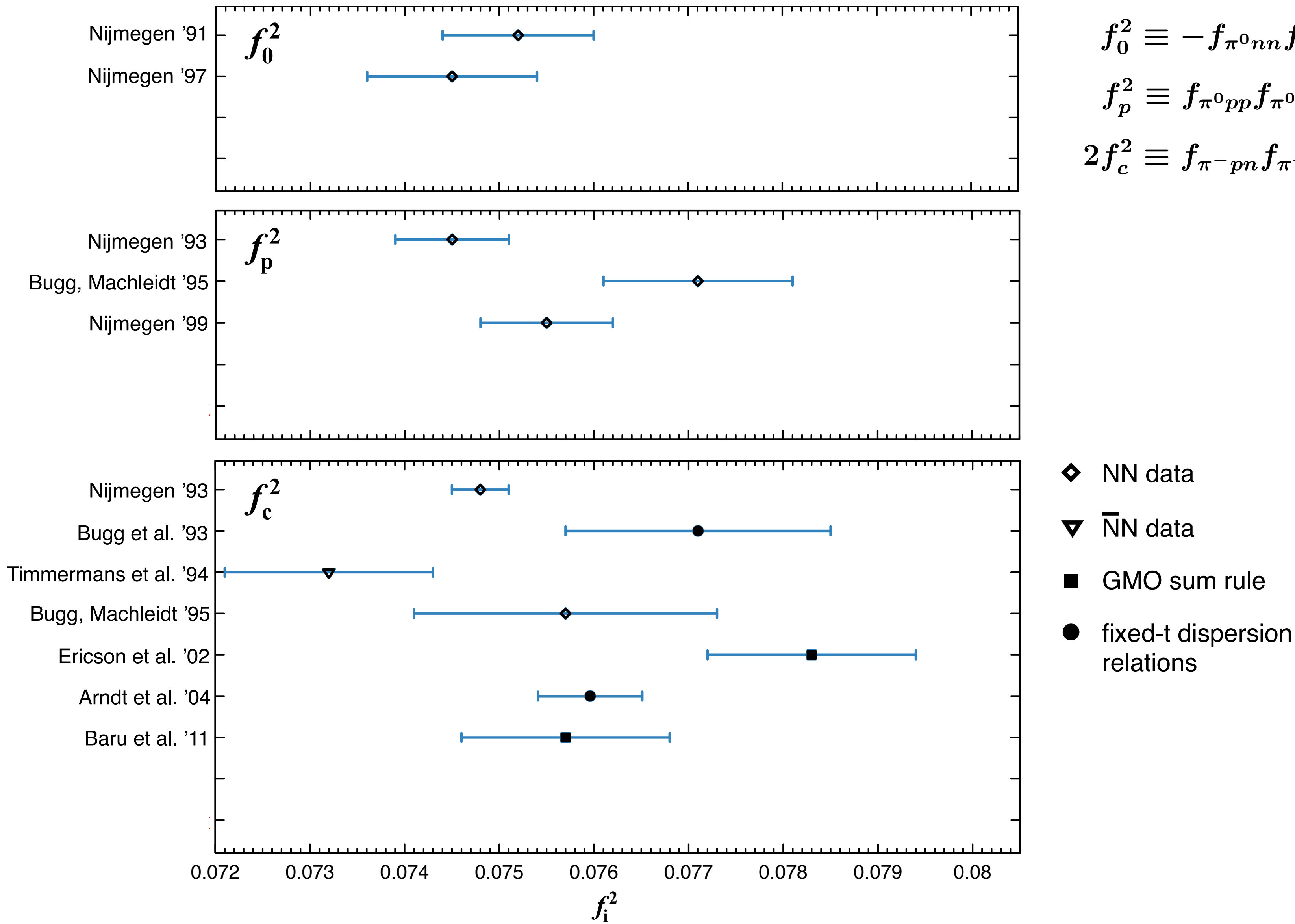
Away from the isospin limit:



Notation: $f_p^2 \equiv f_{\pi^0 pp} f_{\pi^0 pp}$, $f_0^2 \equiv -f_{\pi^0 nn} f_{\pi^0 pp}$, $2f_c^2 \equiv f_{\pi^- pn} f_{\pi^+ np}$

