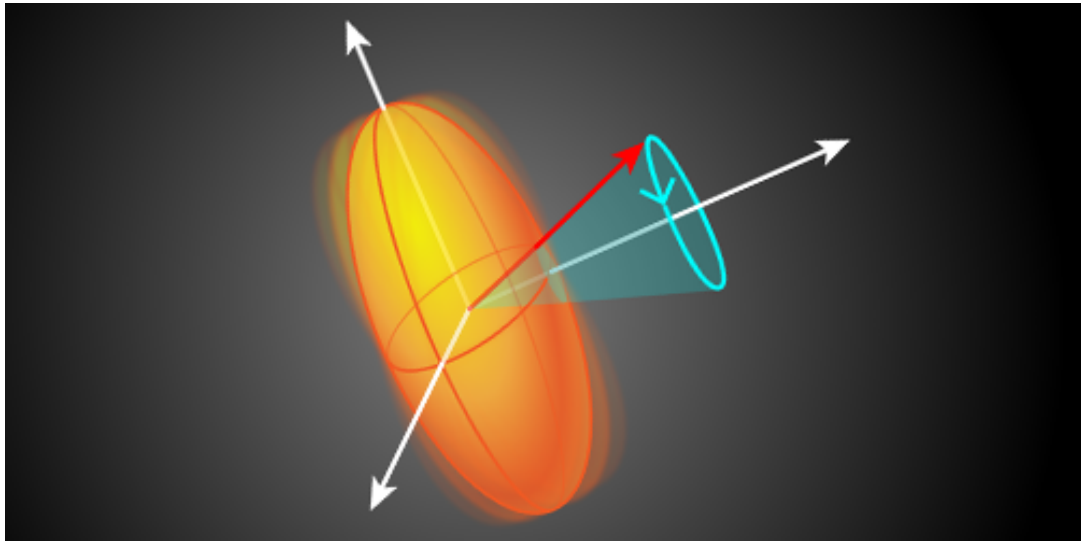
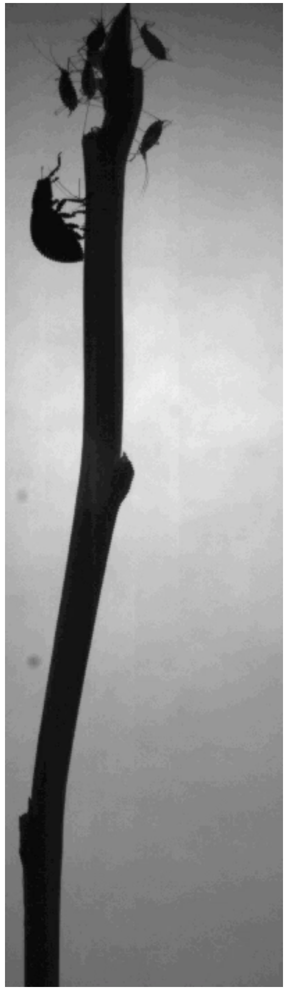
Falling Cats and EFTs for Deformed Nuclei

Agence Nature/Science Source



From APS viewpoint



Ribak, Gish, Weihs & Inbar, Current Biology **23**, PR102 (2013)

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SciDAC
Scientific Discovery through Advanced Computing

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NUCLEI
Nuclear Computational Low-Energy Initiative

Falling Cat Problem

Q: How does a cat change its orientation, i.e. its angular momentum, without an external torque?

A: Changes in its shape (intrinsic degrees of freedom) induce a change in the external orientation.

Q: What does this has to do with odd-mass deformed nuclei?

A: In both cases, non-Abelian gauge potentials arise that describe the internal dynamics and couple it to the overall orientation. (In the nucleus, the odd nucleon causes the internal dynamics.)

