Ab initio treatment of collective correlations in neutrinoless double beta decay

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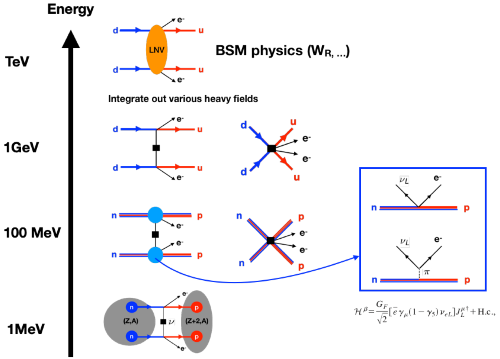
NUCLEI
Nuclear Computational Low-Energy Initiative
A SciDAC-4 Project

Progress in Ab Initio Techniques in Nuclear Physics, TRIUMF, Canada, March 6, 2020

What's neutrinoless double beta decay?

- At nuclear-structure level, it corresponds to the transition

$$^{A}Z \rightarrow ^{A}(Z+2) + 2e^{-}$$



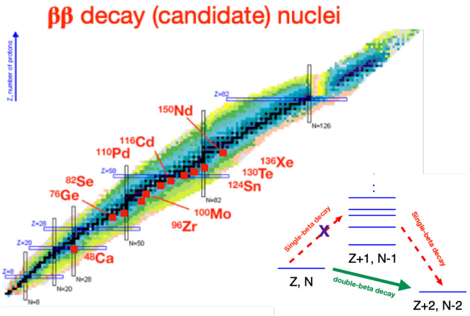
V. Cirigliano+ (2020)

Schechter-Valle theorem (1982): any diagram causing the $0\nu\beta\beta$ decay will generate a Majorana mass term for light neutrinos

1. Beyond SM physics: nonzero neutrino mass
2. Nature of neutrinos: Dirac or Majorana
3. Origin of the matter-antimatter asymmetry: Lepton-number violation

What kind of nuclei to observe the $0\nu\beta\beta$?

- Single-beta decay is energetically forbidden
- Experimental interest
  1. Large $Q_{\beta\beta}$ value
  2. Large isotopic abundance
  3. Low background in the energy region of interest



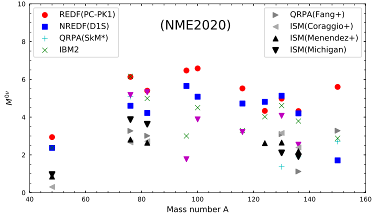
Features of candidate nuclei

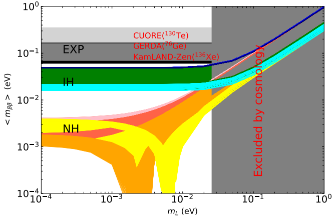
The nuclei evolved in the $0\nu\beta\beta$ are mostly medium-mass open-shell (deformed) nuclei.

Current status on the studies of $0\nu\beta\beta$ decay

Based on the mechanism of exchange light Majorana neutrino, the inverse of half-life of $0\nu\beta\beta$ can be factorized as

$$[T^{0\nu}_{1/2}]^{-1} = g_A^4 G_{0\nu} \left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2 \left|M^{0\nu}\right|^2, \quad \langle m_{\beta\beta}\rangle = |\sum_{i=1,2,3} U^2_{ei} m_i|$$



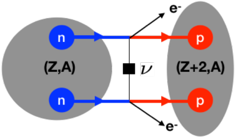


A precise knowledge (from ab initio calculation) of the nuclear matrix element (NME) $M^{0\nu}$ is helpful to determine the neutrino effective mass $\langle m_{\beta\beta}\rangle$, if the process is measured eventually.

