Ab initio treatment of collective correlations in neutrinoless double beta decay

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What’s neutrinoless double beta decay?

- At nuclear-structure level, it corresponds to the transition

\[ ^A Z \rightarrow ^A (Z + 2) + 2e^- \]

V. Cirigliano+ (2020)

Schechter-Valle theorem (1982):
any diagram causing the $0\nu\beta\beta$ decay will generate a Majorana mass term for light neutrinos

1. Beyond SM physics: nonzero neutrino mass
2. Nature of neutrinos: Dirac or Majorana
3. Origin of the matter-antimatter asymmetry: Lepton-number violation
What kind of nuclei to observe the $0\nu\beta\beta$?

- Single-beta decay is energetically forbidden
- Experimental interest
  1. Large $Q_{\beta\beta}$ value
  2. Large isotopic abundance
  3. Low background in the energy region of interest

Features of candidate nuclei

The nuclei evolved in the $0\nu\beta\beta$ are mostly medium-mass open-shell (deformed) nuclei.
Contribution from the short-range operator

Current status on the studies of $0\nu\beta\beta$ decay

Based on the mechanism of exchange light Majorana neutrino, the inverse of half-life of $0\nu\beta\beta$ can be factorized as

$$[T_{1/2}^{0\nu}]^{-1} = g_A^4 G_{0\nu} \left| \frac{\langle m_{\beta\beta} \rangle}{m_e} \right|^2 \left| M^{0\nu} \right|^2, \quad \langle m_{\beta\beta} \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

A precise knowledge (from ab initio calculation) of the nuclear matrix element (NME) $M^{0\nu}$ is helpful to determine the neutrino effective mass $\langle m_{\beta\beta} \rangle$, if the process is measured eventually.
The NME for the $0\nu\beta\beta$ transition from $|0^+_I\rangle$ to $|0^+_F\rangle$

$$M^{0\nu}(0^+_I \rightarrow 0^+_F) = \langle 0^+_F | O^{0\nu} | 0^+_I \rangle$$

- the transition operator: exchange of light neutrinos and with closure approximation

$$O^{0\nu} = \frac{4\pi R}{g_A^2} \int d^3 \vec{r}_1 \int d^3 \vec{r}_2 \int d^3 \vec{q} \frac{e^{i\vec{q} \cdot (\vec{r}_1 - \vec{r}_2)}}{(2\pi)^3 q(q + E_d)} \mathcal{J}_\mu^\dagger(\vec{r}_1) \mathcal{J}_\mu^\dagger(\vec{r}_2)$$

$$= \sum_K \sum_{1,2} H_K(r_{12}, E_d) \tau_1^+ \tau_2^+ S_K$$

(1)

where $S_K = \{1, \sigma_1 \cdot \sigma_2, 3(\sigma_1 \cdot \hat{r}_{12})(\sigma_2 \cdot \hat{r}_{12}) - \sigma_1 \cdot \sigma_2\}$ for $K = \{F, GT, T\}$, respectively. The average excitation energy $E_d = \bar{E} - (E_i + E_f)/2 \sim 1.12A^{1/2}$. Only one-body current $\mathcal{J}_\mu$ is taken into account in the present study.
The wave functions of initial and final nuclei require the calculation from ab initio methods:

1. starts from a bare nucleon-nucleon interaction (fitted to data of NN scattering/few-body systems)
2. solves Schroedinger equation (for the many-body system) with a controllable accuracy of approximations

Benchmark calculations in light nuclei:

- Variational Monte Carlo calculation starting from the Argonne v18 two-nucleon potential and Illinois-7 three-nucleon interaction for light nuclei
  S. Pastore et al. (2017)
- No-core shell model calculations starting from chiral NN+3N interactions for light nuclei
  P. Gysbers et al., R. A. Basili et al. (2019)

Extension to medium-mass candidate nuclei:

- Application of coupled-cluster (S. Novario, G. Hagen, T. Papenbrock et al.) and valence-space in-medium similarity renormalization group (IMSRG) (Antoine Belley, R. Stroberg, J. Holt et al.) method starting from chiral NN+3N interactions for $0 \bar{\nu} \beta \beta$-candidate nuclei
- Merging the multi-reference IMSRG with generator coordinate method (GCM) starting from chiral NN+3N interactions for $0 \bar{\nu} \beta \beta$-candidate nuclei
Contribution from the short-range operator

The method: basic idea of IMSRG

- A set of continuous unitary transformations onto the Hamiltonian

\[ H(s) = U(s)H_0U^\dagger(s) \]

- Flow equation for the Hamiltonian

\[ \frac{dH(s)}{ds} = [\eta(s), H(s)] \]

where the \( \eta(s) = \frac{dU(s)}{ds} U^\dagger(s) \) is the so-called generator chosen to decouple a given reference state from its excitations.