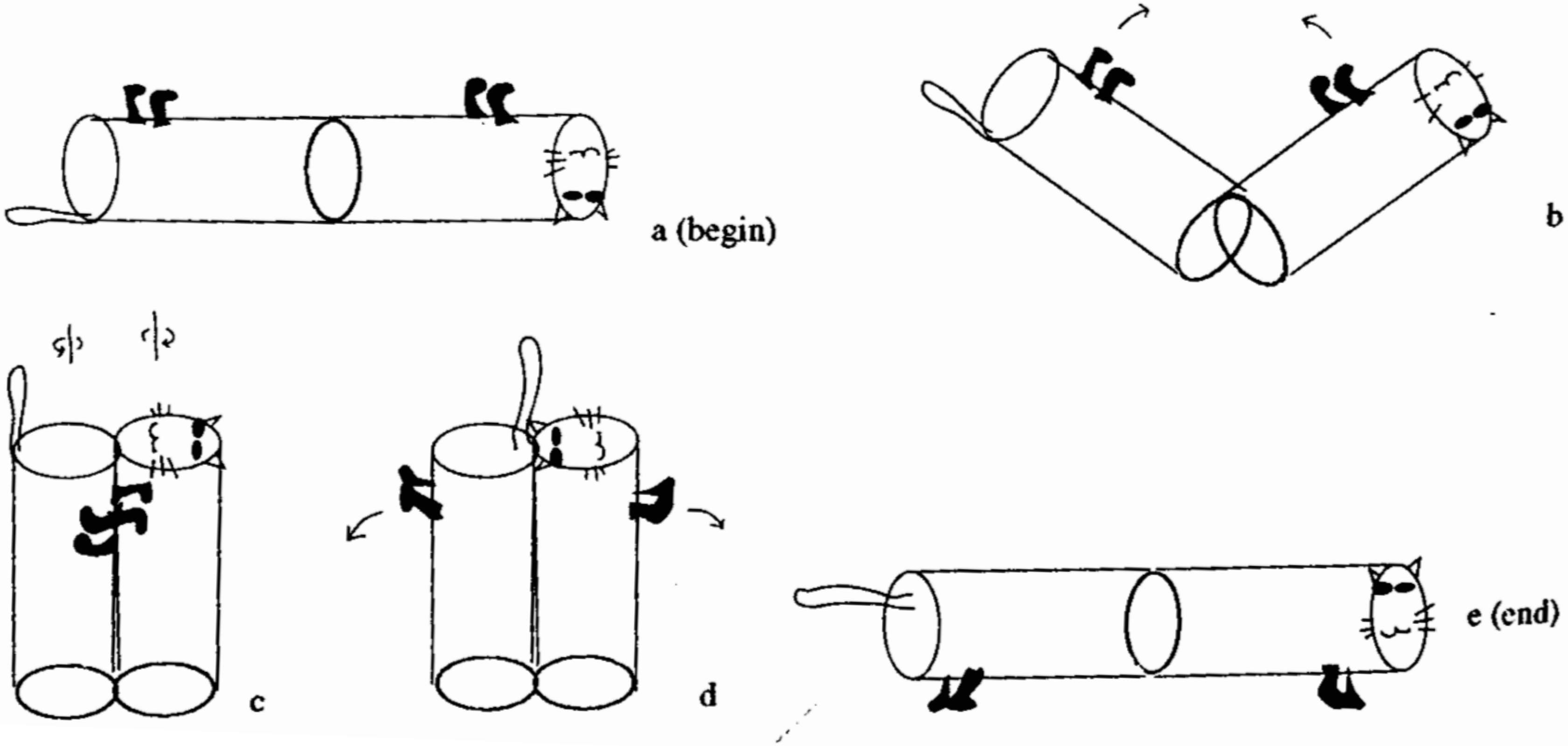
→ Gauge theory of deformable bodies

Shapere & Wilzcek, *Geometric Phases in Physics* (1989); Littlejohn & Reinsch, Rev. Mod. Phys. (1997)

“Gauge theory of the falling cat,” Montgomery (1993)

6. Some Specific Reorientations and Steering Strategies

6.1. A Cartoon. Probably the simplest path resulting in the cat flip is the one depicted below.



“Bend, twist, unbend” makes a closed loop in internal configuration space while leading to a rotation.

Effective field theories in the physics of nuclei

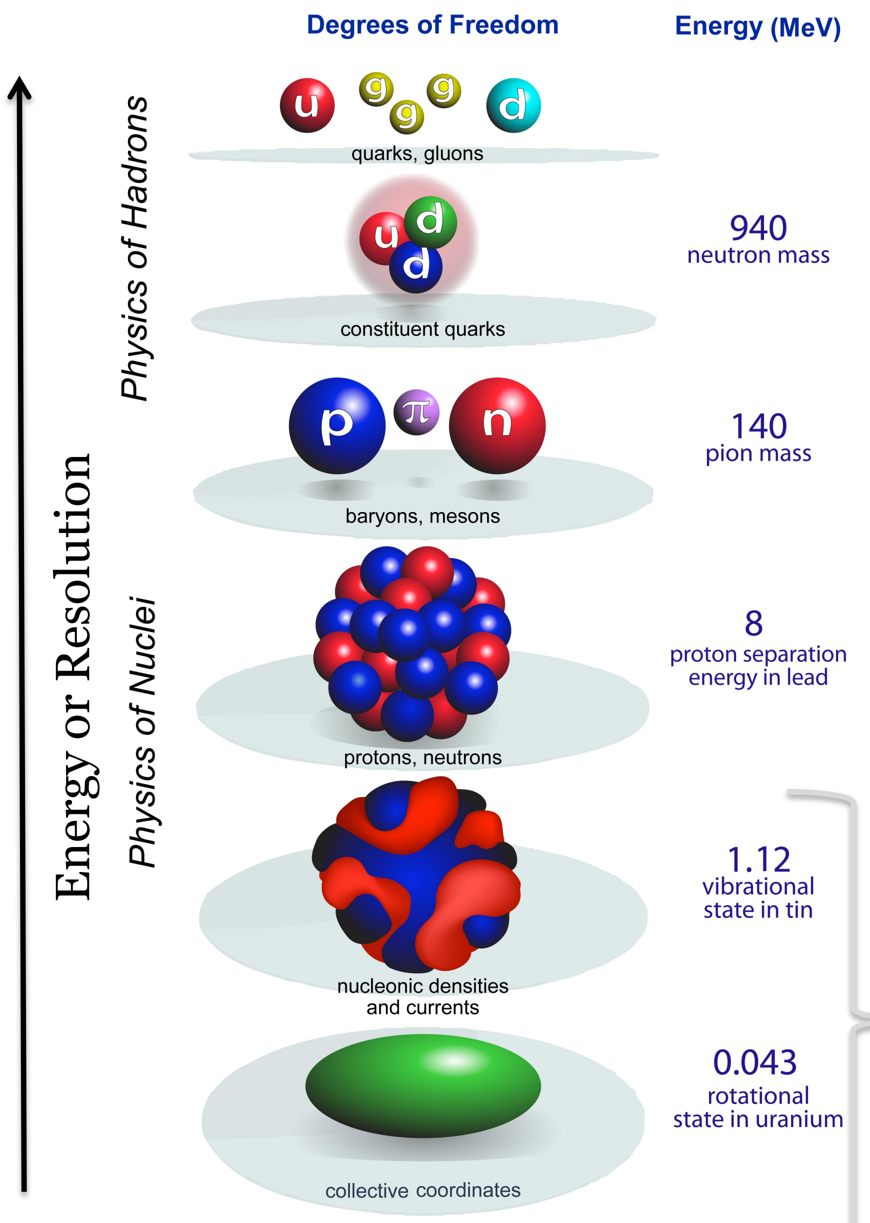
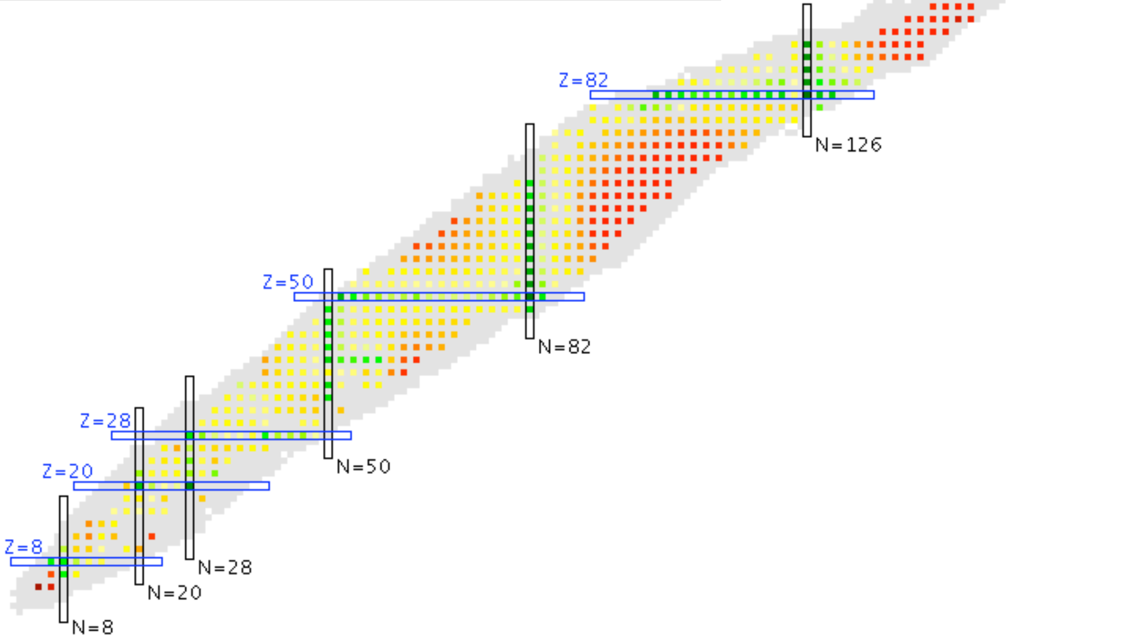