Nuclear matrix element for the $0\nu\beta\beta$ decay

- The NME for the $0\nu\beta\beta$ transition from $|0_I^+\rangle$ to $|0_F^+\rangle$

$$M^{0\nu}(0_I^+ \to 0_F^+) = \langle 0_F^+ | O^{0\nu} | 0_I^+ \rangle$$



  – the transition operator: exchange of light neutrinos and with closure approximation

$$\begin{aligned} O^{0\nu} &= \frac{4\pi R}{g_A^2} \int d^3\vec{r}_1 \int d^3\vec{r}_2 \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}}{q(q+E_d)} \mathcal{J}_\mu^\dagger(\vec{r}_1)\mathcal{J}^{\mu\dagger}(\vec{r}_2) \\ &= \sum_K \sum_{1,2} H_K(r_{12}, E_d)\tau_1^+\tau_2^+ S_K \end{aligned} \quad (1)$$

where $S_K = \{1, \sigma_1 \cdot \sigma_2, 3(\sigma_1 \cdot \hat{r}_{12})(\sigma_2 \cdot \hat{r}_{12}) - \sigma_1 \cdot \sigma_2\}$ for $K = \{F, GT, T\}$, respectively. The average excitation energy $E_d = \bar{E} - (E_i + E_f)/2 \sim 1.12A^{1/2}$.
Only one-body current $\mathcal{J}^\mu$ is taken into account in the present study.

ab initio calculations of nuclear structure

– The wave functions of initial and final nuclei require the calculation from ab initio methods:
  1. starts from a bare nucleon-nucleon interaction (fitted to data of *NN* scattering/few-body systems)
  2. solves Schroedinger equation (for the many-body system) with a controllable accuracy of approximations

- Benchmark calculations in light nuclei:

√ Variational Monte Carlo calculation starting from the Argonne v18 two-nucleon potential and Illinois-7 three-nucleon interaction for light nuclei
S. Pastore et al. (2017)

√ No-core shell model calculations starting from chiral NN+3N interactions for light nuclei P. Gysbers et al., R. A. Basili et al. (2019)

- Extension to medium-mass candidate nuclei:

√ Application of coupled-cluster (S. Novario, G. Hagen, T. Papenbrock et al.) and valence-space in-medium similarity renormalization group (IMSRG) (Antoine Belley, R. Stroberg, J. Holt et al.) method starting from chiral NN+3N interactions for $0\nu\beta\beta$-candidate nuclei

√ Merging the multi-reference IMSRG with generator coordinate method (GCM) starting from chiral NN+3N interactions for $0\nu\beta\beta$-candidate nuclei
JMY, B. Bally, J. Engel, R. Wirth, T. R. Rodríguez, H. Hergert, arXiv:1908.05424

The method: basic idea of IMSRG

- A set of continuous unitary transformations onto the Hamiltonian

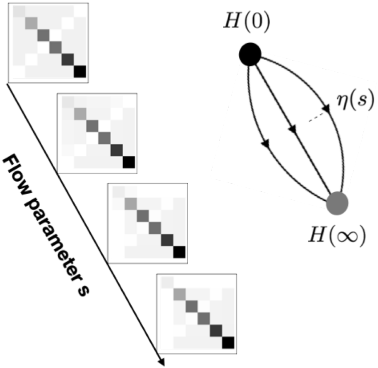
$$H(s) = U(s) H_0 U^\dagger(s)$$

- Flow equation for the Hamiltonian

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

where the $\eta(s) = \frac{dU(s)}{ds} U^\dagger(s)$ is the so-called generator chosen to decouple a given reference state from its excitations.

- Computation complexity scales polynomially with nuclear size

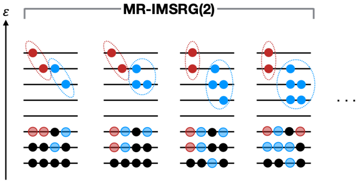


Tsukiyama, Bogner, and Schwenk (2011)
Hergert, Bogner, Morris, Schwenk, Tsukiyama (2016)

Not necessary to construct the H matrix elements in many-body basis !