Determination of the πN constants



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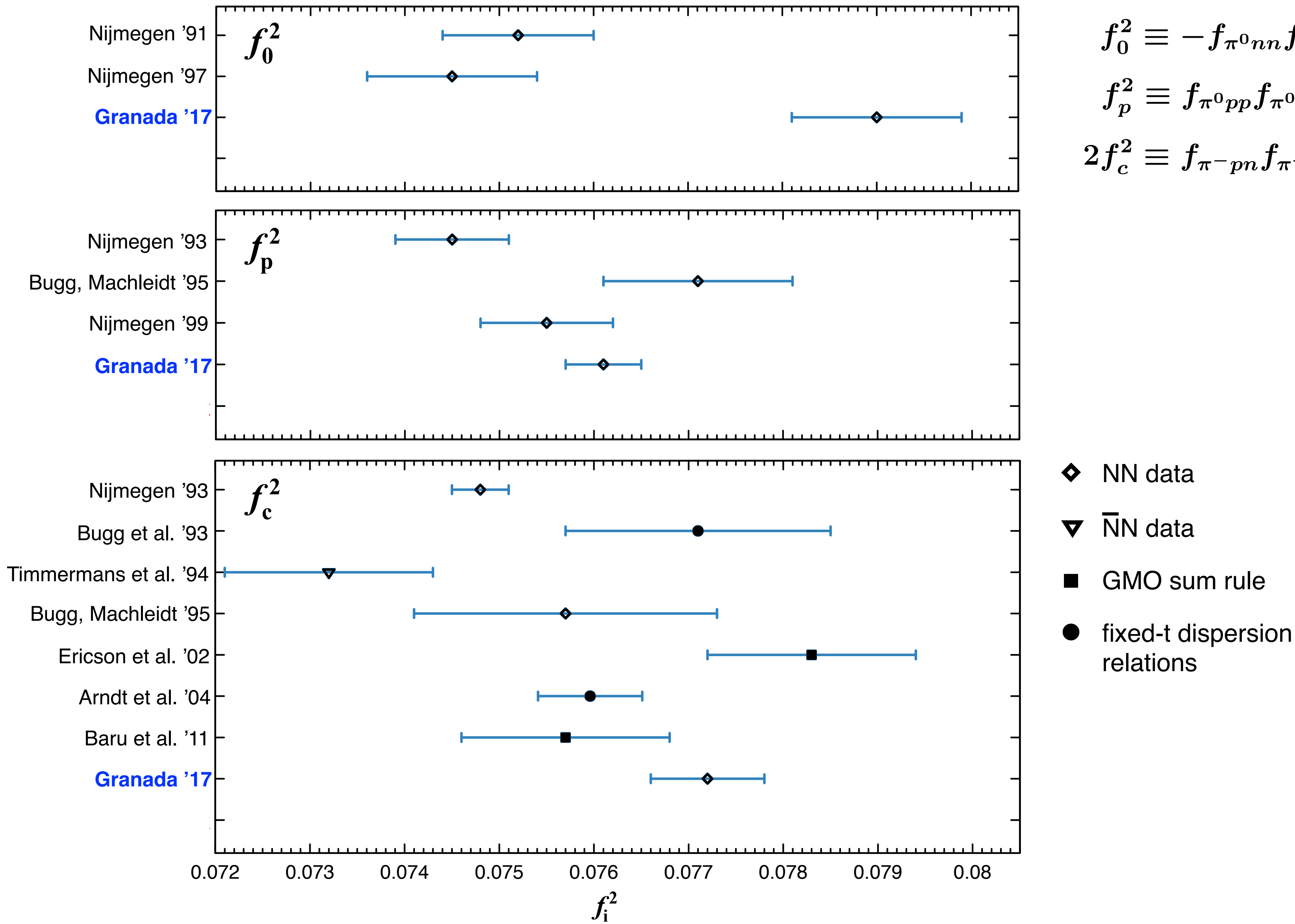