Fig.: Bertsch, Dean, Nazarewicz (2007)

Rotors: $E(4^+)/E(2^+) = 10/3$

Vibrators: $E(4^+)/E(2^+) = 2$



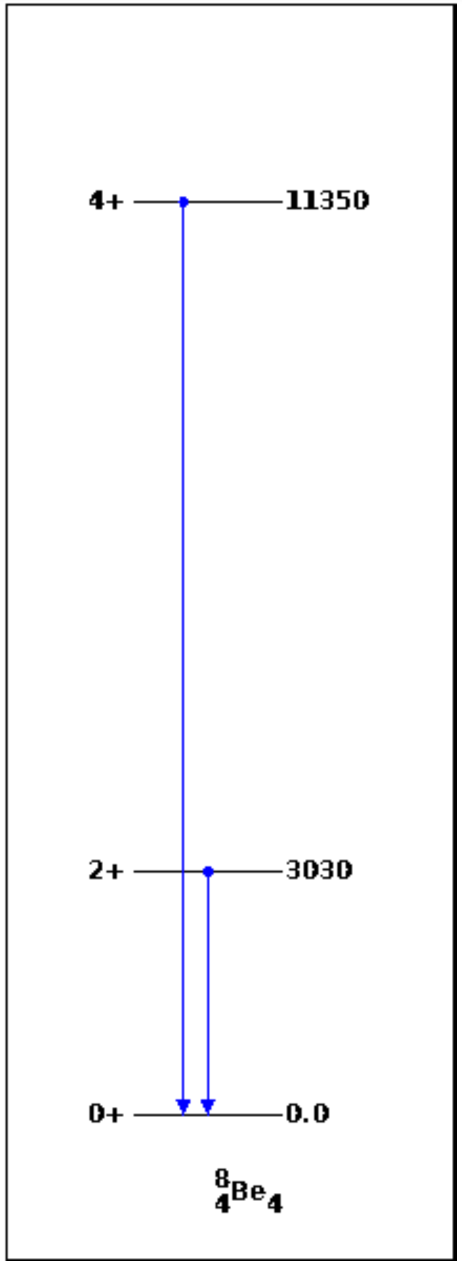
Vibrators: EFT based on linear (Wigner/Weyl) realization [Coello Pérez & TP 2015; 2016; Coello Pérez, Menéndez & Schwenk 2018]

Rotors: EFTs based on non-linear realization of SO(3)

Axially symmetric nuclei: [TP 2011; TP & Weidenmüller 2014; Coello Pérez & TP 2015]

Triaxial deformation: [Chen, Kaiser, Meißner, Meng 2017; 2018]

^{9}Be as a neutron coupled to the rotor ^{8}Be



NNDC spectra: levels up to 12 MeV

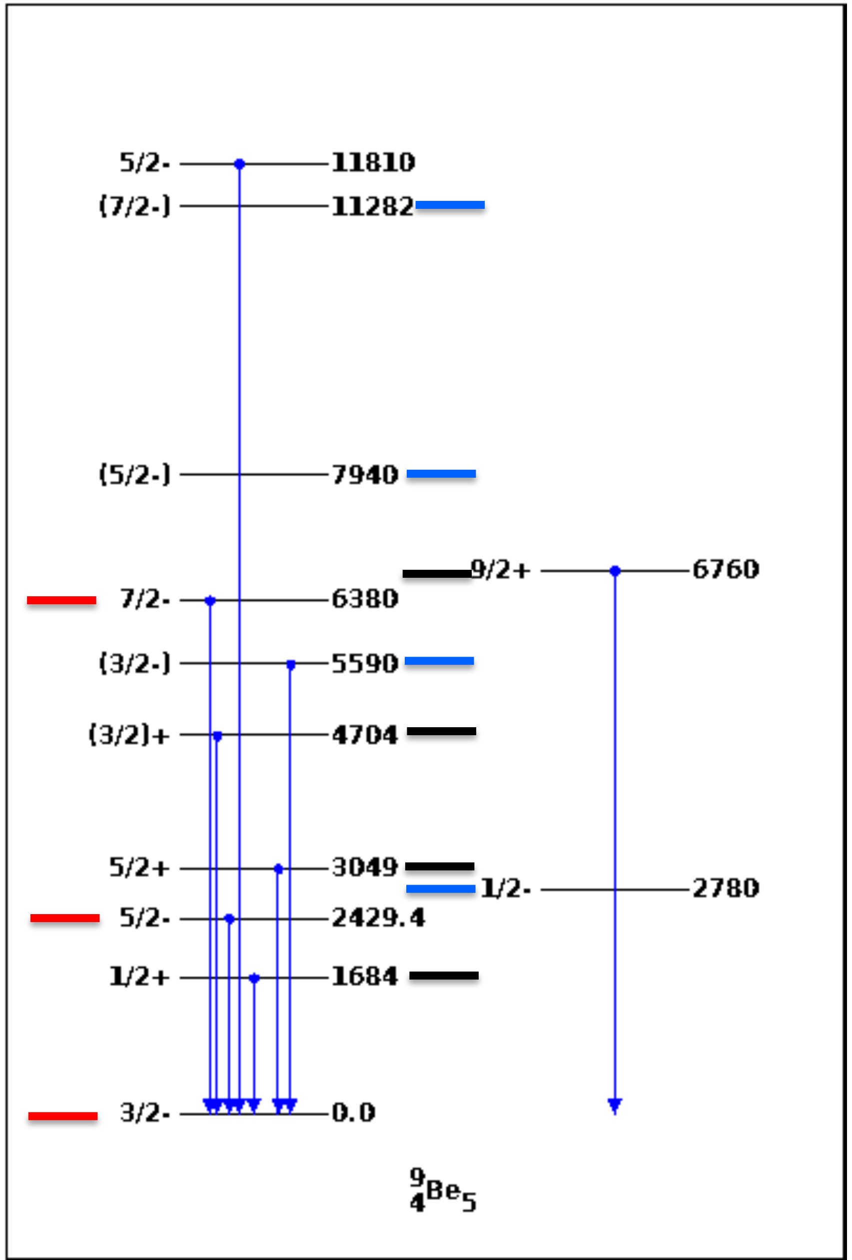
^{8}Be:

- 0$^+$, 2$^+$, 4$^+$ levels form rotational band $E(I) = A\,I(I+1)$ with $A \approx 0.505$ MeV
- 2$^+$ state sets low-energy scale $\xi \approx 3$ MeV
- First non-rotational state (at 16.6 MeV) sets breakdown scale $\Lambda = 16.6$ MeV
- Small expansion parameter is $\frac{\xi}{\Lambda} \approx 0.2$

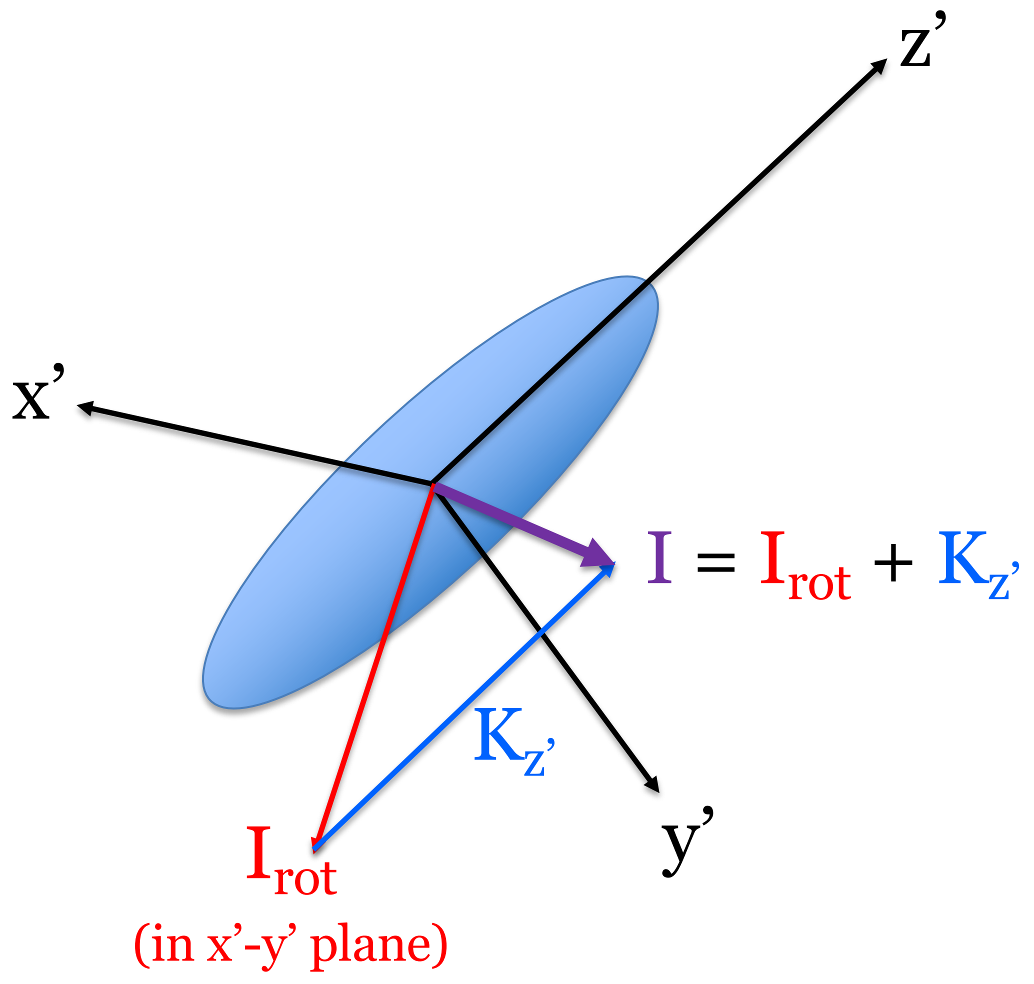
^{9}Be:

- Spectrum looks complicated …
- However: All but one state below 12 MeV explained by three rotational bands built on band heads with $K = {}^3/_2{}^-, ½^+, ½^-$
- $\Delta E(I) = A\left[I(I+1) + a(-1)^{I+\frac{1}{2}}\left(I+\frac{1}{2}\right)\delta_K^{\frac{1}{2}}\right]$