Extension of the IMSRG for the NMEs of $0\nu\beta\beta$: challenges

- open-shell nuclei with collective correlations: mp-mh excitation configurations



MR-IMSRG: build correlations on top of **already correlated** state (e.g., from a method that describes static correlation well)

- different unitary transformation for the initial and final nuclei: $U_I(s) \neq U_F(s)$. Computation of the following matrix element

$$M^{0\nu} = \langle \Phi_F | U_F(s) O^{0\nu} U_I^{\dagger}(s) | \Phi_I \rangle = \langle \Phi_F | e^{\Omega_F(s)} O^{0\nu} e^{-\Omega_I(s)} | \Phi_I \rangle \qquad (2)$$

with truncation error controllable is challenge.

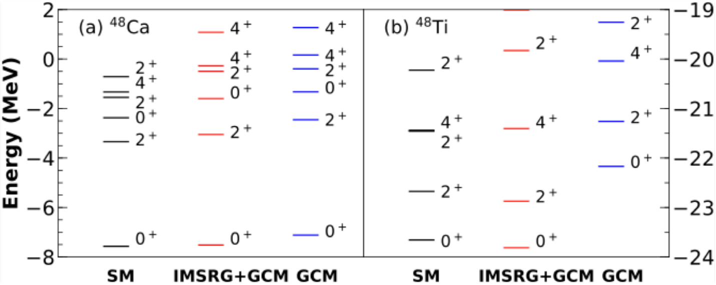
choose the reference state $|\Phi\rangle$ as an ensemble of the initial and final nuclei

The IMSRG+GCM method: procedure

- Generation of a reference state with collective correlations
  1. Constrained deformed HFB calculation with variation after particle-number projection
  2. projection onto the right quantum numbers (NZ,J)
  3. computing many-body density matrices of the reference state
- Normal-ordering all the operators with respect to the reference state and solve the IMSRG flow equation
  1. Ensemble normal-ordering (NO2B)
  2. Computing all the RG evolved operators
- Diagonalization of the evolved Hamiltonian with GCM
  1. Generate a set of non-orthogonal quantum-number projected HFB states with different coll. correlations
  2. mixing of these states with GCM
  3. Computing observables with the GCM wave functions using the corresponding evolved operators

A benchmark of the method

- model space: *pf* shell
- KB3G interaction



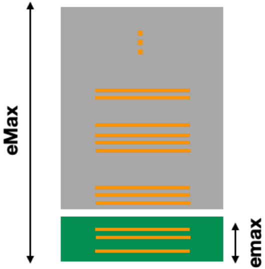
JMY, J. Engel, L.J. Wang, C.F. Jiao, H. Hergert (2018)

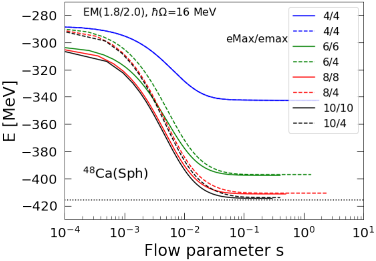
Computation time for the many-body density matrices increases significantly while using chiral interactions in full model space.

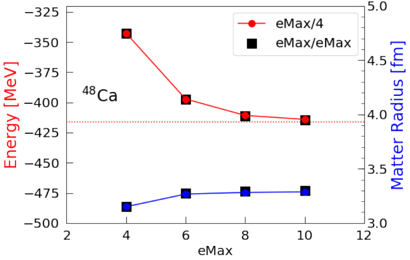
Many-body density matrices in a small model space

Prescription

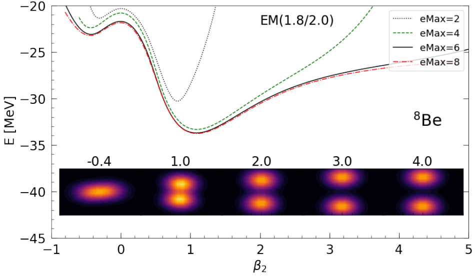
- Construct density matrix elements in a small model space (defined by emax)
- Normalize the H and solve the IMSRG flow in a large model space (eMax)