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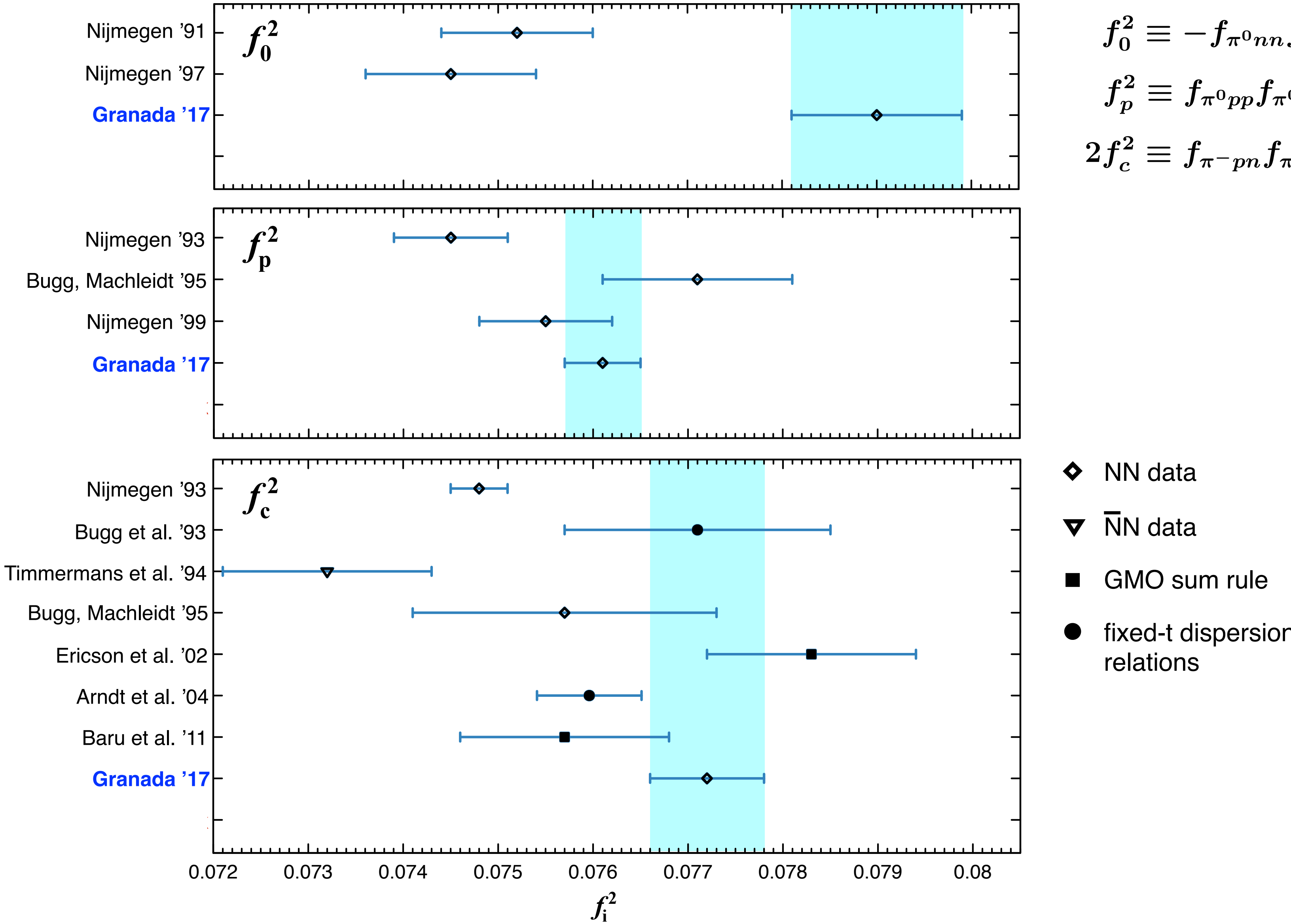

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Granada 2017 [Perez et al., PRC95 (17)]: clear evidence of **charge dependence** of πN constants!

Determination of the πN constants

Consider a fixed database in the energy range from 0…E$_{max}$.