→ ab initio by Caprio et al., arXiv:1912.0008



Nucleon coupled to a deformed even-even nucleus

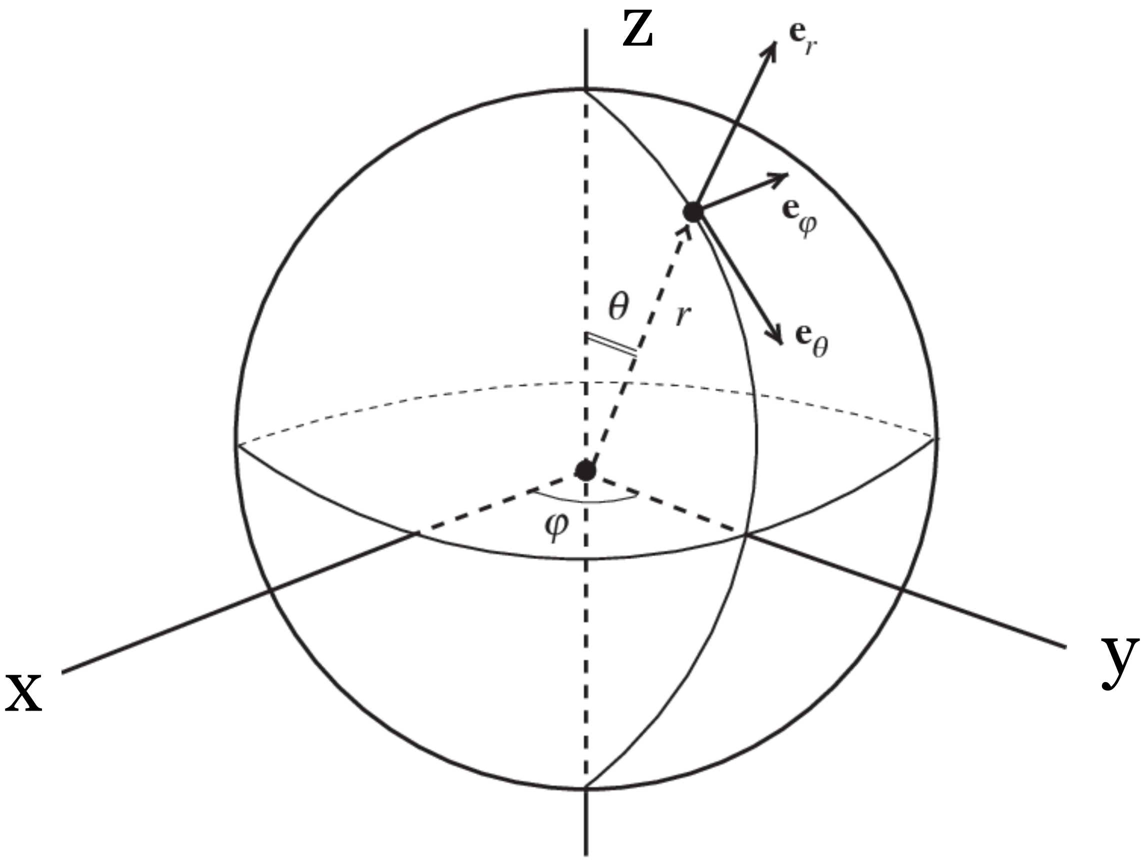


Rotor:
- Deformed, axially symmetric, even-even nucleus
- Symmetry axis defines body-fixed z' axis
- Slow / heavy degree of freedom

Nucleon:
- Fast / light degree of freedom; "instantaneous" in the body-fixed frame
- Strongly coupled to the rotor
- Experiences axially symmetric forces in the body-fixed frame

Rotor: axially symmetric, even-even nucleus

1. Emergent symmetry breaking from SO(3) to SO(2): Nambu-Goldstone modes parametrize the sphere.
2. Body-fixed frame ambiguous in axially symmetric nucleus → “gauge” freedom



Degrees of freedom: θ, ϕ

$$e_{z'}(\theta,\phi) = e_r(\theta,\phi) = \begin{pmatrix} \cos\phi\,\sin\theta \\ \sin\phi\,\sin\theta \\ \cos\theta \end{pmatrix}$$

Angular velocity:

$$\mathbf{v} = \frac{de_r}{dt} = \dot{\theta} e_\theta + \dot{\phi}\,\sin\theta\; e_\phi$$

Simplest (leading order) Lagrangian:

$$L = \frac{C_0}{2}\mathbf{v}\cdot\mathbf{v} = \frac{C_0}{2}\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right)$$

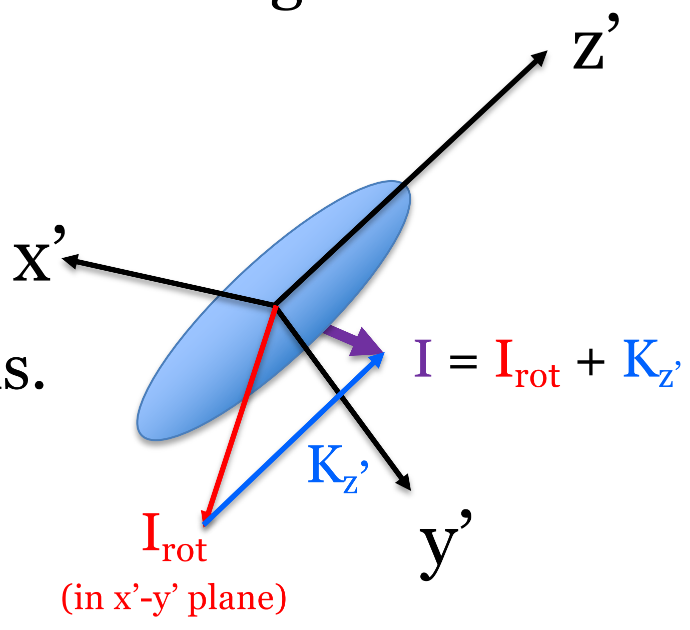
Hamiltonian:

$$H = \frac{\hat{I}^2}{2C_0}$$

Nucleon

Nucleon is two-component spinor $\hat{\Psi}(x') = \begin{pmatrix} \hat{\psi}_{+\frac{1}{2}}(x') \\ \hat{\psi}_{-\frac{1}{2}}(x') \end{pmatrix}$, components along the z' axis.

Total nucleon spin $\hat{\mathbf{K}} = \int \mathrm{d}^3\mathbf{x}' \hat{\Psi}^\dagger(\mathbf{x}') \left(-i\mathbf{x}' \times \nabla' + \hat{\mathbf{S}} \right) \hat{\Psi}(\mathbf{x}')$



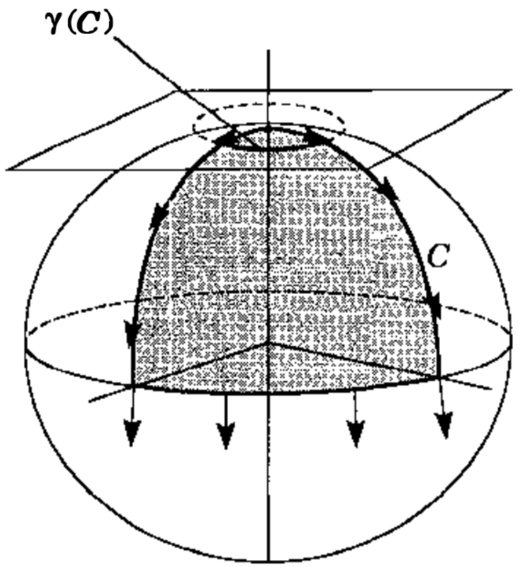
Coupling between the nucleon and the rotor via two mechanisms.

1. **Axially symmetric potential *V* in body-fixed frame**
   a. Lagrangian
   $$L_\Psi = \int \mathrm{d}^3\mathbf{x}' \hat{\Psi}^\dagger(\mathbf{x}') \left(i\partial_t + \frac{\hbar^2 \Delta'}{2m} - V \right) \hat{\Psi}(\mathbf{x}')$$
   b. All nucleon states come in pairs $|\pm K, \alpha\rangle$ related by time-reversal invariance (Kramer's degeneracy); we have $\hat{K}_{z'}|\pm K, \alpha\rangle = \pm K\, |\pm K, \alpha\rangle$

2. **Gauge couplings, i.e. couplings to the rotor's angular velocity**

Nucleon creates gauge field for rotor

Kinematics: Assume that the rotor's symmetry axis moves along a closed path C.

