

Phenomenological Four-Body Interactions in the NCSM

Tobias Wolfgruber and Robert Roth



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Introduction

Improvements in all three many-body methods have led to greater calculations of observables, especially energies and radii. The resulting data are used to constrain the parameters of a few-body potential from experimental data. One general feature is the inclusion of four-body forces, whose signs and values vary. Many-body calculations are highlighted in the present talk.

Explicit inclusion of a Gaussian four-body force in the ^4He ground state energy and the comparison to the ^4He ground state energy and other ground state.

The Gaussian four-body interaction are studied in a model to be substituted in the NCSM.

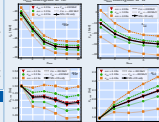
Range of the interactions

Four-body interactions are in the nuclear interaction derived from ab initio effective field theory (chiral EFT).

Explicit inclusion of the ab initio four-body interaction in the NCSM.

Small Component Behavior of Gaussian 4B Interactions

Chiral EFT interaction in ^4He



• An significant effect of the Gaussian 4B interaction on the ^4He ground state energy.

• The small component is a four-body interaction.

Phenomenological Four-Body Interaction

We use a Gaussian four-body interaction (lower of the simple form than results for calculations).

$$V_{4B} = G_{4B} \sum_{i,j,k,l} \delta(\mathbf{r}_i - \mathbf{r}_j) \delta(\mathbf{r}_i - \mathbf{r}_k) \delta(\mathbf{r}_i - \mathbf{r}_l)$$

• We use only to constrain open, not open to longer distances.

• We constrain the Gaussian four-body interaction in the NCSM calculations on the following steps:

• Interaction will fit each ^4He state $|\Psi\rangle$, with $\langle\Psi|V_{4B}|\Psi\rangle = E_{4B}$.

• The variational method allows to fit the three independent and equal radial lengths, with respect to that λ and a symmetry.

$$\langle\Psi|V_{4B}|\Psi\rangle = \int d^3r_1 d^3r_2 d^3r_3 d^3r_4 \Psi^*(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) V_{4B}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)$$

• Here length scales in the four-body interaction function and the coefficient open representation of the interaction and can be solved analytically [1, 2].

$$E_{4B} = \int d^3r_1 d^3r_2 d^3r_3 d^3r_4 \Psi^*(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) V_{4B}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)$$

• Parameters fixed to optimal conditions with harmonic oscillator wavefunction [3, 4, 5, 6, 7].

• Adjusted fit comparison of the experiment 4B ground state energy [8].

• Add spin and isospin components, produce a single parameter (coupling and an explicit antisymmetrization in orbit, NCSM matrix element).

$$\langle\Psi|V_{4B}|\Psi\rangle = \int d^3r_1 d^3r_2 d^3r_3 d^3r_4 \Psi^*(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) V_{4B}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)$$

• All calculations were carried out using the NCSM with ^4He interaction [1].

• NCSM 4B frequency of 4B, 4B and 4B, with four parameters in a 4B fit.

• Systematics of Ground State Energy and Purity Phase Factors



• Gaussian 4B interaction affects energy and purity phase factors.

• The range of the interaction determines the energy and purity phase factors.

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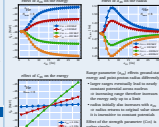
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Phenomenological Behavior of Gaussian 4B Interactions

Effect of G_{4B} on the energy



• An significant effect on ^4He ground state energy.

• The small component is a four-body interaction.

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Why look at four-body interactions?

- ▶ chiral effective field theory (EFT) at $N^3\text{LO}$ gives rise to $4N$ force terms
- ▶ similarity renormalization group (SRG) induces many-body forces

Why use a phenomenological $4N$ interaction?