For normally distributed errors: $p(D|f^2 C\Lambda) = \frac{1}{N} e^{-\frac{1}{2}\chi^2}$

3 πN couplings → f^2; 25 IC + 5 IB short-range LECs → C

Using Bayes' theorem: $p(f^2|D) = \int d\Lambda\, dC\; p(f^2 C\Lambda|D) = \int d\Lambda\, dC\; \frac{p(D|f^2 C\Lambda)\; p(f^2 C\Lambda)}{p(D)}$

Employ independent priors: $p(f^2 C\Lambda) = \underbrace{p(f^2)\, p(\Lambda)}_{\text{uniform}}\, \underbrace{p(C)}_{\text{Gaussian with } \bar{C} = 5}$

However, for any set of f^2 need to evaluate a 30 (C) + 1 (Λ) dimensional integral… 😢

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However, for any set of f^2 need to evaluate a 30 (C) + 1 (Λ) dimensional integral… 😢

Solution: Approximate the likelihood $p(D|f^2 C\Lambda)$ via :

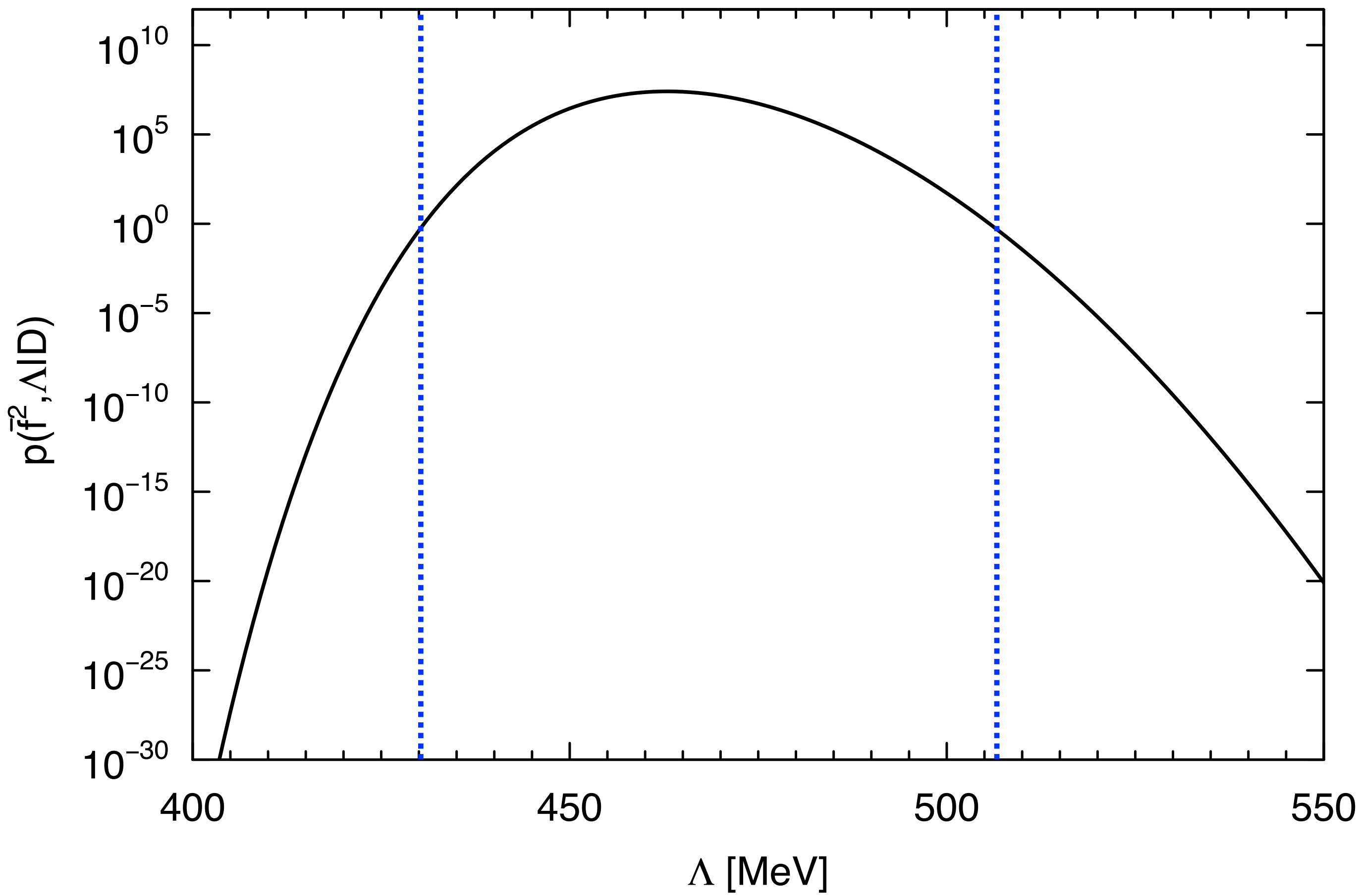
$$p(D|f^2 C\Lambda) \approx \frac{1}{N} e^{-\frac{1}{2}[\chi^2_{\rm min} + \frac{1}{2}(C - C_{\rm min})^T H (C - C_{\rm min})]}$$