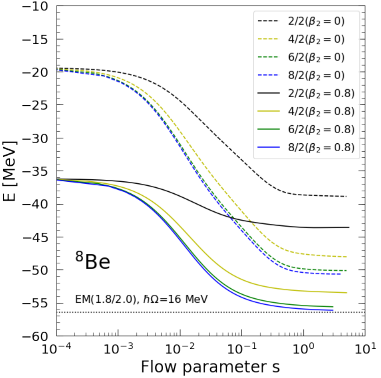






An illustrative calculation for deformed nuclei: ^{8}Be





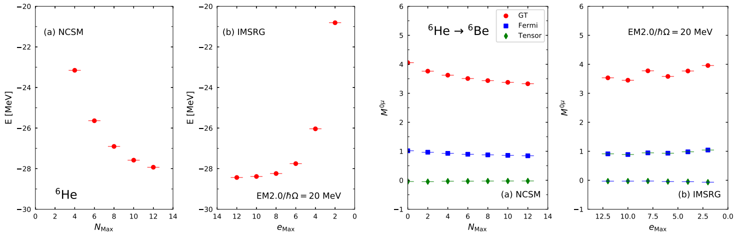
- HFB potential energy surface by the SRG softened chiral interaction EM1.8/2.0($\hbar\Omega = 16$ MeV)
- Starting from the reference state with two-α structure, the IMSRG(2) is converged to ground state.

Applications to neutrinoless double beta decay

- Benchmark calculations of light nuclei:
  1. transition between $\Delta T = 0$ states: ^{6}He $\rightarrow$ ^{6}Be, and ^{10}Be $\rightarrow$ ^{10}C
  2. transition between $\Delta T = 2$ states: ^{8}He $\rightarrow$ ^{8}Be, and ^{22}O $\rightarrow$ ^{22}Ne
- Application to candidate $0\nu\beta\beta$ process ($\Delta T = 2$):
  1. ^{48}Ca $\rightarrow$ ^{48}Ti
  2. ^{76}Ge $\rightarrow$ ^{76}Se

$0\nu\beta\beta$ from ^{6}He and ^{6}Be ($\Delta T = 0$)

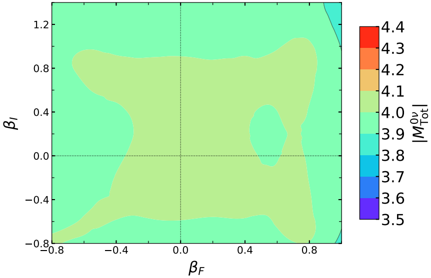
- SRG softened two-body NN interaction: EM2.0/500
- make use of isospin symmetry in the wave functions of initial and final nuclei

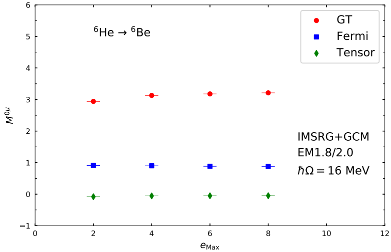


R.A. M Basili, JMY, J. Engel, H. Hergert, M. Lockner, P. Maris, J.P. Vary, arXiv:1909.06501

$0\nu\beta\beta$ from ^{6}He and ^{6}Be ($\Delta T = 0$)

- chiral 2N+3N interaction(EM1.8/2.0), isospin symmetry is NOT assumed in the wfs

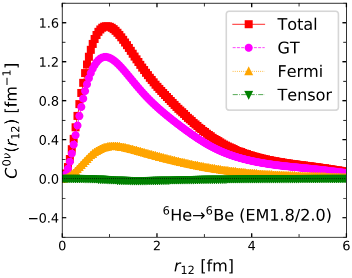




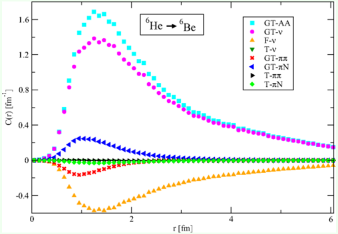
- $M^{0\nu}$ is weakly sensitive to the shapes/deformations of concerned
- $M^{0\nu}$(GT/F/TE)$=$ 3.18/0.88/-0.05
- VMC (AV18+IL17): $M^{0\nu}$(GT/F/TE)=3.688/0.946/-0.025 [S. Pastore+(2018)] discrepancy contributed from both wfs and transition operators.