- Computation complexity scales polynomially with nuclear size

Tsukiyama, Bogner, and Schwenk (2011)
Hergert, Bogner, Morris, Schwenk, Tsukiyama (2016)

Not necessary to construct the H matrix elements in many-body basis!
open-shell nuclei with collective correlations: mp-mh excitation configurations

- different unitary transformation for the initial and final nuclei: $U_I(s) \neq U_F(s)$. Computation of the following matrix element

$$M^{0\nu} = \langle \Phi_F | U_F(s) O^{0\nu} U_I^\dagger(s) | \Phi_I \rangle = \langle \Phi_F | e^{\Omega_F(s)} O^{0\nu} e^{-\Omega_I(s)} | \Phi_I \rangle$$

with truncation error controllable is challenge.

choose the reference state $|\Phi\rangle$ as an ensemble of the initial and final nuclei
The IMSRG+GCM method: procedure

- Generation of a reference state with collective correlations
  1. Constrained deformed HFB calculation with variation after particle-number projection
  2. Projection onto the right quantum numbers (NZ,J)
  3. Computing many-body density matrices of the reference state

- Normal-ordering all the operators with respect to the reference state and solve the IMSRG flow equation
  1. Ensemble normal-ordering (NO2B)
  2. Computing all the RG evolved operators

- Diagonalization of the evolved Hamiltonian with GCM
  1. Generate a set of non-orthogonal quantum-number projected HFB states with different coll. correlations
  2. Mixing of these states with GCM
  3. Computing observables with the GCM wave functions using the corresponding evolved operators
A benchmark of the method

- model space: \( pf \) shell
- KB3G interaction

Computation time for the many-body density matrices increases significantly while using chiral interactions in full model space.
Contribution from the short-range operator

Many-body density matrices in a small model space

Prescription

- Construct density matrix elements in a small model space (defined by $e_{\text{max}}$)
- Normal-order the $H$ and solve the IMSRG flow in a large model space ($e_{\text{Max}}$)
Contribution from the short-range operator

An illustrative calculation for deformed nuclei: $^8$Be

**HFB potential energy surface by the SRG softened chiral interaction EM1.8/2.0($\hbar\Omega = 16$ MeV)**

Starting from the reference state with two-$\alpha$ structure, the IMSRG(2) is converged to ground state.
Contribution from the short-range operator

Applications to neutrinoless double beta decay

Benchmark calculations of light nuclei:

1. transition between $\Delta T = 0$ states: $^6$He $\rightarrow$ $^6$Be, and $^{10}$Be $\rightarrow$ $^{10}$C
2. transition between $\Delta T = 2$ states: $^8$He $\rightarrow$ $^8$Be, and $^{22}$O $\rightarrow$ $^{22}$Ne

Application to candidate $0\nu\beta\beta$ process ($\Delta T = 2$):

1. $^{48}$Ca $\rightarrow$ $^{48}$Ti
2. $^{76}$Ge $\rightarrow$ $^{76}$Se
0$\nu\beta\beta$ from $^6$He and $^6$Be ($\Delta T = 0$)

- SRG softened two-body NN interaction: EM2.0/500
- make use of isospin symmetry in the wave functions of initial and final nuclei

Contribution from the short-range operator $\nu\beta\beta$ from $^6$He and $^6$Be ($\Delta T = 0$)

- Chiral 2N+3N interaction (EM1.8/2.0), isospin symmetry is NOT assumed in the wfs

- $M^{0\nu}$ is weakly sensitive to the shapes/deformations of concerned

- $M^{0\nu}(GT/F/TE) = 3.18/0.88/-0.05$

- VMC (AV18+IL17): $M^{0\nu}(GT/F/TE) = 3.688/0.946/-0.025$ [S. Pastore+(2018)]
  
  discrepancy contributed from both wfs and transition operators.
Contribution from the short-range operator

\[ M^{0\nu} = \int_0^{\infty} C^{0\nu}(r_{12})dr_{12} \]

Note: A factor of \(-g_A^2\) has been multiplied into the Fermi part.

