Falling Cats and EFTs for Deformed Nuclei

From APS viewpoint

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TRIUMF Ab initio workshop, March 3–6, 2020
Work supported by the US Department of Energy
**Falling Cat Problem**

**Q:** How does a cat change its orientation, i.e. its angular momentum, without an external torque?

**A:** Changes in its shape (intrinsic degrees of freedom) induce a change in the external orientation.

**Q:** What does this have to do with odd-mass deformed nuclei?

**A:** In both cases, non-Abelian gauge potentials arise that describe the internal dynamics and couple it to the overall orientation. (In the nucleus, the odd nucleon causes the internal dynamics.)

→ **Gauge theory of deformable bodies**

6. Some Specific Reorientations and Steering Strategies

6.1. A Cartoon. Probably the simplest path resulting in the cat flip is the one depicted below.

“Bend, twist, unbend” makes a closed loop in internal configuration space while leading to a rotation.
Effective field theories in the physics of nuclei

**Rotors:** \( \frac{E(4^+)}{E(2^+)} = \frac{10}{3} \)

**Vibrators:** \( \frac{E(4^+)}{E(2^+)} = 2 \)

**Vibrators:** EFT based on linear (Wigner/Weyl) realization [Coello Pérez & TP 2015; 2016; Coello Pérez, Menéndez & Schwenk 2018]

**Rotors:** EFTs based on non-linear realization of SO(3)

Axially symmetric nuclei: [TP 2011; TP & Weidenmüller 2014; Coello Pérez & TP 2015]

Triaxial deformation: [Chen, Kaiser, Meißner, Meng 2017; 2018]
NNDC spectra: levels up to 12 MeV

\( ^{8}\text{Be} \):
- \( 0^+, 2^+, 4^+ \) levels form rotational band
  \( E(I) = A I(I + 1) \) with \( A \approx 0.505 \) MeV
- \( 2^+ \) state sets low-energy scale \( \xi \approx 3 \) MeV
- First non-rotational state (at 16.6 MeV) sets breakdown scale \( \Lambda = 16.6 \) MeV
- Small expansion parameter is \( \frac{\xi}{\Lambda} \approx 0.2 \)

\( ^{9}\text{Be} \):
- Spectrum looks complicated ...
- However: All but one state below 12 MeV explained by three rotational bands built on band heads with \( K = \frac{3}{2}^-, \frac{1}{2}^+, \frac{1}{2}^- \)
- \( \Delta E(I) = A \left[ I(I + 1) + a(-1)^{I+\frac{1}{2}} (I + \frac{1}{2}) \delta_{K} \right] \)

\( \rightarrow \) ab initio by Caprio et al., arXiv:1912.00008
Nucleon coupled to a deformed even-even nucleus

**Rotor:**
- Deformed, axially symmetric, even-even nucleus
- Symmetry axis defines body-fixed z’ axis
- Slow / heavy degree of freedom

**Nucleon:**
- Fast / light degree of freedom; “instantaneous” in the body-fixed frame
- Strongly coupled to the rotor
- Experiences axially symmetric forces in the body-fixed frame

\[ I = I_{\text{rot}} + K_{z'} \]
1. Emergent symmetry breaking from SO(3) to SO(2): Nambu-Goldstone modes parametrize the sphere.

2. Body-fixed frame ambiguous in axially symmetric nucleus \( \rightarrow \) “gauge” freedom

Degrees of freedom: \( \theta, \phi \)

\[
e_z'(\theta, \phi) = e_r(\theta, \phi) = \begin{pmatrix} \cos \phi \sin \theta \\ \sin \phi \sin \theta \\ \cos \theta \end{pmatrix}
\]

Angular velocity:

\[
v = \frac{de_r}{dt} = \dot{\theta} e_\theta + \dot{\phi} \sin \theta \ e_\phi
\]

Simplest (leading order) Lagrangian:

\[
L = \frac{c_0}{2} v \cdot v = \frac{c_0}{2} (\dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta)
\]

Hamiltonian:

\[
H = \frac{\tilde{J}^2}{2c_0}
\]
Nucleon is two-component spinor \( \Psi(x') = \begin{pmatrix} \Psi_{+1/2}(x') \\ \Psi_{-1/2}(x') \end{pmatrix} \), components along the z' axis.

Total nucleon spin \( \hat{K} = \int d^3x' \hat{\Psi}^\dagger(x') \left( -i x' \times \nabla' + \hat{S} \right) \hat{\Psi}(x') \)

Coupling between the nucleon and the rotor via two mechanisms.

1. **Axially symmetric potential \( V \) in body-fixed frame**
   a. Lagrangian
   \[
   L_{\Psi} = \int d^3x' \hat{\Psi}^\dagger(x') \left( i \partial_t + \frac{\hbar^2 \Delta'}{2m} - V \right) \hat{\Psi}(x')
   \]
   b. All nucleon states come in pairs \( |\pm K, \alpha \rangle \) related by time-reversal invariance (Kramer's degeneracy); we have \( \hat{K}_{z'} |\pm K, \alpha \rangle = \pm K |\pm K, \alpha \rangle \)

2. **Gauge couplings**, i.e. couplings to the rotor's angular velocity
**Kinematics:** Assume that the rotor’s symmetry axis moves along a closed path C.