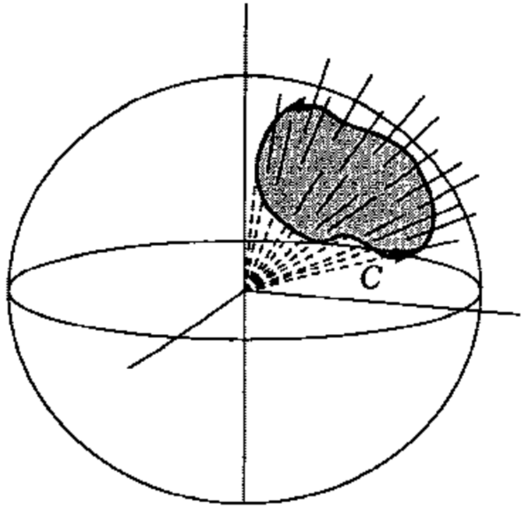


This induces a rotation of the body-fixed frame by an angle γ around the z' axis. This is a geometric effect.

Rotation of nucleon in the body-fixed system $\hat{\Psi}(x') \to e^{-i\gamma \hat{J}_{z'}} \hat{\Psi}(x')$ introduces a Berry phase [Berry 1984] in its wave function.

The angle γ is identical to the solid angle of the enclosed area.

Dynamics: A magnetic monopole field changes the phase by an angle that is identical to the flux (area!) [Wu & Yang 1976]



Vector potential of monopole: $\mathbf{A}_{\rm a}(\theta,\phi) \equiv \mathbf{e}_\phi \cot\theta \hat{K}_{z'}$

Corresponding "magnetic" field: $\mathbf{B}_{\rm a}(\theta,\phi) \equiv \nabla\times\mathbf{A}_{\rm a} = -\mathbf{e}_r \hat{K}_{z'}$

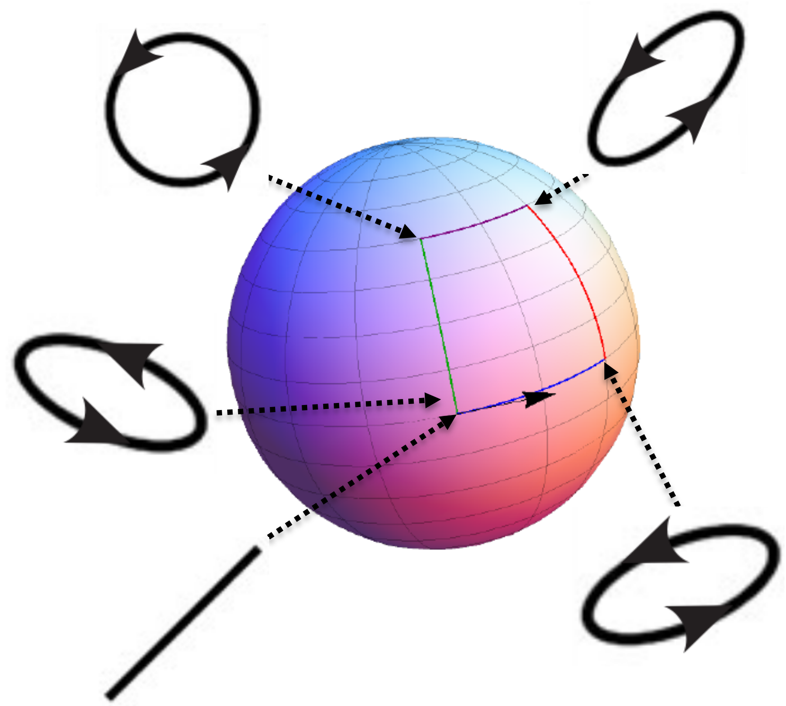
No parameter in singular gauge potential (Dirac's quant. cond.)

Sphere = shape space; closed path in shape space induces rotation.

Images: Zwanziger, Koenig & Pines (1990)

A non-Abelian gauge coupling also possible

Fermion states come in pairs (Kramer's degeneracy).



Kinematics: As the rotor's symmetry axis moves along a closed path on the sphere, the degenerate nucleon states might mix. Original and final states differ by a unitary transformation: $\widehat{\Psi}(x') \to \widehat{U}\,\widehat{\Psi}(x')$

Dynamics: A non-Abelian gauge potential causes such a non-Abelian Berry phase [Wilczek & Zee 1984]. (The amount of SU(2) rotation is not quantized, the gauge potential is not singular, and has an adjustable dimensionless parameter g.)

Non-Abelian gauge potential: $\mathbf{A}_{\mathrm{n}}(\theta,\phi) = g\mathbf{e}_r \times \hat{\mathbf{K}} = g\left(\mathbf{e}_\phi \hat{K}_{x'} - \mathbf{e}_\theta \hat{K}_{y'}\right)$

Corresponding "magnetic" field: $\mathbf{B}_{\mathrm{n}}(\theta,\phi) \equiv \nabla \times \mathbf{A}_{\mathrm{n}} - i\mathbf{A}_{\mathrm{n}} \times \mathbf{A}_{\mathrm{n}}$

$$= g^2 \mathbf{e}_r \hat{K}_{z'}\ .$$

The non-Abelian gauge fields link deformed odd-mass nuclei to falling cats

Gauge potentials, Berry phases, and Coriolis forces

Different interpretations of the velocity–dependent rotor–nucleon couplings

1. **Coriolis forces** enter in rotating frames: Velocity-dependent forces are present in rotating nuclei [Bohr, Kerman, Mottelson, Nilsson 1950s].

2. **Molecular Aharonov-Bohm effect**: In rotating molecules, the nuclei are slow (and the electrons are fast), and the adiabatic decoupling (à la Born Oppenheimer) introduces Berry phases and gauge potentials [Mead & Truhlar 1979; Wilczek & Zee 1984; Kuratsuji & Iida 1985].

3. **Covariant derivative**: In presence of spontaneous symmetry breaking, the rotational symmetry is realized non-linearly for the rotor's degrees of freedom. This introduces a covariant derivative $iD \equiv i\partial_t + \mathbf{v} \cdot A_a$ [Weinberg 1968; Callan, Coleman, Wess & Zumino 1969].

4. **Gauge invariance**: The ambiguities in defining a body-fixed frame, i.e. separating rotational and intrinsic degrees of freedom, imply a gauge invariance [Littlejohn & Reinsch 1997]. Our case: ambiguities regarding rotations around the z' axis.