$0\nu\beta\beta$ from ^{6}He and ^{6}Be ($\Delta T = 0$)

$$M^{0\nu} = \int_0^\infty C^{0\nu}(r_{12}) dr_{12}$$

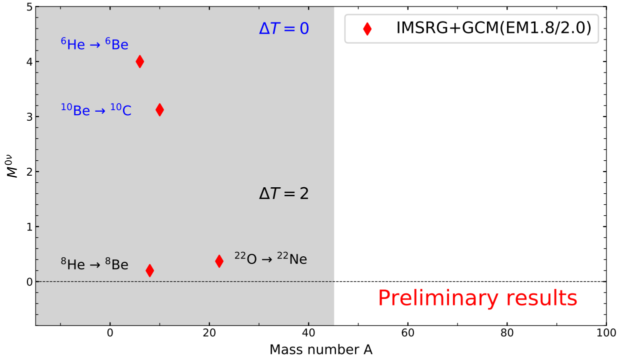


S. Pastore et al (2018)



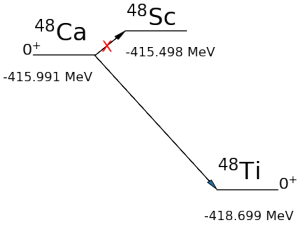
Note: A factor of $-g_A^2$ has been multiplied into the Fermi part.

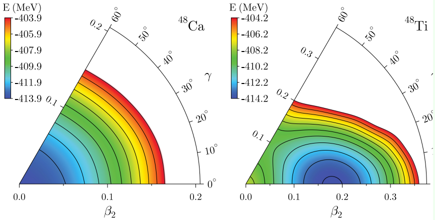
Summary of the NMEs in light nuclei



- consistent with the findings in the "exact" calculations with VMC (AV18+IL17) S. Pastore et al (2018); X.B. Wang (2019) and NCSM (EM1.8/2.0) P. Gysbers et al.

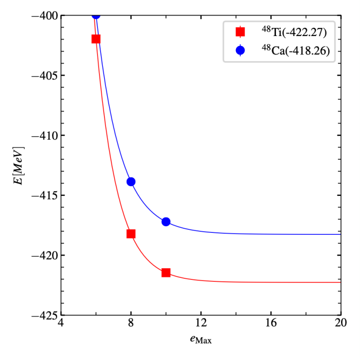
Application: $0\nu\beta\beta$ from ^{48}Ca to ^{48}Ti



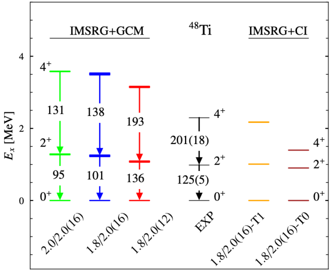


Extrapolation

$$E(e_{\rm Max}) = E(\infty) + a\exp(-b \cdot e_{\rm Max})$$

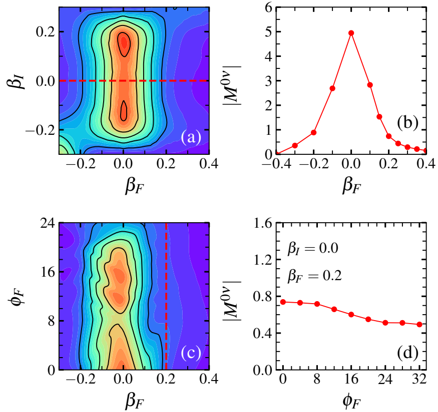


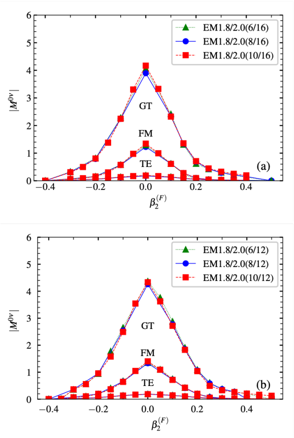
Application: $0\nu\beta\beta$ from ^{48}Ca to ^{48}Ti