This induces a rotation of the body-fixed frame by an angle $\gamma$ around the $z'$ axis. This is a geometric effect.

Rotation of nucleon in the body-fixed system $\Psi(x') \rightarrow e^{-i\gamma \hat{f}_{z'}} \Psi(x')$ introduces a Berry phase [Berry 1984] in its wave function.

The angle $\gamma$ is identical to the solid angle of the enclosed area.

**Dynamics:** A magnetic monopole field changes the phase by an angle that is identical to the flux (area!) [Wu & Yang 1976]

Vector potential of monopole: $A_a(\theta, \phi) \equiv e_\phi \cot \theta \hat{K}_{z'}$

Corresponding “magnetic” field: $B_a(\theta, \phi) \equiv \nabla \times A_a = -e_r \hat{K}_{z'}$

No parameter in singular gauge potential (Dirac’s quant. cond.)

_sphere = shape space; closed path in shape space induces rotation._
A non-Abelian gauge coupling also possible

Fermion states come in pairs (Kramer’s degeneracy).

**Kinematics:** As the rotor’s symmetry axis moves along a closed path on the sphere, the degenerate nucleon states might mix. Original and final states differ by a unitary transformation: \( \Psi(x') \rightarrow \hat{U} \Psi(x') \)

**Dynamics:** A non-Abelian gauge potential causes such a non-Abelian Berry phase [Wilczek & Zee 1984]. (The amount of SU(2) rotation is not quantized, the gauge potential is not singular, and has an adjustable dimensionless parameter \( g \).)

Non-Abelian gauge potential: \( \mathbf{A}_n(\theta, \phi) = g \mathbf{e}_r \times \hat{\mathbf{K}} = g \left( \mathbf{e}_\phi \hat{K}_x' - \mathbf{e}_\theta \hat{K}_y' \right) \)

Corresponding “magnetic” field: \( \mathbf{B}_n(\theta, \phi) \equiv \nabla \times \mathbf{A}_n - i \mathbf{A}_n \times \mathbf{A}_n = g^2 \mathbf{e}_r \hat{K}_z' \).
Gauge potentials, Berry phases, and Coriolis forces

Different interpretations of the velocity–dependent rotor–nucleon couplings

1. **Coriolis forces** enter in rotating frames: Velocity-dependent forces are present in rotating nuclei [Bohr, Kerman, Mottelson, Nilsson 1950s].

2. **Molecular Aharonov-Bohm effect**: In rotating molecules, the nuclei are slow (and the electrons are fast), and the adiabatic decoupling (à la Born Oppenheimer) introduces Berry phases and gauge potentials [Mead & Truhlar 1979; Wilczek & Zee 1984; Kuratsuji & Iida 1985].

3. **Covariant derivative**: In presence of spontaneous symmetry breaking, the rotational symmetry is realized non-linearly for the rotor’s degrees of freedom. This introduces a covariant derivative $iD \equiv i\partial_t + \mathbf{v} \cdot \mathbf{A}$ [Weinberg 1968; Callan, Coleman, Wess & Zumino 1969].

4. **Gauge invariance**: The ambiguities in defining a body-fixed frame, i.e. separating rotational and intrinsic degrees of freedom, imply a gauge invariance [Littlejohn & Reinsch 1997]. Our case: ambiguities regarding rotations around the z’ axis.

[Leutwyler 1994; Roman & Soto 1999; Hofmann 1999; Chandrasekharan et al. 2008; Brauner 2010; TP 2011, TP & Weidenmüller 2014; ... ]
Let us start with the most general Lagrangian quadratic in the velocities:

\[
L = \frac{C_0}{2} \mathbf{v}^2 + \mathbf{v} \cdot (\mathbf{A}_a + \mathbf{A}_n) + L_\Psi
\]

\[
= \frac{C_0}{2} \left( \dot{\varphi}^2 + \dot{\phi}^2 \sin^2 \theta \right) + g \left( \dot{\phi} \sin \theta \hat{K}_{x'} - \dot{\theta} \hat{K}_{y'} \right) + \dot{\phi} \cos \theta \hat{K}_{z'} + L_\Psi
\]

The nucleon part is

\[
L_\Psi = \int d^3x' \hat{\Psi}^\dagger(x') \left( i \partial_t + \frac{\hbar^2 \Delta'}{2m} - V \right) \hat{\Psi}(x')
\]

Low energy constants are \(C_0\), \(g\), and parameters of the axially symmetric potential \(V\).
Hamiltonian
\[ H = \int d^3x' \hat{\Psi}^\dagger(x') \left( -\frac{\hbar^2 \Delta'}{2m} + V \right) \hat{\Psi}(x') \]
\[ + \frac{1}{2C_0} \left( p_\theta + g \hat{K}_{y'} \right)^2 + \frac{1}{2C_0} \left( \frac{p_\phi}{\sin \theta} - \cot \theta \hat{K}_{z'} - g \hat{K}_{x'} \right)^2 \]