[Leutwyler 1994; Roman & Soto 1999; Hofmann 1999; Chandrasekharan et al. 2008; Brauner 2010; TP 2011, TP & Weidenmüller 2014; …]

Leading order Lagrangian

Let us start with the most general Lagrangian quadratic in the velocities:

$$L = \frac{C_0}{2}\mathbf{v}^2 + \mathbf{v}\cdot(\mathbf{A}_\mathrm{a} + \mathbf{A}_\mathrm{n}) + L_\Psi$$
$$= \frac{C_0}{2}\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) + g\left(\dot{\phi}\sin\theta\hat{K}_{x'} - \dot{\theta}\hat{K}_{y'}\right) + \dot{\phi}\cos\theta\hat{K}_{z'} + L_\Psi$$

The nucleon part is

$$L_\Psi = \int \mathrm{d}^3\mathbf{x}'\hat{\Psi}^\dagger(\mathbf{x}')\left(i\partial_t + \frac{\hbar^2\Delta'}{2m} - V\right)\hat{\Psi}(\mathbf{x}')$$

Low energy constants are C_0, g, and parameters of the axially symmetric potential V.

Leading order Hamiltonian

Hamiltonian

$$H = \int d^3\mathbf{x}' \hat{\Psi}^\dagger(\mathbf{x}') \left(-\frac{\hbar^2\Delta'}{2m} + V\right) \hat{\Psi}(\mathbf{x}')$$

$$+\frac{1}{2C_0}\left(p_\theta + g\hat{K}_{y'}\right)^2 + \frac{1}{2C_0}\left(\frac{p_\phi}{\sin\theta} - \cot\theta \hat{K}_{z'} - g\hat{K}_{x'}\right)^2$$

Key: angular momenta

$$\mathbf{I}_{z'} = \mathbf{e}_{z'}\hat{K}_{z'} \qquad \mathbf{I}_\perp = p_\theta \mathbf{e}_{y'} - \left(\frac{p_\phi}{\sin\theta} - \cot\theta\hat{K}_{z'}\right)\mathbf{e}_{x'}$$

Rewrite Hamiltonian (combines particle-rotor and Nilsson models)

$$H = \int d^3\mathbf{x}' \hat{\Psi}^\dagger(\mathbf{x}') \left(-\frac{\hbar^2\Delta'}{2m} + V\right) \hat{\Psi}(\mathbf{x}') + \frac{g^2}{2C_0}\left(\hat{K}_{x'}^2 + \hat{K}_{y'}^2\right)$$

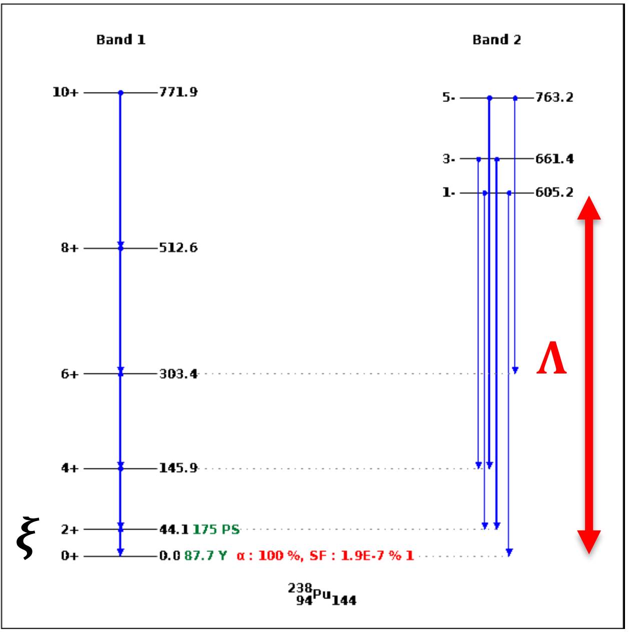
$$+\frac{\mathbf{I}^2 - \hat{K}_{z'}^2}{2C_0} + \frac{g}{C_0}\left(I_{x'}\hat{K}_{x'} + I_{y'}\hat{K}_{y'}\right)$$

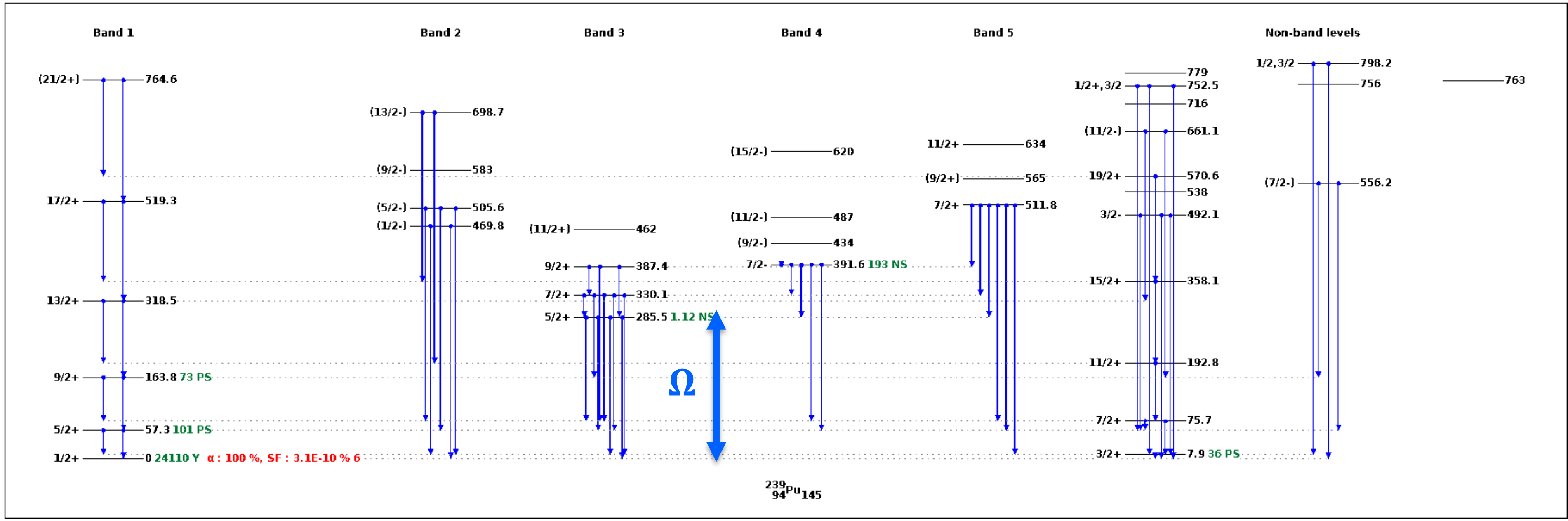
Solution is standard (e.g. via Nilsson orbitals)

$$E(I) = -S_n - \frac{|K|}{2C_0} + \frac{I(I+1) - K^2}{2C_0} \qquad \Delta E(g) = -\frac{g}{C_0}\delta^{\frac{1}{2}}_{|K|}(-1)^{I+\frac{1}{2}}\left(I + \frac{1}{2}\right)$$

Q: How to turn this into an EFT?