where $\chi^2_{\rm min} \equiv \chi^2_{\rm min}(f^2, \Lambda)$ at $C_{\rm min} \equiv C_{\rm min}(f^2, \Lambda)$ and $H_{ij}(f^2, \Lambda) = \frac{\partial^2 \chi^2}{\partial C_i \partial C_j}\Big|_{C = C_{\rm min}}$.

Then, the integral over C can be done analytically. The remaining integral over Λ is done numerically. For the considered grid of f^2, Λ need to perform ~ 10^4 fits of C….

Determination of the πN constants

How strong do the results depend on the employed p(Λ) prior (i.e. cutoff range)?



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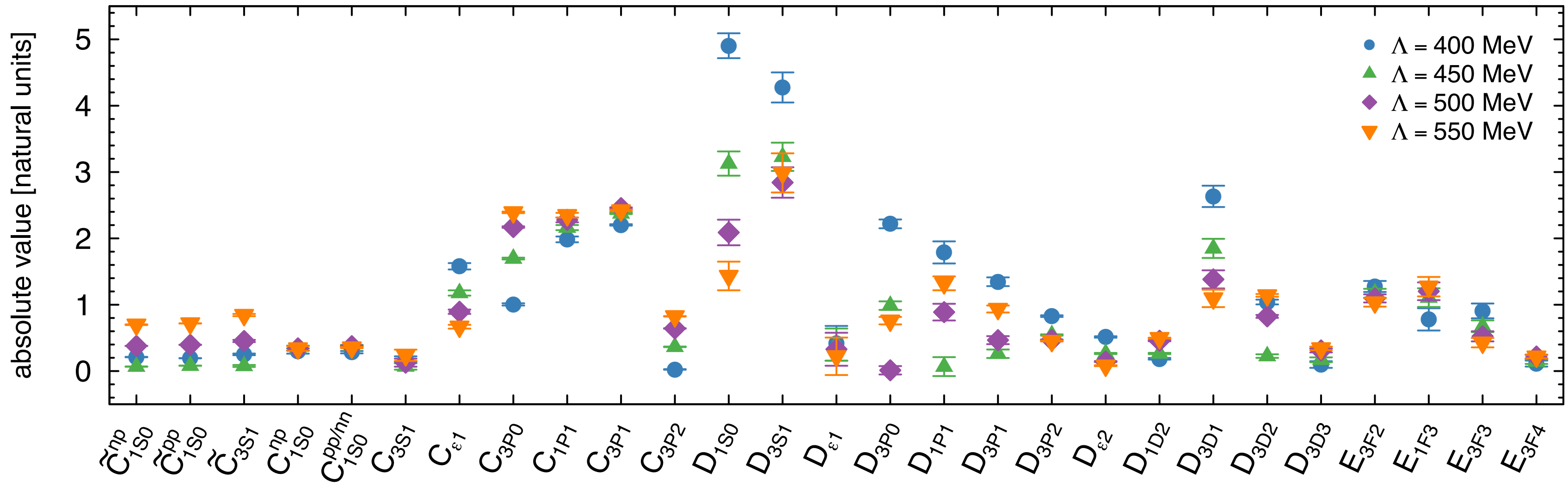

Results for the N⁴LO⁺ of Reinert, Krebs, EE, EPJA 54 (18)

→ Naturalness prior ($\bar{C}$ = 5) plays a very minor role

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Determination of the πN constants

- Performed own (recursive) data selection for the optimal choice of Λ = 463 MeV. Results for E_{max} = {220, 240, 260, 280, 300} MeV are found to be consistent with each other.