- ▶ initial or induced $4N$ forces in general not computationally feasible in many-body calculations
 - ▶ use phenomenological interaction to bypass expensive calculation steps
- ⇒ develop a phenomenological four-body interaction that can be computed on-the-fly during a NCSM calculation

Many-Body Toolchain with Phenomenological 4N Interaction



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Initial Hamiltonian



SRG evolution



Add phenomenological
4N interaction



NCSM calculation

- start with two- and three-body interaction derived from chiral EFT
- calculate SRG evolution and truncate induced forces at the 3N level
- calculate phenomenological 4N matrix elements
$$\langle \{n_i l_i m_i\}_{i=1,\dots,4} | \hat{V}_{4N} | \{n'_i l'_i m'_i\}_{i=1,\dots,4} \rangle$$
- run many-body calculation in harmonic oscillator basis



Why use a Gaussian four-body interaction?

- ▶ more flexible than e.g. contact interaction
- ▶ comparatively intuitive to understand and simple to calculate
- ▶ completely different matrix element calculation without PWD, relative basis, ...

$$\hat{V}_{4N} = C_{4N} \exp \left\{ -\frac{1}{a_{4N}^2} \sum_{\substack{i,j=1 \\ i < j}}^4 \left(\hat{r}_i - \hat{r}_j \right)^2 \right\}$$

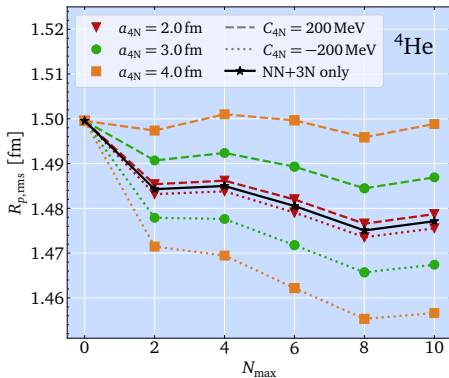
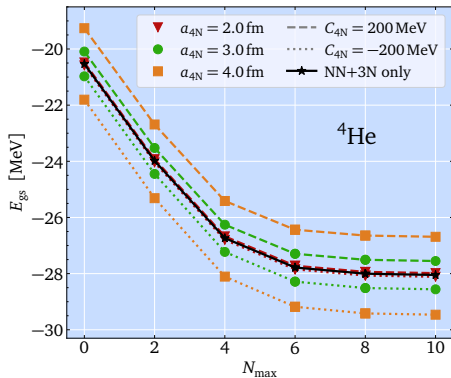
Interaction has two free parameters

- ▶ C_{4N} : strength of the interaction, also determines if attractive/repulsive
- ▶ a_{4N} : range of the interaction

N_{\max} convergence behavior of Gaussian 4N Interaction



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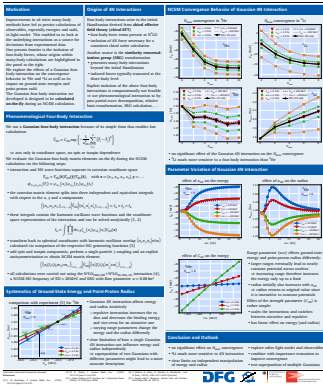


- ▶ $\hbar\Omega = 20$ MeV, $N^3\text{LO}_{\text{EMN},500} + N^3\text{LO}_{\text{NL},500,\text{cD4}} + \text{Gaussian 4N}$
- ▶ no significant effect on N_{\max} convergence
- ▶ attractive interaction increases energy and reduces radius, repulsive vice-versa

- explore other light nuclei
- combine with importance truncation to improve convergence
- calculate other observables besides ground-state energies and radii
- implement ability to use superposition of two Gaussians
- mitigate SRG induced effects and correct energy/radius

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Thank you for your attention!



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► Thanks to my group and collaborators

S. Alexa, T. Hüther, M. Knöll, L. Mertes,
T. Mongelli, J. Müller, **R. Roth**



DFG

computing time:



Hessisches Kompetenzzentrum
für Hochleistungsrechnen



Bundesministerium
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