A: **Introduce a power counting**! Guided by heavy rotors ^{238,239}Pu (shown are all states below 800 keV).





Rotational low energy scale in ^{238}Pu set by 2^+ state at $\xi \approx 44$ keV.

Breakdown scale in ^{238}Pu set by 1^- band head at $\Lambda \approx 606$ keV.

Fermionic single-particle energy in ^{239}Pu set by $5/2^-$ band head at $\Omega \approx 285$ keV.

Separation of scales $\xi \ll \Omega \ll \Lambda$ (last inequality only marginally fulfilled)

Subleading corrections

Leading-order LECs scale as $C_0 \sim \xi^{-1}$, $S_n \sim \Omega$, $g \sim O(1)$; also have $\mathbf{v} \sim \xi$.

$$E(I) = -S_n - \frac{|K|}{2C_0} + \frac{I(I+1) - K^2}{2C_0} \qquad \Delta E(g) = -\frac{g}{C_0}\delta^{\frac{1}{2}}_{|K|}(-1)^{I+\frac{1}{2}}\left(I + \frac{1}{2}\right)$$

Subleading corrections: terms quadratic in **v** and $\widehat{K}$. Independent terms are

$$L_{1\mathrm{a}} = \frac{g_a}{2}\mathbf{v}^2\left(K^2_{x'} + K^2_{y'}\right) \ ,$$
$$L_{1\mathrm{b}} = \frac{g_b}{2}\mathbf{v}^2 K^2_{z'} \ ,$$
$$L_{1\mathrm{c}} = \frac{g_c}{2}\left(\mathbf{v}\cdot\mathbf{K}\right)^2 \ .$$

Power counting implies that $g_{a,b,c} \sim \Lambda^{-1}$ and $L_{1a,1b,1c} \sim \frac{\xi^2}{\Lambda} \ll E(I), \ \Delta E(g)$.
These corrections will make the moment of inertia dependent on the band head. Corrections are suppressed by ξ/Λ, compared to $\left(\frac{\xi}{\Lambda}\right)^2$ for even-even nuclei [Jiang Zhang & TP 2013].

Summary

Effective field theory for odd-mass deformed nuclei

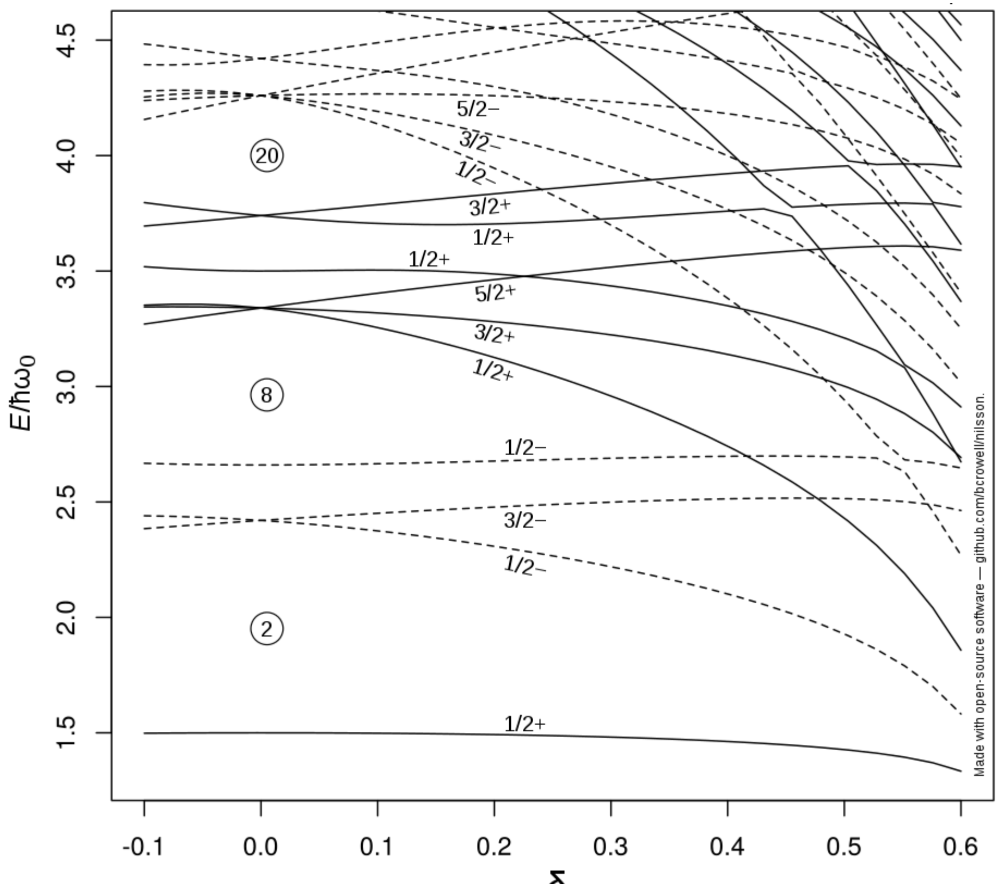
1. Velocity-dependent terms can be viewed as gauge potentials that induce geometric (Berry) phases.
2. Abelian gauge potentials due to the geometry of the sphere; gauge transformations relate different (ambiguous) body-fixed frames.
3. Non-Abelian gauge potential due to Kramer's degeneracy of the intrinsic nucleon degree of freedom.
4. At leading order, the EFT combines the particle-rotor model and the Nilsson model.
5. Next-to-leading-order corrections account for variations in moments of inertia for different rotational bands.

When dropped from height, odd-mass deformed nuclei would land on their feet!

Traditional view in nuclear physics

Internal dynamics / shape space: Nilsson model (axially sym. nucleus)

$$H = \frac{1}{2}m\omega_z^2 z^2 + \frac{1}{2}m\omega_\perp^2(x^2+y^2) - c_1\ell\cdot s - c_2(\ell^2 - \langle\ell^2\rangle_N).$$



Traditional view in nuclear physics

Overall dynamics: Particle-rotor model

$$H = \sum_{k=1}^{2} \frac{(I_k - j_k)^2}{2C_0} = \frac{1}{2C_0}(I^2 - I_3^2) - \frac{1}{2C_0}(I_+ j_- \pm I_- j_+) + \frac{1}{2C_0}(j^2 - j_3^2)$$

I = total angular momentum
j = angular momentum of particle in a Nilsson orbital

First Equation: The overall system is a rigid rotor, and the rotor's angular momentum is equal to the difference between total and the particle's angular momentum.

Second equation: Expand out.