S. Pastore et al (2018)
Summary of the NMEs in light nuclei

- Consistent with the findings in the "exact" calculations with VMC (AV18+IL17) S. Pastore et al (2018); X.B. Wang (2019) and NCSM (EM1.8/2.0) P. Gysbers et al.
Application: $0\nu\beta\beta$ from $^{48}\text{Ca}$ to $^{48}\text{Ti}$

Extrapolation

$$E(e_{\text{Max}}) = E(\infty) + a \exp(-b \cdot e_{\text{Max}})$$
Application: $0\nu\beta\beta$ from $^{48}$Ca to $^{48}$Ti

- IMSRG+GCM: Low-energy structure of $^{48}$Ti is reasonably reproduced (spectrum stretched). Inclusion of non-collective configurations from neutron-proton isoscalar pairing fluctuation can compress the spectra further by about 6%.

- IMSRG+CI(T0 → T1): the spectrum becomes more stretched in a larger model space (more collective correlations).
Application: $0\nu\beta\beta$ from $^{48}\text{Ca}$ to $^{48}\text{Ti}$

Contribution from the short-range operator

Application: $0\nu\beta\beta$ from $^{48}\text{Ca}$ to $^{48}\text{Ti}$

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Ab initio calculation of $0\nu\beta\beta$
Contribution from the short-range operator

Application: $0\nu\beta\beta$ from $^{48}\text{Ca}$ to $^{48}\text{Ti}$

\[ M^{0\nu} = \int dr_{12} C^{0\nu}(r_{12}) \]

- The quadrupole deformation in $^{48}\text{Ti}$ changes both the short and long-range behaviors.
- Neutron-proton isoscalar pairing is mainly a short-range effect.
Contribution from the short-range operator

Application: $0\nu\beta\beta$ from $^{48}$Ca to $^{48}$Ti

The value from Markov-chain Monte-Carlo extrapolation is $M^{0\nu} = 0.61^{+0.05}_{-0.04}$.

The neutron-proton isoscalar pairing fluctuation quenches $\sim 17\%$ further, which might be canceled out partially by the isovector pairing fluctuation.
Contribution from the short-range operator

$0\nu\beta\beta$ from $^{76}\text{Ge}$ to $^{76}\text{Se}$ (preliminary results)

The above $B(E2 : 2^+ \rightarrow 0^+)$ values are evaluated with the evolved one-body E2 operator only.
$0^{\nu}\beta\beta$ from $^{76}$Ge to $^{76}$Se (preliminary results)

**Contribution from the short-range operator**

$0^{\nu}\nu\beta$ from $^{76}$Ge to $^{76}$Se

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**Renormalization effect on the transition operator:**

1. The renormalization effect is mainly contributed from $\Omega^{(2)}$
2. The pp/hh diagrams enhance the NME (GT)

$$
\sum_{ab} O^{0\nu}_{pp',ab} \Omega_{abnn'} (1 - n_a - n_b) + (\Omega \leftrightarrow O^{0\nu})
$$

while the ph diagrams quench the NME.

enhances the transition from $^{48}$Ca to $^{48}$Ti and quenches that from $^{76}$Ge to $^{76}$Se.
The unpublished results of VS-IMSRG are from (A. Belley, R. Stroberg, J. Holt et al.)

Uncertainties from different sources (model truncation, chiral expansion, contact operator, two-body currents) are to be included.
Summary and outlook

- The mass ordering of neutrinos is expected to be disclosed with the development of ton-scale $0\nu\beta\beta$ decay experiments in the next few years, depending on the values of the NMEs.

- The NMEs governing the $0\nu\beta\beta$ are essential to determine the neutrino effective mass. Several ab initio methods have begun to calculate the NMEs starting from first principles.

- We develop a novel multi-reference framework of IMSRG+GCM which opens a door to modeling deformed nuclei with realistic nuclear forces.

- The NMEs for several unphysical process in light nuclei and those for candidate process in medium-mass nuclei $^{48}$Ca and $^{76}$Ge (preliminary) are calculated. The NMEs in both cases are smaller than the predictions by (most) phenomenological models.

- More benchmarks among different ab initio calculations for the NMEs are underway.

- Quantification of uncertainties from different sources: systematic and statistic (open for comments/suggestions)
Contribution from the short-range operator

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