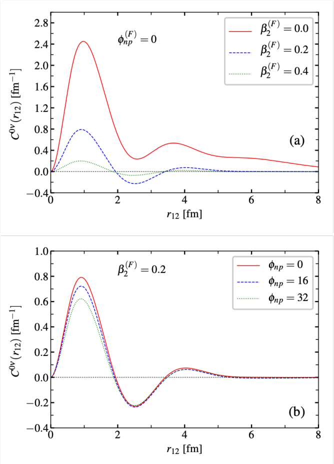
- IMSRG+GCM: Low-energy structure of ^{48}Ti is reasonably reproduced (spectrum stretched). Inclusion of non-collective configurations from neutron-proton isoscalar pairing fluctuation can compress the spectra further by about 6%.
- IMSRG+CI(T0 $\rightarrow$ T1): the spectrum becomes more stretched in a larger model space (more collective correlations).

Application: $0\nu\beta\beta$ from ^{48}Ca to ^{48}Ti





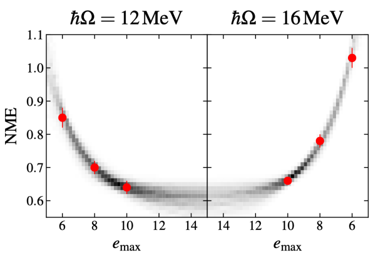
Application: $0\nu\beta\beta$ from ^{48}Ca to ^{48}Ti

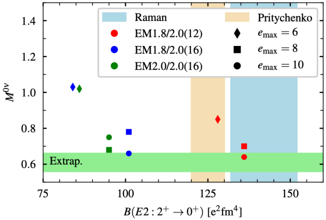


$$M^{0\nu} = \int dr_{12}\, C^{0\nu}(r_{12})$$

- The quadrupole deformation in ^{48}Ti changes both the short and long-range behaviors
- Neutron-proton isoscalar pairing is mainly a short-range effect

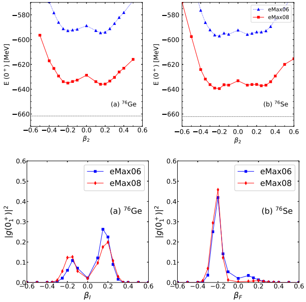
Application: $0\nu\beta\beta$ from ^{48}Ca to ^{48}Ti

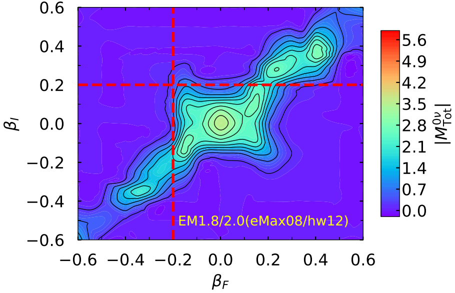


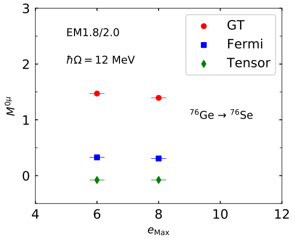


- The value from Markov-chain Monte-Carlo extrapolation is $M^{0\nu} = 0.61^{+0.05}_{-0.04}$
- The neutron-proton isoscalar pairing fluctuation quenches $\sim$17% further, which might be canceled out partially by the isovector pairing fluctuation.