Key: angular momenta
\[ I_{z'} = e_{z'} \hat{K}_{z'} \quad I_\perp = p_\theta e_{y'} - \left( \frac{p_\phi}{\sin \theta} - \cot \theta \hat{K}_{z'} \right) e_{x'} \]

Rewrite Hamiltonian (combines particle-rotor and Nilsson models)
\[ H = \int d^3x' \hat{\Psi}^\dagger(x') \left( -\frac{\hbar^2 \Delta'}{2m} + V \right) \hat{\Psi}(x') + \frac{g^2}{2C_0} \left( \hat{K}_{x'}^2 + \hat{K}_{y'}^2 \right) \]
\[ + \frac{I^2 - \hat{K}_{z'}^2}{2C_0} + \frac{g}{C_0} \left( I_{x'} \hat{K}_{x'} + I_{y'} \hat{K}_{y'} \right) \]

Solution is standard (e.g. via Nilsson orbitals)
\[ E(I) = -S_n - \frac{|K|}{2C_0} + \frac{I(I + 1) - K^2}{2C_0} \quad \Delta E(g) = -\frac{g}{C_0} \delta_{|K|}^{\frac{1}{2}} (-1)^{I + \frac{1}{2}} \left( I + \frac{1}{2} \right) \]
Q: How to turn this into an EFT?

A: Introduce a power counting! Guided by heavy rotors $^{238,239}$Pu (shown are all states below 800 keV).

Rotational low energy scale in $^{238}$Pu set by $2^+$ state at $\xi \approx 44$ keV.
Breakdown scale in $^{238}$Pu set by $1^-$ band head at $\Lambda \approx 606$ keV.
Fermionic single-particle energy in $^{239}$Pu set by $5/2^-$ band head at $\Omega \approx 285$ keV.

Separation of scales $\xi \ll \Omega \ll \Lambda$ (last inequality only marginally fulfilled)
Subleading corrections

**Leading-order** LECs scale as $C_0 \sim \xi^{-1}$, $S_n \sim \Omega$, $g \sim O(1)$; also have $v \sim \xi$.

$$E(I) = -S_n - \frac{|K|}{2C_0} + \frac{I(I+1) - K^2}{2C_0} \quad \Delta E(g) = -\frac{g}{C_0} \delta \left( K \right)^\frac{1}{2} (-1)^I \left( I + \frac{1}{2} \right)$$

**Subleading corrections**: terms quadratic in $v$ and $\mathbf{K}$. Independent terms are

$$L_{1a} = \frac{g_a}{2} v^2 \left( K_{x'}^2 + K_{y'}^2 \right)$$

$$L_{1b} = \frac{g_b}{2} v^2 K_{z'}^2$$

$$L_{1c} = \frac{g_c}{2} (v \cdot K)^2$$

Power counting implies that $g_{a,b,c} \sim \Lambda^{-1}$ and $L_{1a,1b,1c} \sim \frac{\xi^2}{\Lambda} \ll E(I), \Delta E(g)$. These corrections will make the moment of inertia dependent on the band head. Corrections are suppressed by $\xi / \Lambda$, compared to $\left( \frac{\xi}{\Lambda} \right)^2$ for even-even nuclei [Jiang Zhang & TP 2013].
Effective field theory for odd-mass deformed nuclei

1. Velocity-dependent terms can be viewed as gauge potentials that induce geometric (Berry) phases.

2. Abelian gauge potentials due to the geometry of the sphere; gauge transformations relate different (ambiguous) body-fixed frames.

3. Non-Abelian gauge potential due to Kramer’s degeneracy of the intrinsic nucleon degree of freedom.

4. At leading order, the EFT combines the particle-rotor model and the Nilsson model.

5. Next-to-leading-order corrections account for variations in moments of inertia for different rotational bands.

When dropped from height, odd-mass deformed nuclei would land on their feet!
Traditional view in nuclear physics

Internal dynamics / shape space: Nilsson model (axially sym. nucleus)

\[ H = \frac{1}{2} m \omega_z^2 z^2 + \frac{1}{2} m \omega_{\perp}^2 (x^2 + y^2) - c_1 \ell \cdot s - c_2 (\ell^2 - \langle \ell^2 \rangle_N). \]
Traditional view in nuclear physics

Overall dynamics: Particle-rotor model

\[ H = \sum_{k=1}^{2} \frac{(I_k - j_k)^2}{2C_0} = \frac{1}{2C_0} (I^2 - I_3^2) - \frac{1}{2C_0} (I_+j_- \pm I_-j_+) + \frac{1}{2C_0} (j^2 - j_3^2) \]

I = total angular momentum
j = angular momentum of particle in a Nilsson orbital

First Equation: The overall system is a rigid rotor, and the rotor’s angular momentum is equal to the difference between total and the particle’s angular momentum.

Second equation: Expand out.