Resulting database below 280 MeV:	2096 pp data	+	2836 np data
Granada-2017:	2083 pp data	+	2859 np data

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Resulting database below 280 MeV:	2096 pp data	+	2836 np data
Granada-2017:	2083 pp data	+	2859 np data

- Performed weighted averaging of the marginal PDFs over E_{max} = 220…300 MeV

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Resulting database below 280 MeV: 2096 pp data + 2836 np data
Granada-2017: 2083 pp data + 2859 np data

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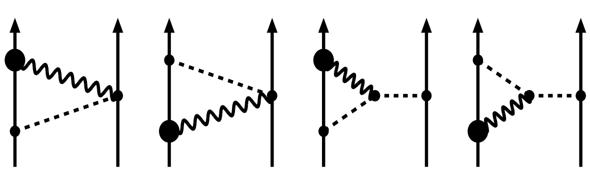
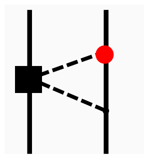
- To estimate the uncertainty from the truncation of the EFT expansion performed weighted averaging of 3 models which differ by N^5LO terms:

Subleading πγ-exchange (1/m-corrections)



[Kaiser 2006]

IB πN coupling constant in the subleading TPE



Marginalized PDFs (preliminary)

Statistical validity checks

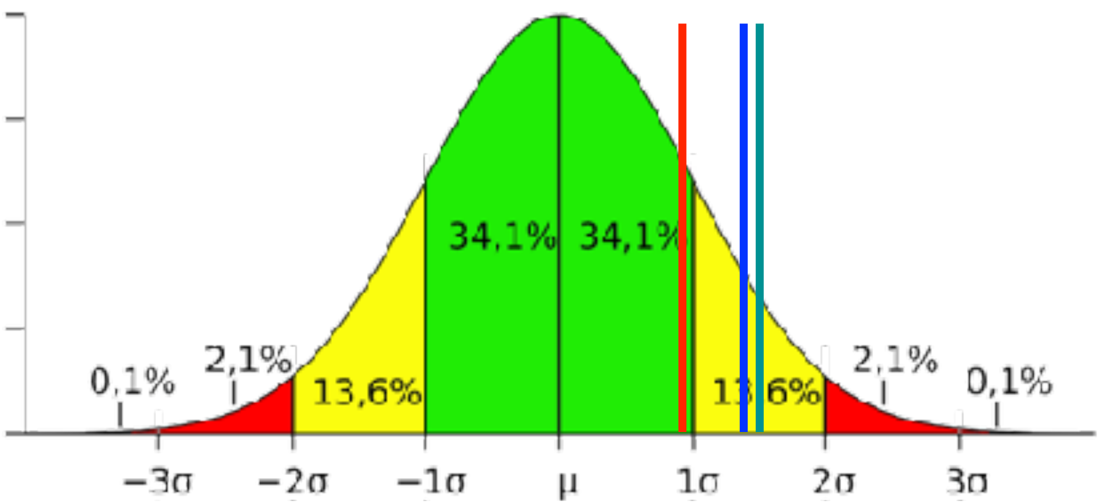
- Published RKE potential: incomplete IB effects, Granada 2013 database, E_{max} = 280 MeV:

  χ^2 = 5003 for 4895 pp+nn data ⟶ $\chi^2 / N_{dat} = 1.022$; $\chi^2 / (N_{dat} - \underbrace{N_{par}}_{27+1}) = 1.028$

- Complete treatment of IB effects, own data selection, E_{max} = 280 MeV:

- Granada 2017 PWA, E_{max} = 350 MeV:

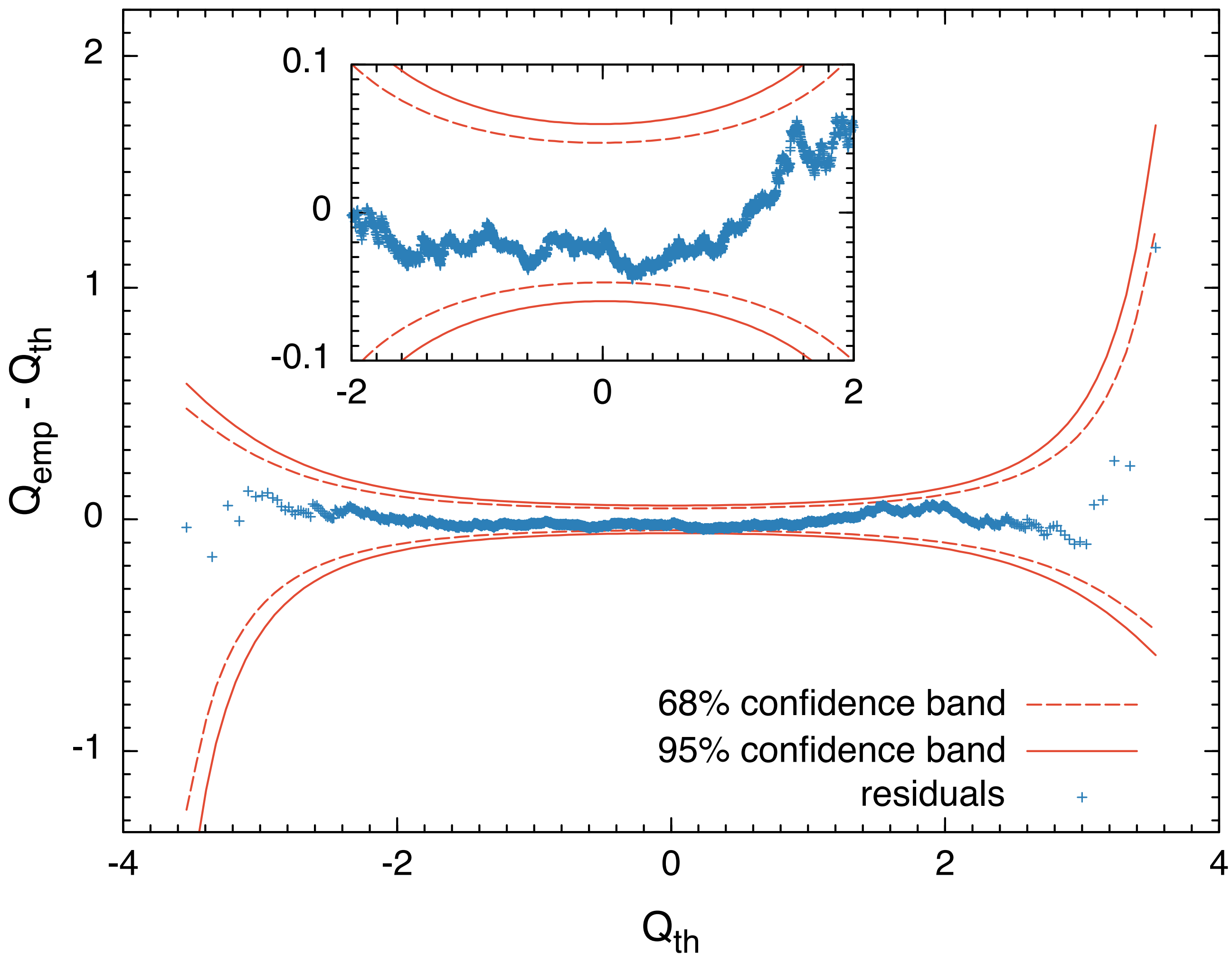
  χ^2 = 6856 for 6741 pp+nn data ⟶ $\chi^2 / N_{dat} = 1.017$; $\chi^2 / (N_{dat} - \underbrace{N_{par}}_{55}) = 1.025$



Normality checks

Do the outcoming residuals follow a normal distribution?

Tail-sensitive rotated quantile-quantile plot (Λ = 463 MeV, E_{max} = 280 MeV)



Results for the πN constants (preliminary)

Summary and outlook

- Chiral EFT was used to determine πN coupling constants from a combined analysis of np and pp scattering data at a per-cent level with fully controlled uncertainties

  → **new reference values for πN coupling constants**

- Differently to Granada 2017, no evidence is found for charge dependence of πN coupling constants

To be done/work in progress:

- finalize error analysis of πN couplings
- IB effects in NN phase shifts and observables
- systematic investigation of IB effects in few-nucleon systems