$0\nu\beta\beta$ from ^{76}Ge to ^{76}Se (preliminary results)







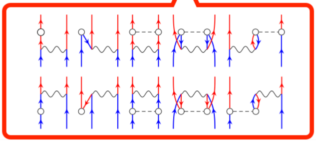
	^{76}Ge		^{76}Se	
eMax	E2 (e^2fm^4)	$E_x(2_1^+)$ (MeV)	E2 (e^2fm^4)	$E_x(2_1^+)$ (MeV)
6	255(3)	1.03(4)	412(11)	0.73(1)
8	287(3)	1.17(7)	468(6)	0.70(1)
Exp.	547(6)	0.563	864(22)	0.559

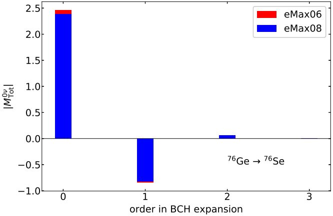
The above $B(E2: 2^+ \rightarrow 0^+)$ values are evaluated with the evolved one-body E2 operator only.

$0\nu\beta\beta$ from ^{76}Ge to ^{76}Se (preliminary results)

$0\nu\beta\beta$ transition operator in IMSRG(2)

$$O^{0\nu}(s) \equiv e^{\Omega(s)} O^{0\nu} e^{-\Omega(s)} = O^{0\nu} + [\Omega, O^{0\nu}] + \frac{1}{2}[\Omega, [\Omega, O^{0\nu}]] + \dots.$$





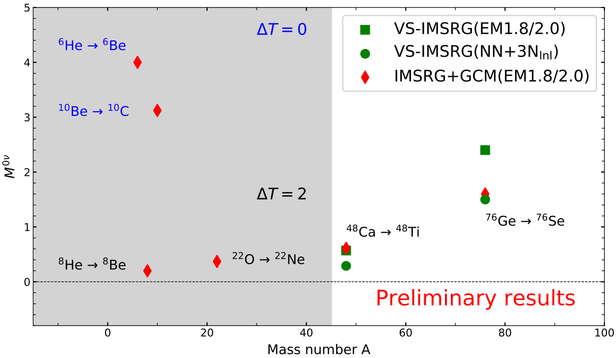
- renormalization effect on the transition operator:
  1. The renormalization effect is mainly contributed from $\Omega^{(2)}$
  2. The pp/hh diagrams enhance the NME (GT)

$$\sum_{ab} O^{0\nu}_{pp'ab} \Omega_{abnn'} (1 - n_a - n_b) + (\Omega \leftrightarrow O^{0\nu}) \quad (3)$$

while the ph diagrams quench the NME.

enhances the transition from ^{48}Ca to ^{48}Ti and quenches that from ^{76}Ge to ^{76}Se.

Summary of NMEs from IMSRG calculations



- The unpublished results of VS-IMSRG are from (A. Belley, R. Stroberg, J. Holt et al.)
- Uncertainties from different sources (model truncation, chiral expansion, contact operator, two-body currents) are to be included.

Summary and outlook

- The mass ordering of neutrinos is expected to be disclosed with the development of ton-scale $0\nu\beta\beta$ decay experiments in the next few years, depending on the values of the NMEs.
- The NMEs governing the $0\nu\beta\beta$ are essential to determine the neutrino effective mass. Several ab initio methods have begun to calculate the NMEs starting from first principles.
- We develop a novel multi-reference framework of IMSRG+GCM which opens a door to modeling deformed nuclei with realistic nuclear forces.
- The NMEs for several unphysical process in light nuclei and those for candidate process in medium-mass nuclei ^{48}Ca and ^{76}Ge(preliminary) are calculated. The NMEs in both cases are smaller than the predictions by (most) phenomenological models.
- More benchmarks among different *ab initio* calculations for the NMEs are underway.
- Quantification of uncertainties from different sources: systematic and statistic (open for comments/suggestions)

Collaborators and acknowledgement

Michigan State University

- Scott Bogner
- Heiko Hergert
- Roland Wirth

University of North Carolina at Chapel Hill

- Jonanthan Engel

Universidad Autónoma de Madrid

- Benjamin Bally
- Tomas R. Rodríguez

Iowa State University

- Robert A. Basili
- M. Lockner
- P. Maris
- James P. Vary

Southwest University

- Longjun Wang

San Diego State University

- Changfeng Jiao

Thank your for your attention!